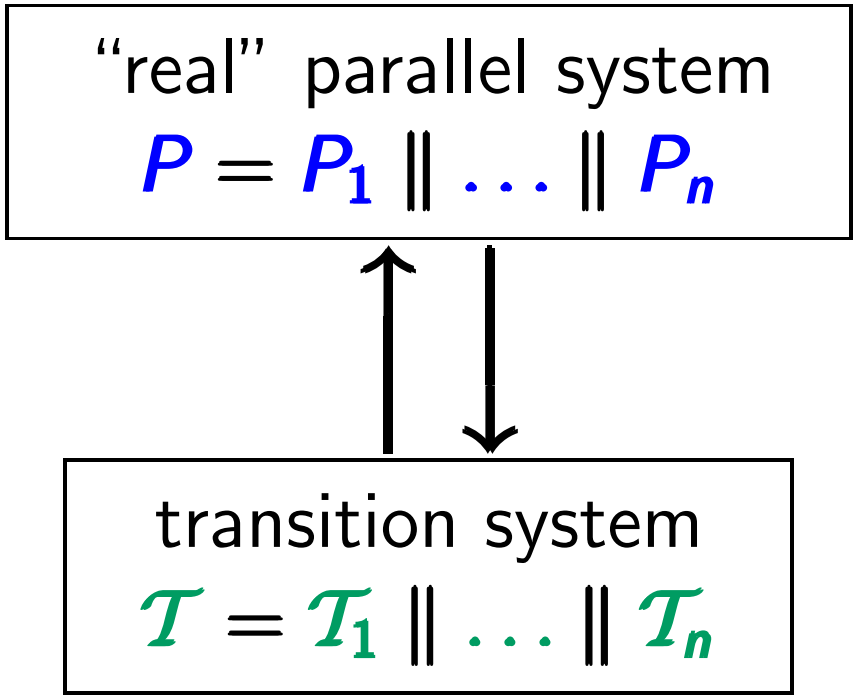
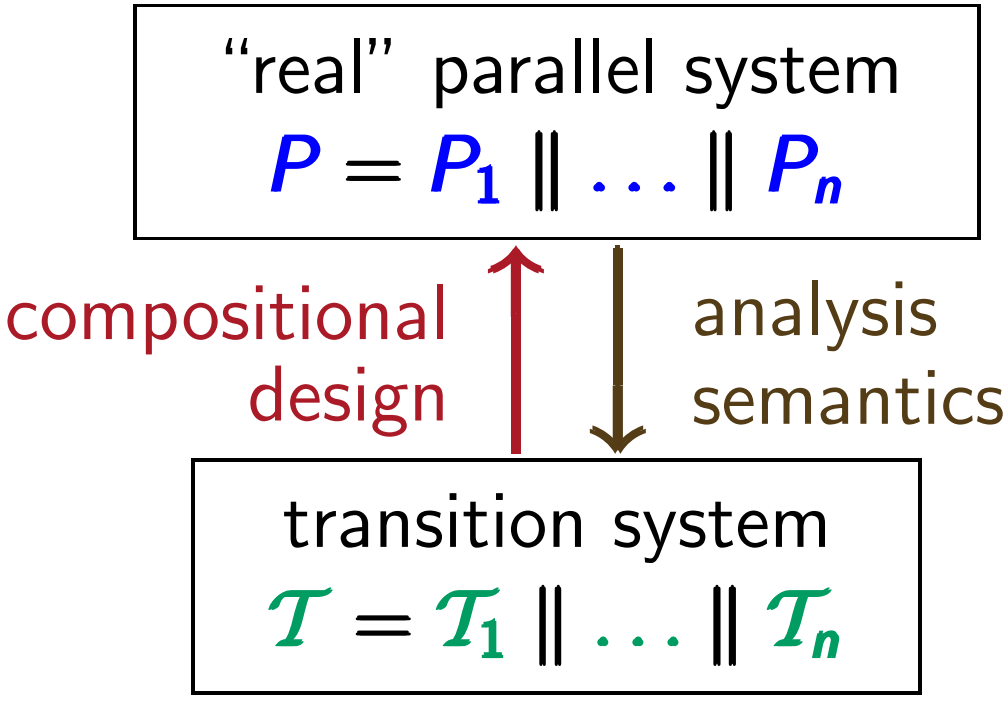
Overview

Parallelism and communication



Parallelism and communication



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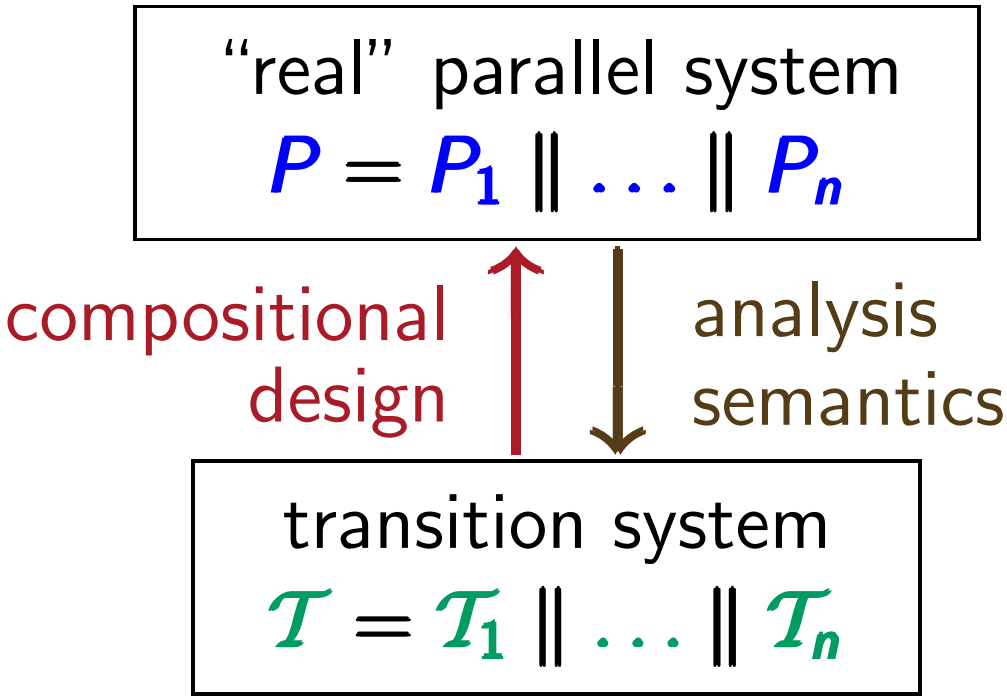

goal: define semantic parallel operators on transition systems or program graphs that model "real" parallel operators

Interleaving operator ||| for TS

- interleaving of concurrent, independent actions of parallel processes (modelled by TS)
- representation by nondeterministic choice:

  "which subprocess performs the next step?"

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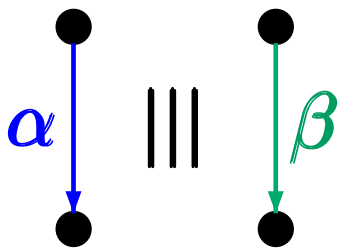

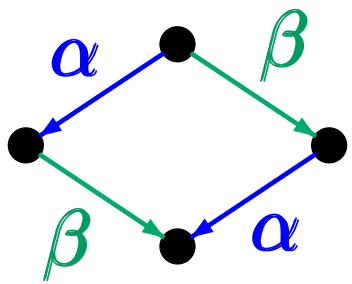

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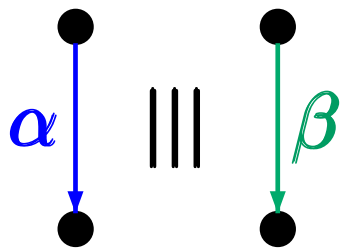

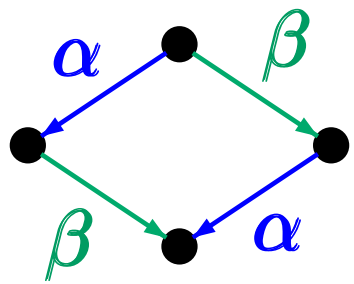

parallel execution of α and β on *two processors*

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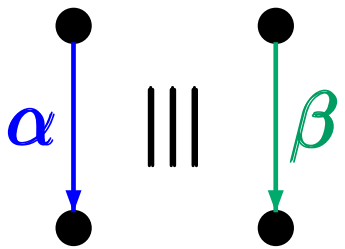

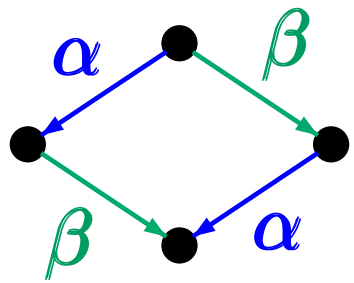

parallel execution of α and β on *two processors* $\;\widehat{=}\;$ serial execution on a *single processor* in arbitrary order

Interleaving operator $|||$ for TS

$$\mathcal{T}_1 = (S_1, Act_1, \longrightarrow_1, S_{0,1}, AP_1, L_1)$$

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The transition system $\mathcal{T}_1 ||| \mathcal{T}_2$ is defined by:

$$\mathcal{T}_1 ||| \mathcal{T}_2 = (S_1 \times S_2, Act_1 \cup Act_2, \longrightarrow, S_{0,1} \times S_{0,2}, AP, L)$$

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$$\frac{s_1 \xrightarrow{\alpha}_1 s_1'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle} \qquad \frac{s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle}$$

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atomic propositions: $AP = AP_1 \uplus AP_2$

labeling function: $L(\langle s_1, s_2 \rangle) = L_1(s_1) \cup L_2(s_2)$

SOS-notation (structured operational semantics)

just a simple notation for operational semantics

$$\frac{\textbf{premise}}{\textbf{conclusion}}$$

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E.g., *"the relation $\longrightarrow$ is given by ..."*

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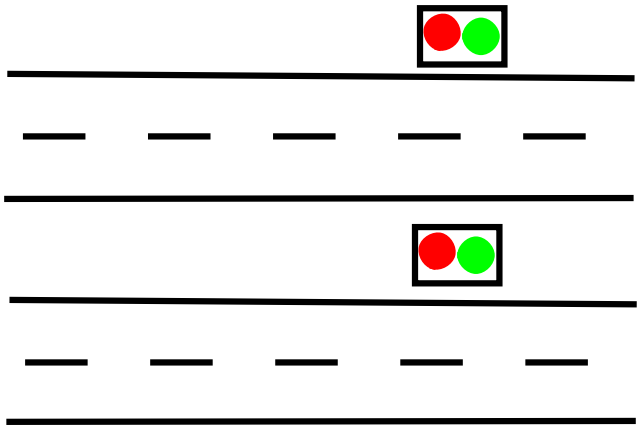
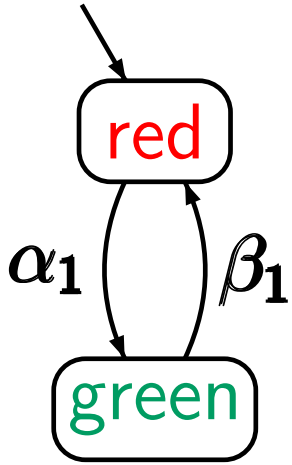
means that $\longrightarrow$ is the smallest relation such that:

(1) If $s_1 \xrightarrow{\alpha}_1 s_1'$, then $\langle s_1, s_2\rangle \xrightarrow{\alpha} \langle s_1', s_2\rangle$

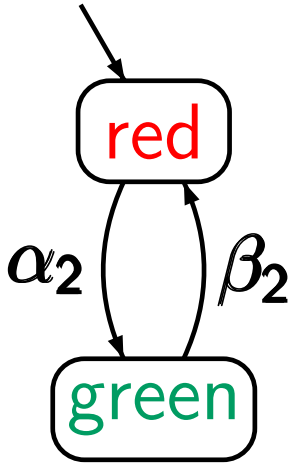
(2) If $s_2 \xrightarrow{\alpha}_2 s_2'$, then $\langle s_1, s_2\rangle \xrightarrow{\alpha} \langle s_1, s_2'\rangle$

Useless lights for non-crossing streets



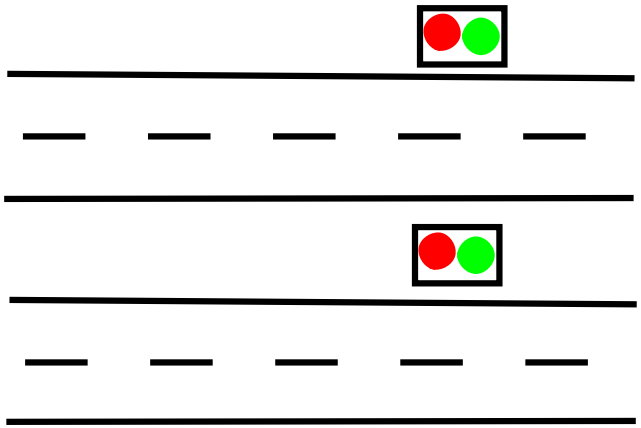


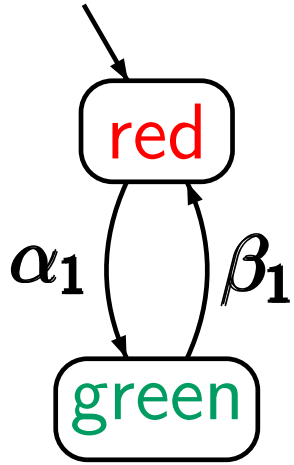
light 1



light 2

Useless lights for non-crossing streets





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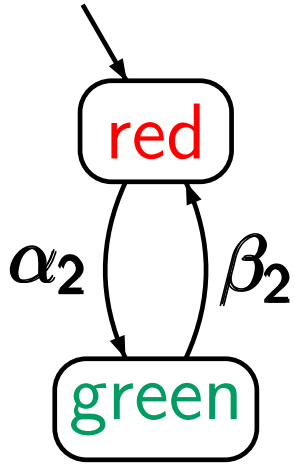
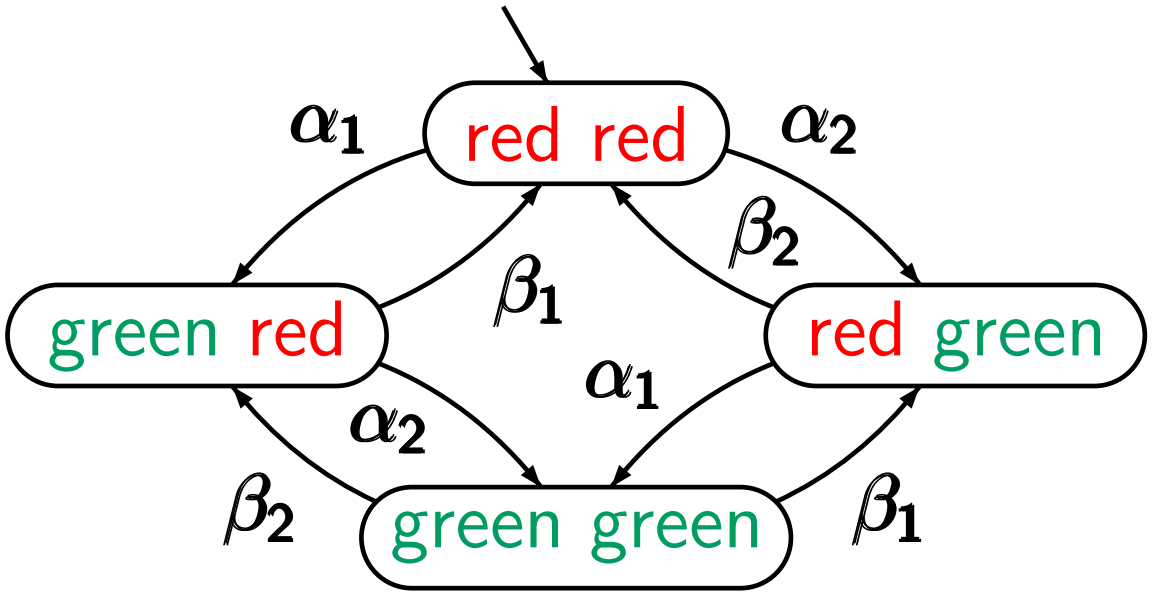


light 2

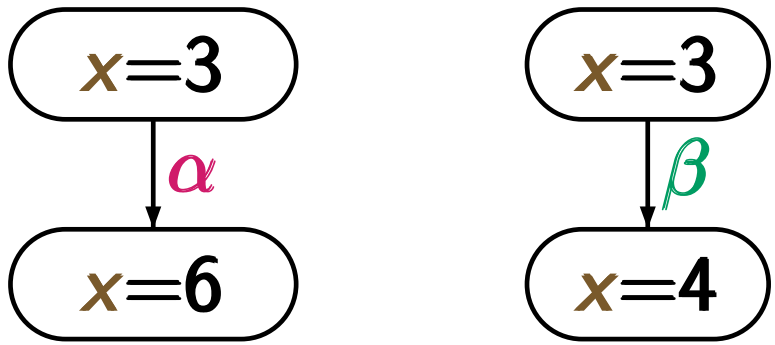
light 1 ||| light 2



Dependent actions

dependent actions $\alpha \mathrel{\widehat{=}} x{:=}2x$ and $\beta \mathrel{\widehat{=}} x{:=}x{+}1$

