Relating plays in game arenas

Sophie Pinchinat
LogicA/IRISA, University of Rennes 1

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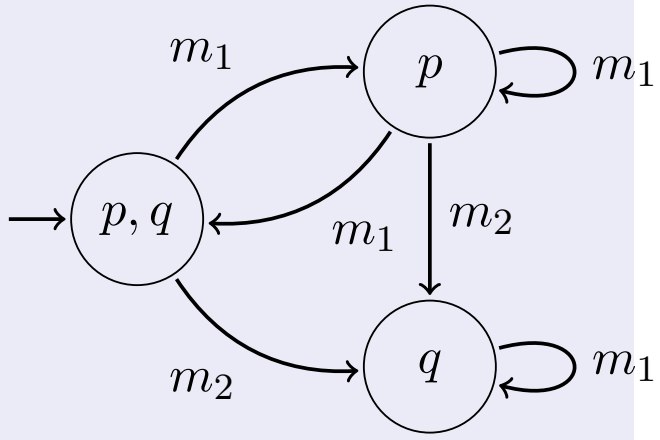
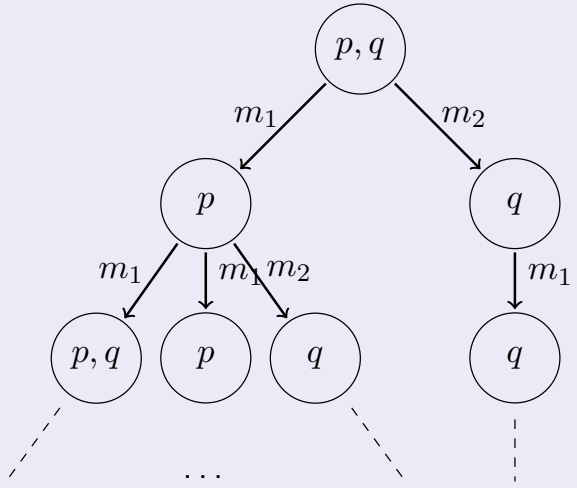
Models for Dynamics

Transition systems $\rightsquigarrow$ Computation trees (unfold)

Game arenas $\mathcal{G}$



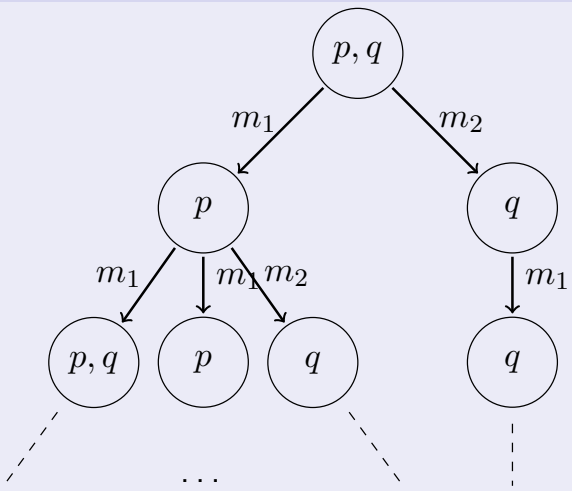
Game trees $\mathcal{T}_{\mathcal{G}}$ (unfoldings)



Turned-based game arenas, Concurrent game structures, Plants (in control theory), Domains (in automated planning) ...

Logics for Strategic reasoning

Game trees $\mathcal{T}$



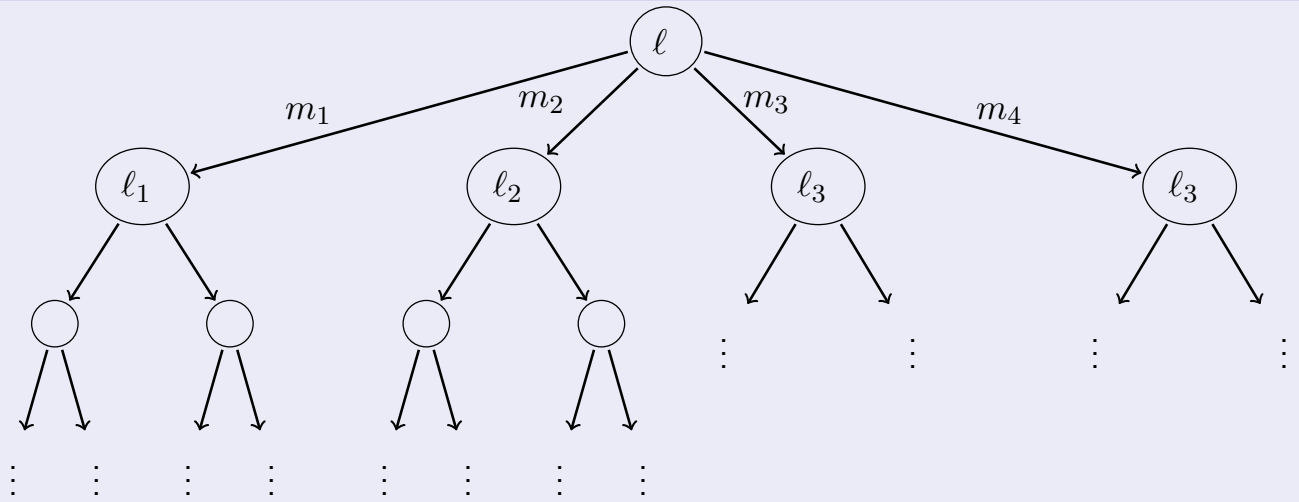
On the basis of temporal logics LTL, CTL, CTL*, L$_\mu$, ...

Logics for strategic reasoning:

- Built-in formalisms: ATL, ATL*, AMC, ATL$_{sc}$...
- Strategies as FO objects: SL, variants of SL...
- Strategies as SO objects: MSO, QL$_\mu$, Bisim$Q$L$_\mu$, ...

