Relating plays in game arenas

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Models for Dynamics

Transition systems $\sim$ Computation trees (unfold)

Game arenas $\mathcal{G}$

Game trees $\mathcal{T}_{\mathcal{G}}$ (unfoldings)

Turned-based game arenas, Concurrent game structures, Plants (in control theory), Domains (in automated planning) ...
Logics for Strategic reasoning

Game trees $\mathcal{T}$

On the basis of temporal logics LTL, CTL, CTL*, $L_\mu$, ...

Logics for strategic reasoning:

- Built-in formalisms: ATL, ATL*, AMC, ATL_{sc}...
- Strategies as FO objects: SL, variants of SL...
- Strategies as SO objects: MSO, $QL_\mu$, Bisim$QL_\mu$, ...
Game trees

Game trees $\mathcal{T}$
Game trees

What is a node label? $\ell \in 2^{Prop}$

Game trees $\mathcal{T}$

Linear-time Logic and Branching-time Logic, LTL, CTL, CTL*
Game trees

What is an edge label?
\( \alpha \in \text{Act} \)

Game trees \( \mathcal{T} \)

Propositional modal logics PDL, \( L_\mu \)
Game trees

What is an edge label?

\[ \alpha \in \text{Act} \]

\[ \vec{j} \in \text{Moves}_1 \times \text{Moves}_2 \times \ldots \times \text{Moves}_N \]

Game trees \( \mathcal{T} \)

Propositional modal logics PDL, \( L_\mu \)
Alternating-time Logics, ATL, ATL\(^*\), ATL\(\text{sc} \)
Adding uncertainty in the game trees

- Epistemic temporal logics: LTLK, CTLK, \( L_\mu K \)...
- Strategy logics with imperfect info: ATL\(_i\), SL\(_i\), ESL...

Common feature:
Indistinguishability relation on finite paths of game arenas
- memoryless, bounded memory, perfect recall
- synchronous, asynchronous...

Role:
- Give a semantics to the knowledge modality
- Constraint the class of strategies to reason about

For instance: would you be able to tell if ATL\(_i\) \( \prec \) L\(_\mu\)K? But for which semantics of K? Memoryless should work ... hum!

Considering relations between paths in game arenas shed lights on the landscape of strategic logics mixing time and knowledge.
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Considering relations between paths in game arenas shed lights on the landscape of strategic logics mixing time and knowledge.
Available information is $\text{Prop}$ and $\text{Moves}$.

A play is a word in $\Pi = 2^{\text{Prop}}(\text{Moves} \times 2^{\text{Prop}})^*$

e.g., $\{p, q\}m_1\{p\}m_1\{p, q\}$
Relating plays in game arenas over $Prop$ and $Moves$,

$$\Pi = 2^{Prop}(Moves \times 2^{Prop})^*$$

$$\leadsto \subseteq \Pi \times \Pi$$

an *indistinguishability* relation for some agent.
Relating plays in game arenas over \( \text{Prop} \) and \( \text{Moves} \)

\[
\Pi = 2^{\text{Prop}}(\text{Moves} \times 2^{\text{Prop}})^* \]

\[\xRightarrow{\sim} \subseteq \Pi \times \Pi\]

an indistinguishability relation for some agent.

Take the node \( x \in \mathcal{T} \)

\[
\text{trace}(x) = \{p, q\}m_1\{p\}m_1\{p, q\}
\]

Say that \( x \xRightarrow{\sim} y \) if \( \text{trace}(x) \xRightarrow{\sim} \text{trace}(y) \)
Relating plays in game arenas over $\textit{Prop}$ and $\textit{Moves}$

\[ \Pi = 2^{\textit{Prop}}(\textit{Moves} \times 2^{\textit{Prop}})^* \]

\[ \bowtie \subseteq \Pi \times \Pi \]

an \textit{indistinguishability} relation for some agent.
Relating plays in game arenas over $Prop$ and $Moves$

$$\Pi = 2^{Prop}(Moves \times 2^{Prop})^*$$

$$\preceq \subseteq \Pi \times \Pi$$

an *indistinguishability* relation for some agent.

$$\{p, q\}m_1\{p\}m_1\{p, q\} \preceq \{p, q\}m_1\{p\}m_1\{p\}$$

a synchronous perfect recall agent, who only observes $p$
Extension of game trees

Just as abstractions of interpreted systems, i.e., the models of ETL
Extension of game trees

The framework of e.g., $L_\mu$ is extended as the jumping $\mu$-calculus

$L_\mu$

$\varphi ::= X \mid p \mid \neg \varphi \mid \varphi \lor \varphi \mid \Box \varphi \mid \mu X. \varphi(X) \mid \Diamond \varphi$

where $p \in Prop$ and $m \in Moves$. 
Extension of game trees

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where $p \in Prop$ and $m \in Moves$.

It captures all epistemic temporal logics.

Jumping Alternating Tree Automata $\equiv L_\mu$ [Maubert+P. 2013]
Extension of game trees
Extension of game trees

Remarkable classes of relations

- \( \sim \) is rational if recognized by a 2-tape automaton.
- \( \sim \) is regular if recognized by a synchronous 2-tape automaton.
- \( \sim \) is recognizable if a word automaton suffices:
  \( \{u \# v \mid u \sim v\} \) is a regular language.