representations in transition systems



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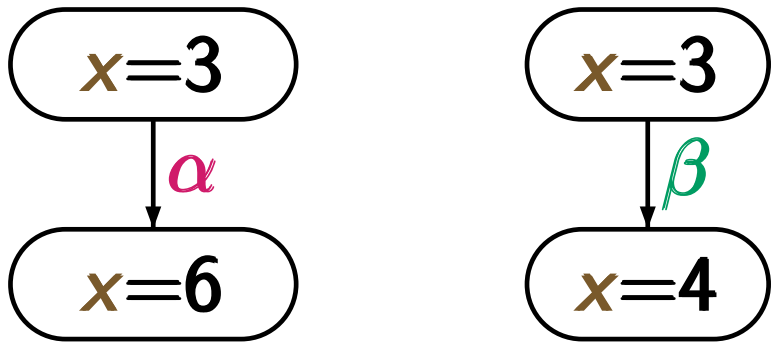
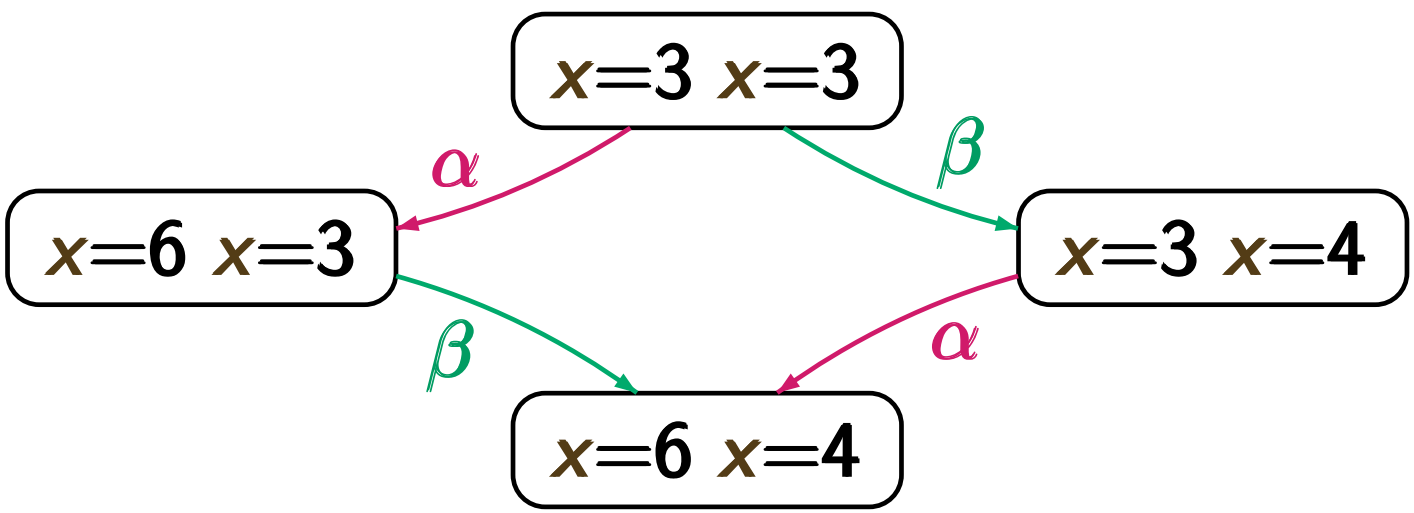


interleaving operator $|||$



Interleaving fails for dependent actions

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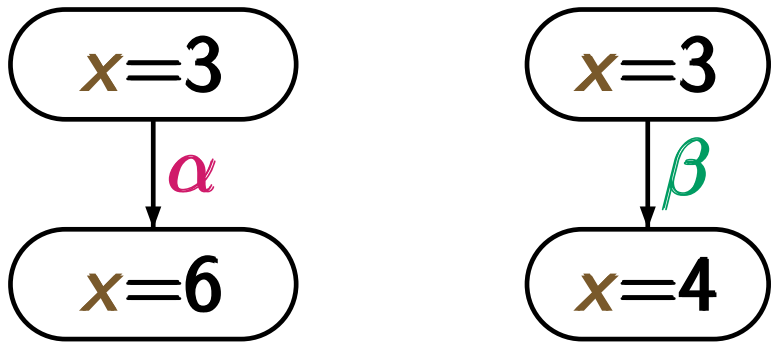
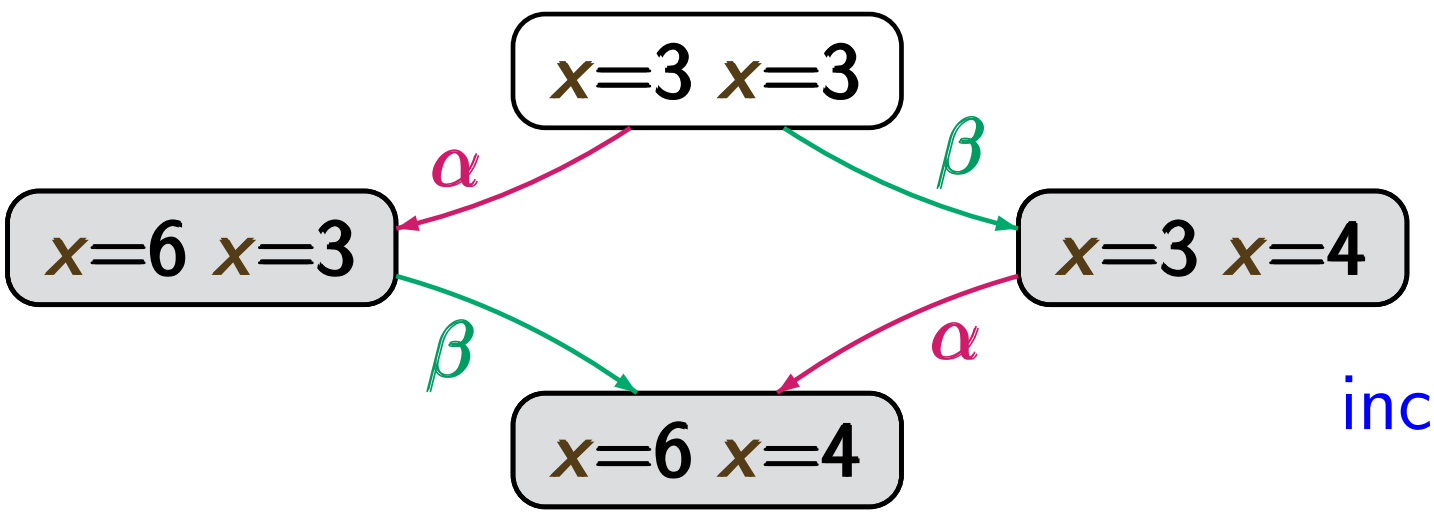
representations in transition systems



interleaving operator $|||$ for transition systems "fails"



inconsistent states

Interleaving for program graphs

Interleaving for program graphs

. . . for modeling parallel systems with
subprocesses communicating via shared variables

Interleaving for program graphs

program graph $\mathcal{P}_1$
$(Loc_1, \ldots, \hookrightarrow_1, \ldots)$

program graph $\mathcal{P}_2$
$(Loc_2, \ldots, \hookrightarrow_2, \ldots)$

interleaving operator

$$\mathcal{P}_1 \,|||\, \mathcal{P}_2 \;=\; (Loc_1 \times Loc_2, \ldots, \hookrightarrow, \ldots)$$

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Example: interleaving for PG

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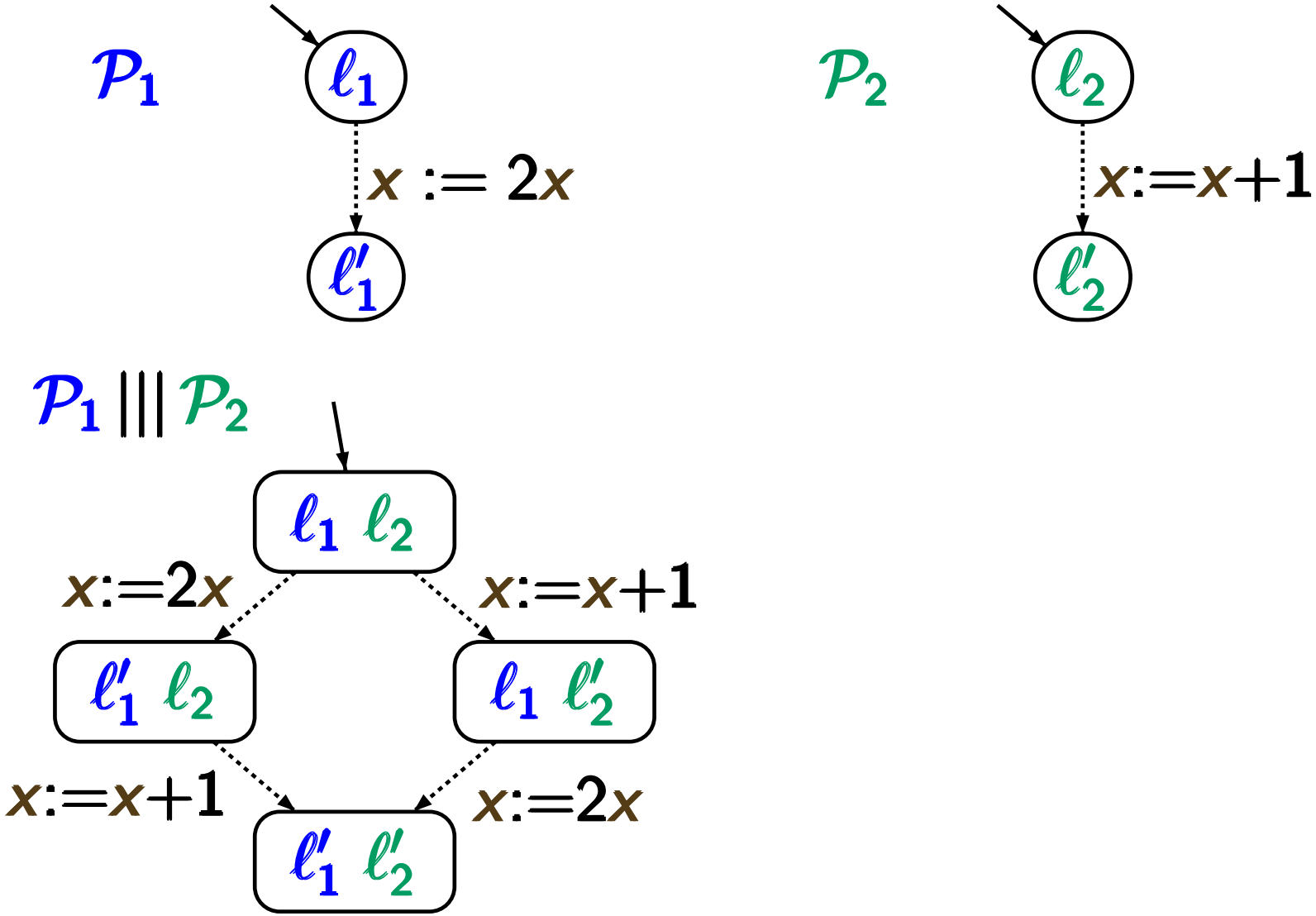

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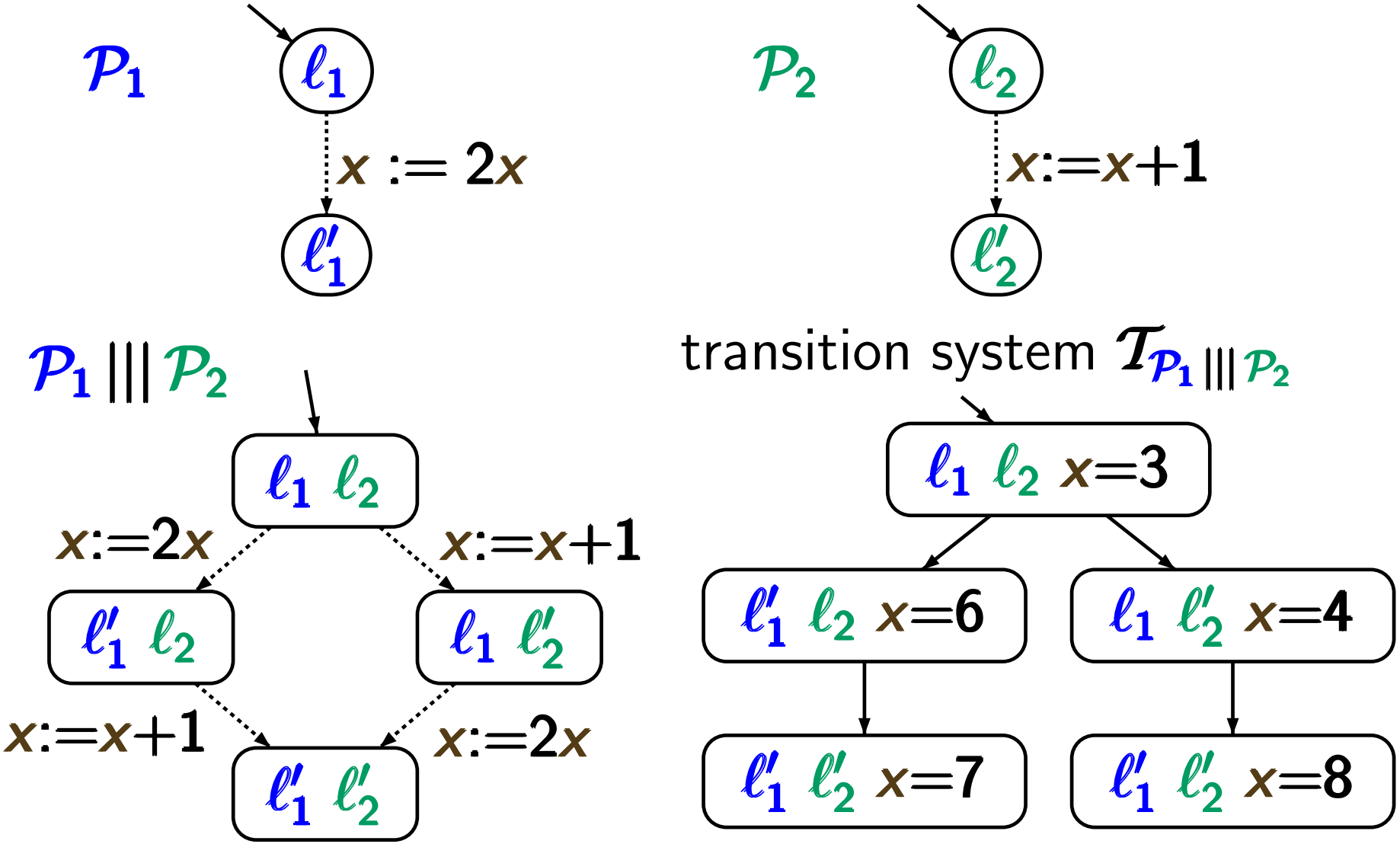

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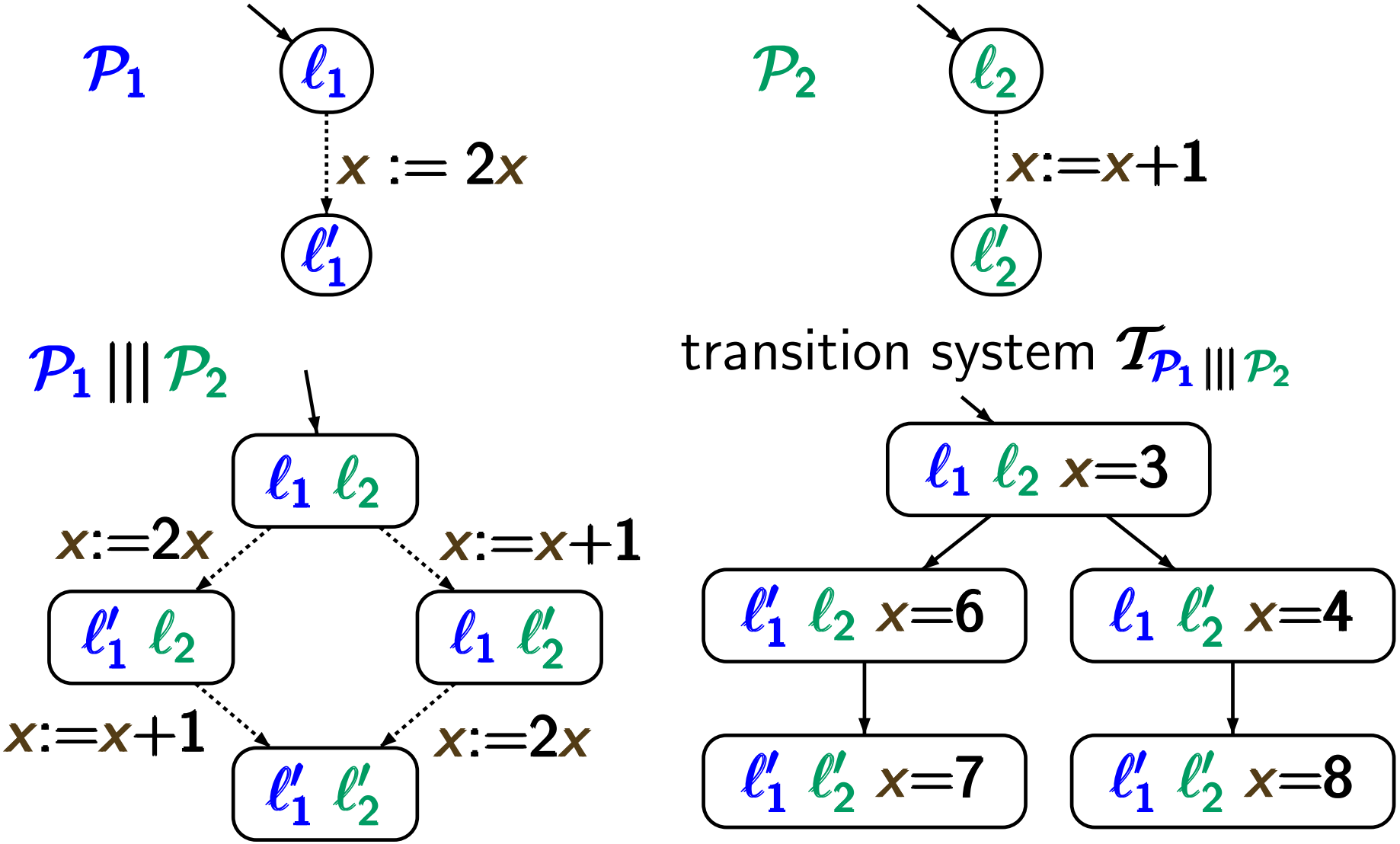
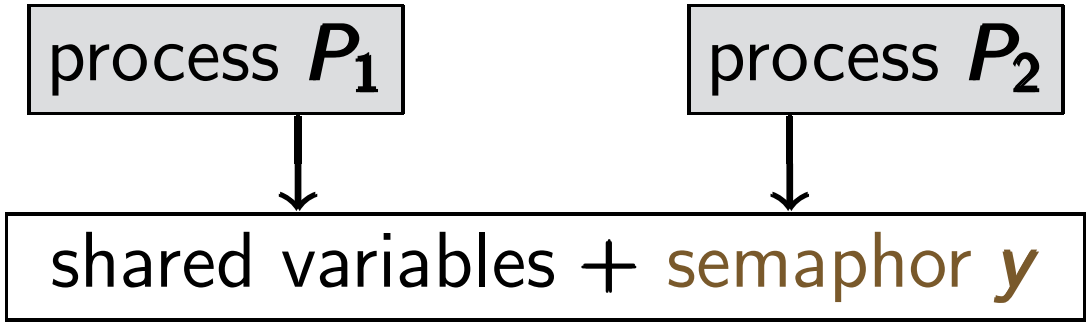


note: $\mathcal{T}_{\mathcal{P}_1 ||| \mathcal{P}_2} \neq \mathcal{T}_{\mathcal{P}_1} ||| \mathcal{T}_{\mathcal{P}_2}$