Game trees

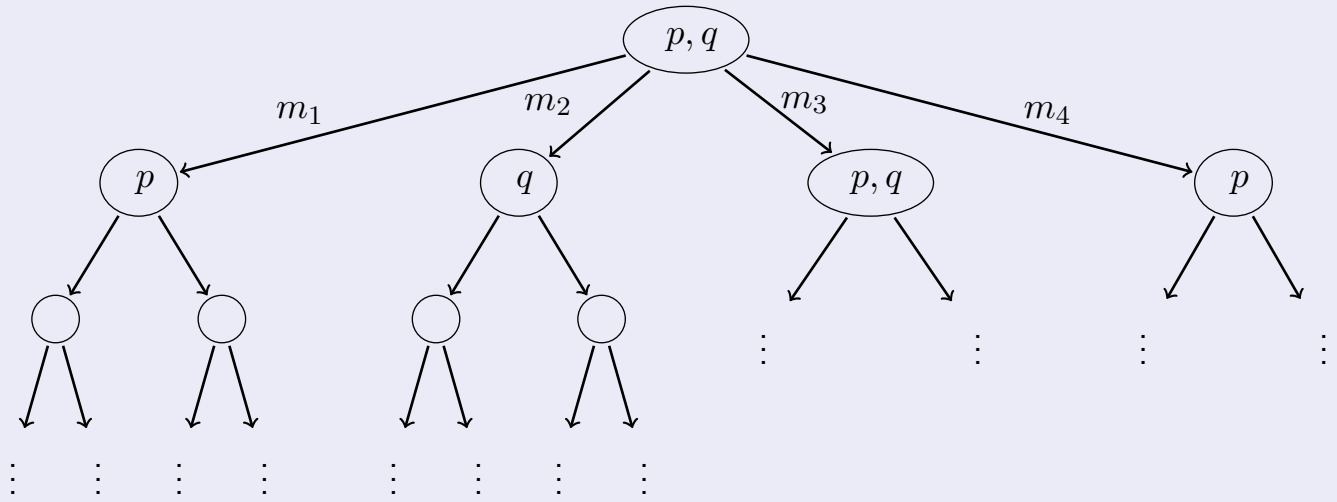
Game trees $\mathcal{T}$



Game trees

What is a node label? $\ell \in 2^{Prop}$

Game trees $\mathcal{T}$



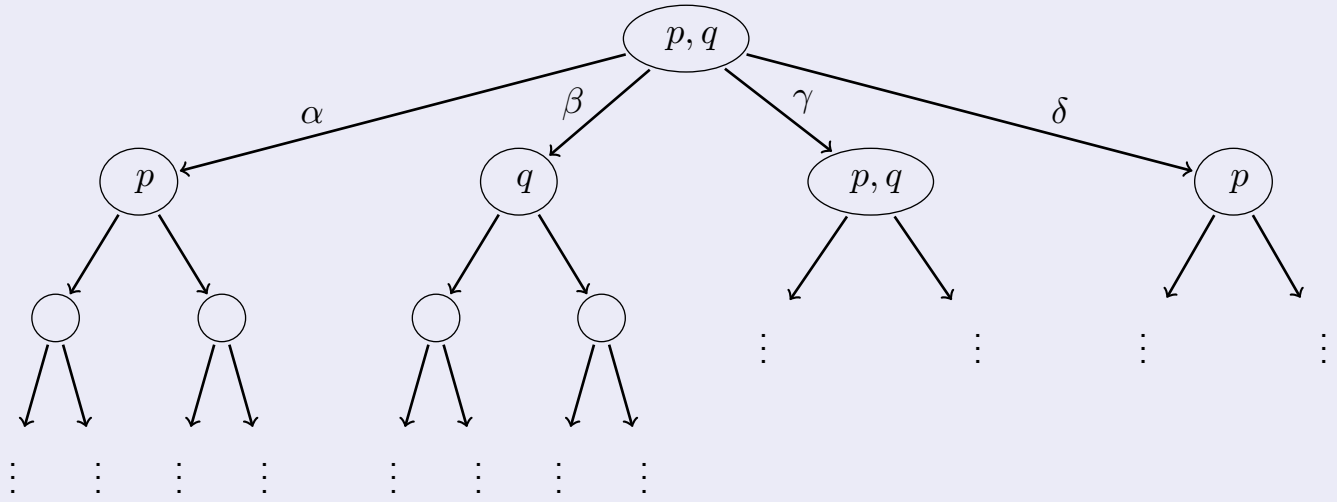
Linear-time Logic and Branching-time Logic, LTL, CTL, CTL*

Game trees

What is an edge label?

$\alpha \in \mathcal{A}ct$ abstract

Game trees $\mathcal{T}$



Propositional modal logics PDL, L_μ

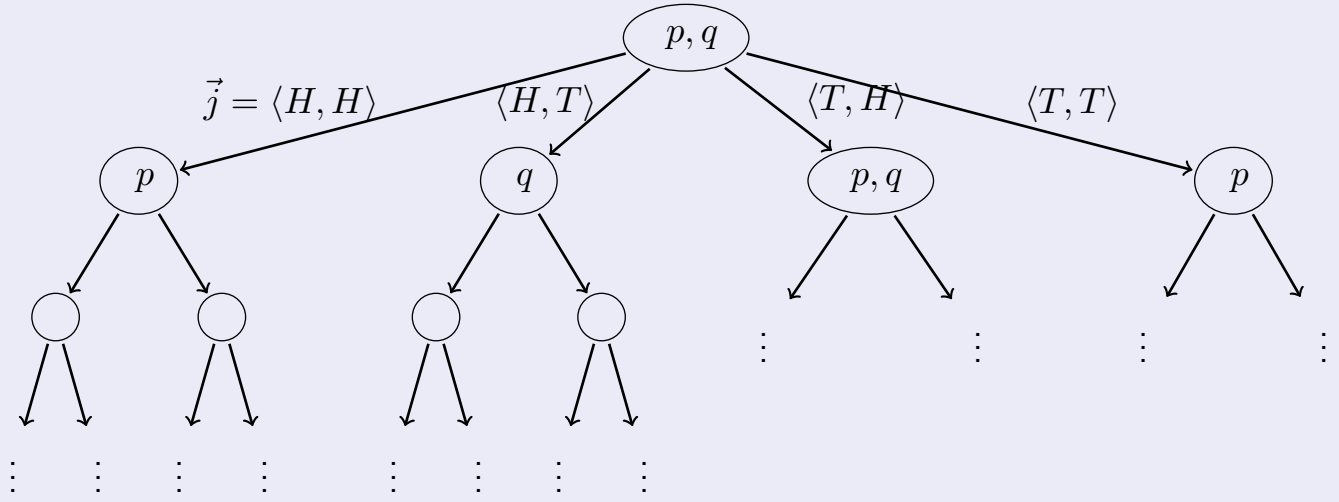
Game trees

What is an edge label?

$\alpha \in \mathcal{A}ct$ abstract

$\vec{j} \in Moves_1 \times Moves_2 \times \ldots \times Moves_N$ more structured

Game trees $\mathcal{T}$



Propositional modal logics PDL, L_μ

Alternating-time Logics, ATL, ATL*, ATL_{sc}

Adding uncertainty in the game trees

- Epistemic temporal logics: LTLK, CTLK, $L_\mu K$...
- Strategy logics with imperfect info: ATL_i, SL_i, ESL...

Common feature:

Indistinguishability relation on finite paths of game arenas

- memoryless, bounded memory, perfect recall
- synchronous, asynchronous...

Role:

- Give a semantics to the knowledge modality
- Constraint the class of strategies to reason about

For instance: would you be able to tell if $ATL_i \prec L_\mu K$? But for which semantics of K? Memoryless should work ... hum!

Considering relations between paths in game arenas shed lights on the landscape of strategic logics mixing time and knowledge.