<table>
<thead>
<tr>
<th>Recognizable</th>
<th>( \subsetneq )</th>
<th>Regular</th>
<th>( \subsetneq )</th>
<th>Rational</th>
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Bounded memory \( \subseteq \) Perf rec synch. \( \subseteq \) Perf rec asynch.
Known results for game trees

- \( L^{\mu} \preceq \text{MSO}^\preceq \), and \( L^{\mu} \) is invariant under bisimulation.
- If \( \preceq \) is recognizable, \( \text{MSO}^\preceq_{\text{bisim}} \equiv \text{MSO}^\preceq_{\text{bisim}} \equiv L^{\mu} \).
  Therefore, \( \text{ATL}_i \preceq L^{\mu} \). But no idea of the effective translation, as the expansion law for fix-points does not apply already for memoryless semantics [Bulling+Jamroga 2011]

**Theorem [Dima+Maubert+P. 2015]**

There is a regular relation \( \preceq \) for which \( L^{\mu} \prec \text{MSO}^\preceq_{\text{bisim}} \).

Proof: the property is “the existence of a winning strategy for (blind) Player 0 in an imperfect information reachability game for synchronous perfect recall”.

**Corollary (from the proof)**

*When* \( \preceq = \text{synchronous perfect recall}, \text{ATL}_i \not\preceq L^{\mu} \).
Back to the class of game trees: generalities

**Proposition**

For $G$ a finite game and $\rightarrow$ regular, $T_G$ is an **automatic structure**.

Namely, the nodes of $T_G$ can be encoded as finite words over some finite alphabet $\Sigma$, and the relations (here $\downarrow$ and $\rightarrow$) can be described by synchronous automata.

**Corollary (of Blumensath 1999)**

The FO theory of $T_G$ is decidable.
Back to the class of game trees: generalities

Proposition

For \( G \) a finite game and \( \rightsquigarrow \) regular, \( T_G \) is an automatic structure.

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Proposition

For $\mathcal{G}$ a finite game and $\leadsto$ regular, $\mathcal{T}_G$ is an automatic structure.

Corollary (of Blumensath 1999)

The FO theory of $\mathcal{T}_G$ is decidable.

Proposition

$\mathcal{T}_G$ is even a regular automatic tree (*).

(*) With the most natural encoding, ie the traces $we_1e_2\ldots e_n$. 
DEL structures are game trees

\[(M, E) \text{ a DEL presentation } \sim \mathcal{T}(M, E) \text{ (one-player game tree).}\]

Intrinsically synchronous (\(\sim\) is horizontal)

**Definition**

\(E\) is propositional if for every \(e \in E\), formula \(\text{pre}(e)\) is propositional. \(E\) has effects if for some \(e \in E\), \(\text{post}(e)\) is not trivial.
Proposition (Douéneau+P.+Schwarzentruber 2016, Aucher+Maubert+P. 2014)

If $\mathcal{E}$ is propositional, then $T_{(\mathcal{M},\mathcal{E})}$ is a regular automatic tree.

Corollary

The DEL planning problem is decidable for propositional $\mathcal{E}$, and translating the goal $\varphi$ into $\hat{\varphi}(x) \in FO$ gives the set of all plans.

Theorem (Douéneau 2016)

The $MSO_{\text{chain}}$ theory of $T_{(\mathcal{M},\mathcal{E})}$ is decidable.

Corollary

The DEL protocol problem for $MSO_{\text{chain}}$-definable goals is decidable. This captures $CTL^* K_n$. 
Less predictable $\mathcal{T}_{(\mathcal{M}, \mathcal{E})}$: non-propositional $\mathcal{E}$

$\mathcal{E}$ has $md = k$ (resp. $ad = k$) depth $k$ if for every $e \in \mathcal{E}$, $md(\text{pre}(e)) \leq k$ (resp. $ad(\text{pre}(e)) \leq k$) and equals $k$ for some $e$.

**Corollary (of Bolander+Anderson 2011)**

*For $\mathcal{E}$ with $md = 1$ and effects, $\mathcal{T}_{(\mathcal{M}, \mathcal{E})}$ can be non-automatic.*

**Corollary (of Charrier+Maubert+Schwarzentruber 2016)**

*For $\mathcal{E}$ with $alt = 2$, $\mathcal{T}_{(\mathcal{M}, \mathcal{E})}$ can be non-automatic.*

**Open**

The case $\mathcal{E}$ with $md = 1$ and no effects.

- For $\mathcal{E}$ public announcements, $\mathcal{T}_{(\mathcal{M}, \mathcal{E})}$ is a regular automatic tree.
- Context-free, context-sensitive natural encodings $we_1e_2 \ldots e_n$, but $\mathcal{T}_{(\mathcal{M}, \mathcal{E})}$ might be automatic for another encoding ...
Wrap up

- Classes of game trees are central mathematical objects.
- Decidable fragments of MSO (SO quantification provides the strategy quantifiers).
- The subclasses of automatic structures, that we called regular automatic trees, might be a track to follow.

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