Overview

Verifying ω-regular properties

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ω-regular property E

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requires techniques for checking
persistence properties in finite TS

Persistence property

Persistence property

Let E be an LT-property, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$

E is called a persistence property if there exists a propositional formula Φ over AP such that

$$E = \left\{ \text{set of all infinite words } A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^{\omega} \text{ s.t. } \overset{\infty}{\forall}\, i \geq 0.\ A_i \models \Phi \right\}$$

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"from some moment on Φ"
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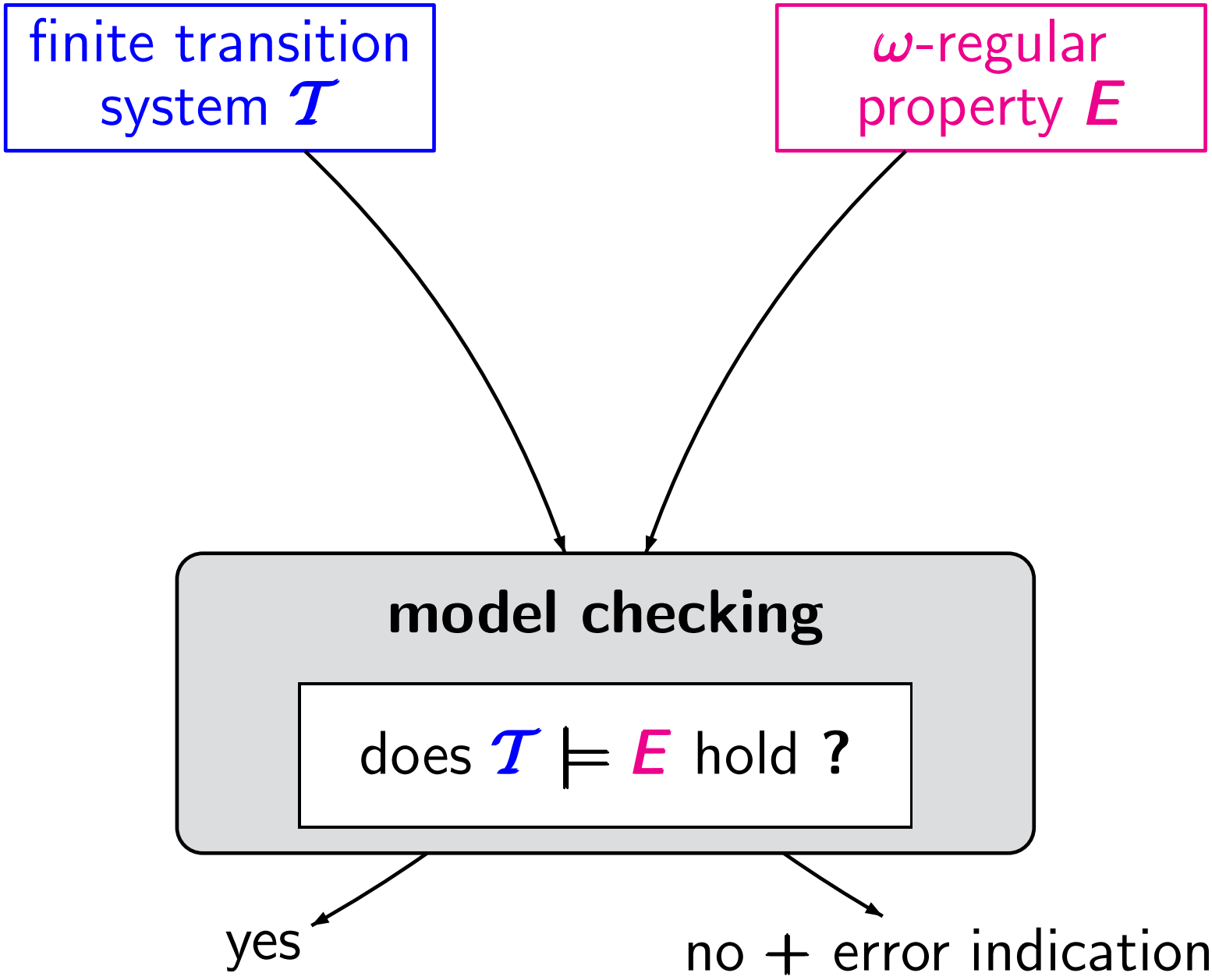
Checking ω-regular properties



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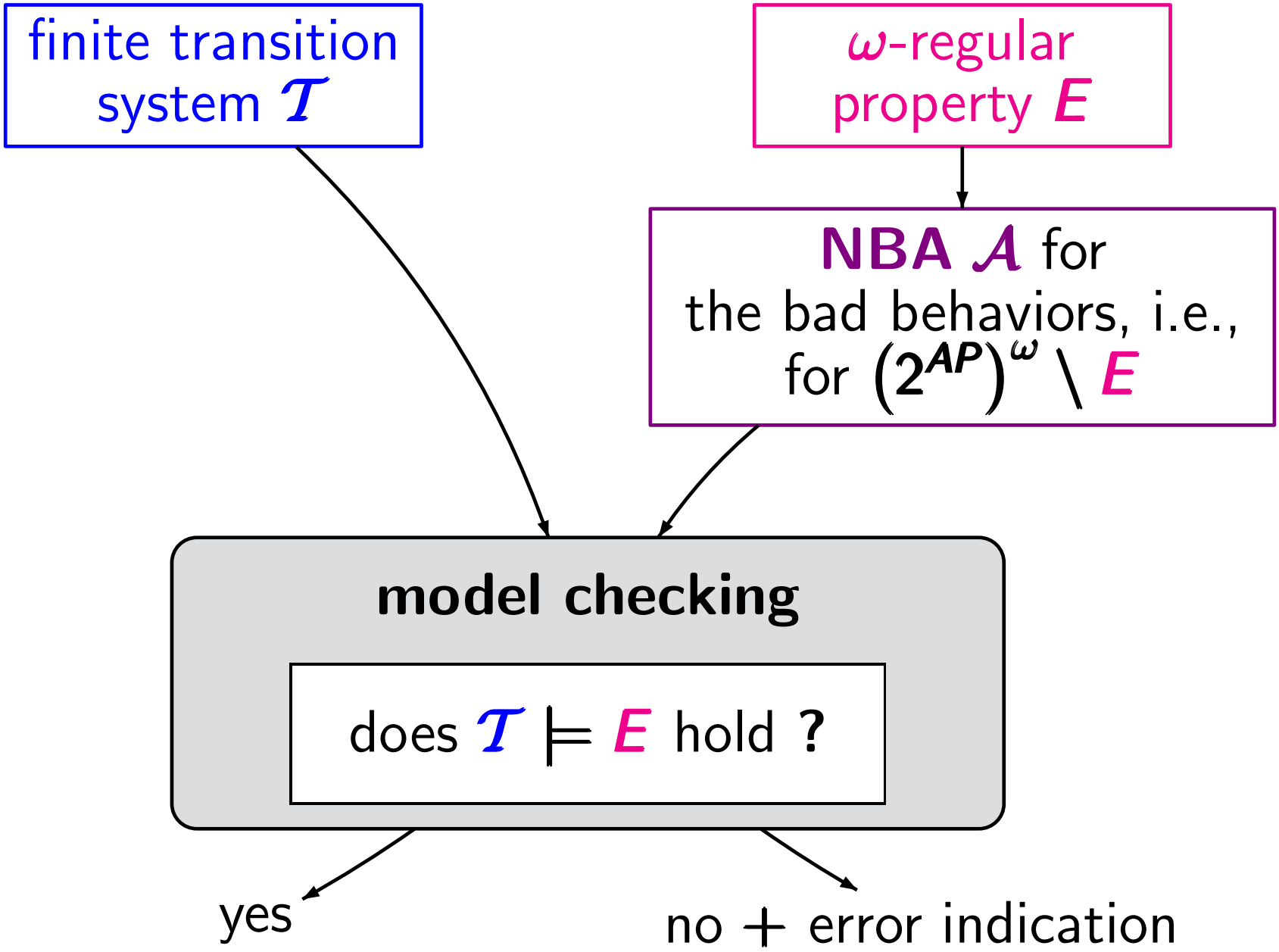

