Overview

Regular LT properties

Idea: define regular LT properties to be those languages of infinite words over the alphabet 2^{AP} that have a representation by a finite automata

- regular safety properties:
  NFA-representation for the bad prefixes
- other regular LT properties:
  representation by ω-automata, i.e., acceptors for infinite words

Overview

Recall: definition of safety properties

Let E be a LT property over AP, i.e., $E \subseteq (2^{AP})^\omega$.

E is called a safety property if for all words

$$\sigma \ = \ A_0 \, A_1 \, A_2 \ldots \in (2^{AP})^\omega \setminus E$$

there exists a finite prefix $A_0 \, A_1 \ldots A_n$ of σ such that *none* of the words $A_0 \, A_1 \ldots A_n \, B_{n+1} \, B_{n+2} \, B_{n+3} \ldots$ belongs to E, i.e.,

$$E \cap \left\{ \sigma' \in (2^{AP})^\omega : A_0 \ldots A_n \text{ is a prefix of } \sigma' \right\} = \varnothing$$

Such words $A_0 \, A_1 \ldots A_n$ are called bad prefixes for E.

$$BadPref \ \stackrel{\text{def}}{=} \ \text{set of bad prefixes for } E \ \subseteq (2^{AP})^+$$

Regular safety properties

Let $E \subseteq (2^{AP})^\omega$ be a safety property.

E is called regular iff the language

$BadPref$ = set of all bad prefixes for $E \subseteq (2^{AP})^+$

$BadPref = \mathcal{L}(\mathcal{A})$ for some NFA $\mathcal{A}$ over the alphabet 2^{AP}

is regular.

Nondeterministic finite automata (NFA)

NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

- Q finite set of states
- Σ alphabet
- $\delta : Q \times \Sigma \rightarrow 2^Q$ transition relation
- $Q_0 \subseteq Q$ set of initial states
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run for a word $A_0 A_1 \ldots A_{n-1} \in \Sigma^*$:

state sequence $\pi = q_0\, q_1 \ldots q_n$ where $q_0 \in Q_0$
and $q_{i+1} \in \delta(q_i, A_i)$ for $0 \leq i < n$

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run π is called accepting if $q_n \in F$

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accepted language $\mathcal{L}(\mathcal{A}) \subseteq \Sigma^*$ is given by:

$\mathcal{L}(\mathcal{A})$ = set of finite words over Σ that have an accepting run in $\mathcal{A}$

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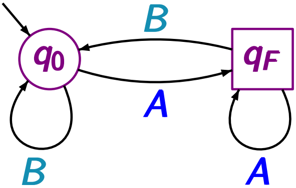
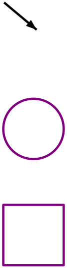
Notations in pictures for NFA

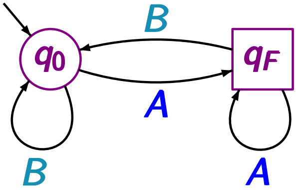


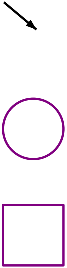


NFA $\mathcal{A}$ with state space $\{q_0, q_F\}$

- q_0 initial state
- q_F final state
- alphabet $\Sigma = \{A, B\}$

Notations in pictures for NFA





accepted language $\mathcal{L}(\mathcal{A})$:

set of all finite words over $\{A, B\}$ ending with letter A

Symbolic notations

NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ over the alphabet $\Sigma = 2^{AP}$

symbolic notation for the labels of transitions:

> If Φ is a propositional formula over AP then
>
> $q \xrightarrow{\Phi} p$ stands for the set of transitions $q \xrightarrow{A} p$
>
> where $A \subseteq AP$ such that $A \models \Phi$

Example: if $AP = \{a, b, c\}$ then

$$q \xrightarrow{a \wedge \neg b} p \mathrel{\widehat{=}} \left\{ q \xrightarrow{A} p : A{=}\{a, c\} \text{ or } A{=}\{a\} \right\}$$

$$q \xrightarrow{true} p \mathrel{\widehat{=}} \left\{ q \xrightarrow{A} p : A \subseteq AP \right\}$$