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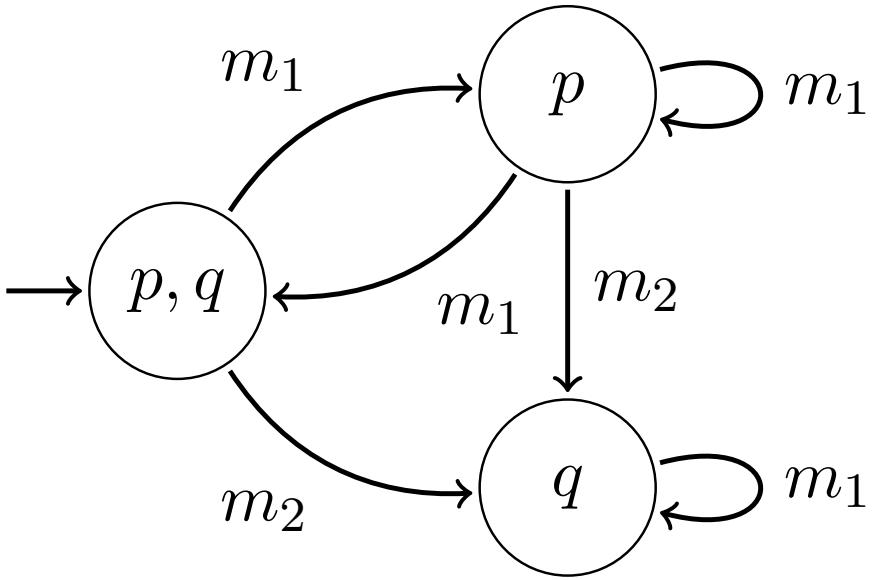
Relating plays in game arenas over $Prop$ and $\mathcal{M}oves$



Available information is $Prop$ and $\mathcal{M}oves$.

A *play* is a word in $\Pi = 2^{Prop}(\mathcal{M}oves \times 2^{Prop})^*$

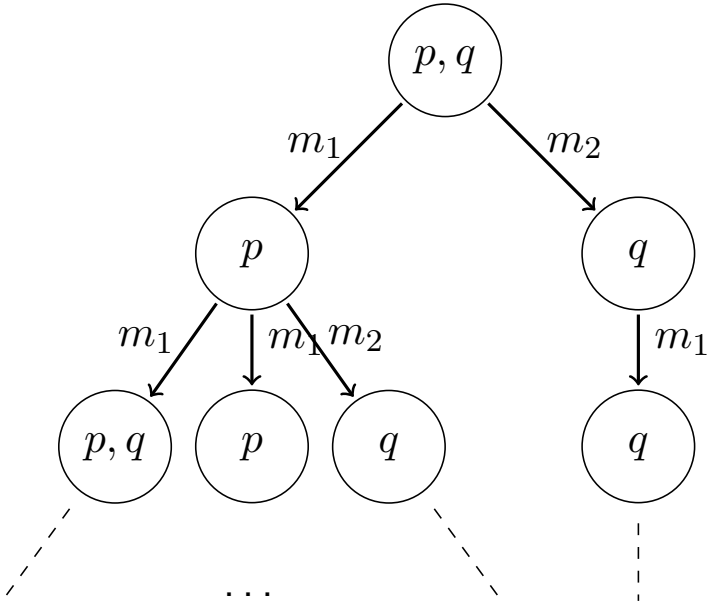
e.g., $\{p, q\}m_1\{p\}m_1\{p, q\}$

Relating plays in game arenas over $Prop$ and $\mathcal{M}oves$

$$\Pi = 2^{Prop}(\mathcal{M}oves \times 2^{Prop})^*$$

$$\sim \subseteq \Pi \times \Pi$$

an *indistinguishability* relation for some agent.



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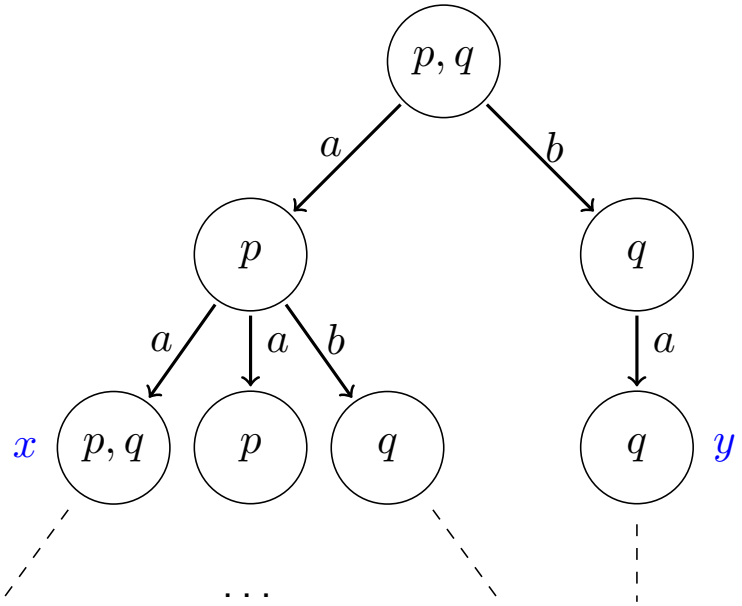

Take the node $x \in \mathcal{T}$

$trace(x) = \{p, q\}m_1\{p\}m_1\{p, q\}$

Say that $x \rightsquigarrow y$ if $trace(x) \rightsquigarrow trace(y)$

Relating plays in game arenas over $Prop$ and $\mathcal{M}oves$

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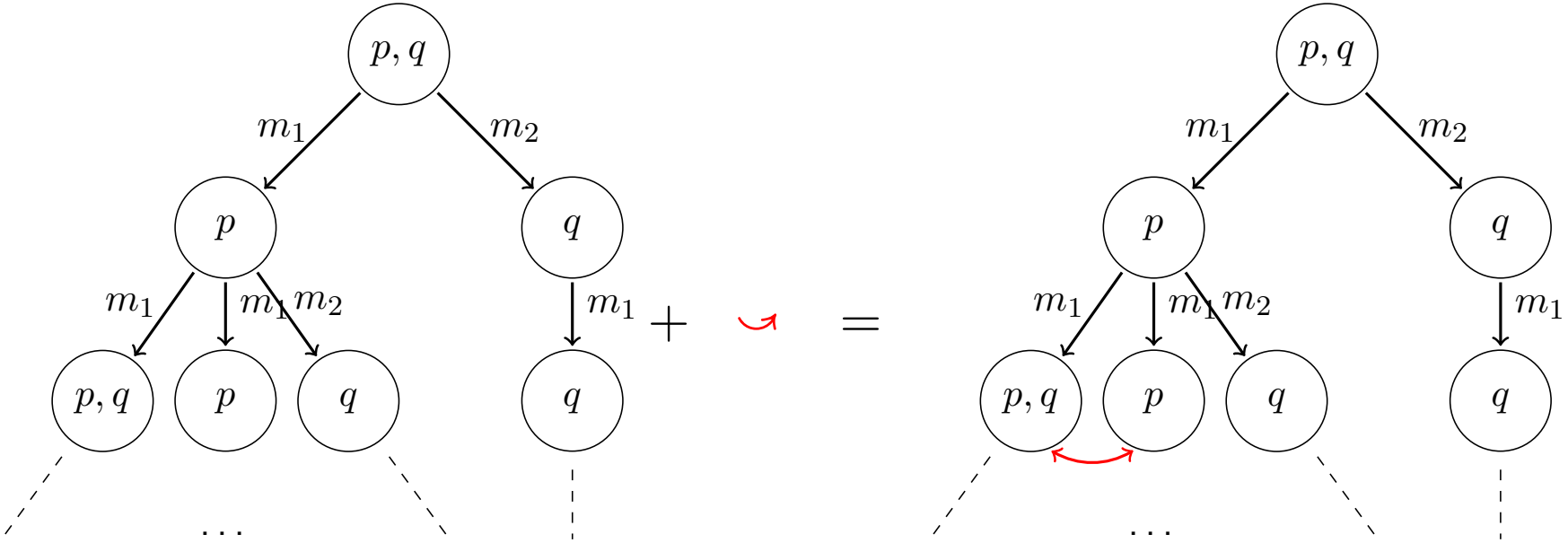