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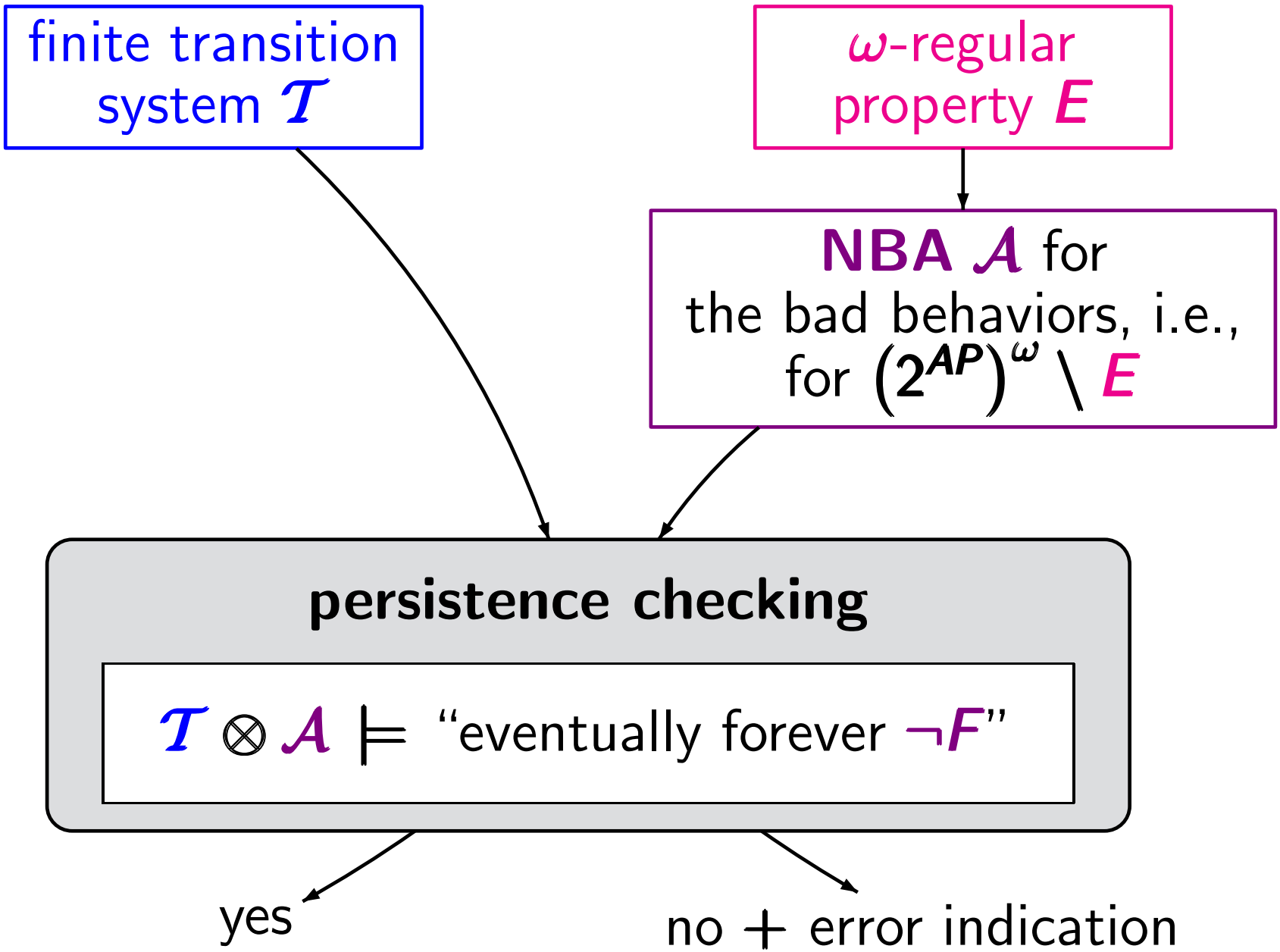

Recall: product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

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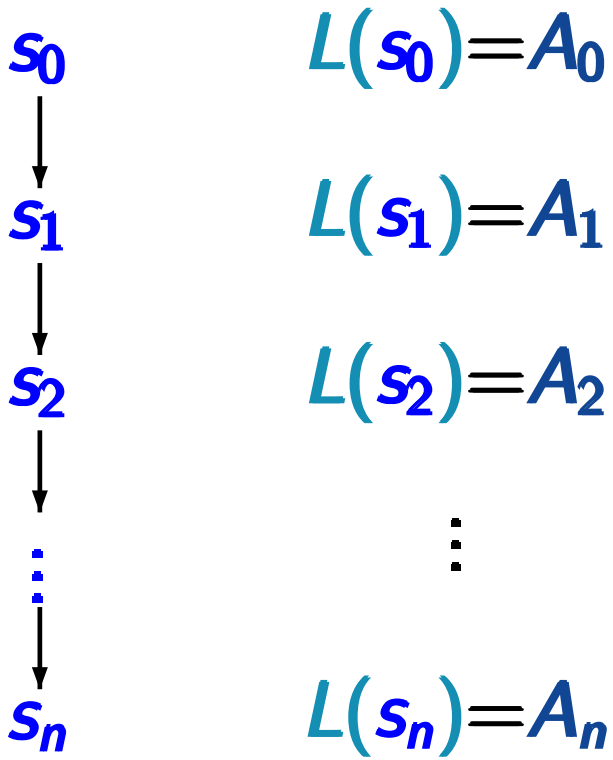

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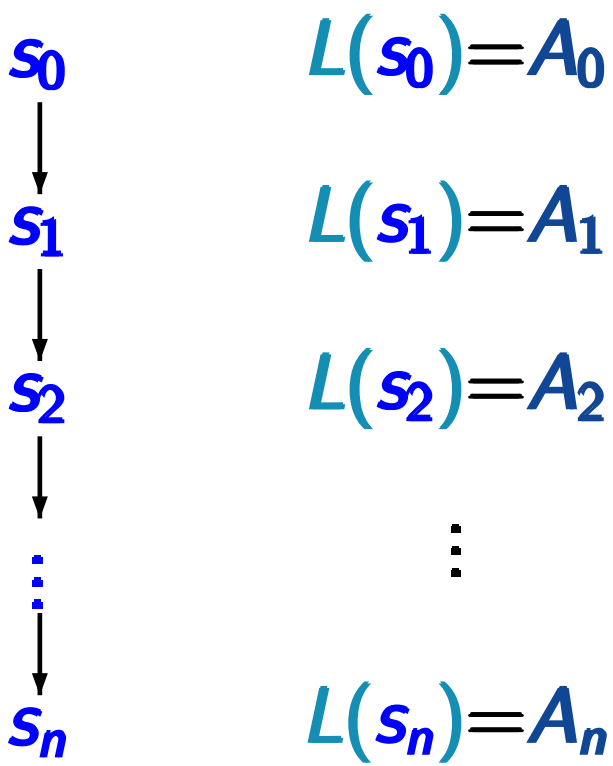

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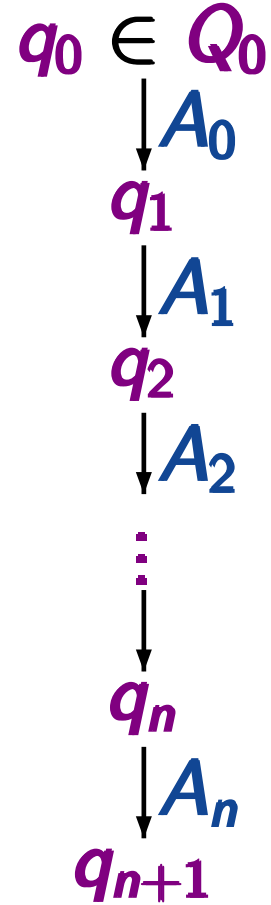

run for $trace(\widehat{\pi})$

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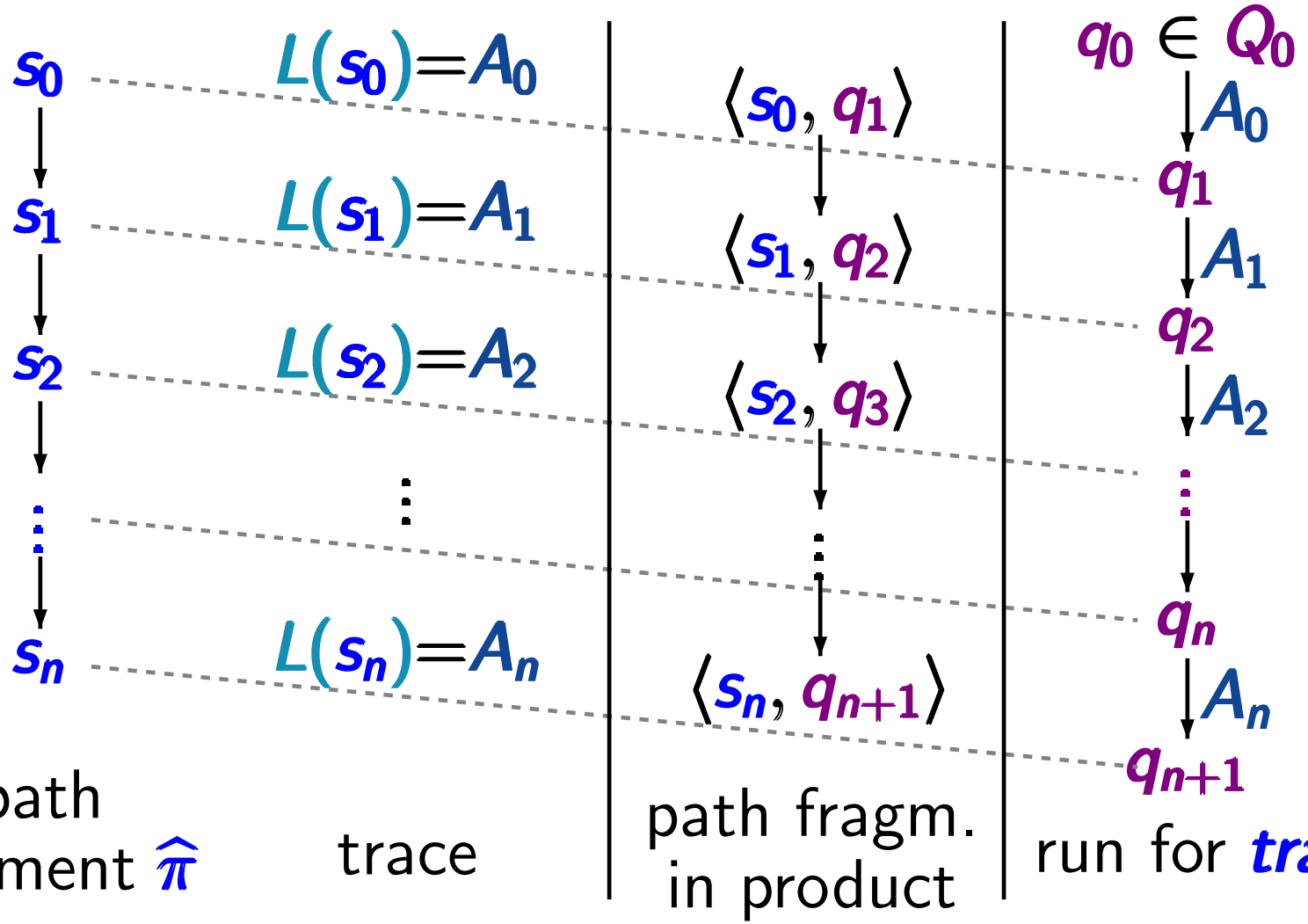