Mutual exclusion with semaphore



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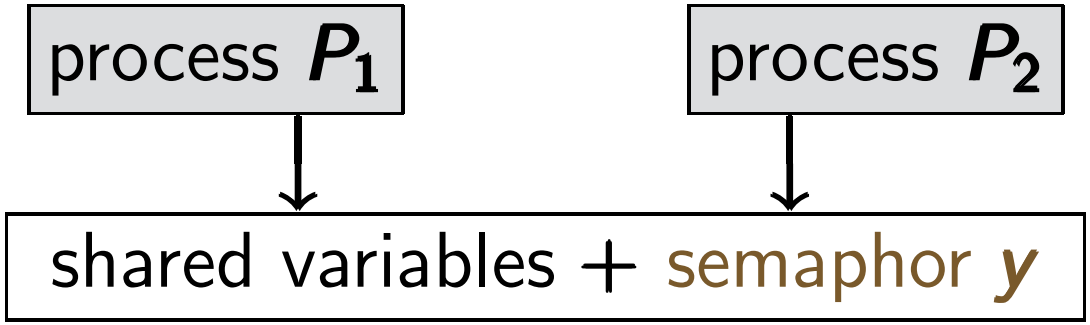

protocol for process P_i

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LOOP FOREVER
  noncritical actions;
  AWAIT y > 0 DO
        y:=y−1
  OD
  critical actions;
  y:=y+1
END LOOP
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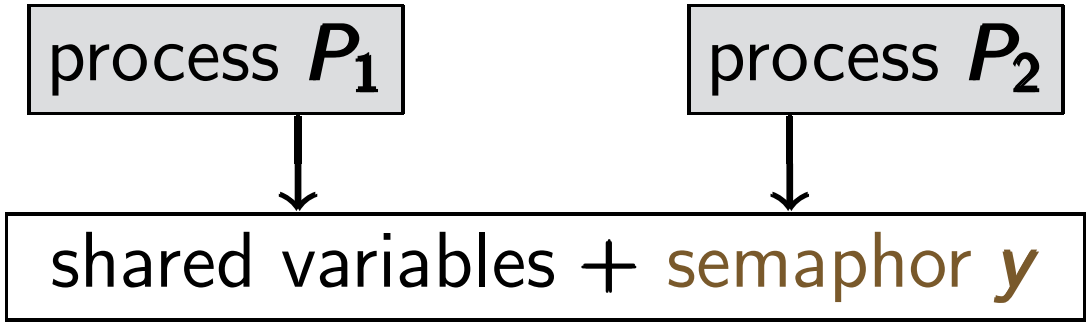

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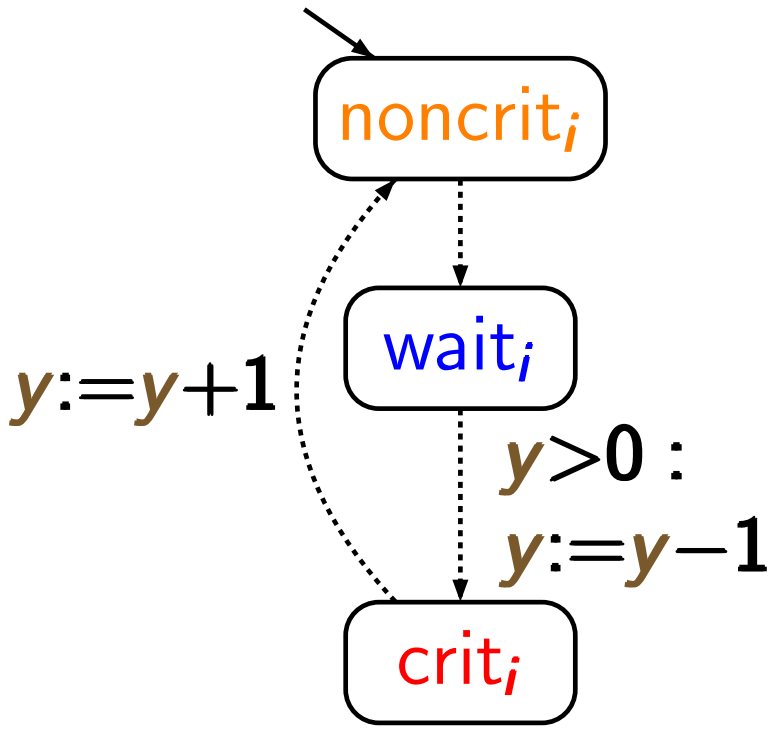
program graph $\mathcal{P}_i$



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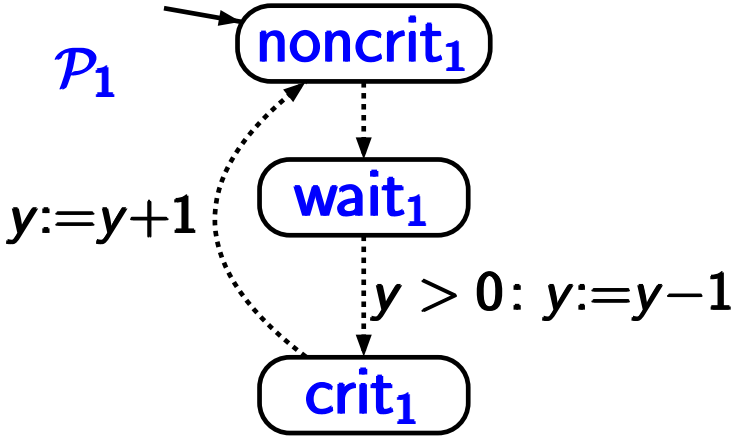

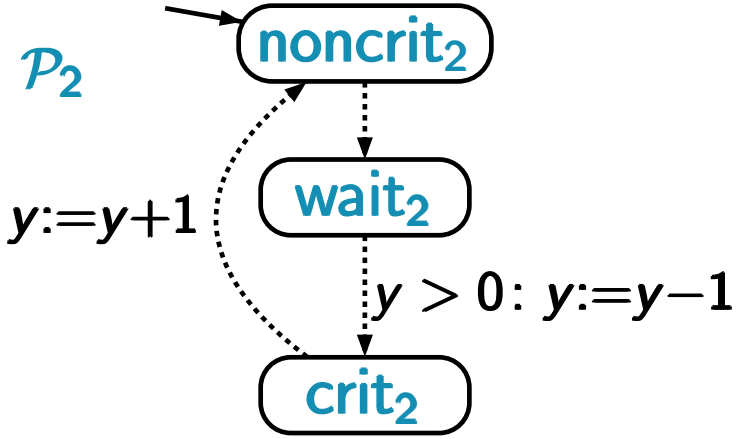

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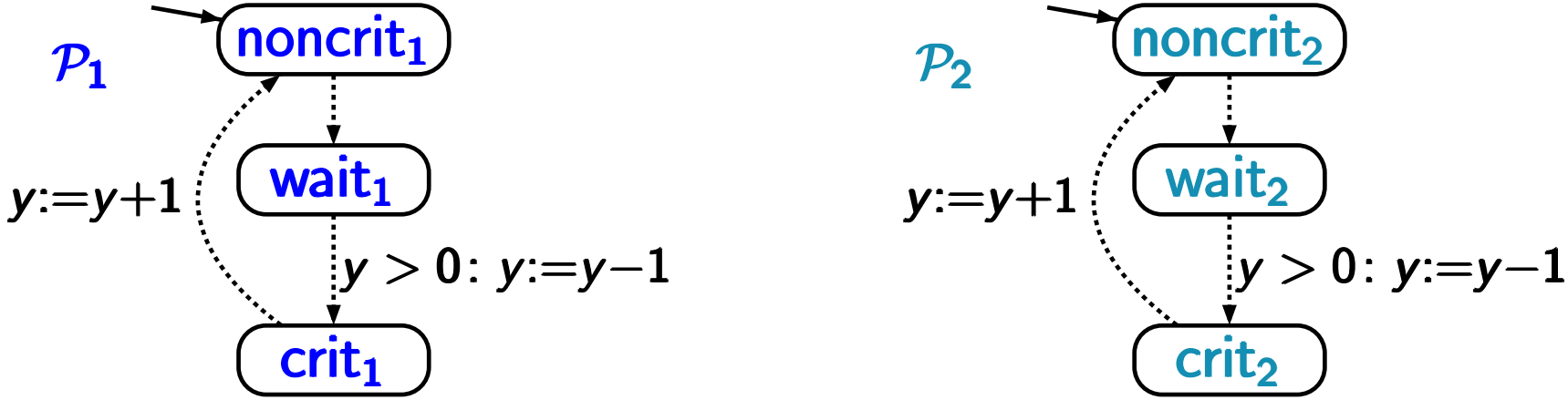
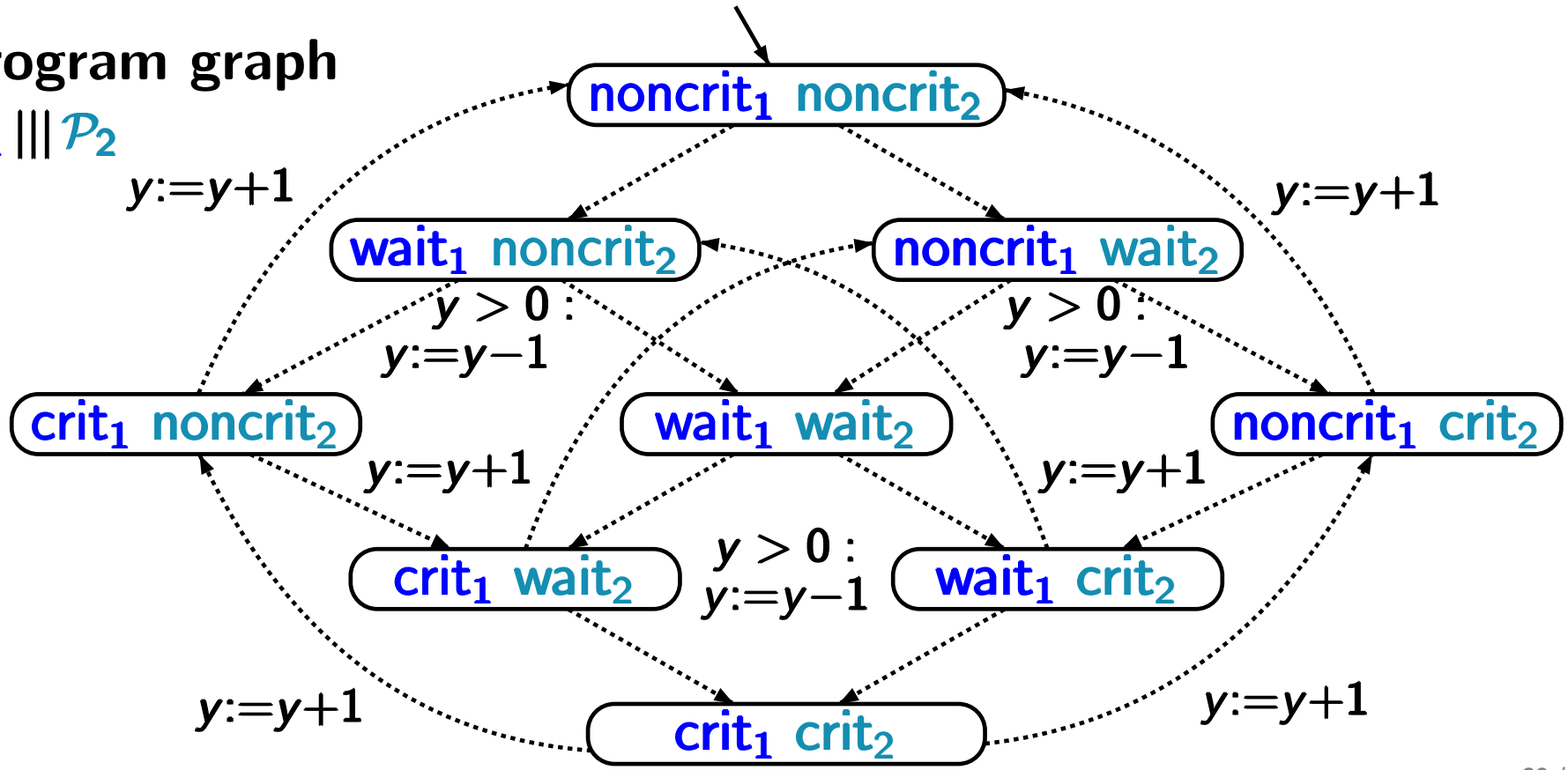


program graph

$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



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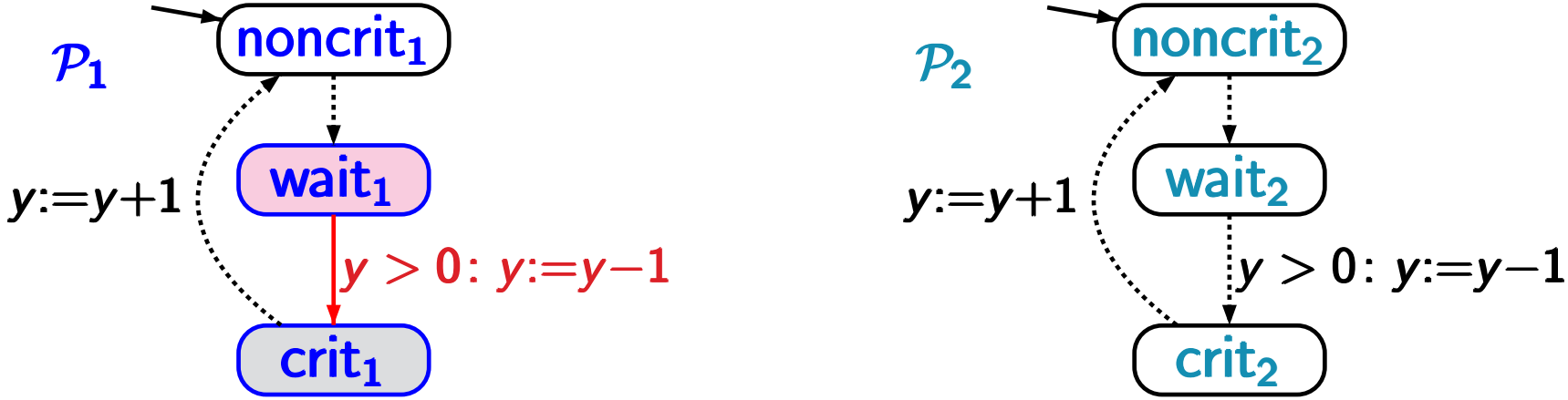