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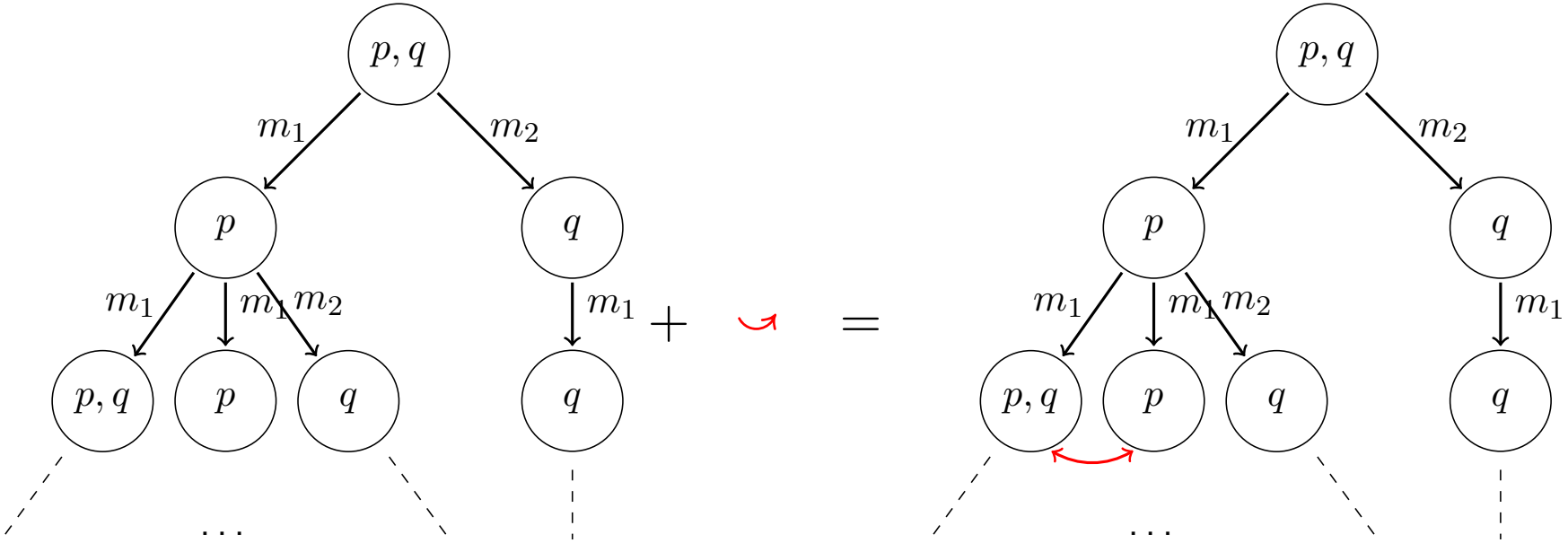
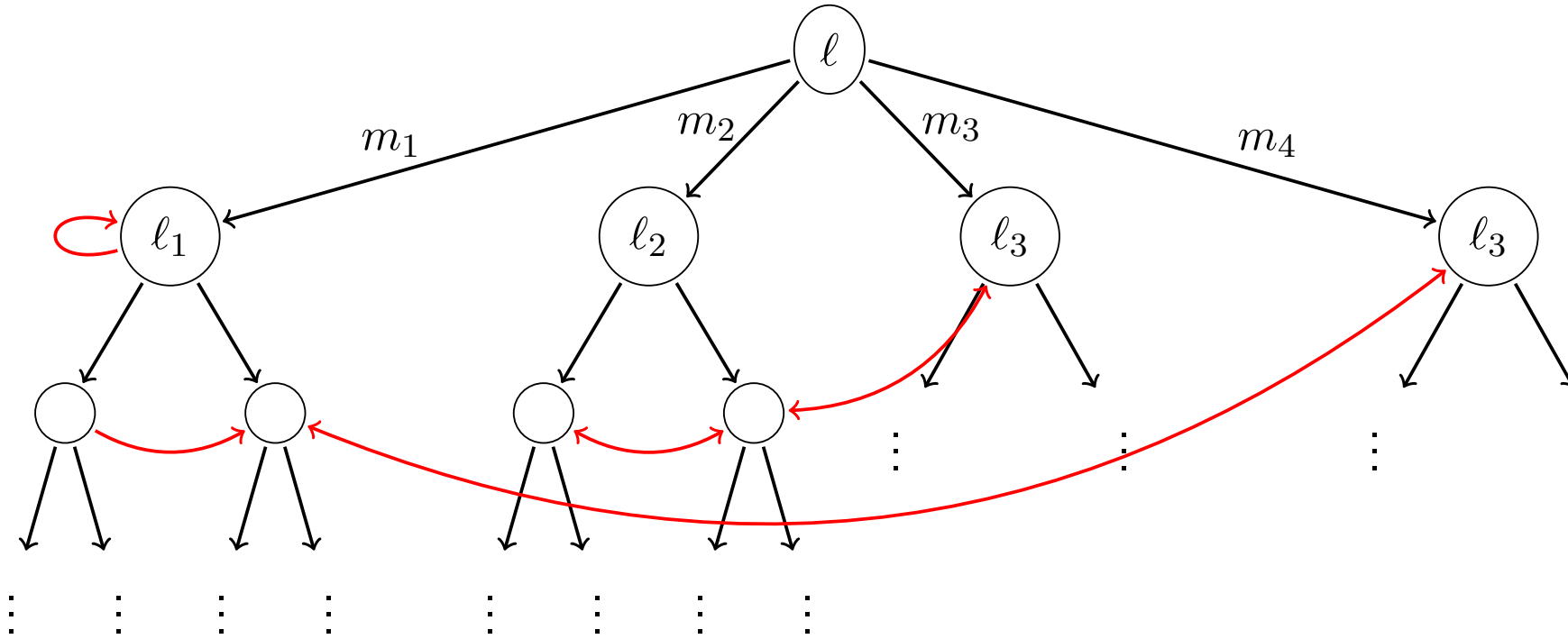