Product transition system

recall: definition of the product of a **TS** and **NFA**

Product transition system

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product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

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labeling function: $L'(\langle s, q \rangle) = \{q\}$

Product transition system

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$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA $\leftarrow$ same definition for **NBA**

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Product of a TS and NBA

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relies on a reduction to the persistence checking problem

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$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ finite transition system without terminal states

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representing the bad behaviors of an ω-regular LT-property E

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Example: ω-regular model checking

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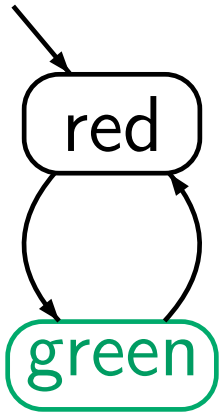

LT property: "infinitely often green"

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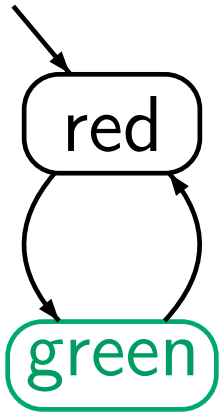

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NBA $\mathcal{A}$ for the complement

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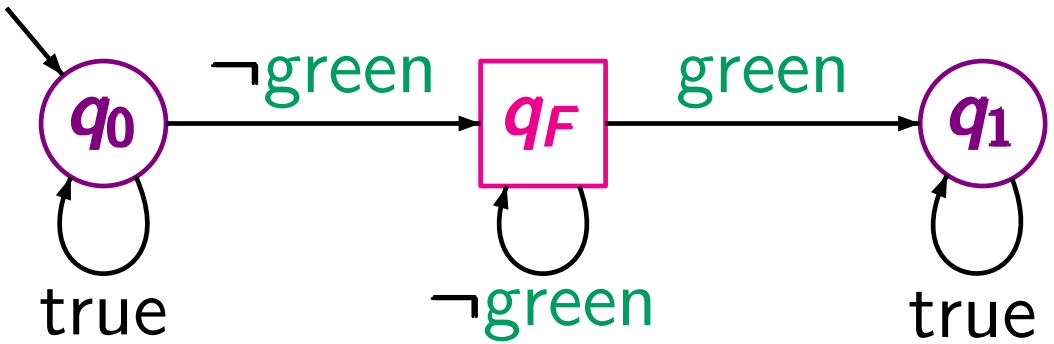

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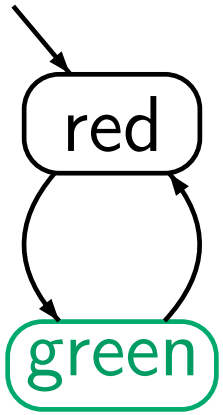

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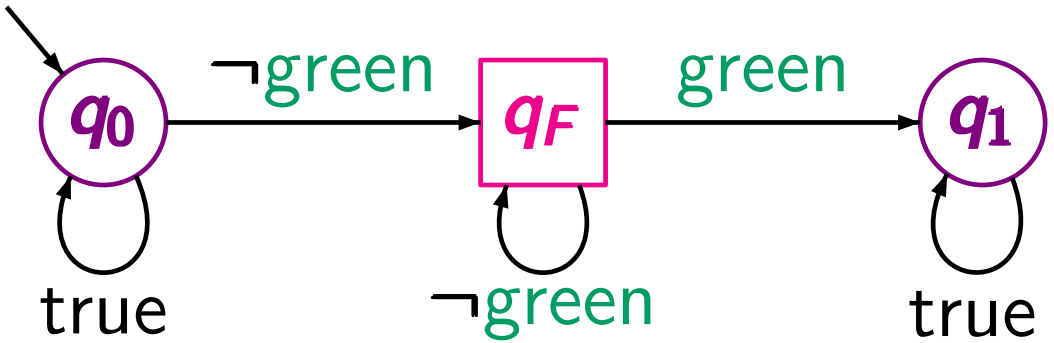

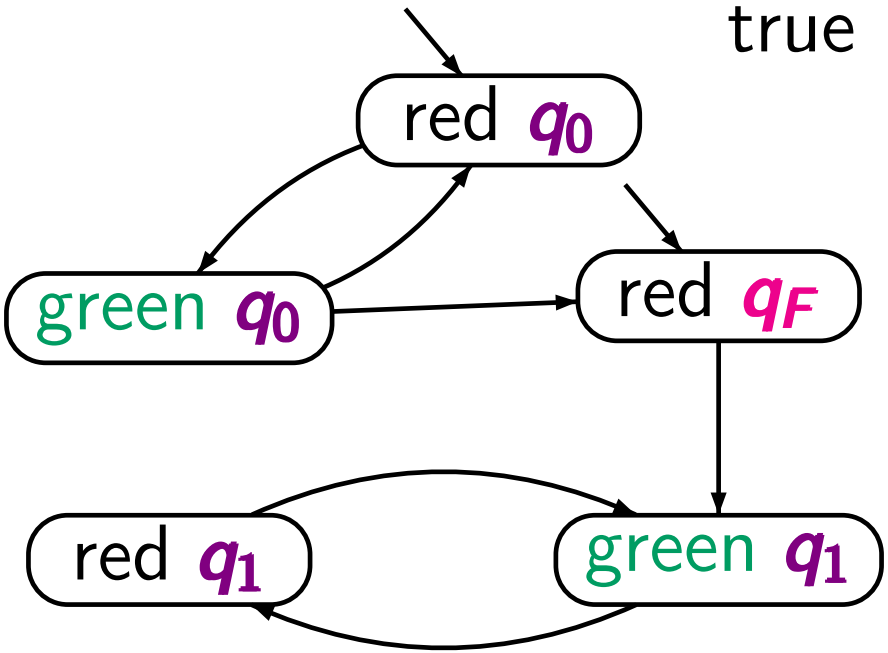

reachable fragment of the product TS $\mathcal{T} \otimes \mathcal{A}$

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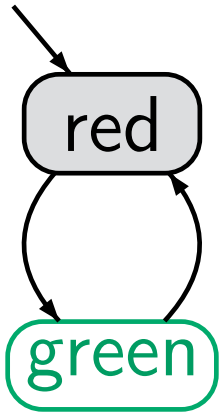

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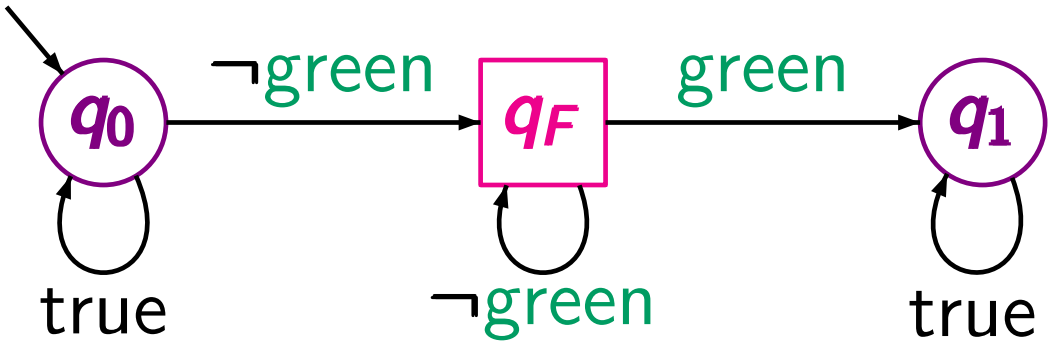

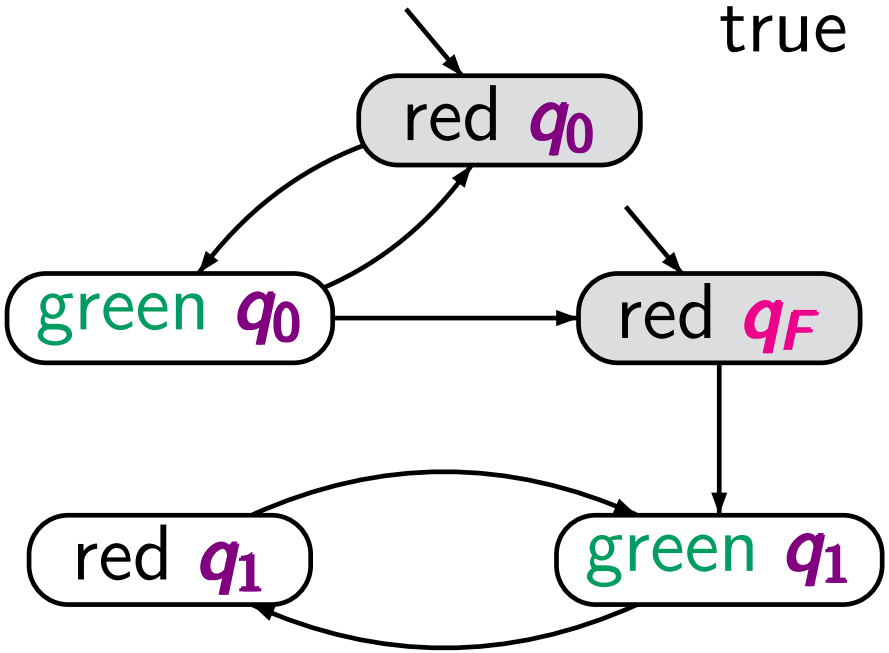

initial states:

$\langle \text{red}, q \rangle$ where

$$q \in \delta(q_0, L(\text{red})) = \delta(q_0, \varnothing) = \{q_0, q_F\}$$

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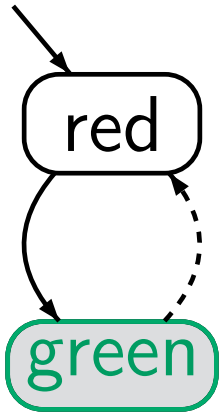

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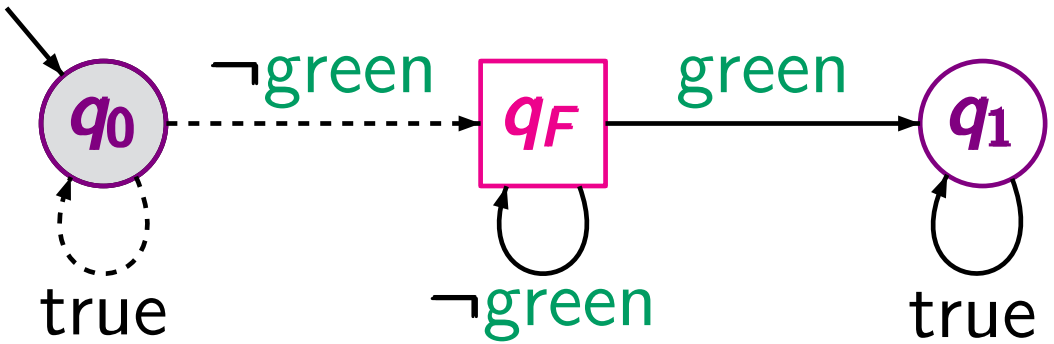

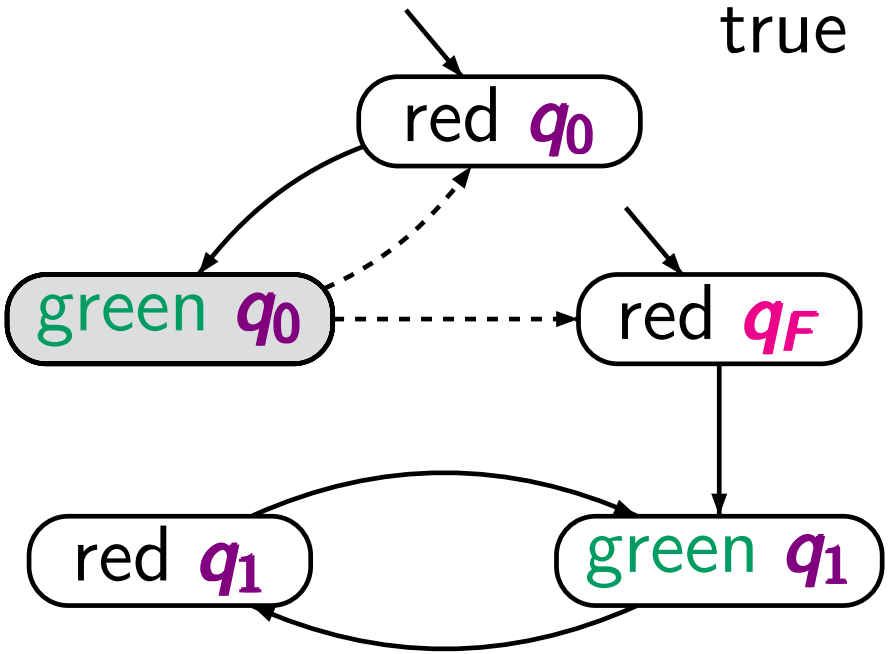

transition

$\langle \text{green}, q_0 \rangle \longrightarrow \langle \text{red}, q \rangle$

$q \in \delta(q_0, L(\text{red}))$

$= \delta(q_0, \varnothing)$

$= \{q_0, q_F\}$

Example: ω-regular model checking

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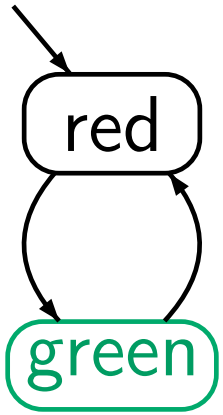

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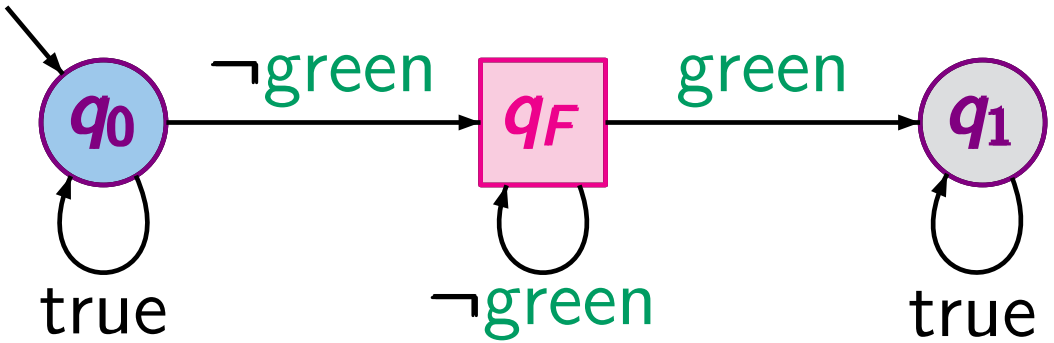

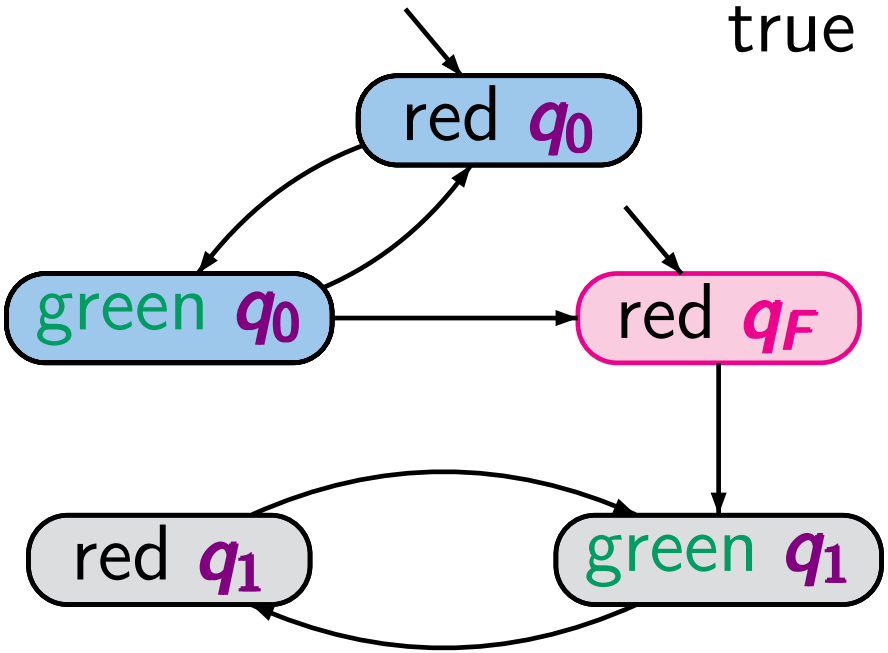

atomic propositions
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obvious labeling function

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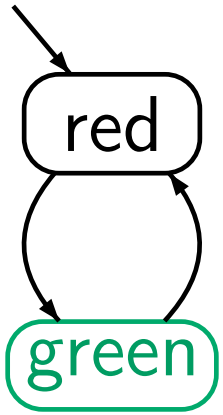

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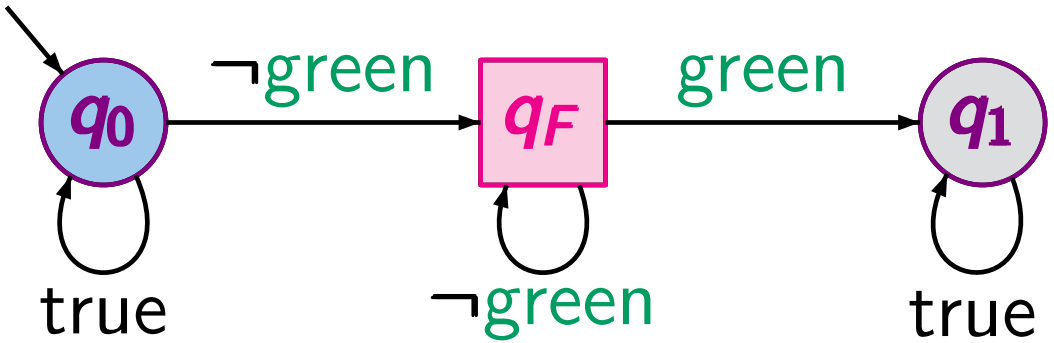