Regular safety properties

A safety property $E \subseteq (2^{AP})^\omega$ is called regular iff

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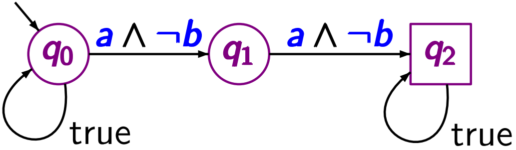

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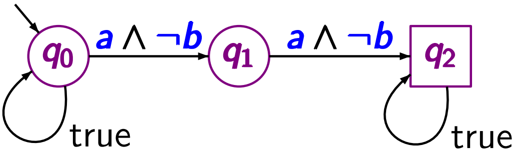

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symbolic notation:

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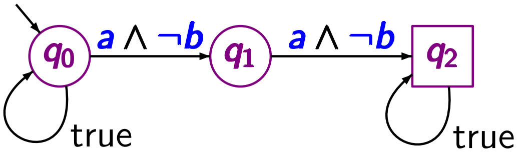

$AP = \{a, b\}$

symbolic notation:

$a \wedge \neg b \mathrel{\widehat{=}} \{a\}$

safety property E: "$a \wedge \neg b$ never holds twice in a row"

Example: regular safety property

"Every red phase is preceded by a yellow phase"

Example: regular safety property

"Every red phase is preceded by a yellow phase"

set of all infinite words $A_0\, A_1\, A_2 \ldots$ s.t. for all $i \geq 0$:

$$red \in A_i \implies i \geq 1 \text{ and } yellow \in A_{i-1}$$

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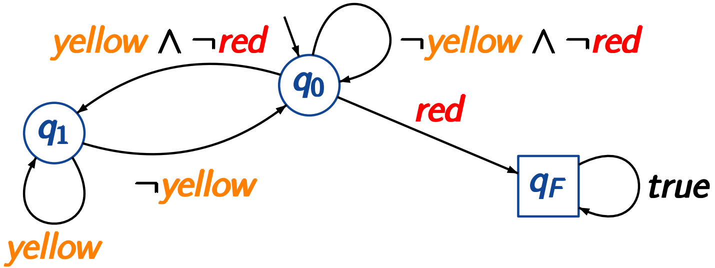
DFA for all (possibly non-minimal) bad prefixes



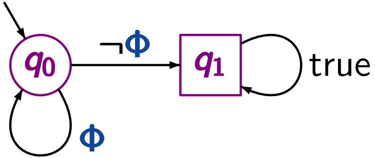
Correct or wrong?

Every invariant is regular.

Every invariant is regular.

correct.

Let E be an invariant with invariant condition Φ



is a DFA for the language of all bad prefixes

Correct or wrong?

Every safety property is regular.

Correct or wrong?

Every safety property is regular.

wrong. e.g., $AP = \{pay, drink\}$

E = set of alle infinite words $A_0\, A_1\, A_2 \ldots \in (2^{AP})^\omega$ such that for all $j \in \mathbb{N}$:

$$\big|\{i \leq j : pay \in A_i\}\big| \geq \big|\{i \leq j : drink \in A_i\}\big|$$

- E is a safety property, but
- the language of (minimal) bad prefixes is *not* regular

Verifying regular safety properties

Verifying regular safety properties

given: finite TS $\mathcal{T}$
regular safety property E
(represented by an **NFA** for its bad prefixes)

question: does $\mathcal{T} \models E$ hold **?**

Verifying regular safety properties

given: finite TS $\mathcal{T}$
regular safety property E
(represented by an **NFA** for its bad prefixes)

question: does $\mathcal{T} \models E$ hold **?**

method: relies on an analogy between the tasks:

- checking language inclusion for **NFA**
- model checking regular safety properties

language inclusion for NFA	verification of regular safety properties
$\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ **?**	$Traces(\mathcal{T}) \subseteq E$ **?**