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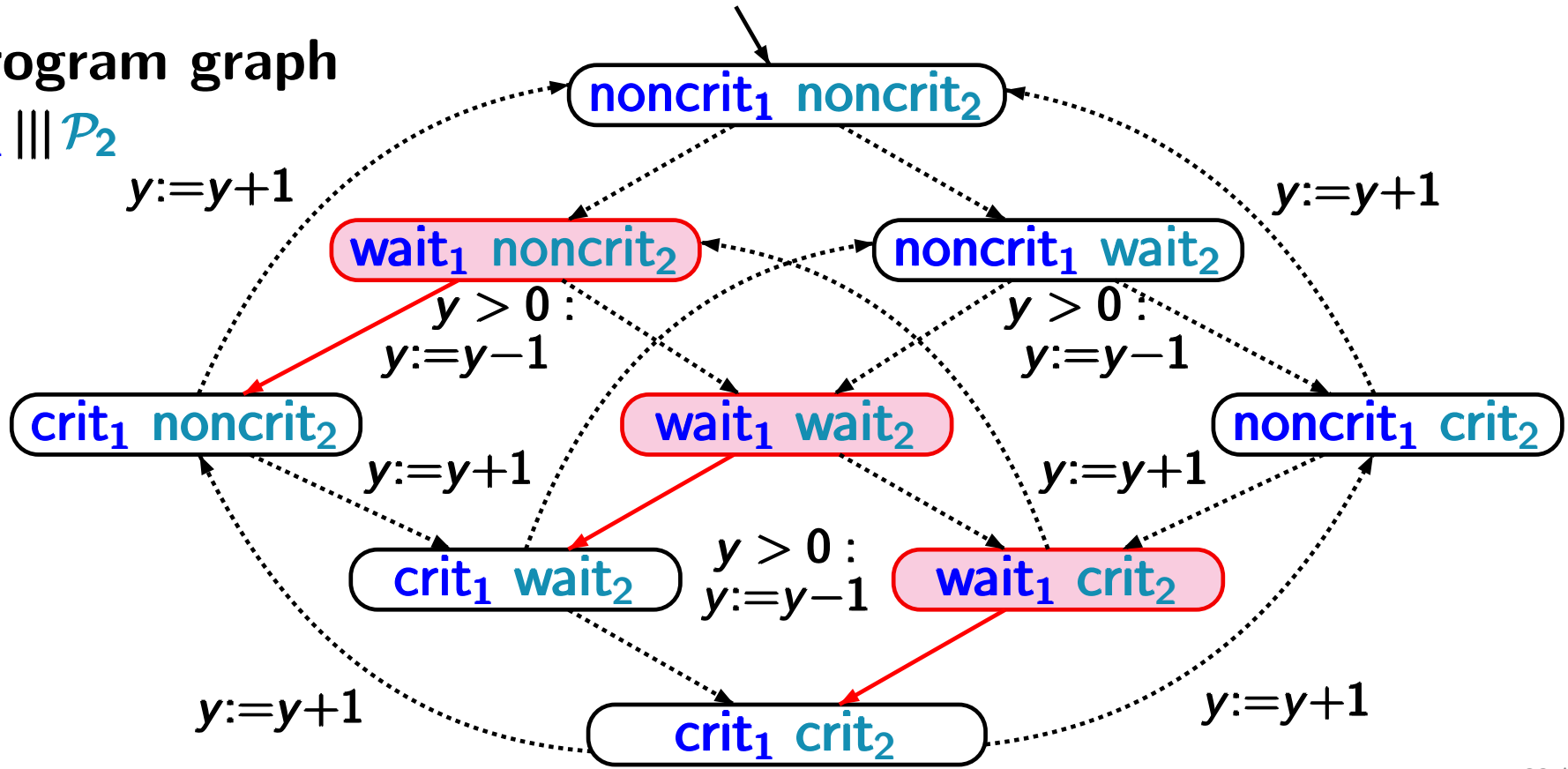

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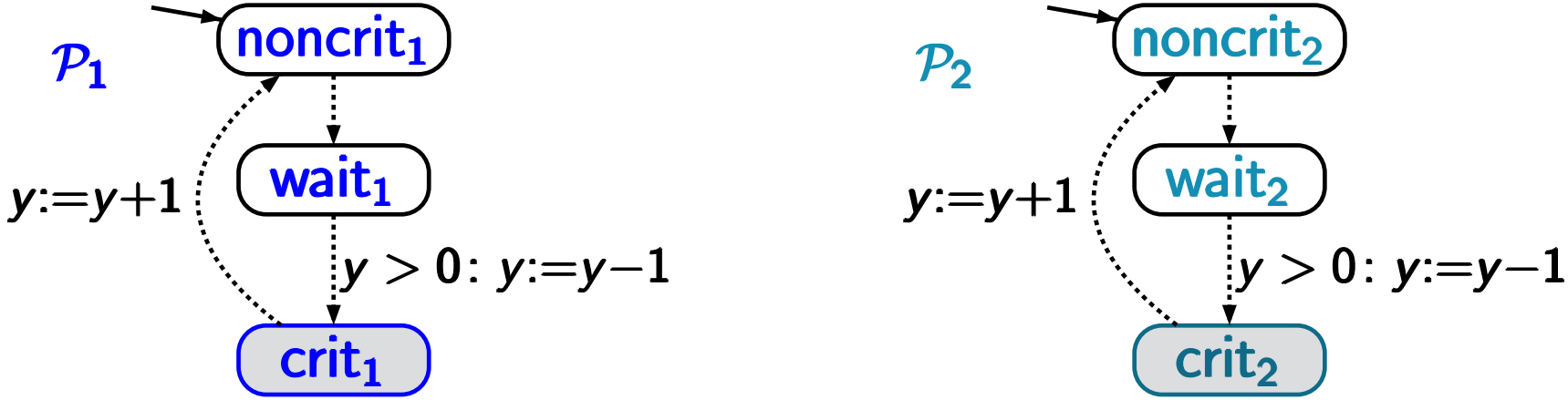

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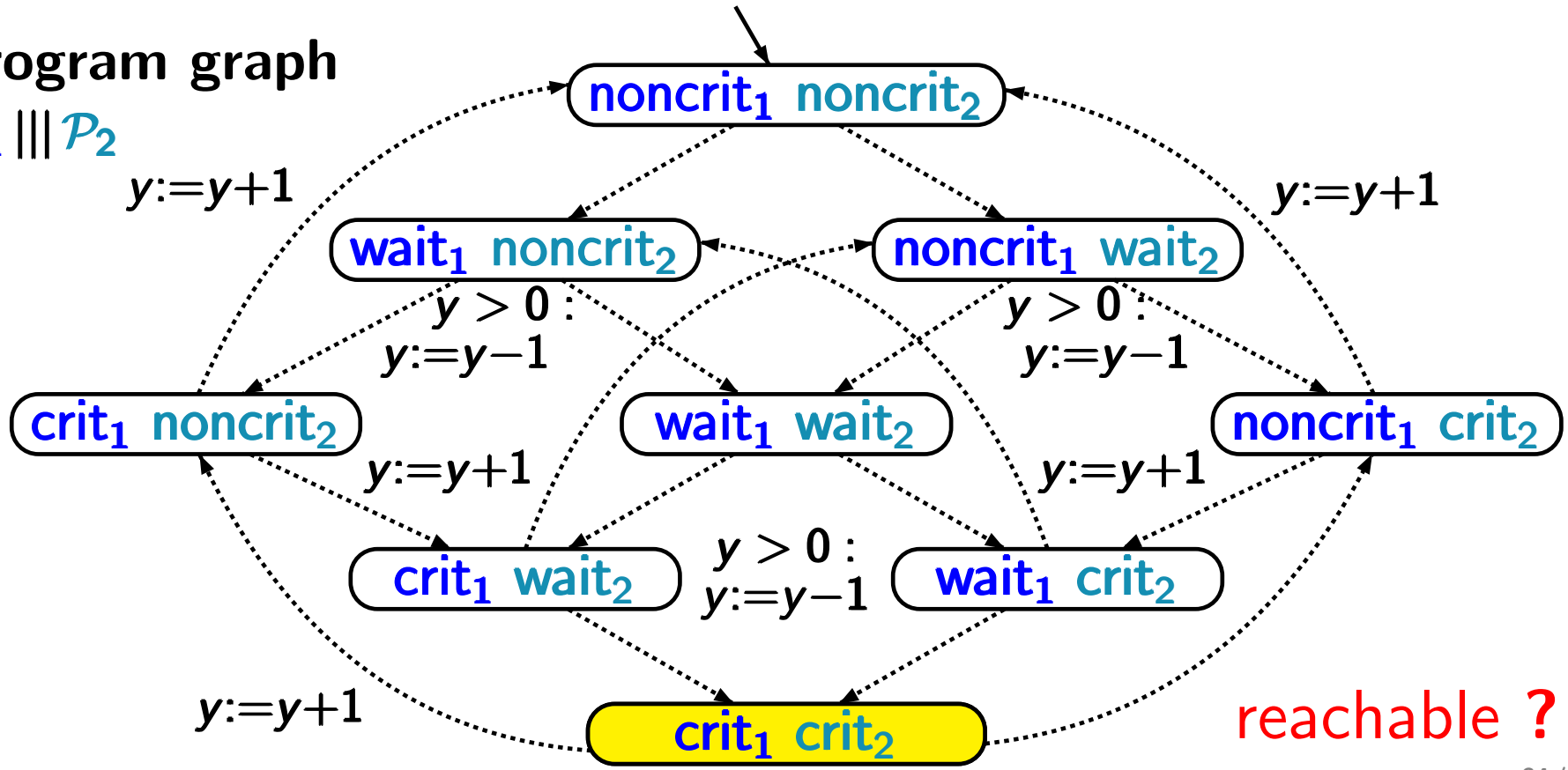
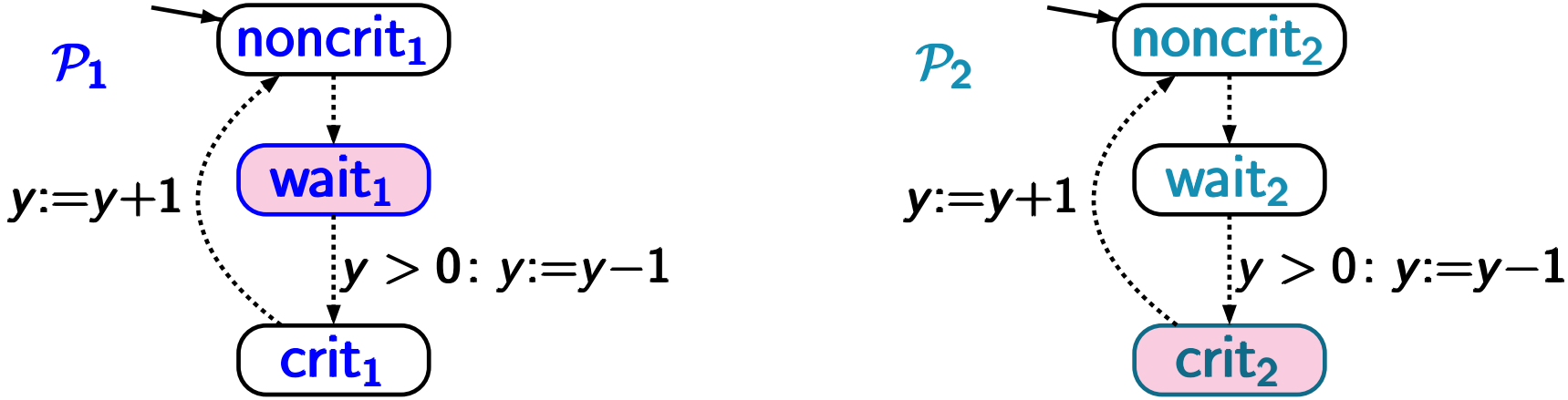


reachable ?

Mutual exclusion with semaphore



program graph

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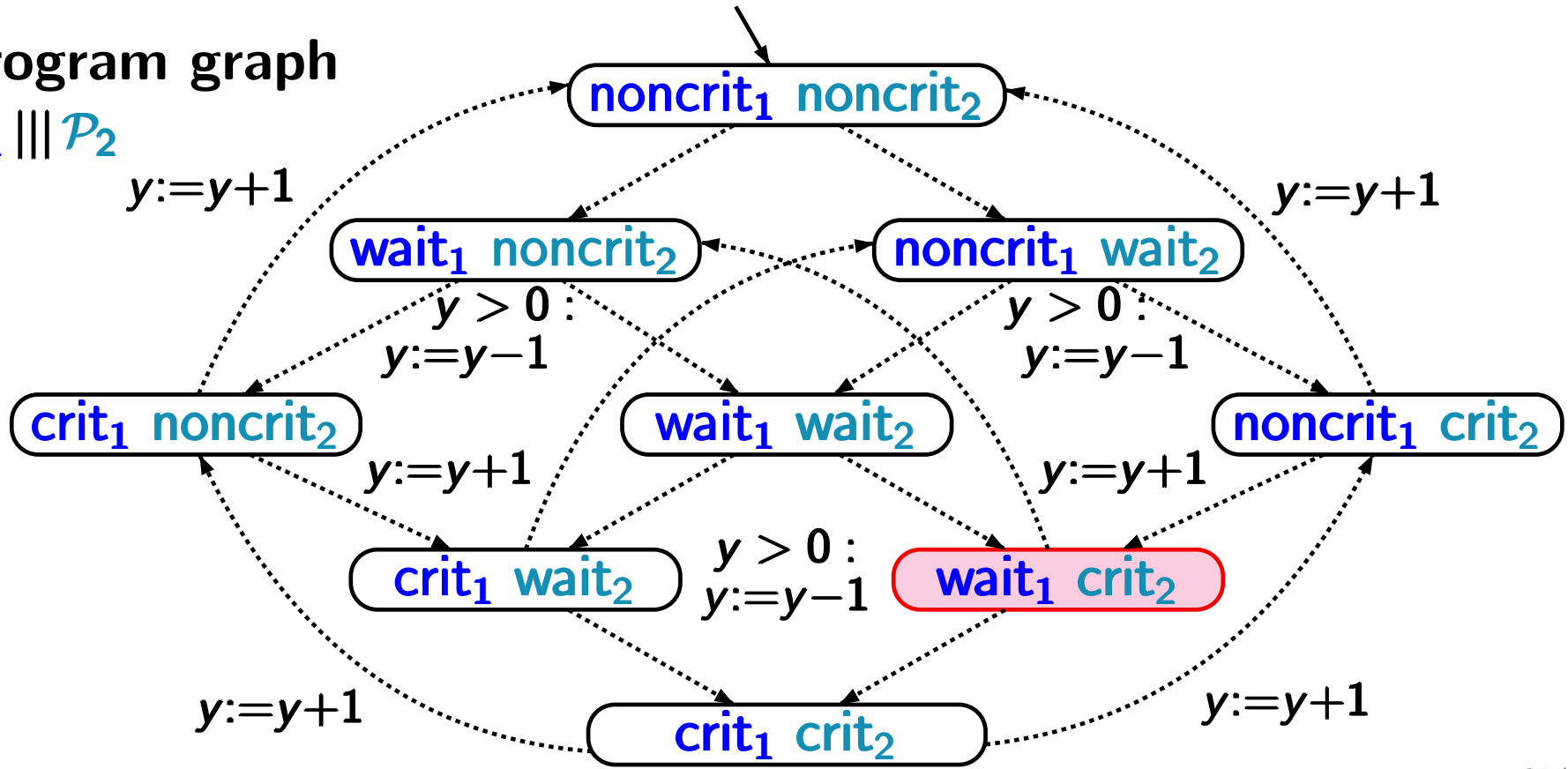
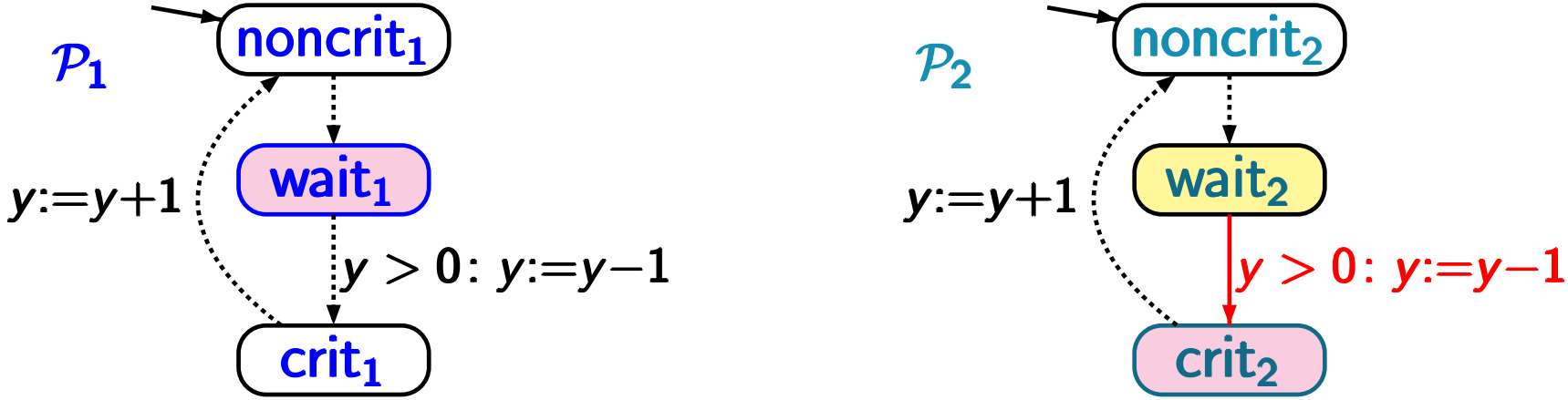
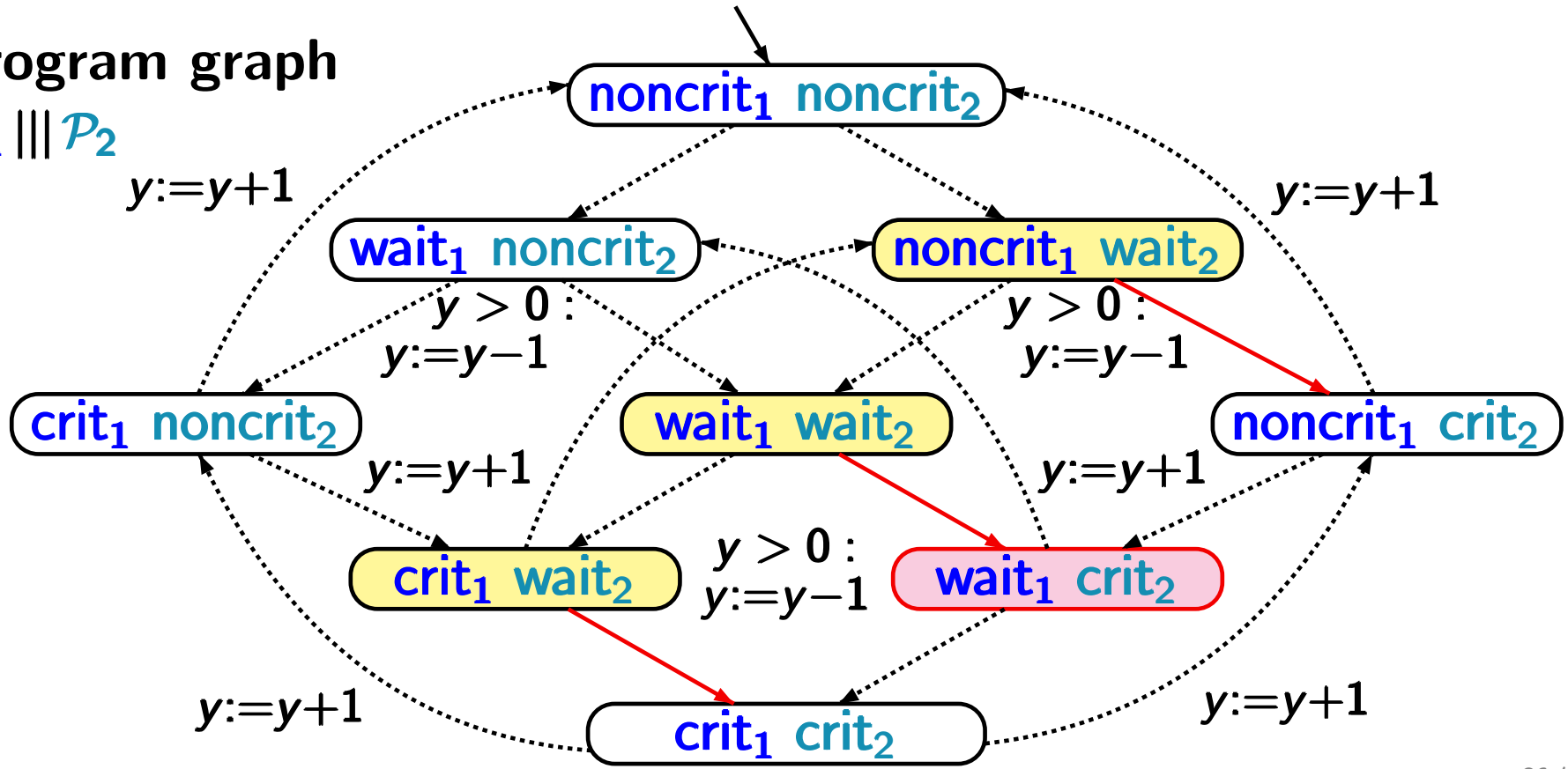