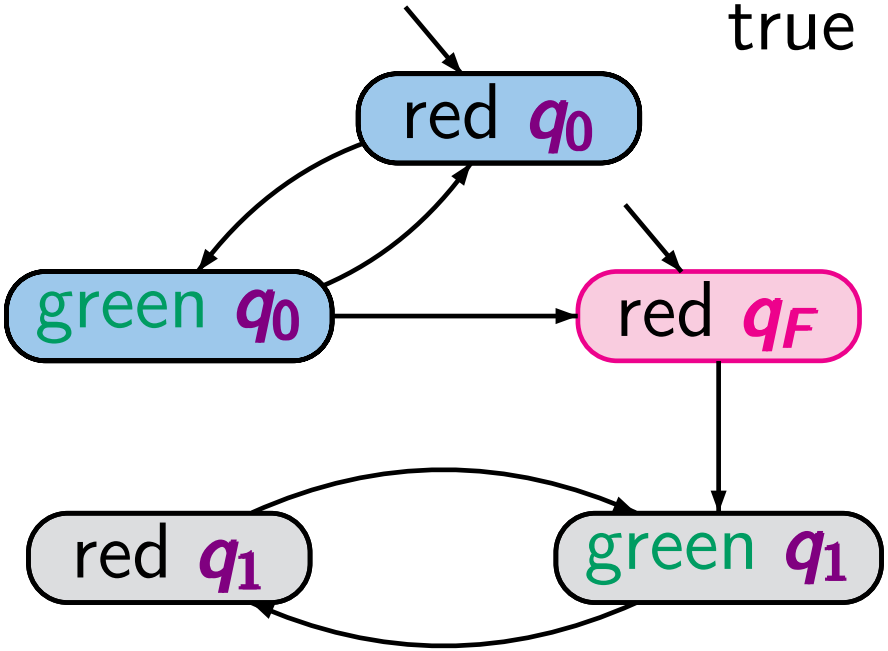

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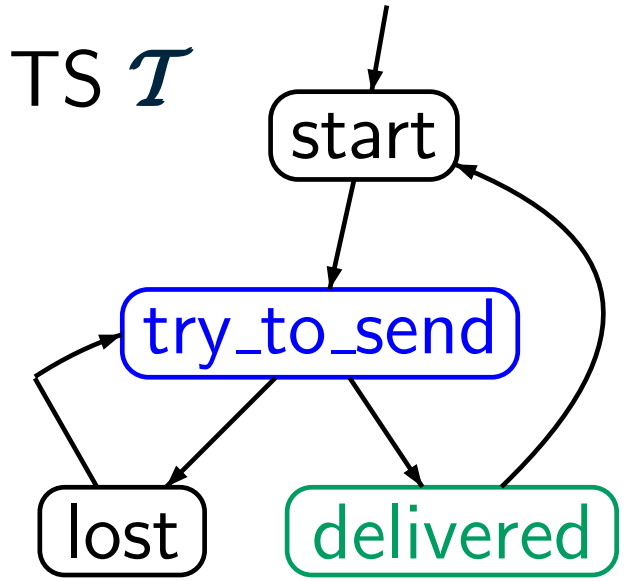

ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

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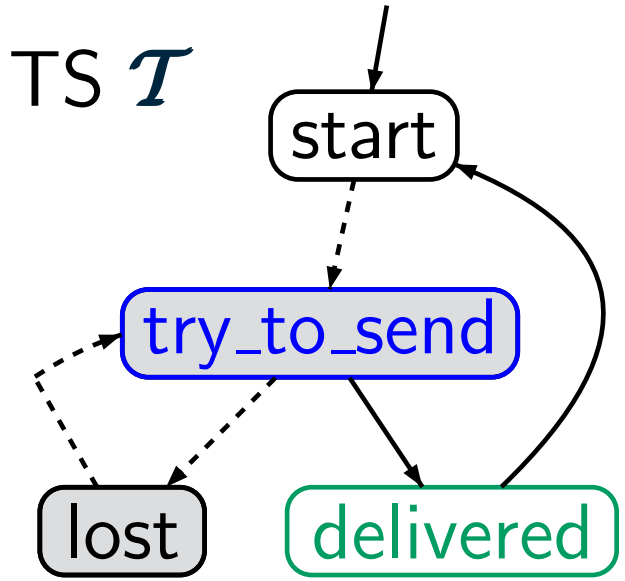

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$\mathcal{T} \not\models E$

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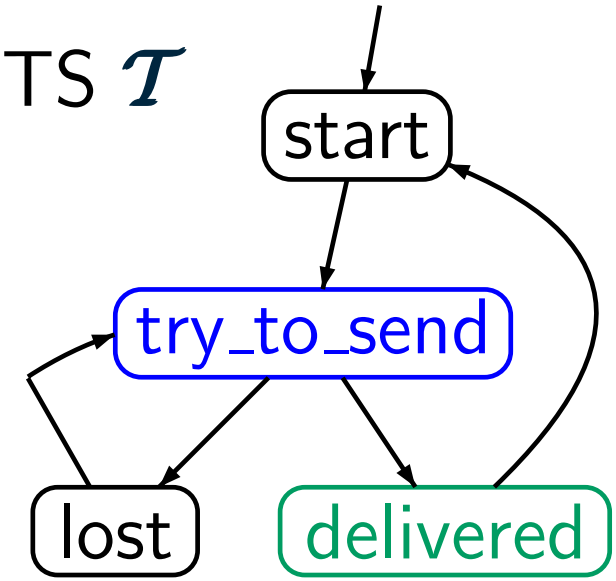
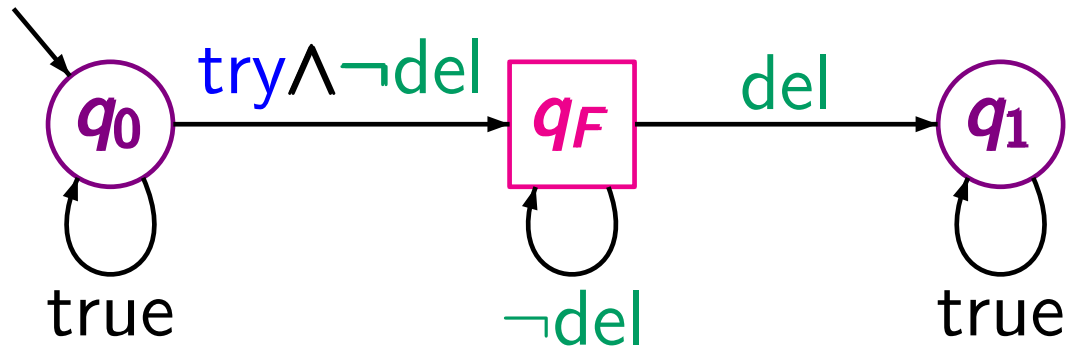


NBA $\mathcal{A}$ for the bad behaviors



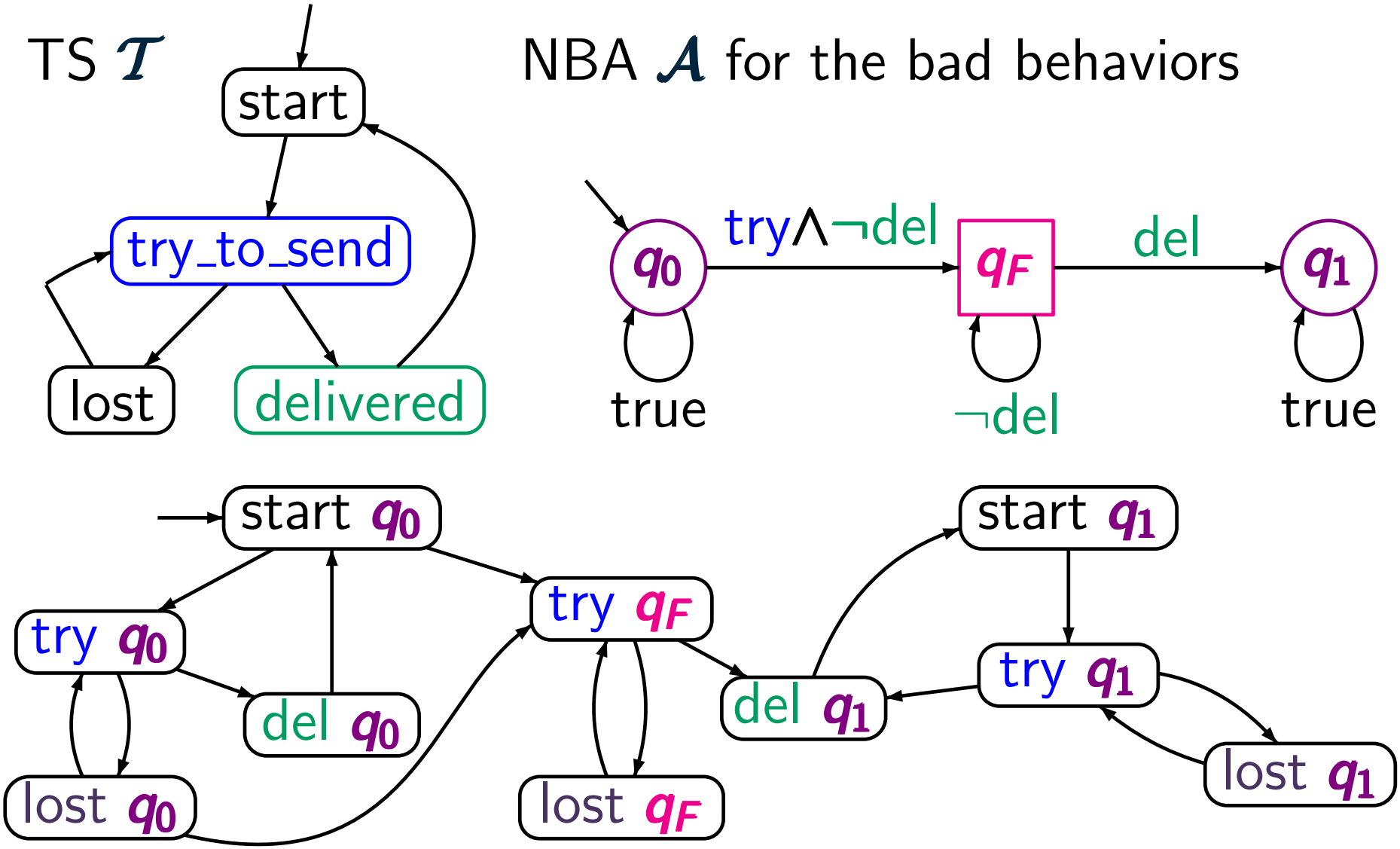
ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