language inclusion for NFA	verification of regular safety properties
$\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ **?**	$\mathit{Traces}(\mathcal{T}) \subseteq E$ **?**
check whether $\mathcal{L}(\mathcal{A}_1) \cap (\Sigma^* \setminus \mathcal{L}(\mathcal{A}_2))$ is empty	
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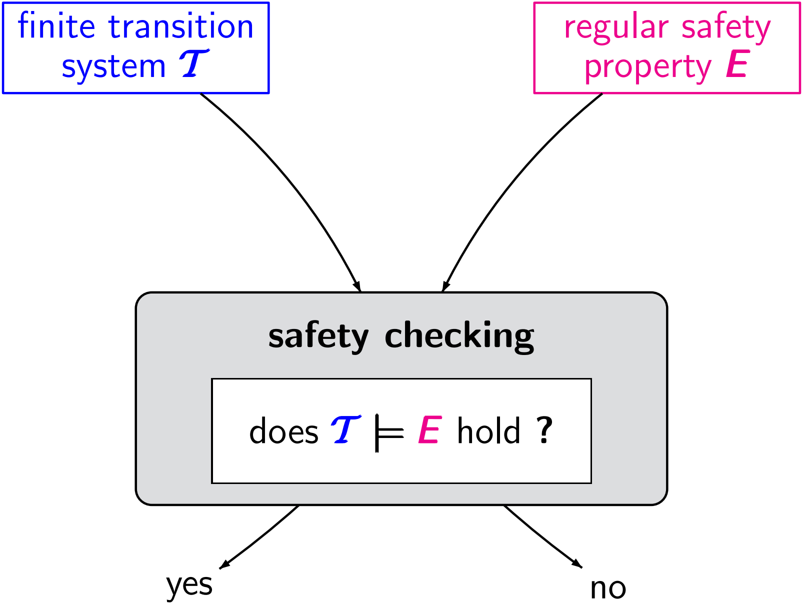
Checking regular safety properties



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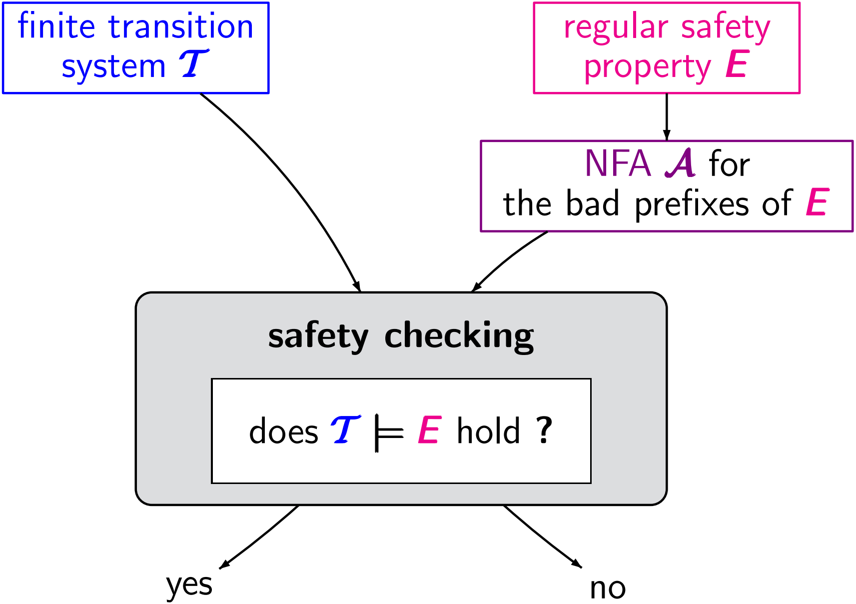

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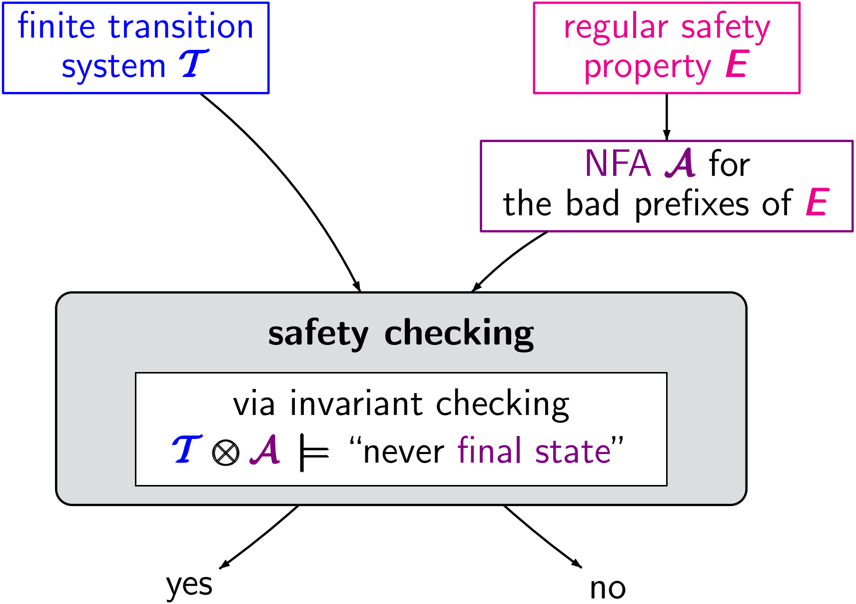