Mutual exclusion with semaphore



program graph

$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



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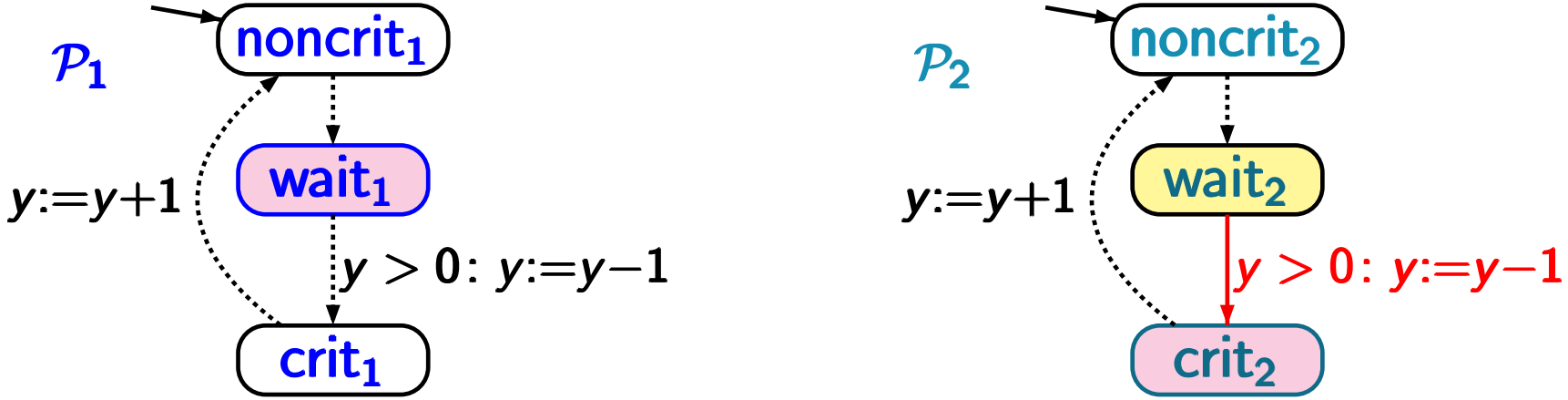

program graph

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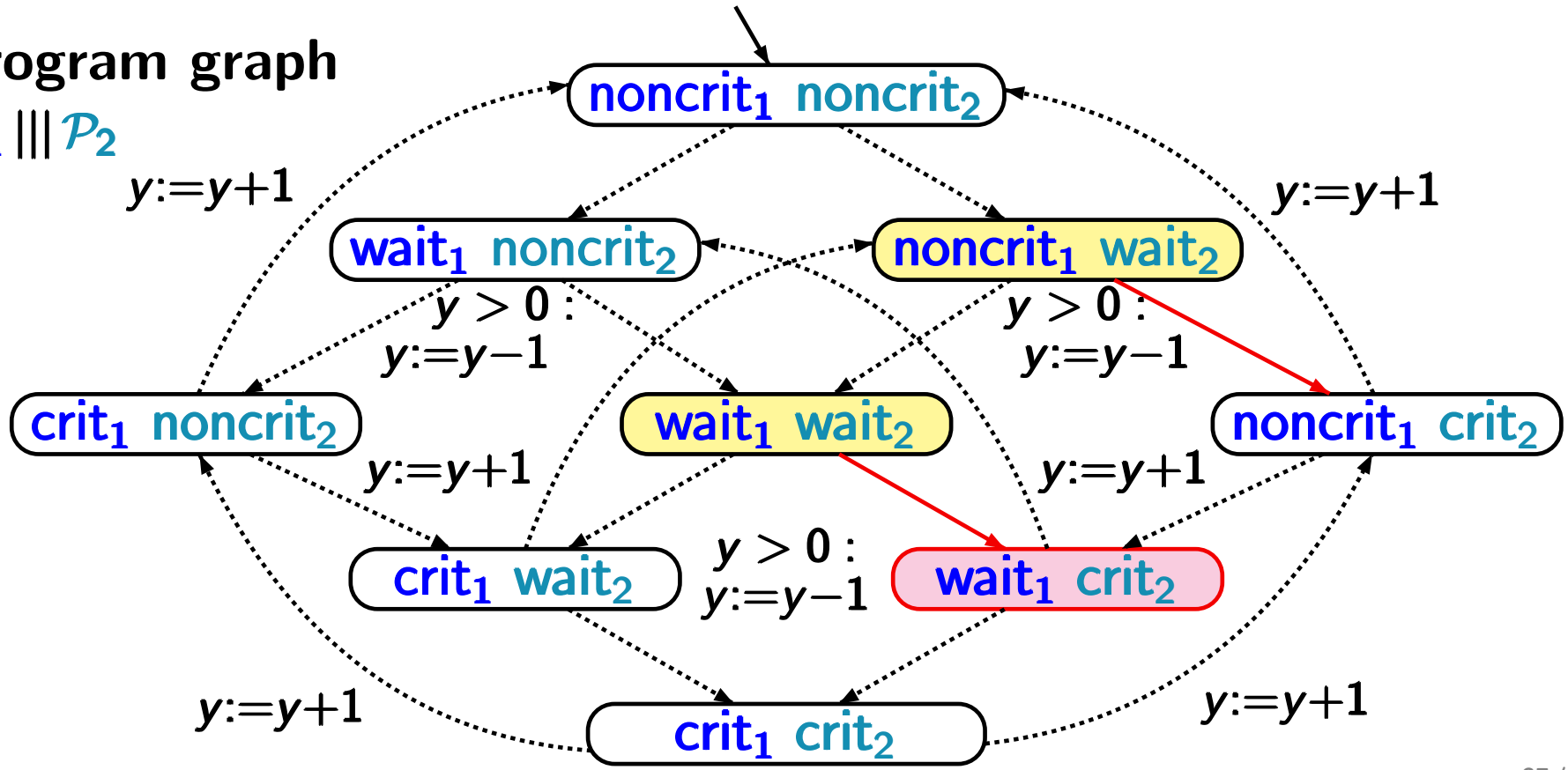
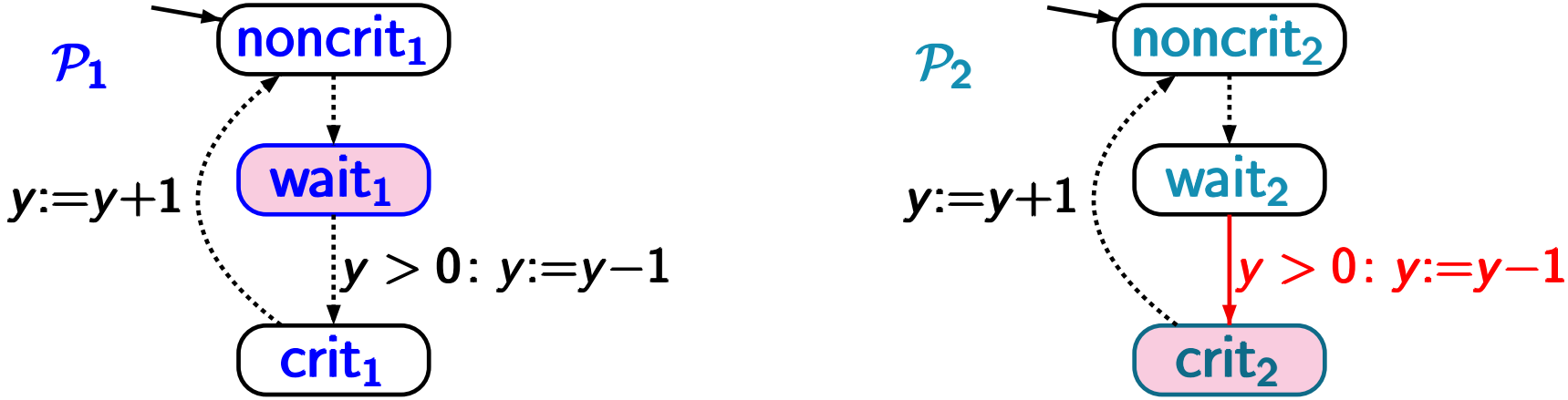
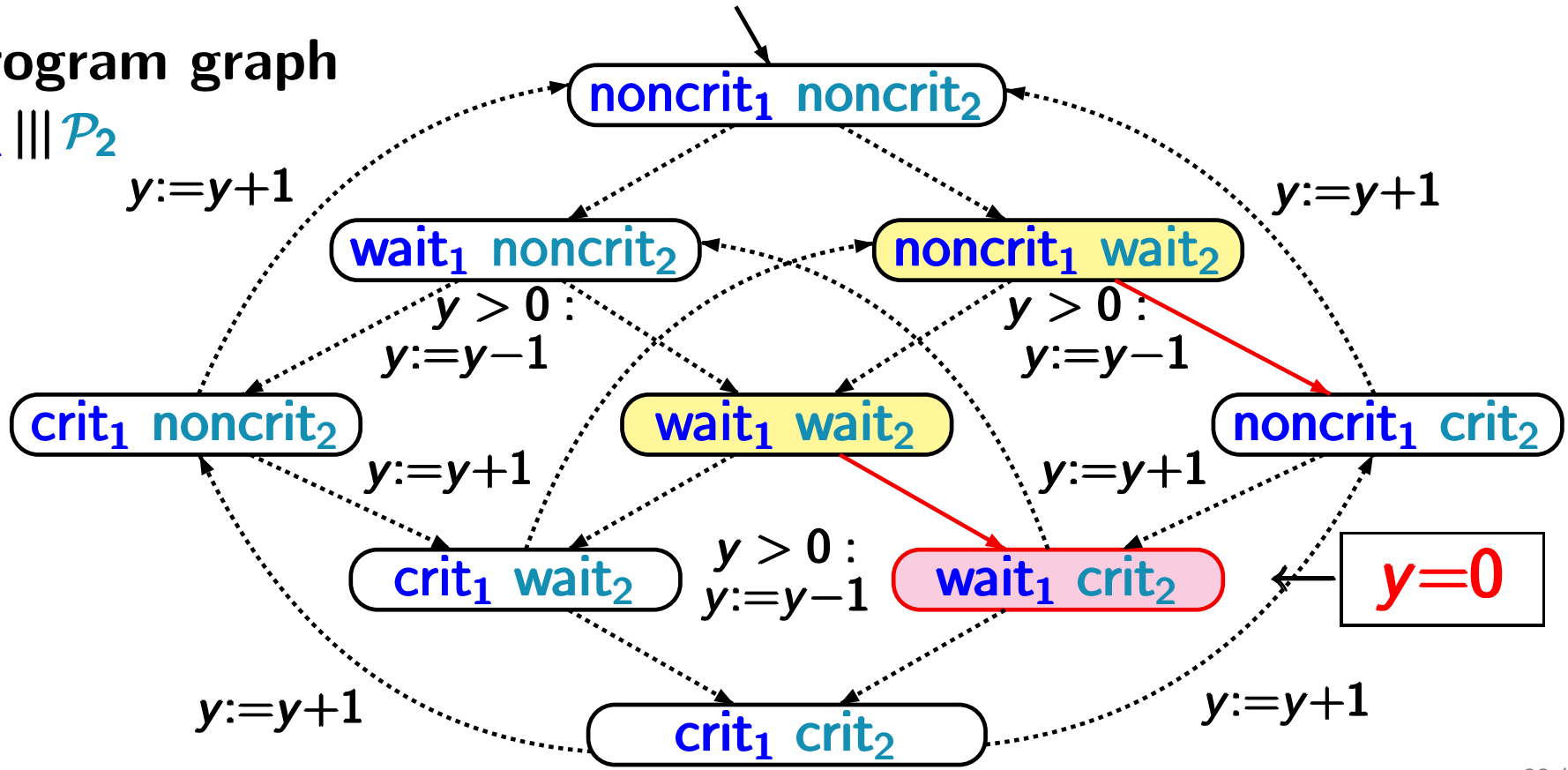