$$\{p,q\}m_1\{p\}m_1\{p,q\} \sim \{p,q\}m_1\{p\}m_1\{p\}$$

a synchronous perfect recall agent, who only observes p

Extension of game trees

Just as abstractions of *interpreted systems*, i.e., the models of ETL



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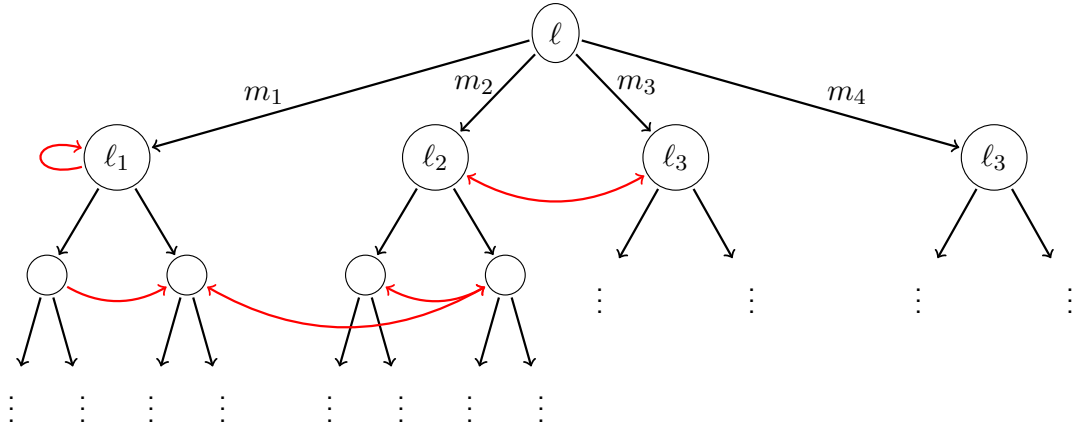

The framework of e.g., L_μ is extended as the jumping μ-calculus

$\mathsf{L}_\mu^{\circlearrowright}$

$\varphi ::= X \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle m \rangle \varphi \mid \mu X.\varphi(X) \mid \langle \circlearrowright \rangle \varphi$
where $p \in Prop$ and $m \in \mathcal{M}oves$.

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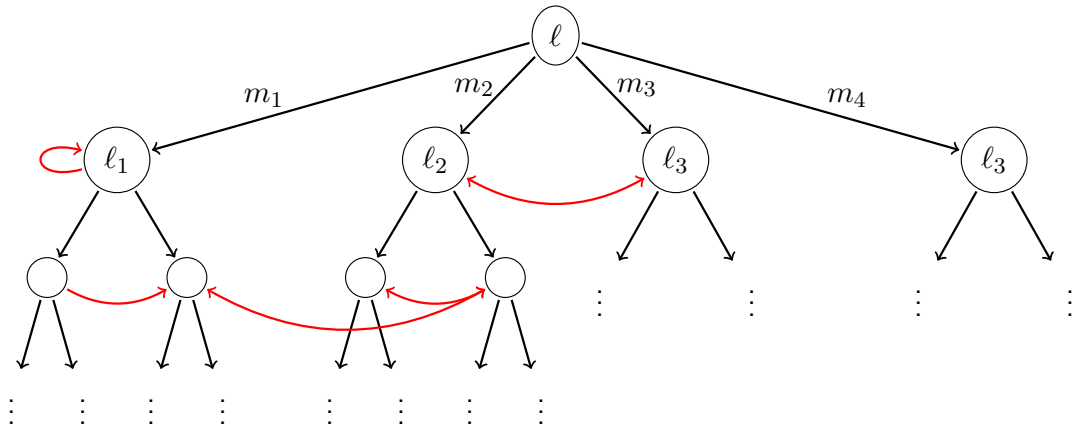

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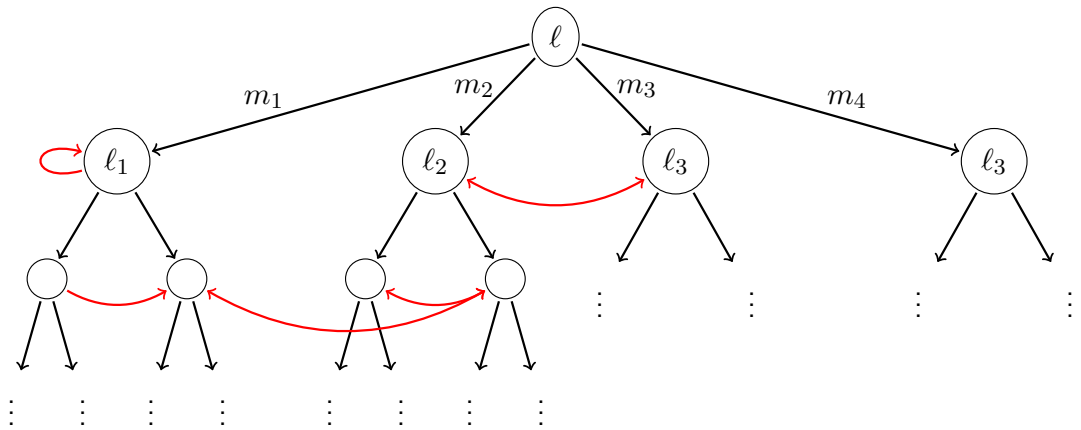
where $p \in Prop$ and $m \in \mathcal{M}oves$.

It captures all epistemic temporal logics.