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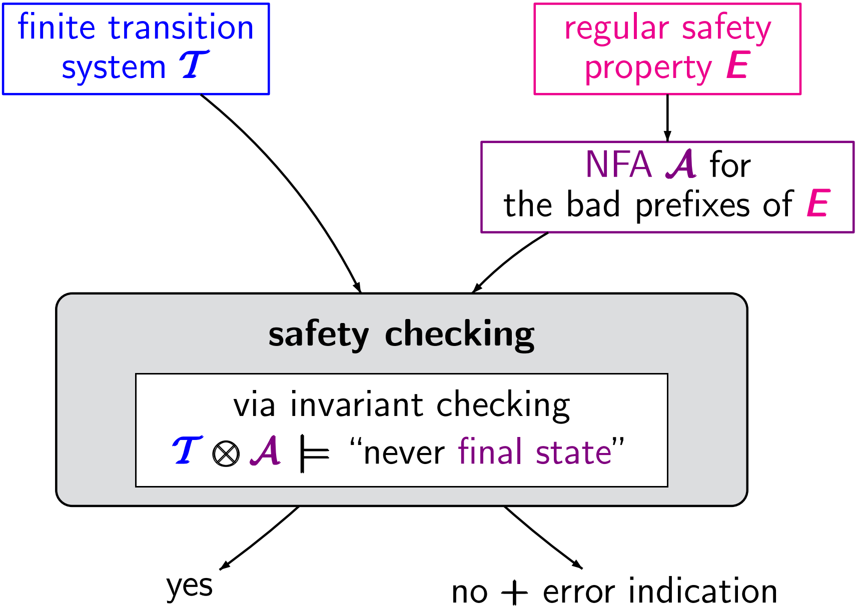

Product of a TS and an NFA

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finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

path
fragment $\widehat{\pi}$

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s_0	$L(s_0) = A_0$
$\downarrow$	
s_1	$L(s_1) = A_1$
$\downarrow$	
s_2	$L(s_2) = A_2$
$\downarrow$	
$\vdots$	$\vdots$
$\downarrow$	
s_n	$L(s_n) = A_n$
path fragment $\widehat{\pi}$	trace

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$$s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \cdots \rightarrow s_n$$

$L(s_0)=A_0$

$L(s_1)=A_1$

$L(s_2)=A_2$

$\vdots$

$L(s_n)=A_n$

$$q_0 \in Q_0 \xrightarrow{A_0} q_1 \xrightarrow{A_1} q_2 \xrightarrow{A_2} \cdots \rightarrow q_n \xrightarrow{A_n} q_{n+1}$$

path fragment $\widehat{\pi}$ | trace | run for $trace(\widehat{\pi})$

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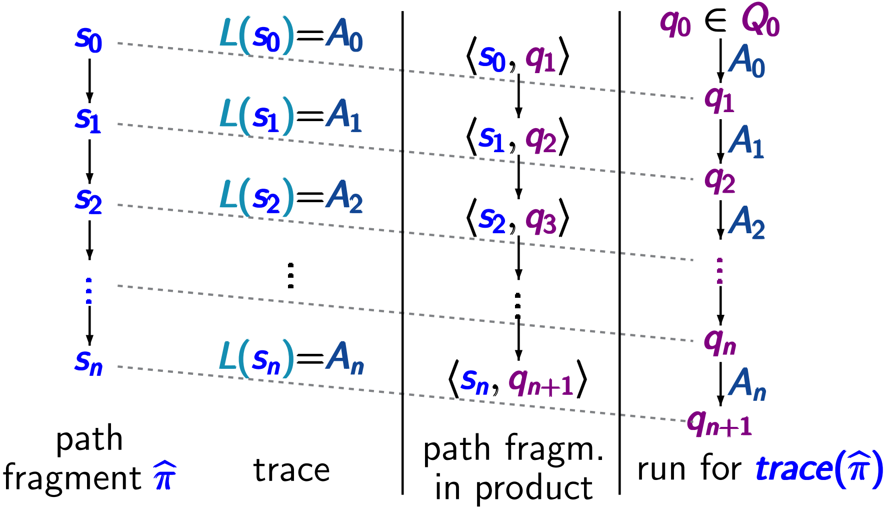