Mutual exclusion with semaphore

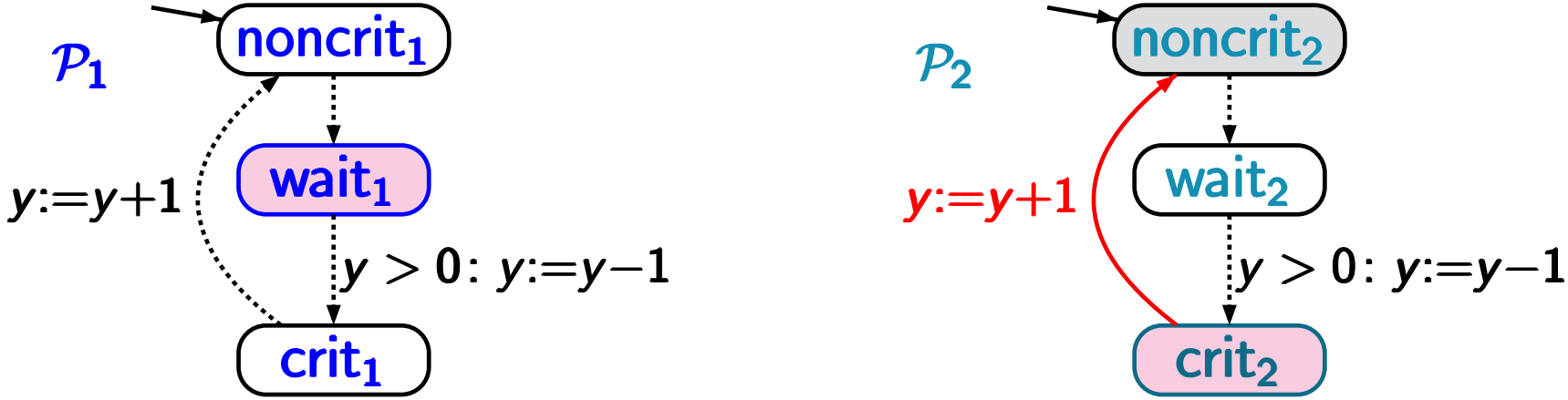


program graph

$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

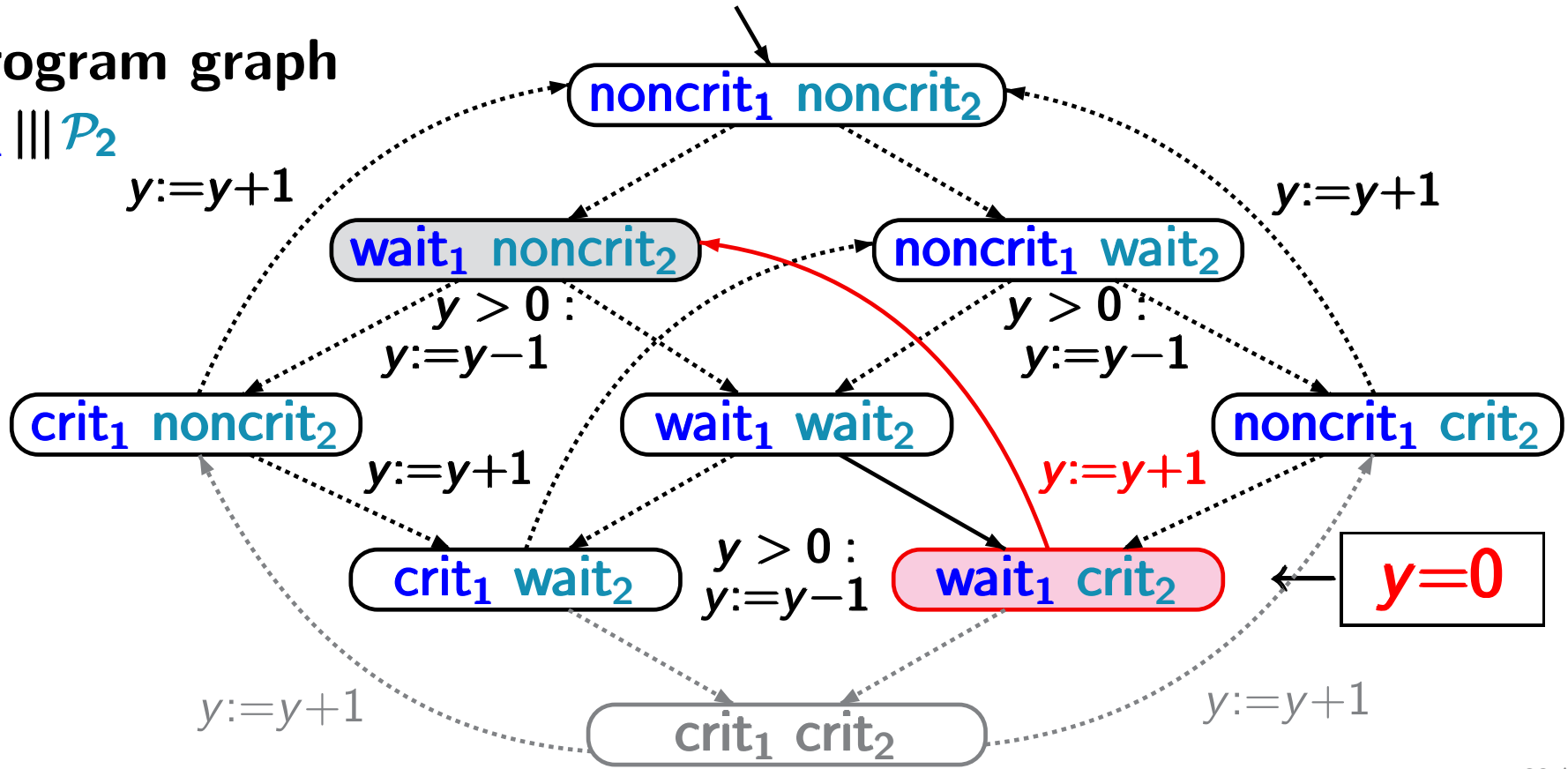


Mutual exclusion with semaphore

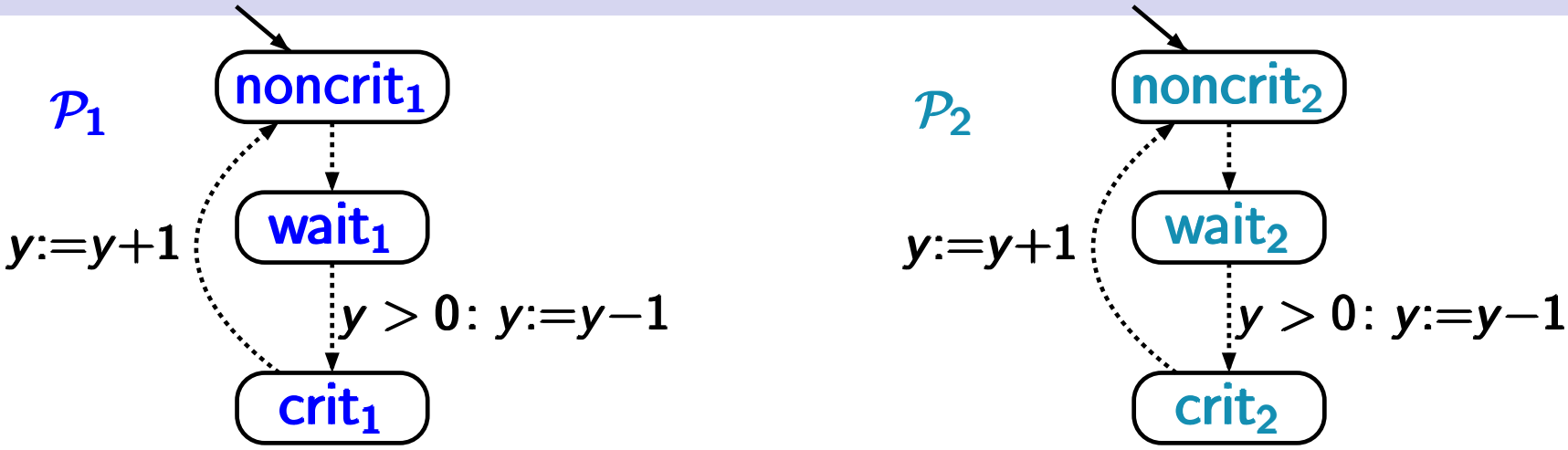


program graph

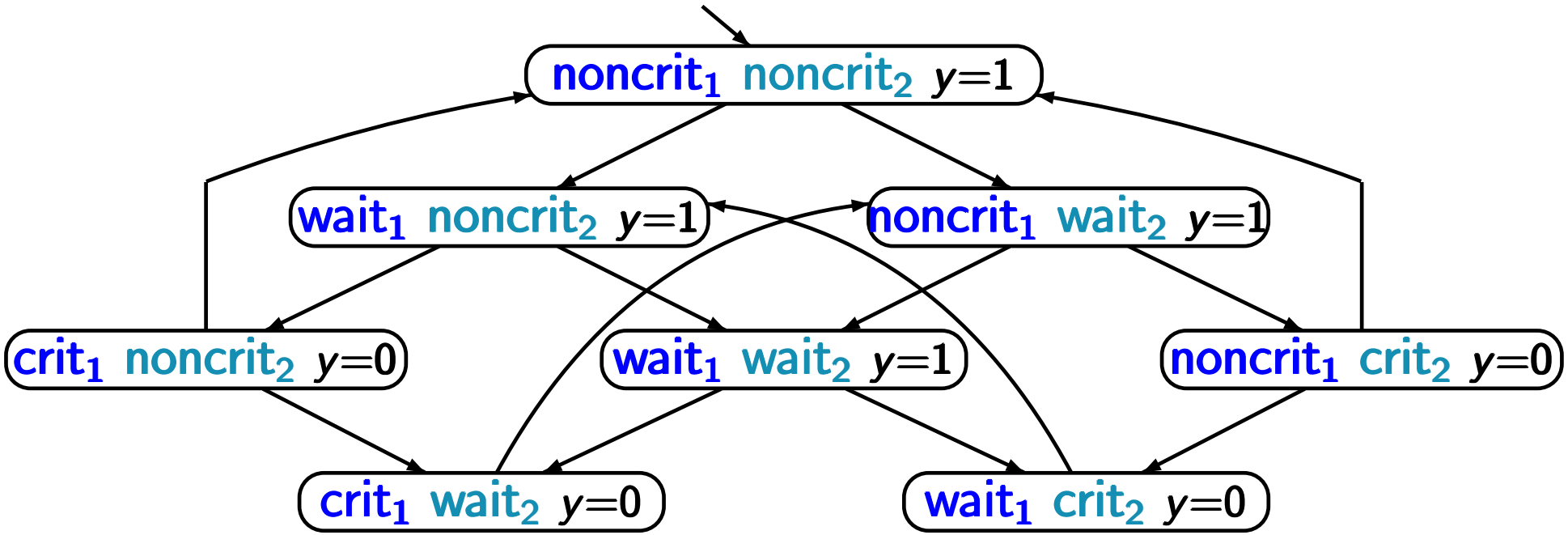
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



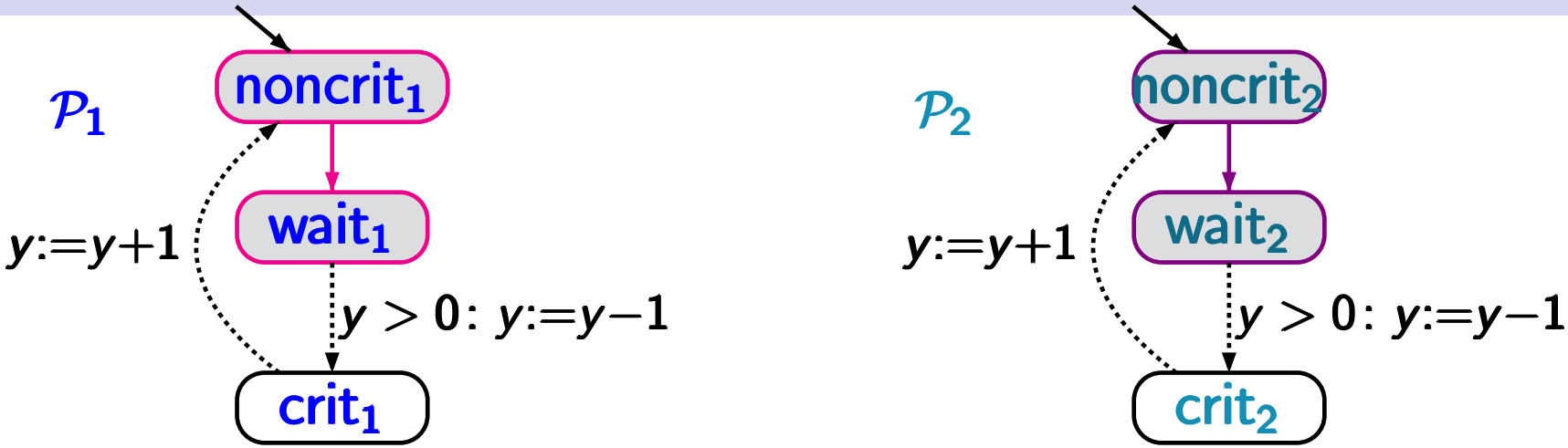
TS for mutual exclusion with semaphore



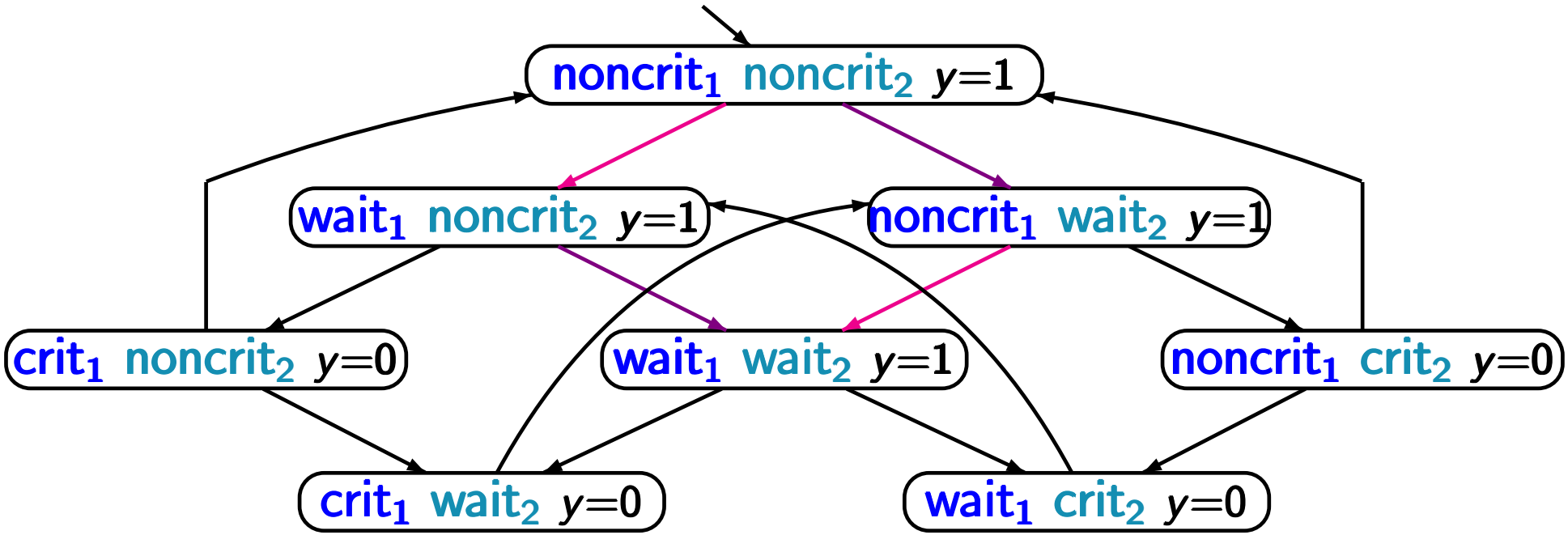
reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



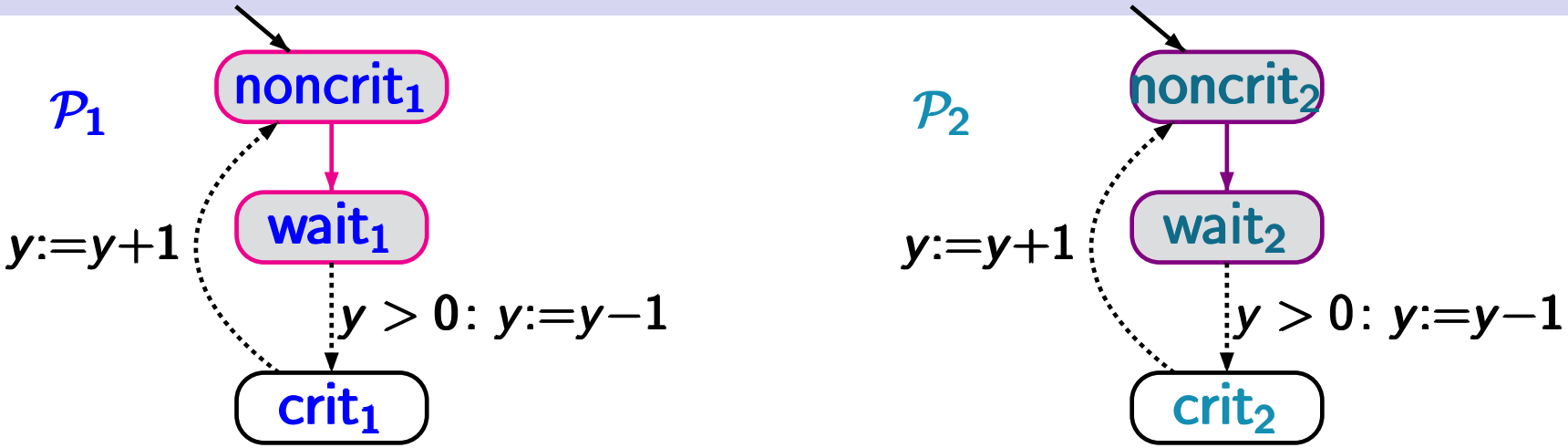
Concurrency of the request actions



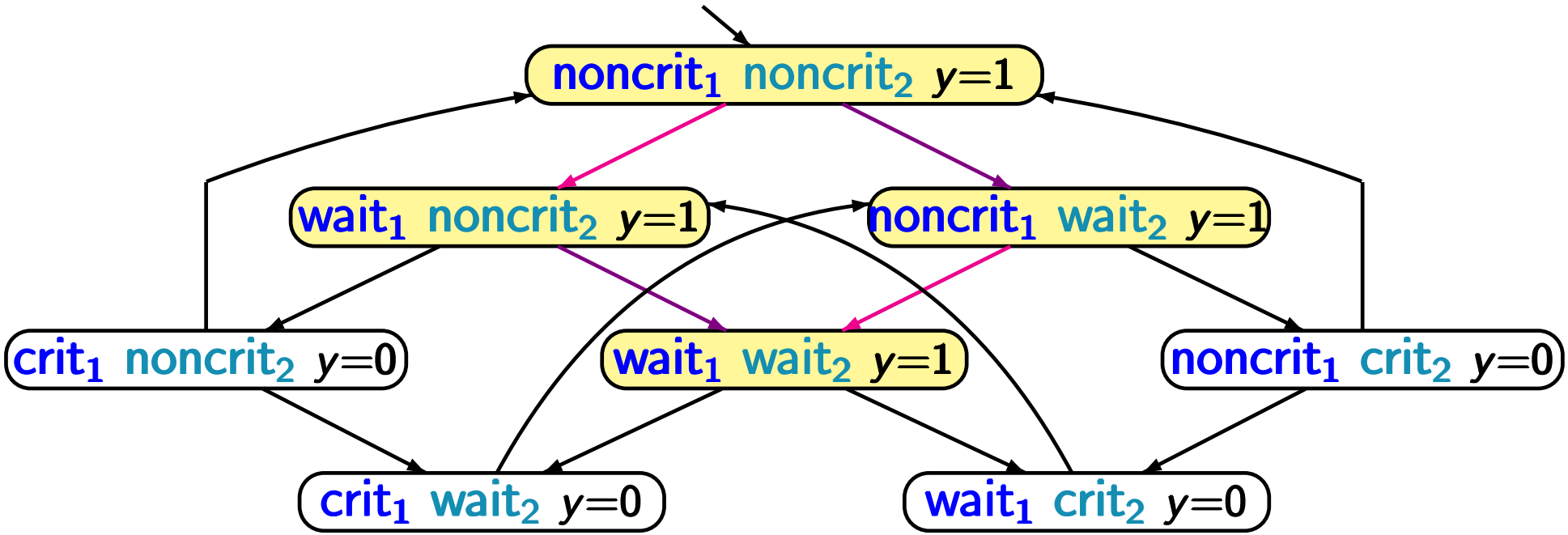
reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