Jumping Alternating Tree Automata $\equiv \mathsf{L}_\mu^{\circlearrowright}$ [Maubert+P. 2013]

Extension of game trees



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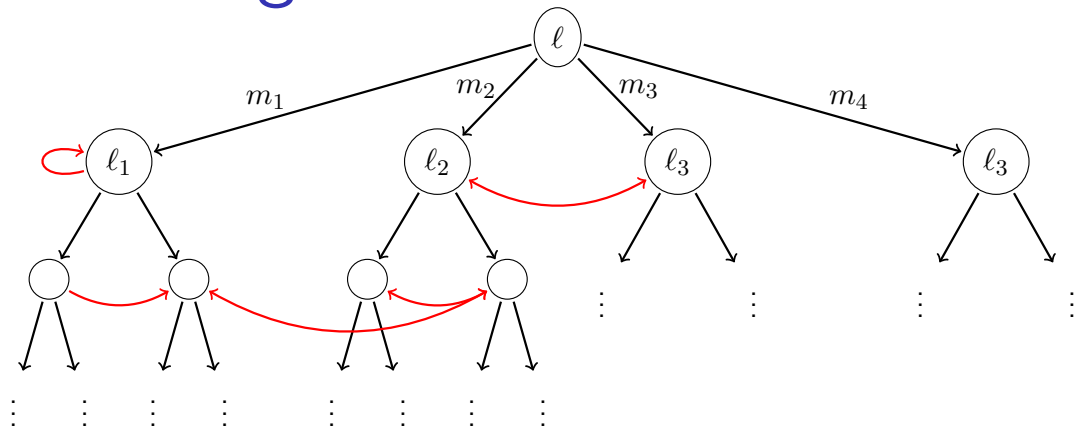


Remarkable classes of relations

- $\rightsquigarrow$ is rational if recognized by a 2-tape automaton.
- $\rightsquigarrow$ is regular if recognized by a synchronous 2-tape automaton.
- $\rightsquigarrow$ is recognizable if a word automaton suffices : $\{u\#v \mid u \rightsquigarrow v\}$ is a regular language.

Recognizable	$\subsetneq$	Regular	$\subsetneq$	Rational
$\cup$		$\cup$		$\cup$
Bounded memory		Perf rec synch.		Perf rec asynch.

Known results for game trees

- $\mathsf{L}_\mu^{\rightsquigarrow} \preceq \mathsf{MSO}^{\rightsquigarrow}$, and $\mathsf{L}_\mu^{\rightsquigarrow}$ is invariant under bisimulation.
- If $\rightsquigarrow$ is recognizable, $\mathsf{MSO}^{\rightsquigarrow}_{\mathsf{bisim}} \equiv \mathsf{MSO}_{\mathsf{bisim}} \equiv \mathsf{L}_\mu$. Therefore, $\mathsf{ATL}_i \preceq \mathsf{L}_\mu^{\rightsquigarrow}$. But no idea of the effective translation, as the expansion law for fix-points does not apply already for memoryless semantics [Bulling+Jamroga 2011]

Theorem [Dima+Maubert+P. 2015]

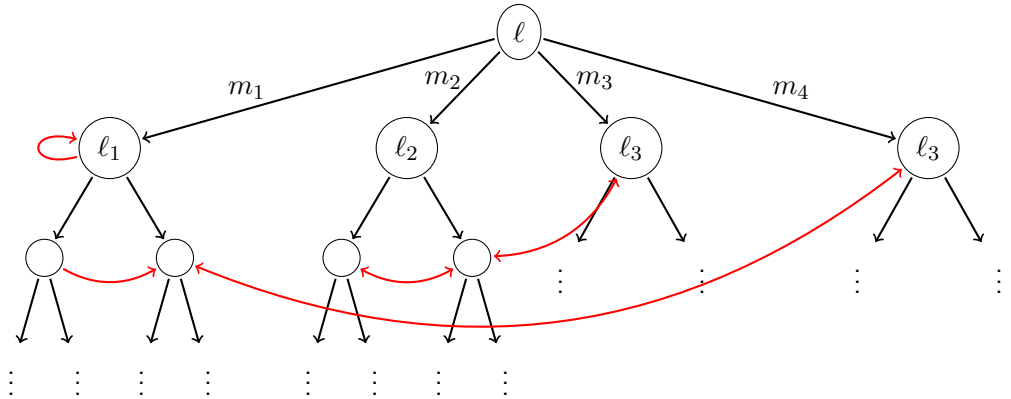
There is a regular relation $\rightsquigarrow$ for which $\mathsf{L}_\mu^{\rightsquigarrow} \prec \mathsf{MSO}^{\rightsquigarrow}_{\mathsf{bisim}}$.

Proof: the property is "the existence of a winning strategy for (blind) Player 0 in an imperfect information reachability game for synchronous perfect recall".

Corollary (from the proof)

When $\rightsquigarrow$ = synchronous perfect recall, $ATL_i \not\preceq L_\mu^{\rightsquigarrow}$.

Back to the class of game trees: generalities



Proposition

For $\mathcal{G}$ a finite game and $\curvearrowright$ regular, $\mathcal{T}_{\mathcal{G}}^{\curvearrowright}$ is an automatic structure.

Namely, the nodes of $\mathcal{T}_{\mathcal{G}}^{\curvearrowright}$ can be encoded as finite words over some finite alphabet Σ, and the relations (here $\downarrow$ and $\curvearrowright$) can be described by synchronous automata.

Corollary (of Blumensath 1999)

The FO theory of $\mathcal{T}_{\mathcal{G}}^{\curvearrowright}$ is decidable.

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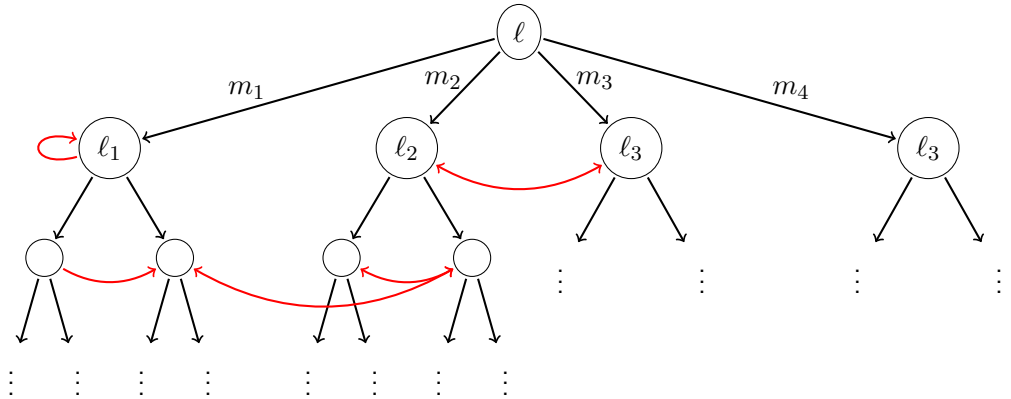

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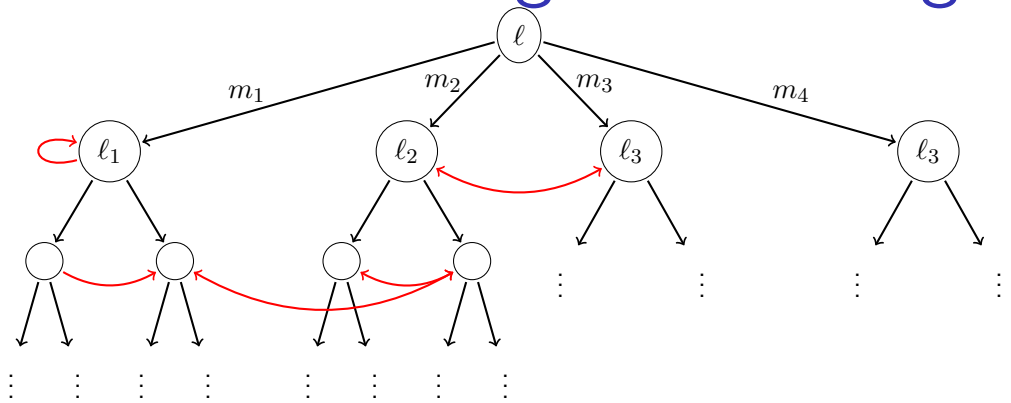