complement of E, i.e., LT property for the bad behaviors:

"never delivered after some trial"

Example: ω-regular model checking



reachable fragment of the product-TS

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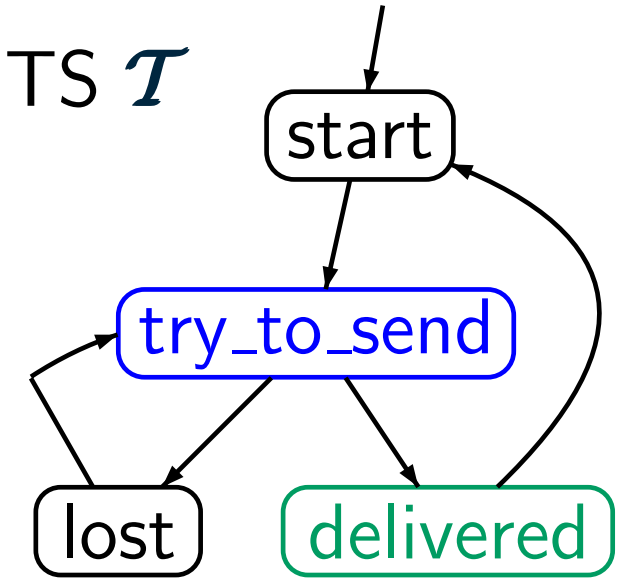
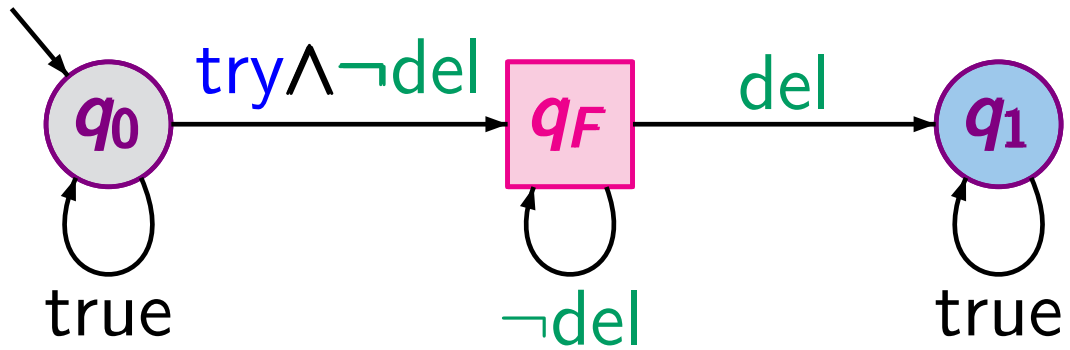
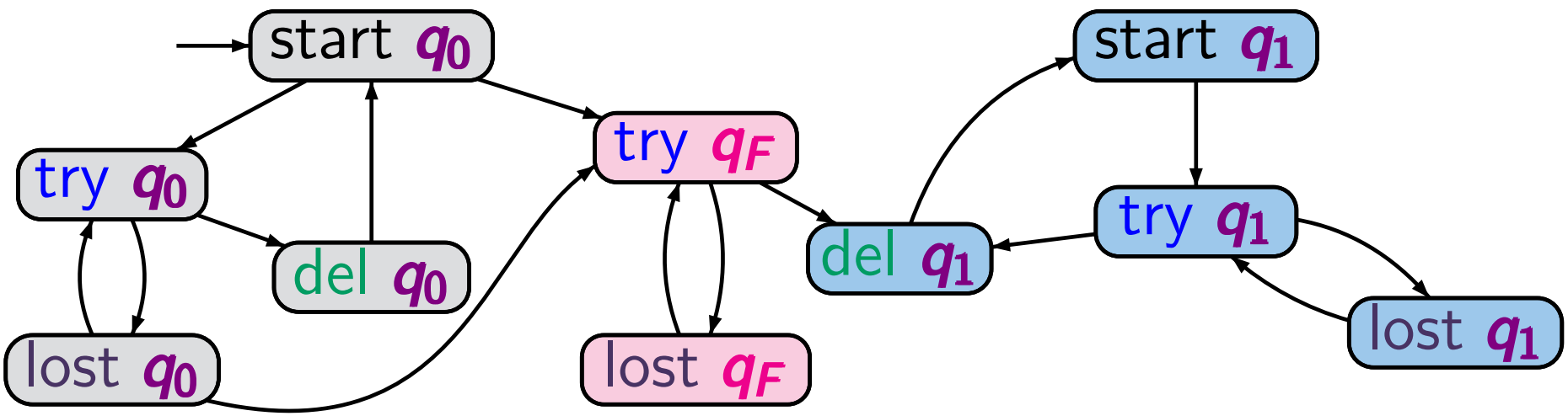
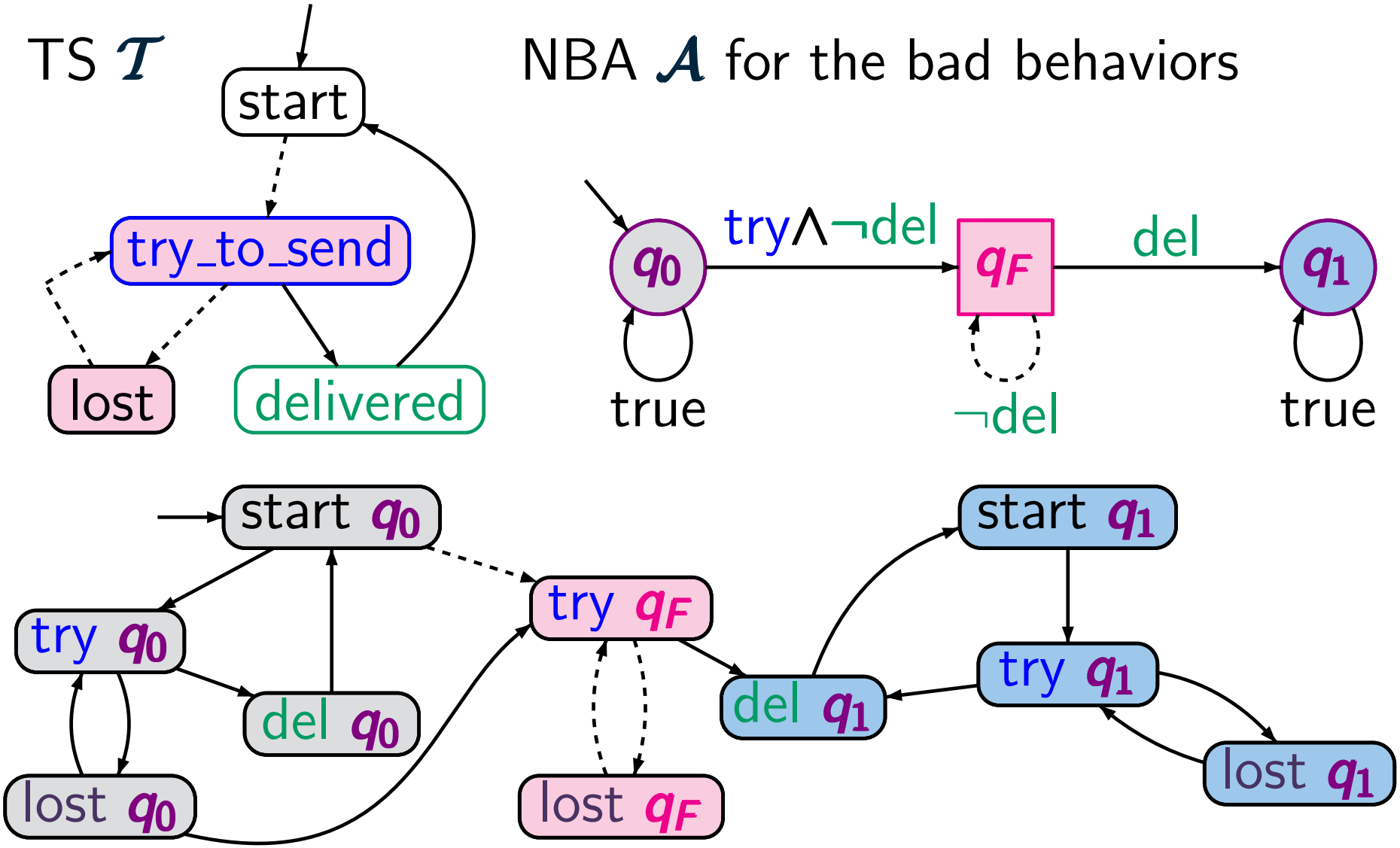






set of atomic propositions $AP' = \{q_0, q_1, q_F\}$

Example: ω-regular model checking



$\mathcal{T} \otimes \mathcal{A} \not\models$ "eventually forever $\neg F$"

