A theory of interfaces

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Agents, Groups, etc...

How agents interact with each other and this is all that matters to reason about groups ⇒ an interface between the agent and the external world.

Agents can form groups, and groups are somehow (meta) agents.

Given an interface (for say a group) can we combine agents to form “this” group?
The formal framework

Fix an alphabet $\text{Act}$ of events.

- **Agents** = Deterministic finite state machines
  
  $$a = (Q_a, \text{Act}, q_a^0, \delta_a)$$

  We use $a, b, c, a_1, \ldots$

- **Interfaces** over the alphabet $\text{Act} = \{e, f, g\}$

We use $A, B, C, A_1, \ldots$

```
0 e f
  ↓
1 e f
  ↓
2 e
  ↓
3 e, f
```

We use $A, B, C, A_1, \ldots$

```
--→  “may e”

  e →  “Must e”
```
Fix an interface $A$ (over $Act$).

- In each state $q$ of $A$, we have
  - two subsets $\text{may}(A)(q)$, $\text{Must}(A)(q) \subseteq Act$
  - the subset $\text{maynot}(A)(q) := Act \setminus \text{may}(A)(q)$

If $\text{Must}(A)(q) \cap \text{maynot}(A)(q) \neq \emptyset$, state $q$ is inconsistent, which we may write as $\bot$. 
Assume for the moment that all states of $A$ are consistent.

- **Agent $a$ satisfies interface $A$, written $a \models A$, or $a \in Mod(A)$, whenever $a$ is obtained from $A$ by cutting may-transitions or making them solid.**

  ![Diagram of agent $A$ and interface $a$]

- **Agents $\not\rightarrow$ Interfaces**

  $a \leftrightarrow a^*$ in a natural way, ie $a \in Mod(a^*)$.

- **$Mod(a^*)$ contains only $a$, up to bisimulation.**
Refinement, as logical implication

\[ A \sqsubseteq B \] “A refines B”, whenever there exists \( \rho \subseteq Q_A \times Q_B \) such that

\begin{enumerate}
  \item \((q_A^0, q_B^0) \in \rho,\)
  \item may-transitions in \( A \) are reflected in \( B \), and
  \item must-transitions in \( B \) are reflected in \( A \).
\end{enumerate}

**Alternating simulation/refinement in games structures** [AHKV98].

**Proposition**

\[ A \sqsubseteq B \text{ if, and only if, } Mod(A) \subseteq Mod(B). \]

In particular, \( a \in Mod(A) \) can be rephrased as \( a^* \sqsubseteq A \).

**Proposition [AHKV98]**

We can decide in PTIME whether \( A \sqsubseteq B \).
We want to define $A \land B$ so that

**Proposition**

$$\text{Mod}(A \land B) = \text{Mod}(A) \cap \text{Mod}(B)$$

As a corollary $A \land B \sqsubseteq A$ and it is the $\sqsubseteq$-greatest (lower bound)

**Definition of $A \land B$**

$$A \land B := (Q_A \times Q_B, (q_A^0, q_B^0), \ldots) \text{ with}$$

- $\text{may}(A \land B)(q_A, q_B) := \text{may}(A)(q_A) \cap \text{may}(B)(q_B)$
- $\text{Must}(A \land B)(q_A, q_B) := \text{Must}(A)(q_A) \cup \text{Must}(B)(q_B)$

Must increases whereas may decreases $\Rightarrow$ inconsistent states.
About consistency

**Proposition**

\[ \text{Mod}(A) \neq \emptyset \text{ if, and only if, no Must-path reaching } \bot. \]

- You may prune your interface to remove inconsistent states.
- The empty interface \( \bot \) corresponds to "false" in the underlying logic.
- The interface \( \top \) for "true" is the one state + dashed flower structure.