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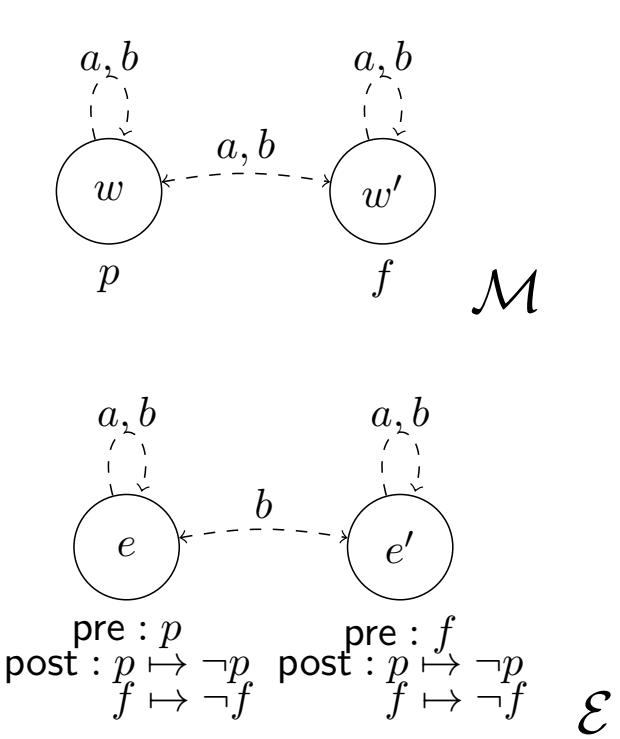
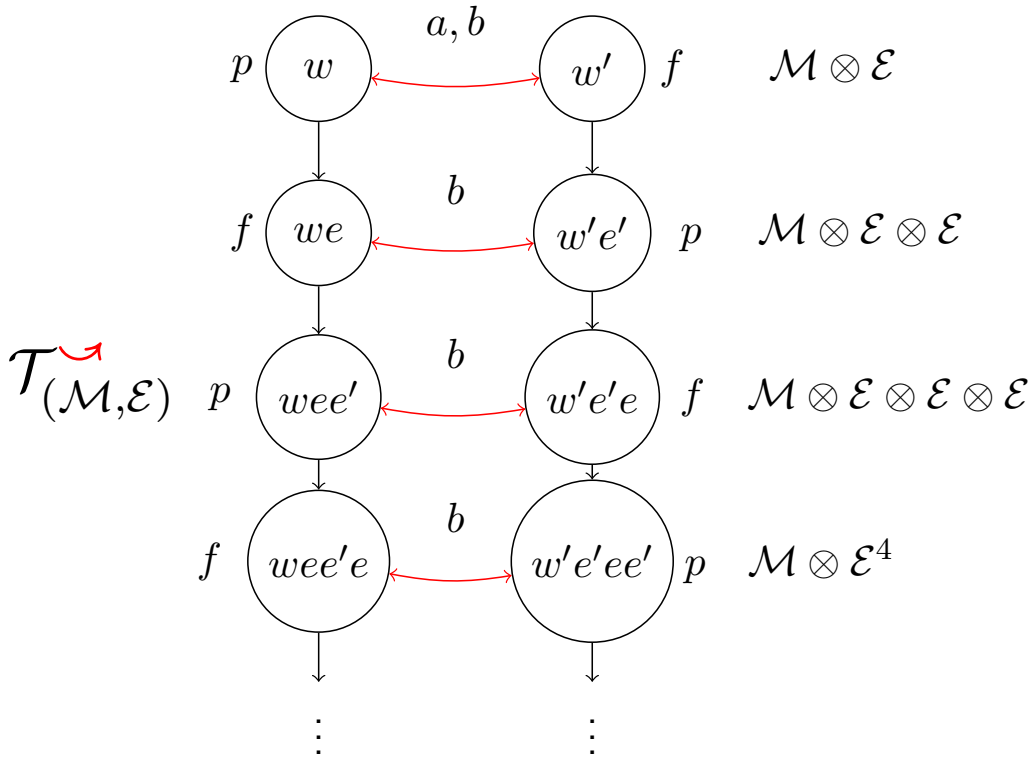
Proposition

$\mathcal{T}_{\mathcal{G}}^{\circlearrowright}$ is even a regular automatic tree (*).

(*) With the most natural encoding, ie the traces $we_1e_2\dots e_n$.

DEL structures are game trees





$(\mathcal{M}, \mathcal{E})$ a *DEL presentation* $\rightsquigarrow \mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ (one-player game tree).

Intrinsically synchronous ($\circlearrowright$ is horizontal)

Definition

$\mathcal{E}$ is *propositional* if for every $e \in \mathcal{E}$, formula $pre(e)$ is propositional.
$\mathcal{E}$ *has effects* if for some $e \in \mathcal{E}$, $post(e)$ is not trivial.

$\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ with propositional $\mathcal{E}$

Proposition (Douéneau+P.+Schwarzentruber 2016, Aucher+Maubert+P. 2014)

If $\mathcal{E}$ is propositional, then $\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ is a regular automatic tree.

Corollary

The DEL planning problem is decidable for propositional $\mathcal{E}$, and translating the goal φ into $\widehat{\varphi}(x) \in$ FO gives the set of all plans.

Theorem (Douéneau 2016)

The $MSO^{\circlearrowright}_{chain}$ theory of $\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ is decidable.

Corollary

The DEL protocol problem for $MSO^{\circlearrowright}_{chain}$-definable goals is decidable. This captures CTL^*K_n.

Less predictable $\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$: non-propositional $\mathcal{E}$

$\mathcal{E}$ has $md = k$ (resp. $ad = k$) depth k if for every $e \in \mathcal{E}$, $md(pre(e)) \leq k$ (resp. $ad(pre(e)) \leq k$) and equals k for some e.

Corollary (of Bolander+Anderson 2011)

For $\mathcal{E}$ with $md = 1$ and effects, $\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ can be non-automatic.