Checking safety and ω-regular properties

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F = set of final states in $\mathcal{A}$

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persistence checking

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given: finite transition system $\mathcal{T}$ over AP
persistence condition $a \in AP$

question: does $\mathcal{T} \models$ "eventually forever a" hold **?**

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SCC: strongly connected component, i.e., maximal set of states that are reachable from each other

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A SCC is called non-trivial if it has at least one edge.
"either 1 state with a self-loop or 2 or more states"

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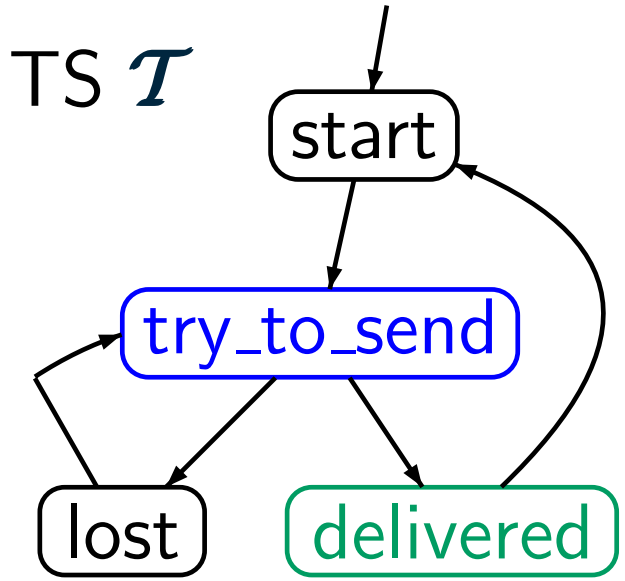
method: calculate and analyze the **SCCs**

Example: ω-regular model checking



ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

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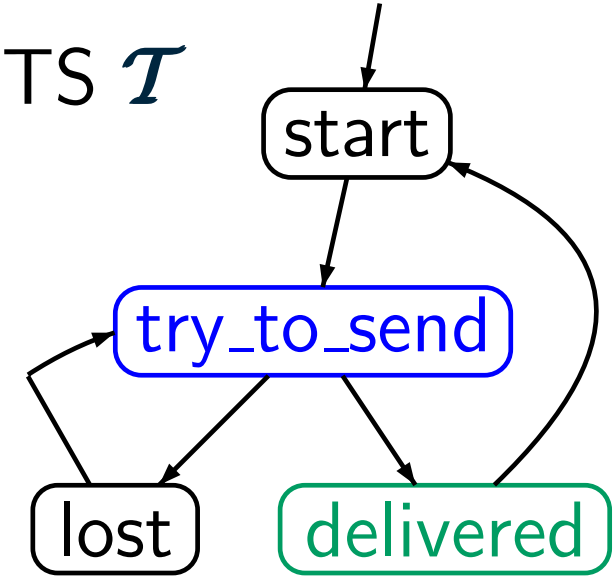
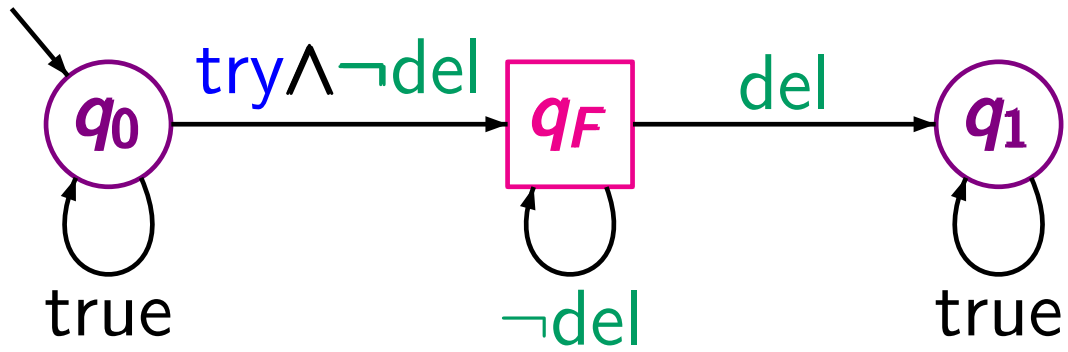
TS $\mathcal{T}$



NBA $\mathcal{A}$ for the bad behaviors



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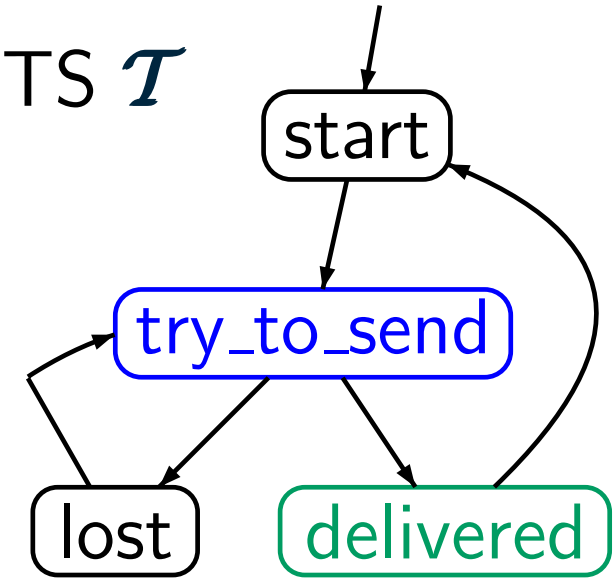

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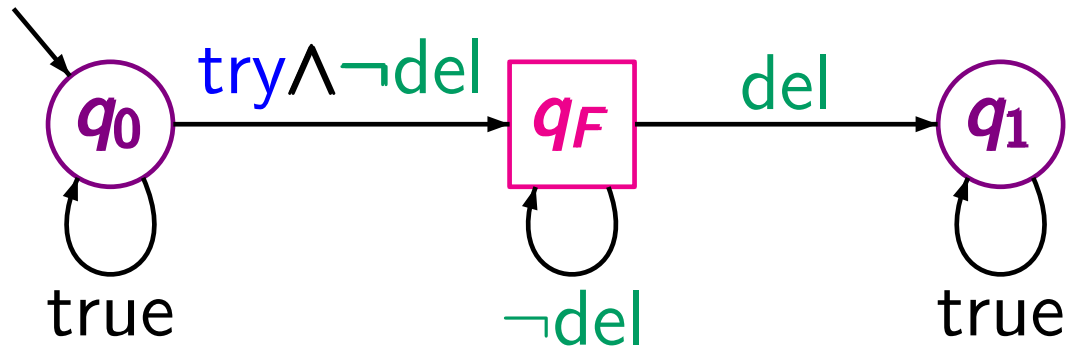

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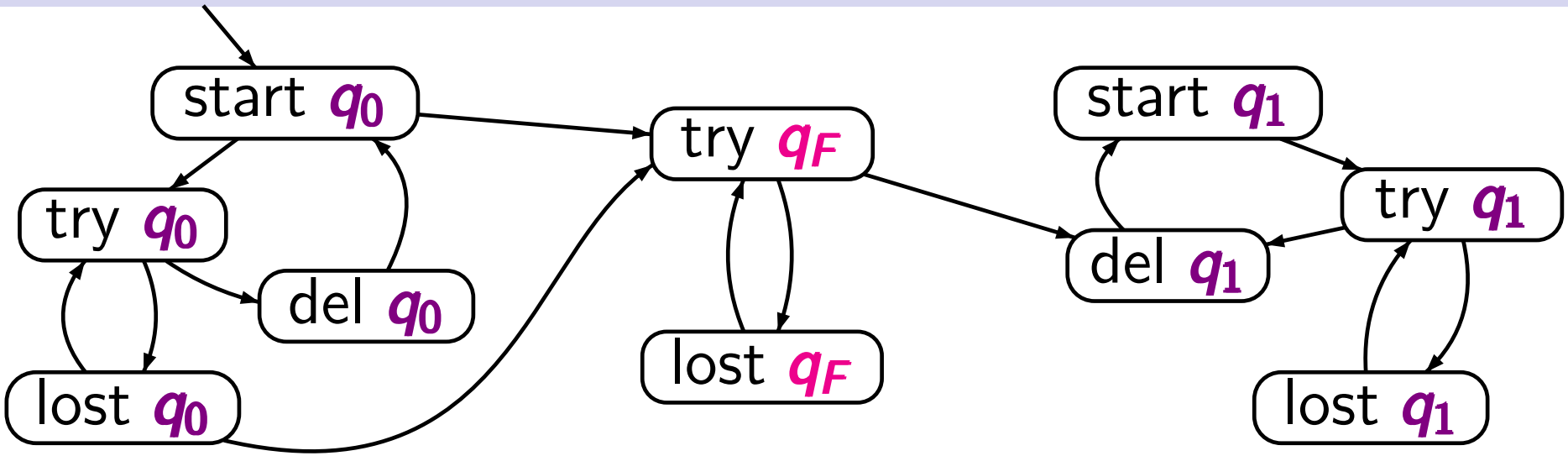
... analysis of the **SCCs** in product $\mathcal{T} \otimes \mathcal{A}$...