Product transition system

IS2.5-25

Product transition system

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for $P \subseteq Q$ and $A \subseteq AP$: $\quad \delta(P, A) = \bigcup_{p \in P} \delta(p, A)$

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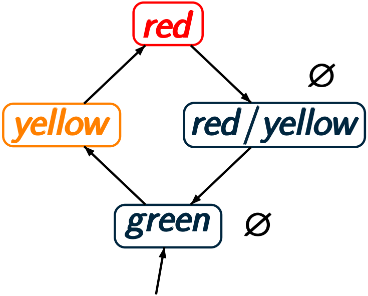
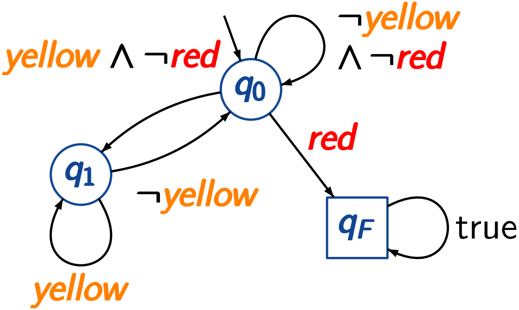
labeling function: $L'(\langle s, q \rangle) = \{q\}$

Example: product-TS



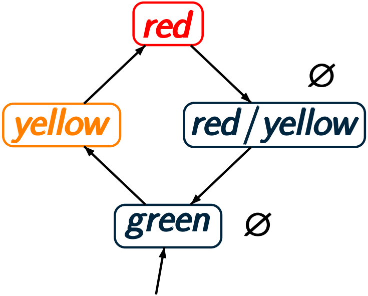


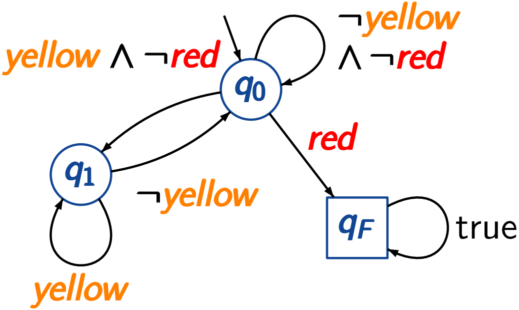
transition system $\mathcal{T}$ over
$AP = \{red, yellow\}$

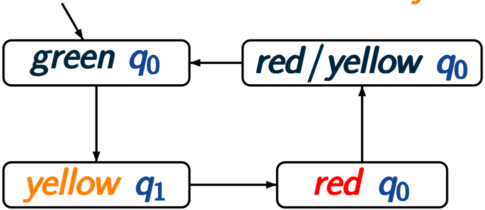
DFA $\mathcal{A}$ for the
bad prefixes for E

> $\mathcal{T}$ satisfies the safety property E
> *"every red phase is preceded by a yellow phase"*

Example: product-TS



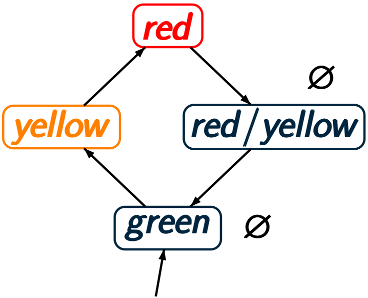


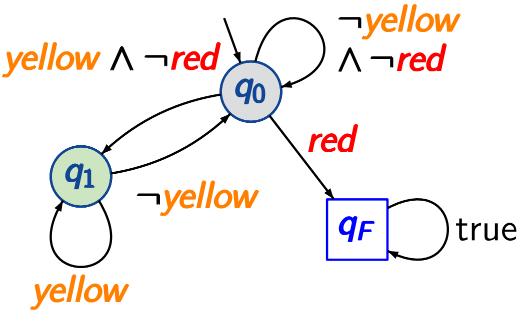


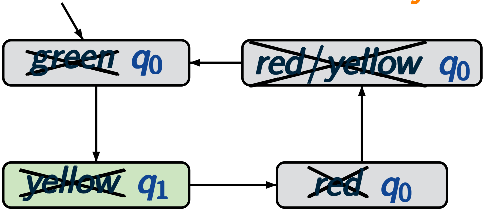
product-TS
$\mathcal{T} \otimes \mathcal{A}$