Corollary (of Charrier+Maubert+Schwarzentruber 2016)

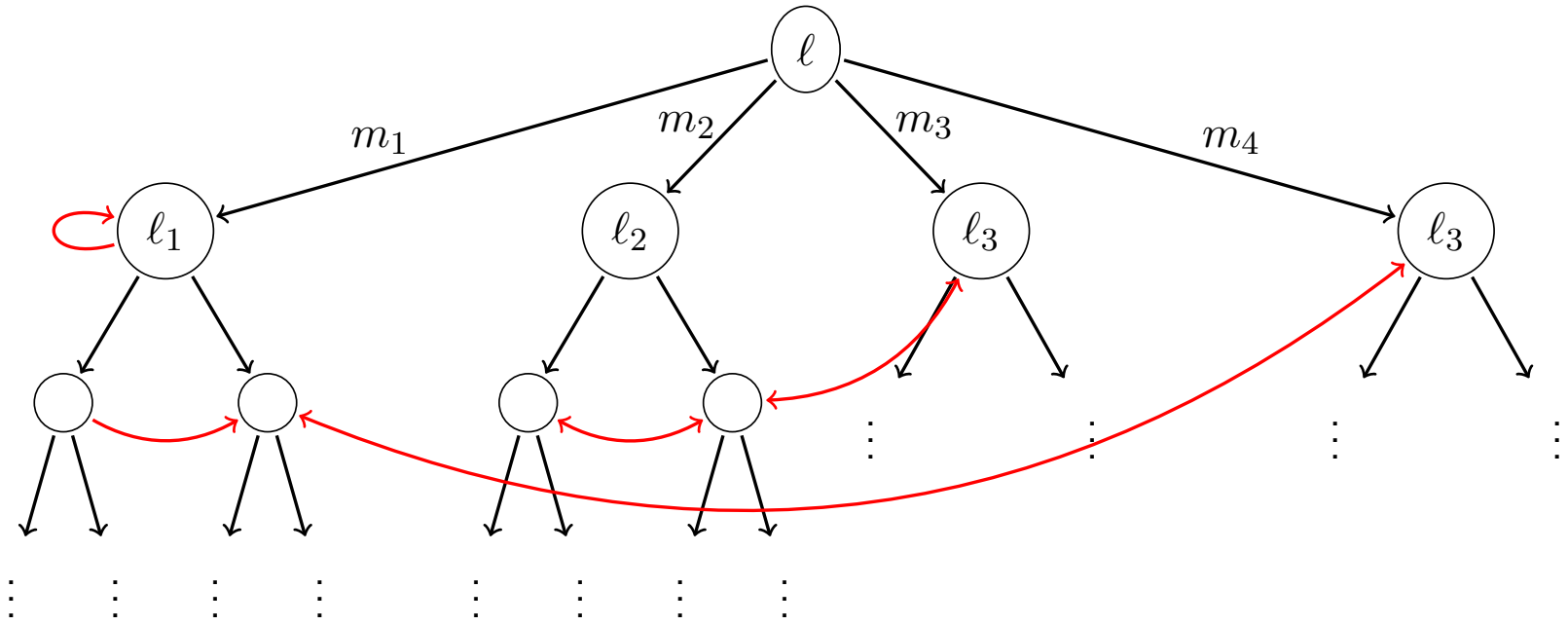
For $\mathcal{E}$ with $alt = 2$, $\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ can be non-automatic.

Open

The case $\mathcal{E}$ with $md = 1$ and no effects.

- For $\mathcal{E}$ public announcements, $\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ is a regular automatic tree.
- Context-free, context-sensitive natural encodings $we_1e_2 \ldots e_n$, but $\mathcal{T}^{\circlearrowright}_{(\mathcal{M},\mathcal{E})}$ might be automatic for another encoding ...

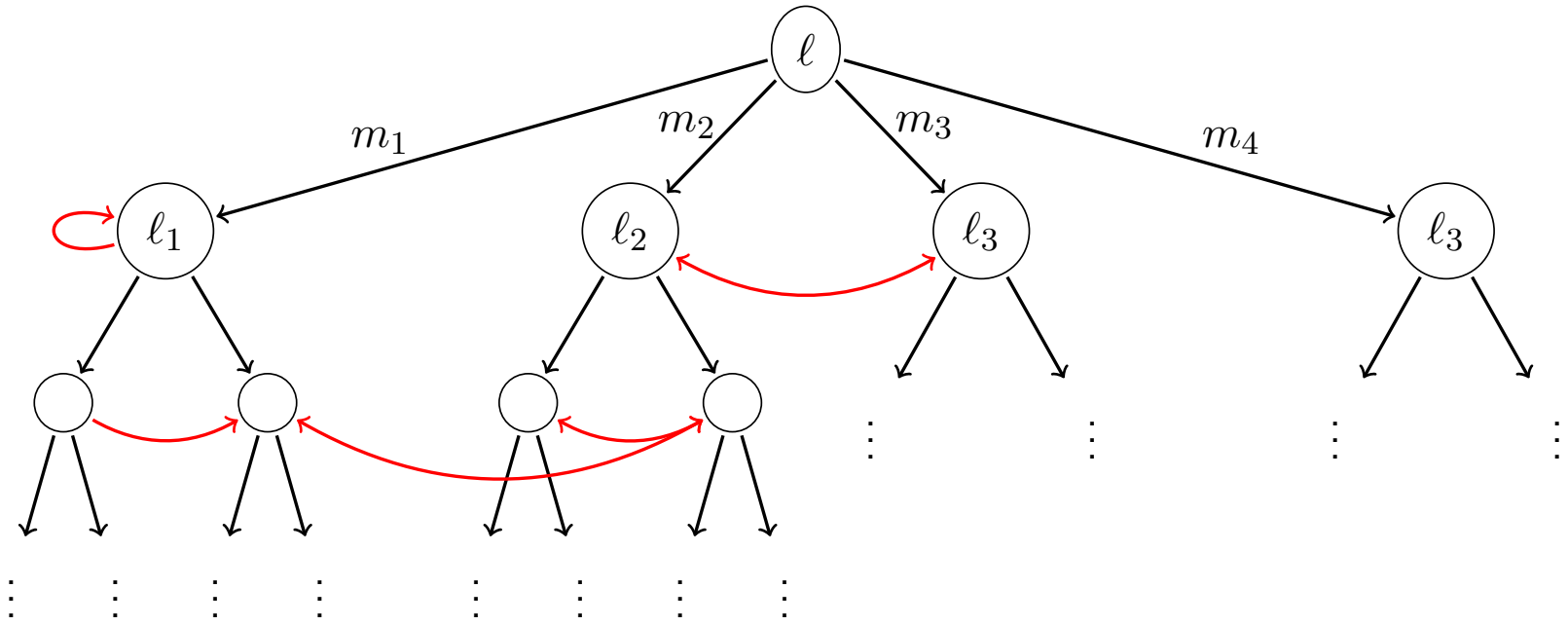
Wrap up



- Classes of game trees are central mathematical objects.
- Decidable fragments of MSO (SO quantification provides the strategy quantifiers).
- The subclasses of automatic structures, that we called *regular automatic trees*, might be a track to follow.

The End of the Beginning

Wrap up



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