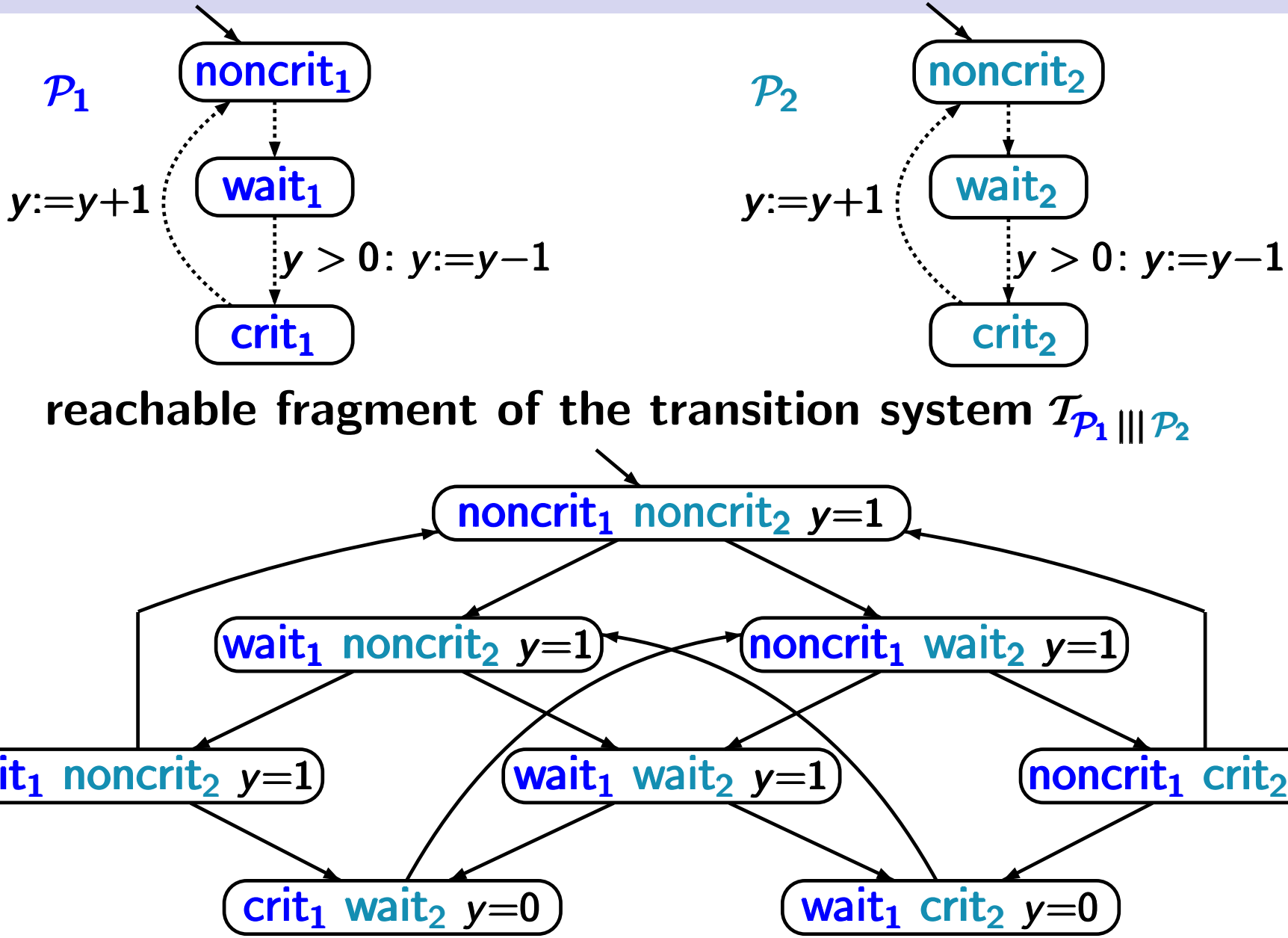
Concurrency of the request actions



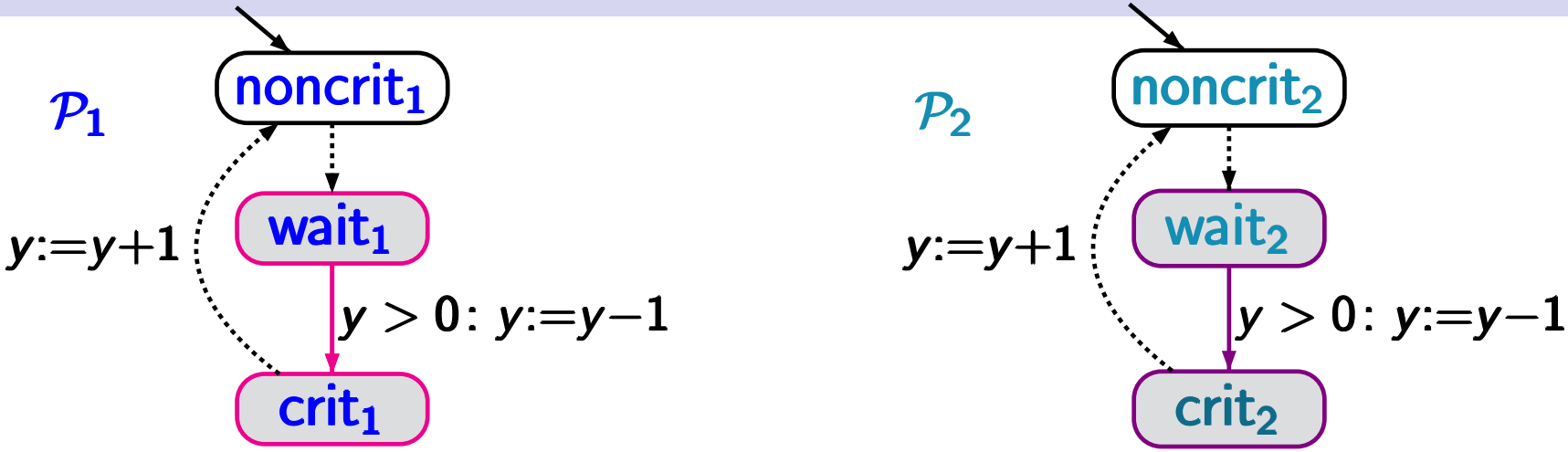
interleaving of the independent request actions