$4 * 3 = 12$ states, but
just 4 reachable states

Example: product-TS







set of propositions
$AP' = \{q_0, q_1, q_F\}$

invariant condition $\neg q_F$ holds
for all reachable states

Technical remark on the product-TS

definition of the product of

- a transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

- an NFA $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

then the product $\mathcal{T} \otimes \mathcal{A} = (S \times Q, Act, \rightarrow', \ldots)$ is a TS

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- $\mathcal{A}$ is non-blocking, i.e.,

$$Q_0 \neq \varnothing \ \wedge \ \forall q \in Q \, \forall A \in 2^{AP}. \, \delta(q, A) \neq \varnothing$$

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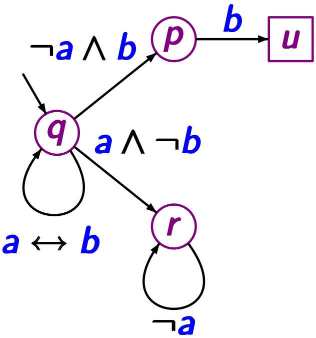
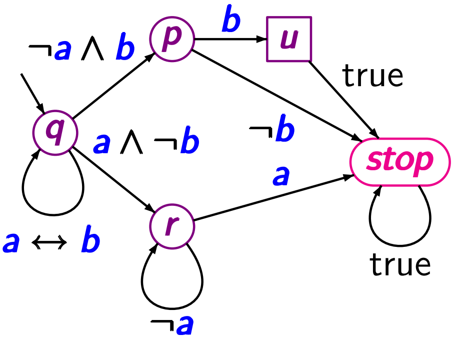
- no initial state of $\mathcal{A}$ is final, i.e., $Q_0 \cap F = \varnothing$

Non-blocking NFA

NFA $\mathcal{A}$ $\rightsquigarrow$ equivalent NFA $\mathcal{A}'$



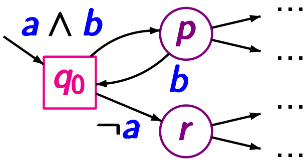


blocks for input
$\{a\}\,\varnothing\,\{a\}$

non-blocking

NFA where no initial state is final

NFA $\mathcal{A}$ with $Q_0 \cap F \neq \varnothing$



NFA where no initial state is final

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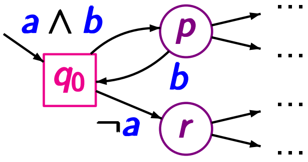
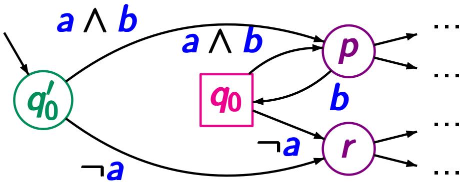
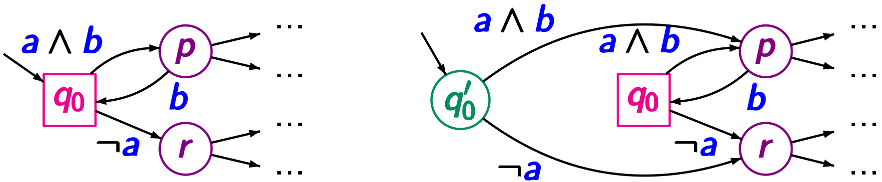




$$\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}) \setminus \{\varepsilon\}$$

NFA where no initial state is final

NFA $\mathcal{A}$ with $Q_0 \cap F \neq \varnothing$ $\leadsto$ NFA $\mathcal{A}'$ with $Q_0 \cap F = \varnothing$



$$\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}) \setminus \{\varepsilon\}$$

note: if $\mathcal{A}$ is an NFA for the bad prefixes of a safety property then

$$\varepsilon \notin \mathcal{L}(\mathcal{A}) = \mathit{BadPref}$$

Model checking regular safety properties

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system (without terminal states)