**Proposition**

- Consistency can be decided in LOGSPACE (Reachability).
- If \( A \) is consistent it has a minimal and a maximal model.
Make agents work together

- Standard synchronous product of agents
  (= product of deterministic finite state machines)

\[ a \times b := (Q_a \times Q_b, (q_a^0, q_b^0), \ldots) \]

- Abstract from which agents in particular

\[ A \otimes B := (Q_A \times Q_B, (q_A^0, q_B^0), \ldots) \]

**Proposition**

\[ \text{Mod}(A) \otimes \text{Mod}(B) \subseteq \text{Mod}(A \otimes B) \text{ (strict inclusion in general).} \]
Definition of $A \otimes B$

$$A \otimes B := (Q_A \times Q_B, (q^0_A, q^0_B), \ldots) \text{ with}$$

- $\text{may}(A \otimes B)(q_A, q_B) := \text{may}(A)(q_A) \cap \text{may}(B)(q_B)$
- $\text{Must}(A \otimes A)(q_A, q_B) := \text{Must}(A)(q_A) \cap \text{Must}(B)(q_B)$

\[\begin{array}{|c|c|c|c|}
\hline
\otimes & e & e & e \\
\hline
e & \rightarrow & e & e \\
\hline
e & \rightarrow & e & e \\
\hline
\rightarrow & e & \rightarrow & e \\
\hline
\rightarrow & e & \rightarrow & e \\
\hline
\rightarrow & e & \rightarrow & e \\
\hline
\end{array}\]
Properties of $\otimes$

- $\text{Mod}(a^* \otimes b^*)$ contains only $a \times b$ (up to bisimulation).

- $\otimes$ is commutative and associative.

- Neutral element: one state $+$ solid flower.

- $\otimes$ is monotonic: $A \sqsubseteq B$ implies $A \otimes C \sqsubseteq B \otimes C$
Suppose we have to find $X_1, X_2, \ldots, X_k$ such that

$$X_1 \otimes X_2 \otimes \ldots \otimes X_k \sqsubseteq A$$

where the $X_i$’s range over \{\(A_1, A_2, \ldots, A_n\}\}.

How can we proceed? We define a quotient $\ominus$ such that

$$A_1 \otimes A_2 \sqsubseteq A \text{ if, and only if } A_2 \sqsubseteq A \ominus A_1$$
Definition of $A \otimes B$

$A \otimes B := (Q_A \times Q_B, (q_A^0, q_B^0), \ldots)$ with

\[
\begin{array}{|c|c|c|c|}
\hline
\otimes & e & e & e \\
\hline
 e & e & e & T \\
\hline
 e & \bot & e & \bot \\
\hline
 e & e & e & T \\
\hline
\end{array}
\]

Proposition

$(A \otimes A_1) \otimes A_2 \equiv A \otimes (A_1 \otimes A_2)$
Achieving interfaces

There are many ways to think of it. I give here one example. From

\[ X_1 \otimes X_2 \otimes \ldots \otimes X_k \subseteq A \]

where each \( X_i \in A := \{ A_1, A_2, \ldots, A_n \} \).

1. \( C := A \)
2. Select \( A_{i_1} \in A \) and compute \( C := C \ominus A_{i_1} \);
3. Select \( A_{i_2} \in A \) and compute \( C := C \ominus A_{i_2} \);
4. \ldots
5. Select \( A_{i_k} \in A \) and compute \( C := C \ominus A_{i_k} \);

At the end, if \( C \equiv \top \) then done, otherwise \( C \) is the needed mediator interface.
The logical view

- Interfaces as a fragment of the $\mu$-calculus [Kozen83]:
  - $\longrightarrow$ is $[e]$ and $\stackrel{e}{\longrightarrow}$ is $\langle e \rangle$.
  - + conjunction, greatest fixed-points and but no outermost negation.

Interfaces $\hookrightarrow L_\mu$: $A \hookrightarrow \alpha$

- Quotient between mu-calculus formulas exists ([AVW03],...):
  - $P \models \varphi/\psi$ if and only if $\exists C \models \psi$, $P \times C \models \varphi$, constructive procedure on the tree automata of the formulas.
  - We can then compute $\alpha \otimes \beta$ as $\neg(\neg\alpha/\beta)$ but @exptime because of two complementations.

However, our quotient is polynomial.
Slight extensions with “local” disjunctions, eg

\[
Acc(A)(q) = \{\{e, f\}, \{e, g\}\}
\equiv (\langle e \rangle \top \land \langle f \rangle \top) \text{XOR}(\langle e \rangle \top \land \langle g \rangle \top)
\]

It generalizes may and Must sets and the entire theory works well.

• Quotienting make things bigger.

• Selection criteria?

• Extension to capture, eg interdependency, reliability, prudence, ....

Eg, \(\sim^e\) for “I cannot do \(e\) but I can follow a companion”. 

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