Example: persistence checking $\mathcal{T} \otimes \mathcal{A}$



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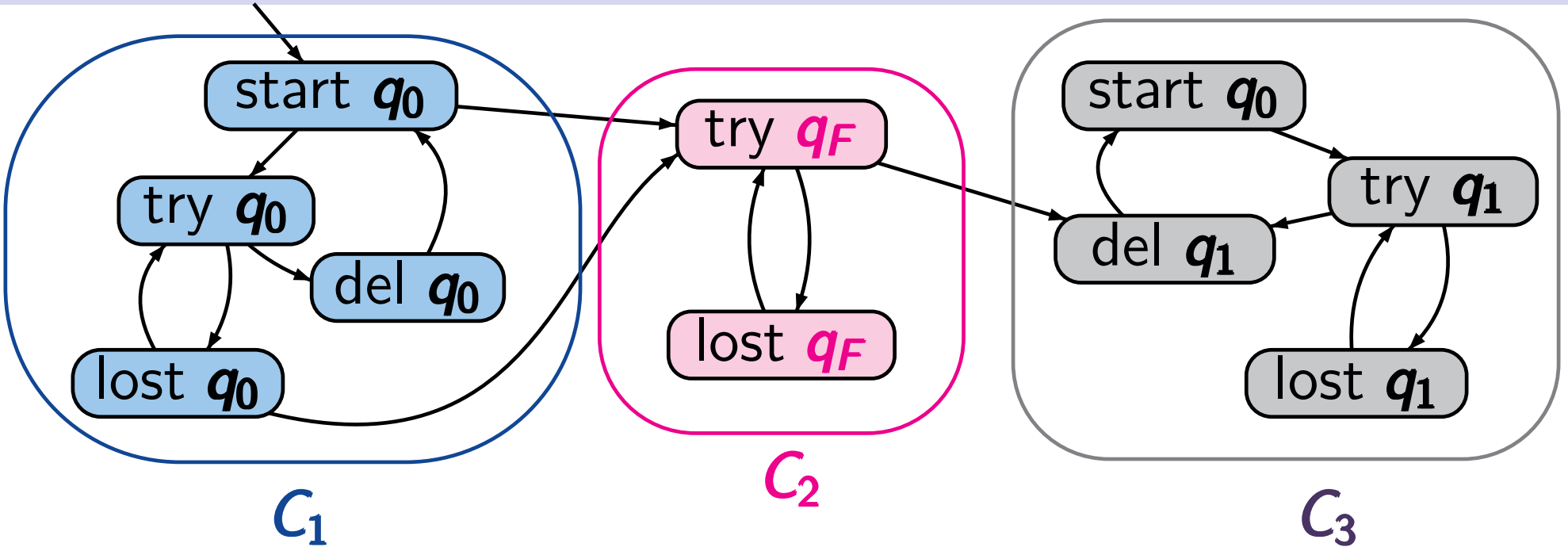

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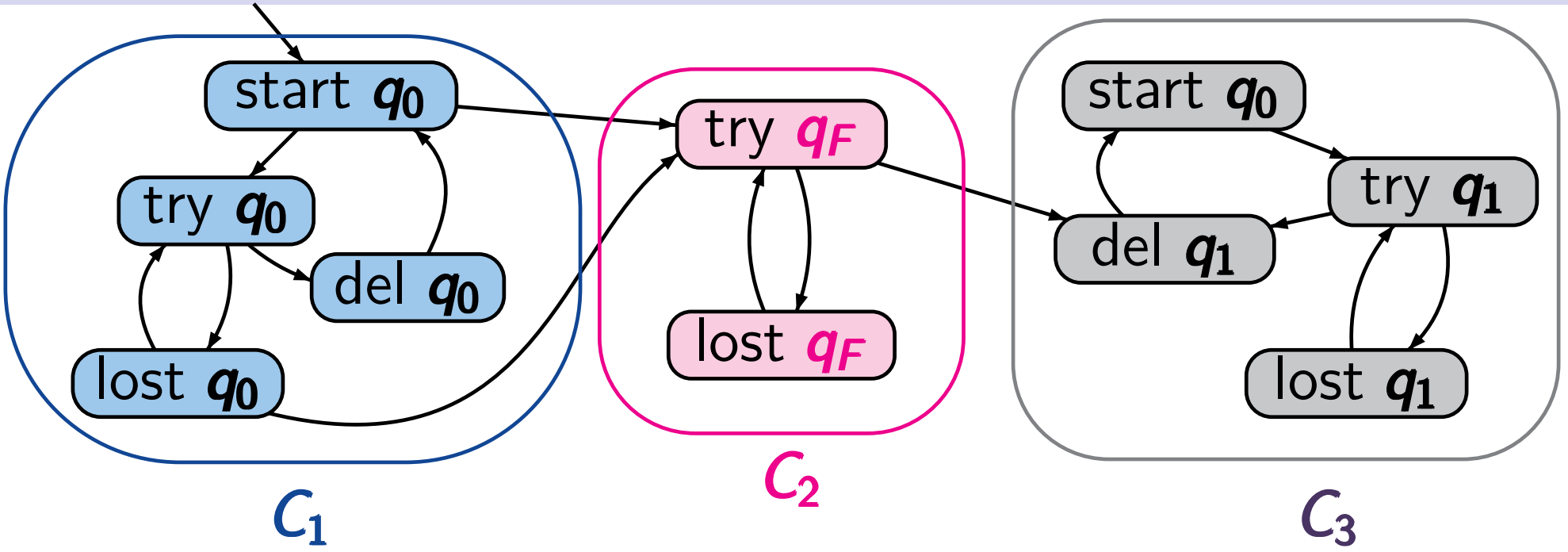

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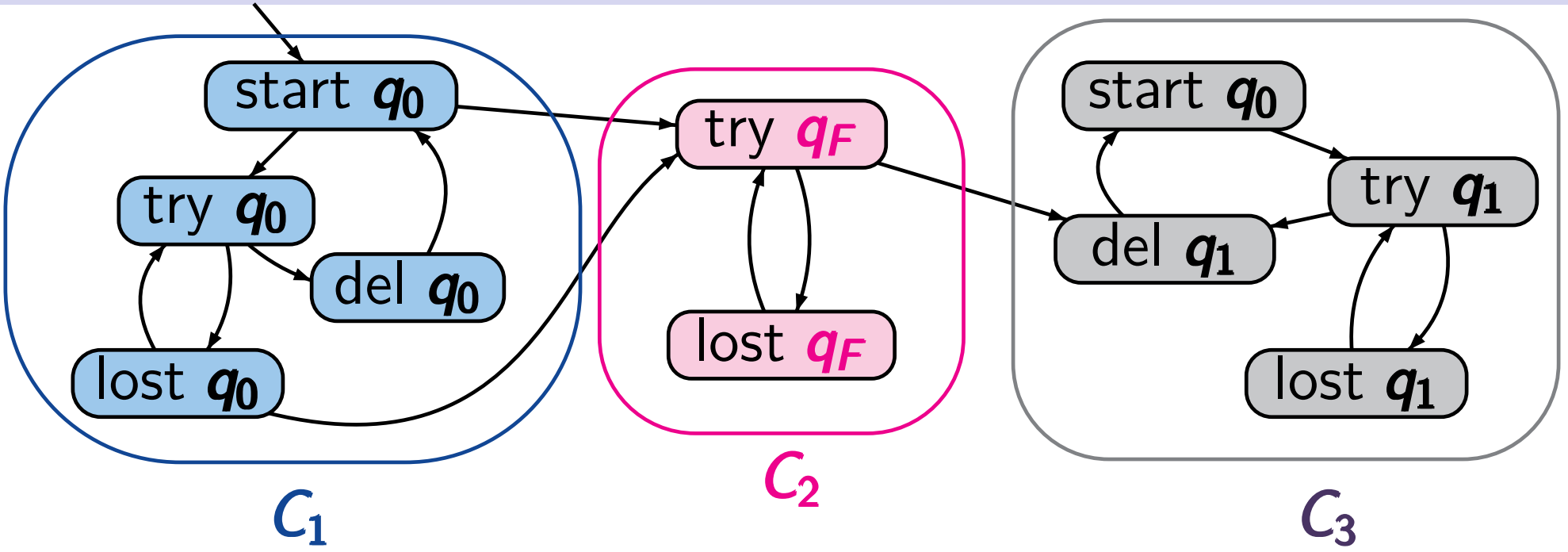

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method 2: **DFS**-based search for **backward edges**

DFS-based cycle check

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Let G be a finite directed graph and s a node in G.

The following statements are equivalent:

(1) G is cyclic

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(1) s belongs to a cycle $s\, s_1\, s_2 \ldots s_k\, s$

(2) The DFS started with s finds a backward edge $s' \longrightarrow s$.

Cycle check for fixed node: "does s belong to a cycle?"

- perform by a DFS with starting node s
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initially all states are unmarked

REPEAT

  choose an unmarked reachable state s with $s \not\models a$;
  mark s;
  IF *CYCLE_CHECK*(s) THEN
    return "no"
  FI

UNTIL all reachable states s with $s \not\models a$ are marked;
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Persistence checking ⟵ Nested DFS

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