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ be an NFA for the bad prefixes of a regular safety property E (non-blocking and $Q_0 \cap F = \varnothing$)

The following statements are equivalent:

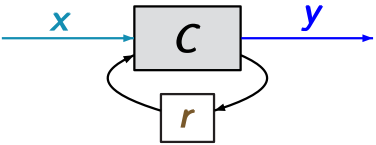
(1) $\mathcal{T} \models E$

(2) $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \varnothing$

(3) $\mathcal{T} \otimes \mathcal{A} \models$ invariant "always $\neg F$"

where "$\neg F$" denotes $\bigwedge_{q \in F} \neg q$

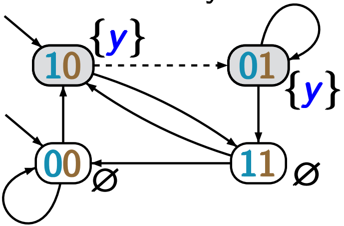
Example: sequential circuit



$\lambda_y = \delta_r = x \oplus r$

initially $r = 0$

transition system $\mathcal{T}$



$\mathcal{T} \not\models E$

error indication, e.g.,

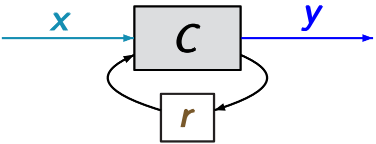
$\langle 10 \rangle \langle 01 \rangle$

bad prefix: $\{y\}\{y\}$

safety property E

The circuit will never ouput two ones after each other

Example: sequential circuit



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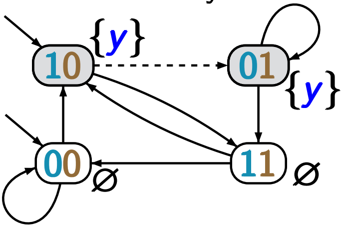

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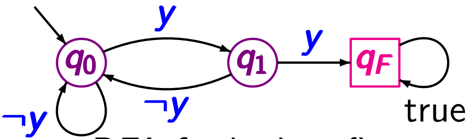

DFA for bad prefixes

safety property E

The circuit will never ouput two ones after each other

Example: product-TS

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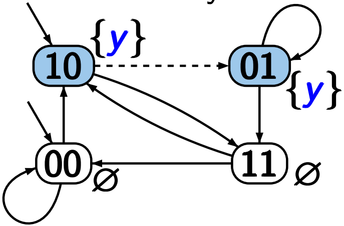
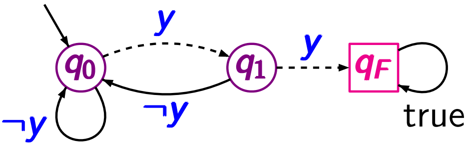
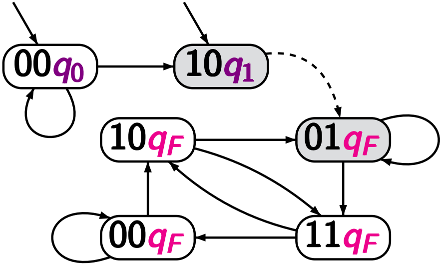


safety property E

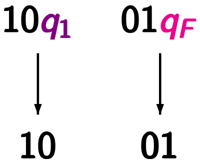
... never two ones in a row ...





error indication for $\mathcal{T} \nvDash E$

error indication for $\mathcal{T} \otimes \mathcal{A} \nvDash$ "never q_F"



Model checking regular safety properties

input: finite TS $\mathcal{T}$,
NFA $\mathcal{A}$ for the bad prefixes of E

output: "yes" if $\mathcal{T} \models E$
otherwise "no" + error indication

construct product transition system $\mathcal{T} \otimes \mathcal{A}$
check whether $\mathcal{T} \otimes \mathcal{A} \models$ "always $\neg F$"
if so, then return "yes"
if not, then return "no" $\longleftarrow$ and an error indication

where F = set of final states in $\mathcal{A}$

Model checking regular safety properties

construct product transition system $\mathcal{T} \otimes \mathcal{A}$

IF $\mathcal{T} \otimes \mathcal{A} \models$ "always $\neg F$"

THEN return "yes"

ELSE compute a counterexample for $\mathcal{T} \otimes \mathcal{A}$ and the invariant "always $\neg F$",

i.e., an initial path fragment in the product

$$\langle s_0, p_0 \rangle \langle s_1, p_1 \rangle \ldots \langle s_n, p_n \rangle \text{ where } p_n \in F$$

return "no" and $s_0\, s_1 \ldots s_n$

FI

time complexity: $\mathcal{O}\big(size(\mathcal{T}) \cdot size(\mathcal{A})\big)$

Correct or wrong?

If $\mathcal{T}$ is a finite transition system then $Traces_{fin}(\mathcal{T})$ is regular.

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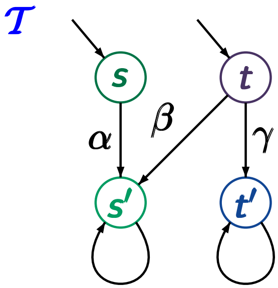

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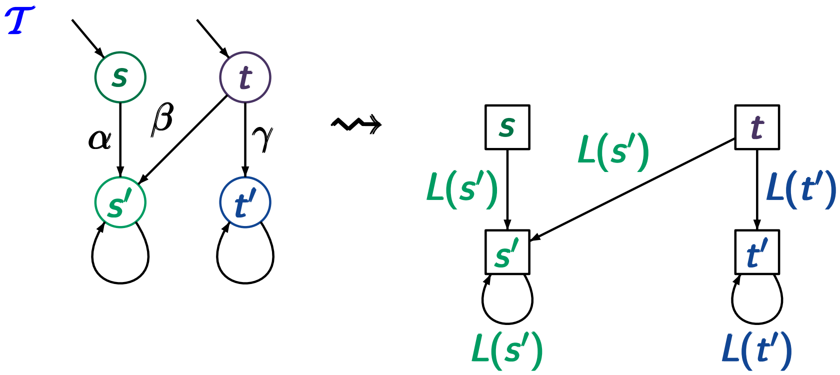

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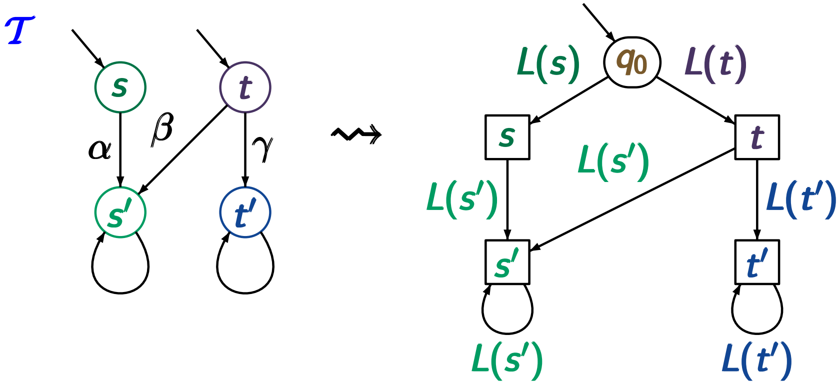

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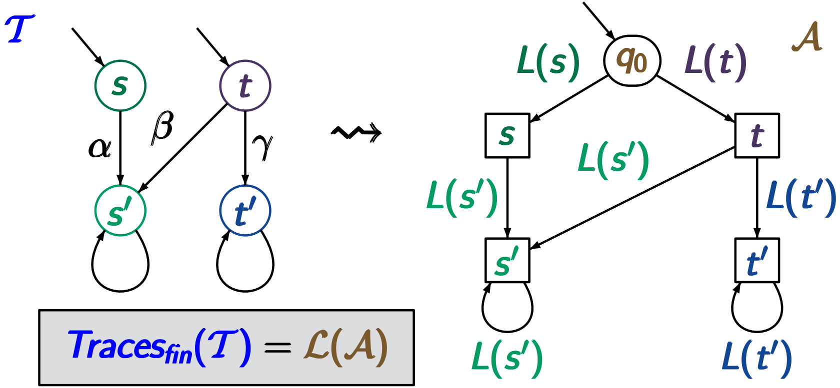

$$Traces_{fin}(\mathcal{T}) = \mathcal{L}(\mathcal{A})$$