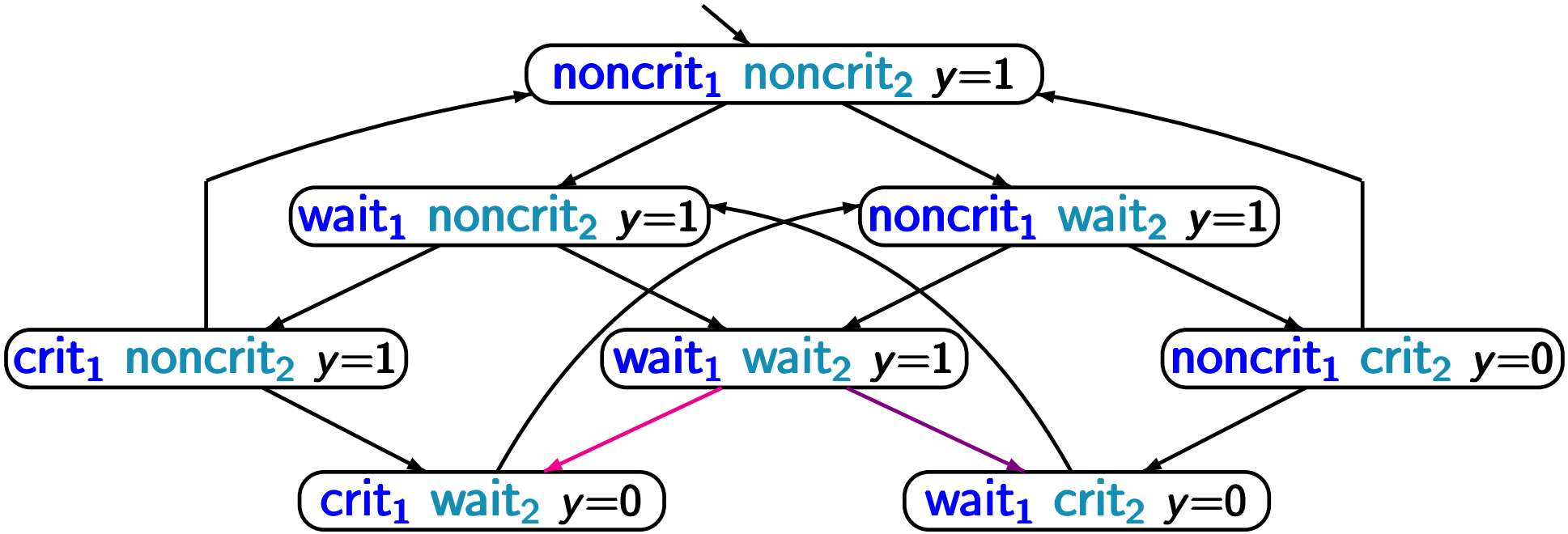
Competition



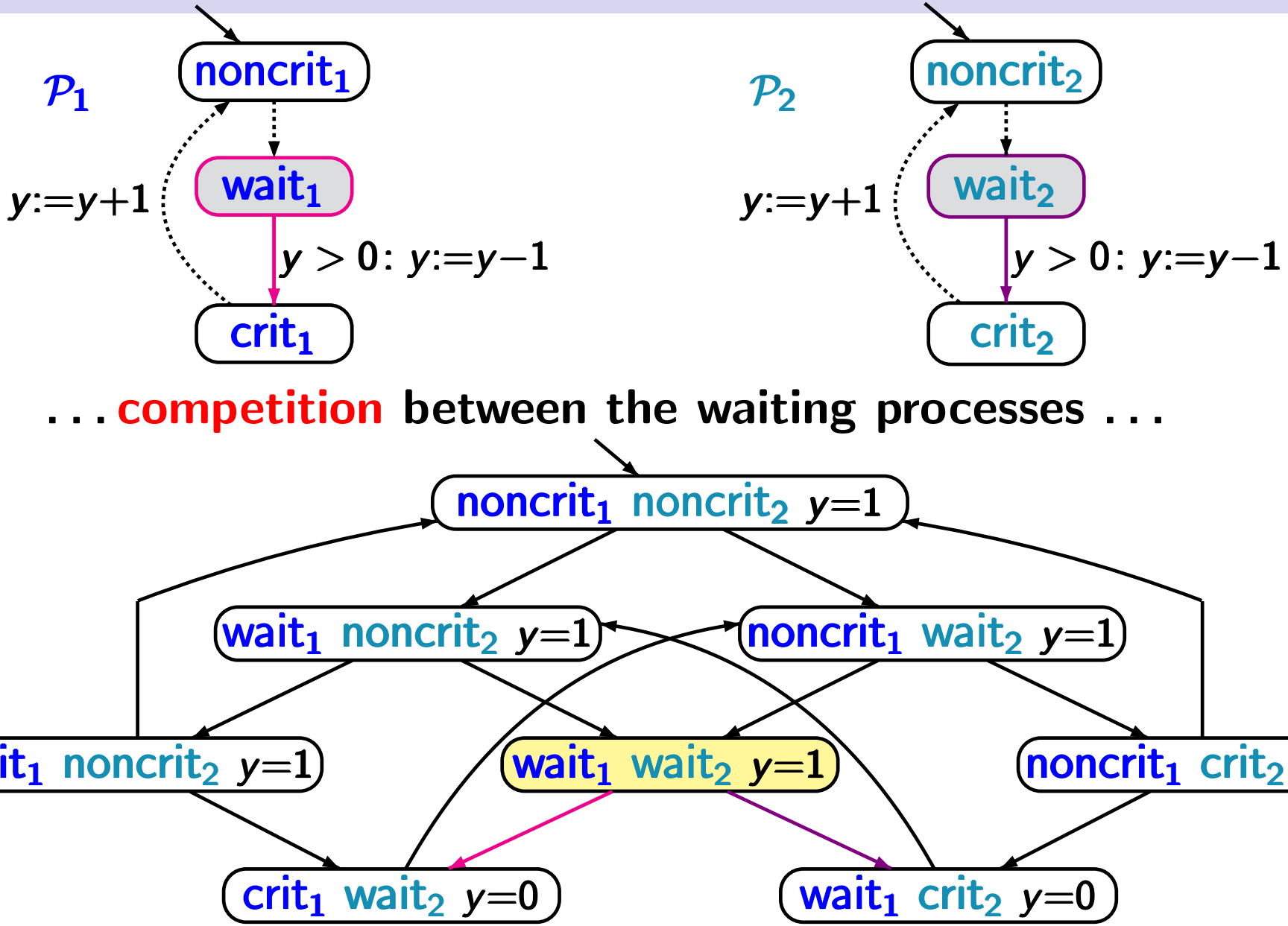
Competition



reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$

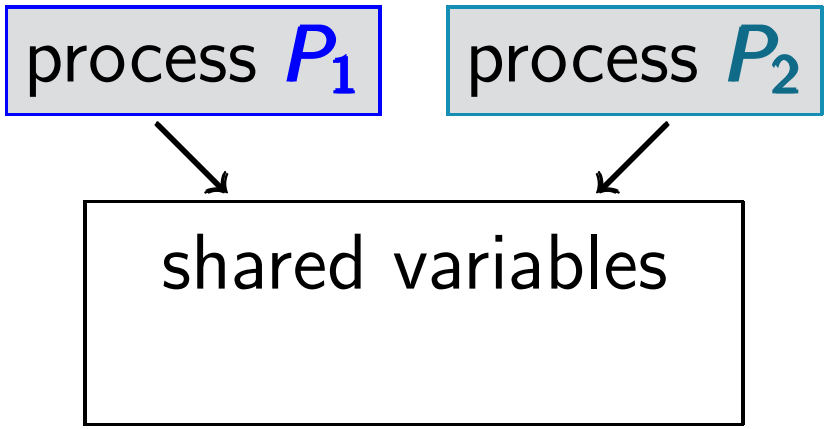


Competition

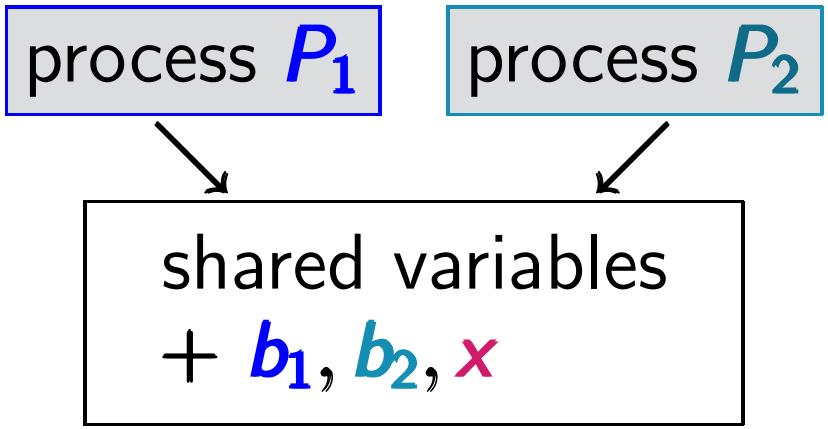


Peterson algorithm for mutual exclusion

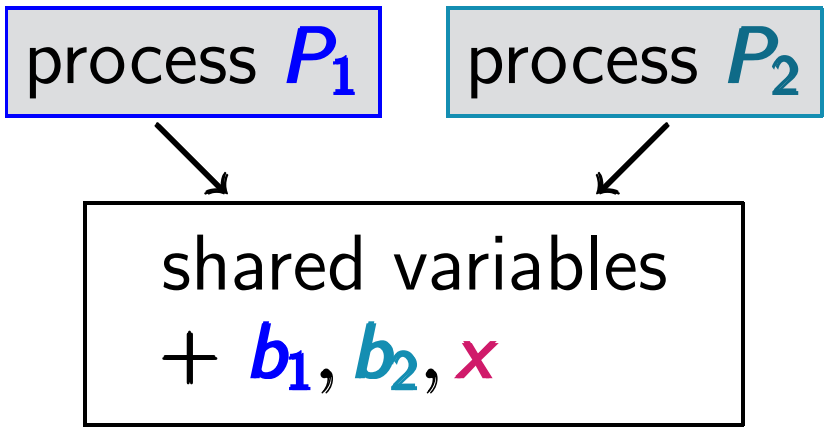
Peterson algorithm for mutual exclusion



Peterson algorithm for mutual exclusion

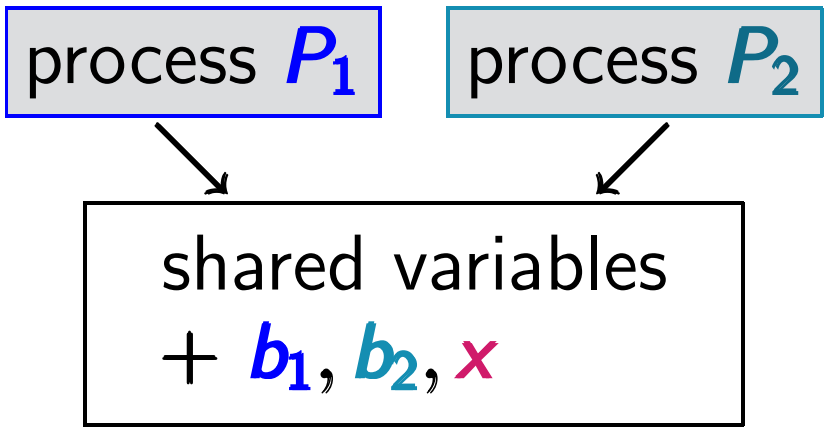


Peterson algorithm for mutual exclusion



b_1, b_2 Boolean variables, $x \in \{1, 2\}$

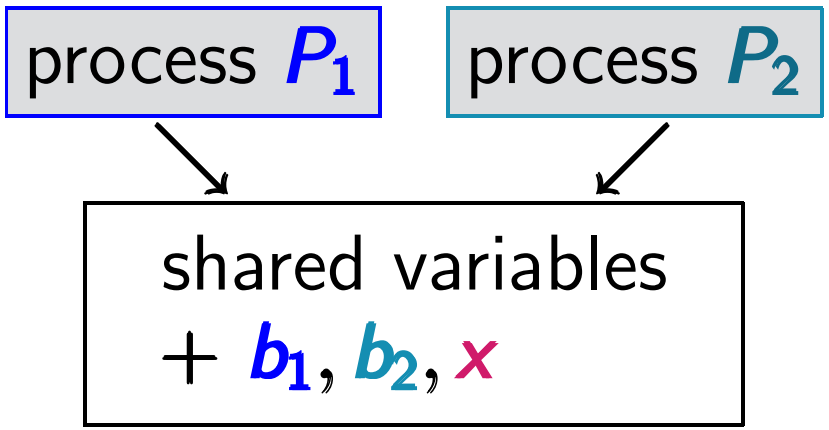
Peterson algorithm for mutual exclusion

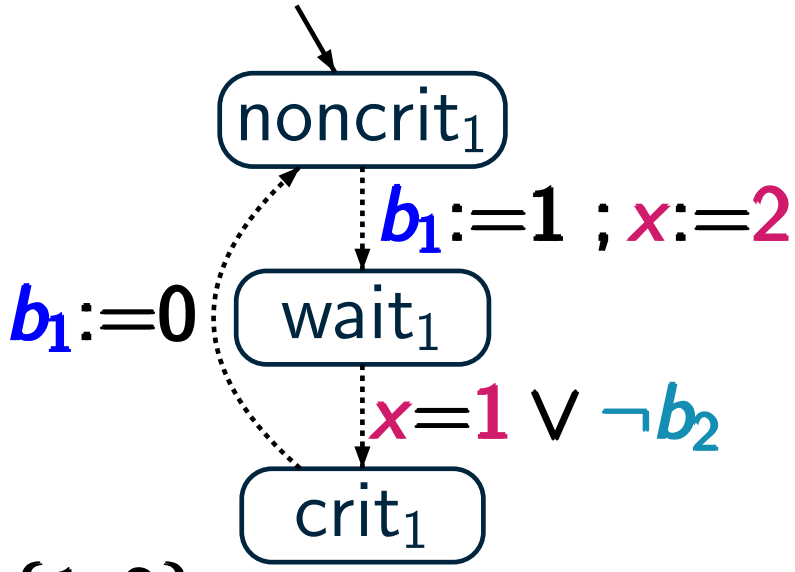


b_1, b_2 Boolean variables, $x \in \{1, 2\}$

```
LOOP FOREVER                                   (* protocol for P1 *)
    noncritical actions;
    b1:=1 ; x:=2;
    AWAIT x=1 ∨ ¬b2 DO critical section OD
    b1:=0
END LOOP
```

Peterson algorithm for mutual exclusion

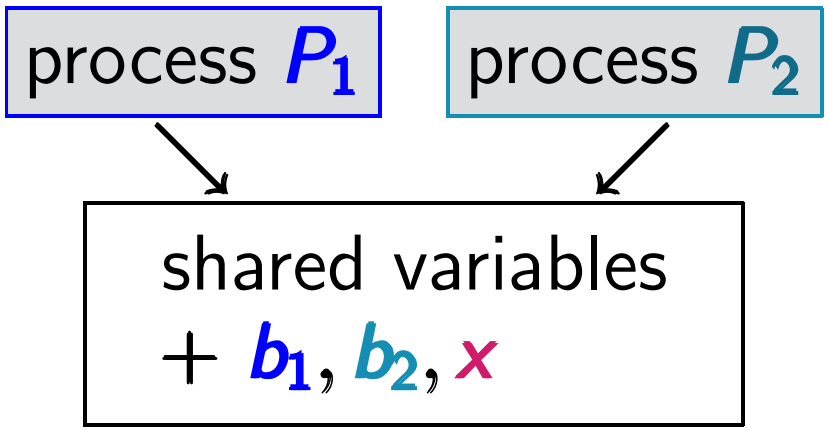


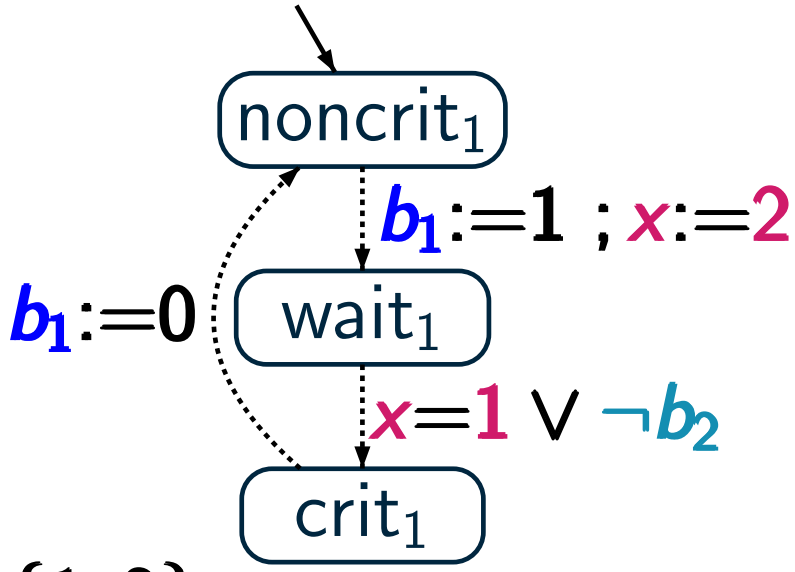


b_1, b_2 Boolean variables, $x \in \{1, 2\}$

```
LOOP FOREVER                                   (* protocol for P1 *)
    noncritical actions;
    b1:=1 ; x:=2;
    AWAIT x=1 ∨ ¬b2 DO critical section OD
    b1:=0
END LOOP
```

Peterson algorithm for mutual exclusion

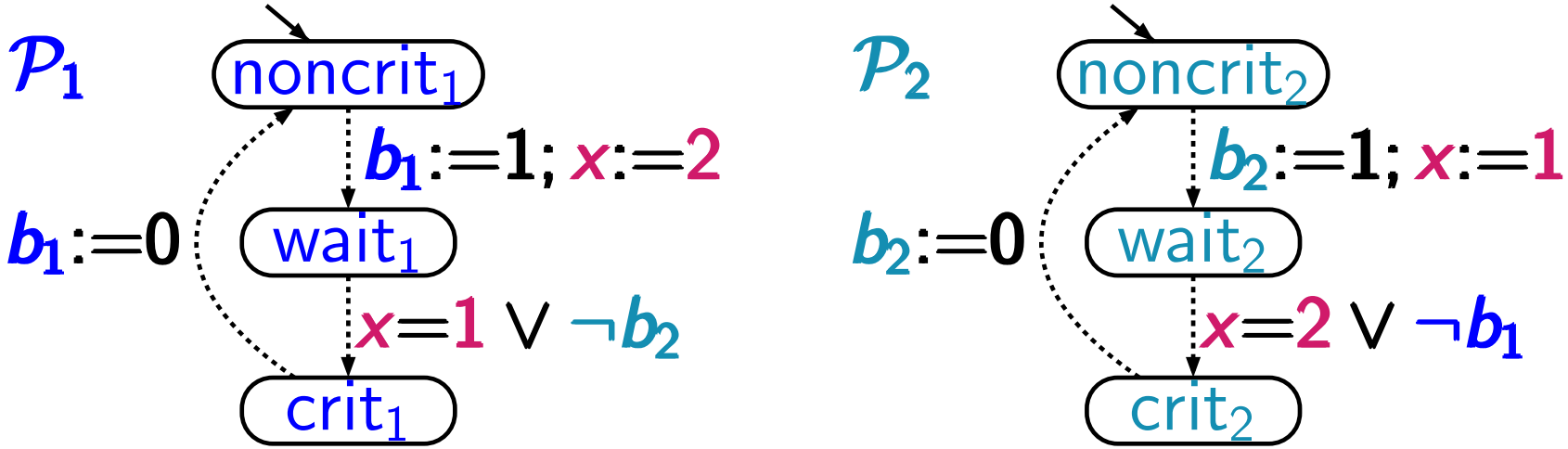




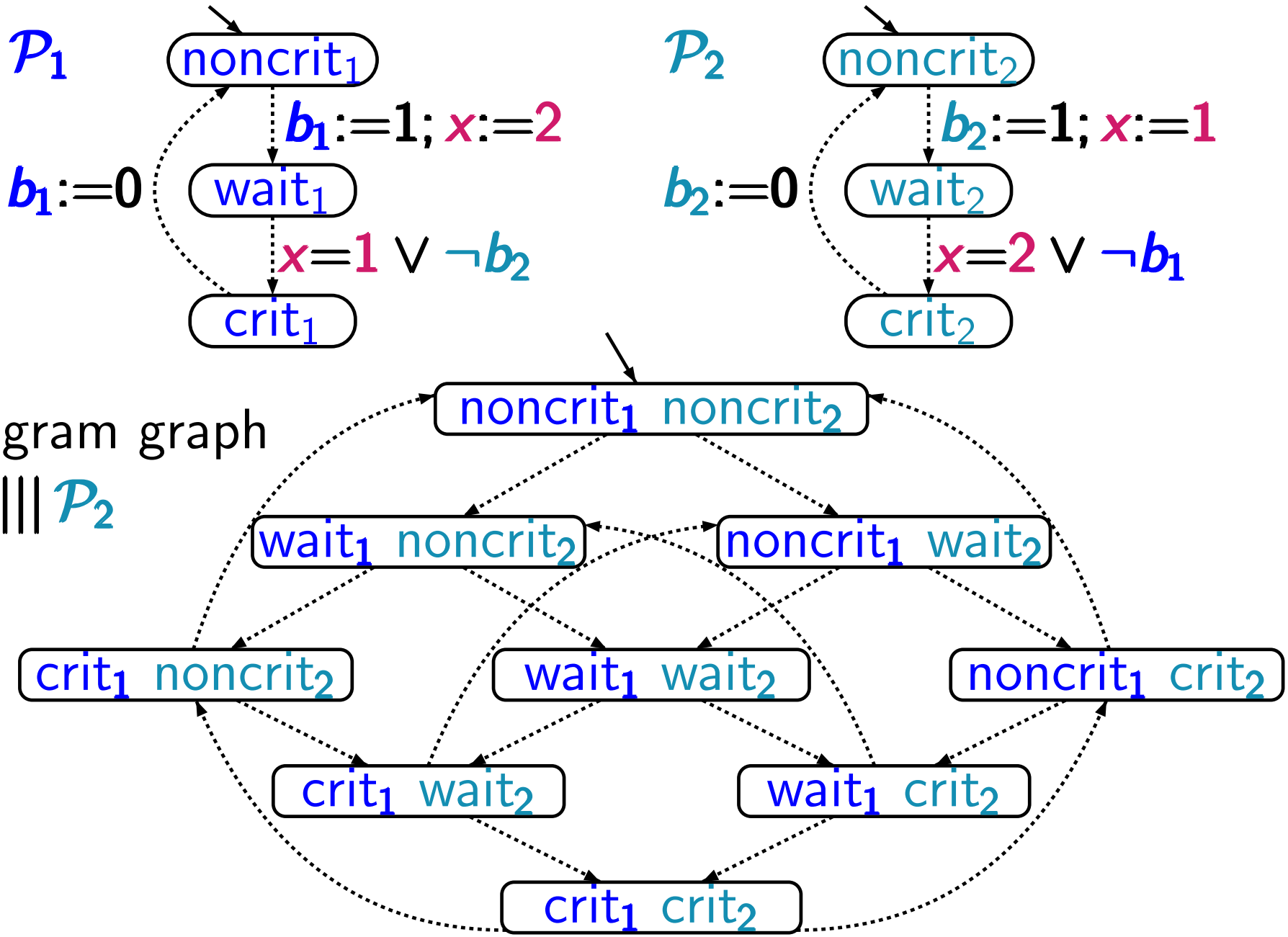
b_1, b_2 Boolean variables, $x \in \{1, 2\}$

```
LOOP FOREVER                          (* protocol for P1 *)
    noncritical actions;
    atomic{b1:=1 ; x:=2};
    AWAIT x=1 ∨ ¬b2 DO critical section OD
    b1:=0
END LOOP
```

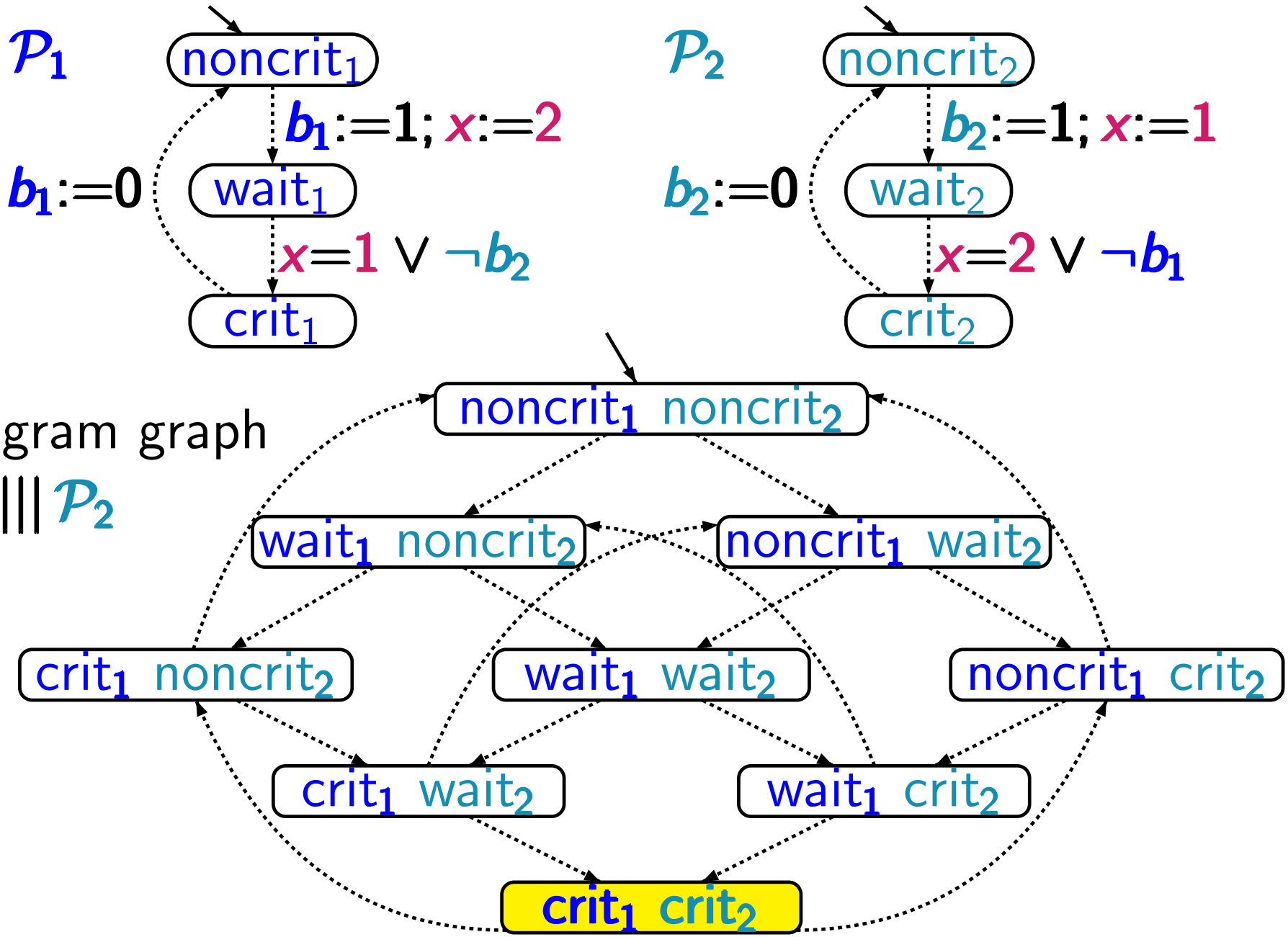
Program graphs for Peterson algorithm



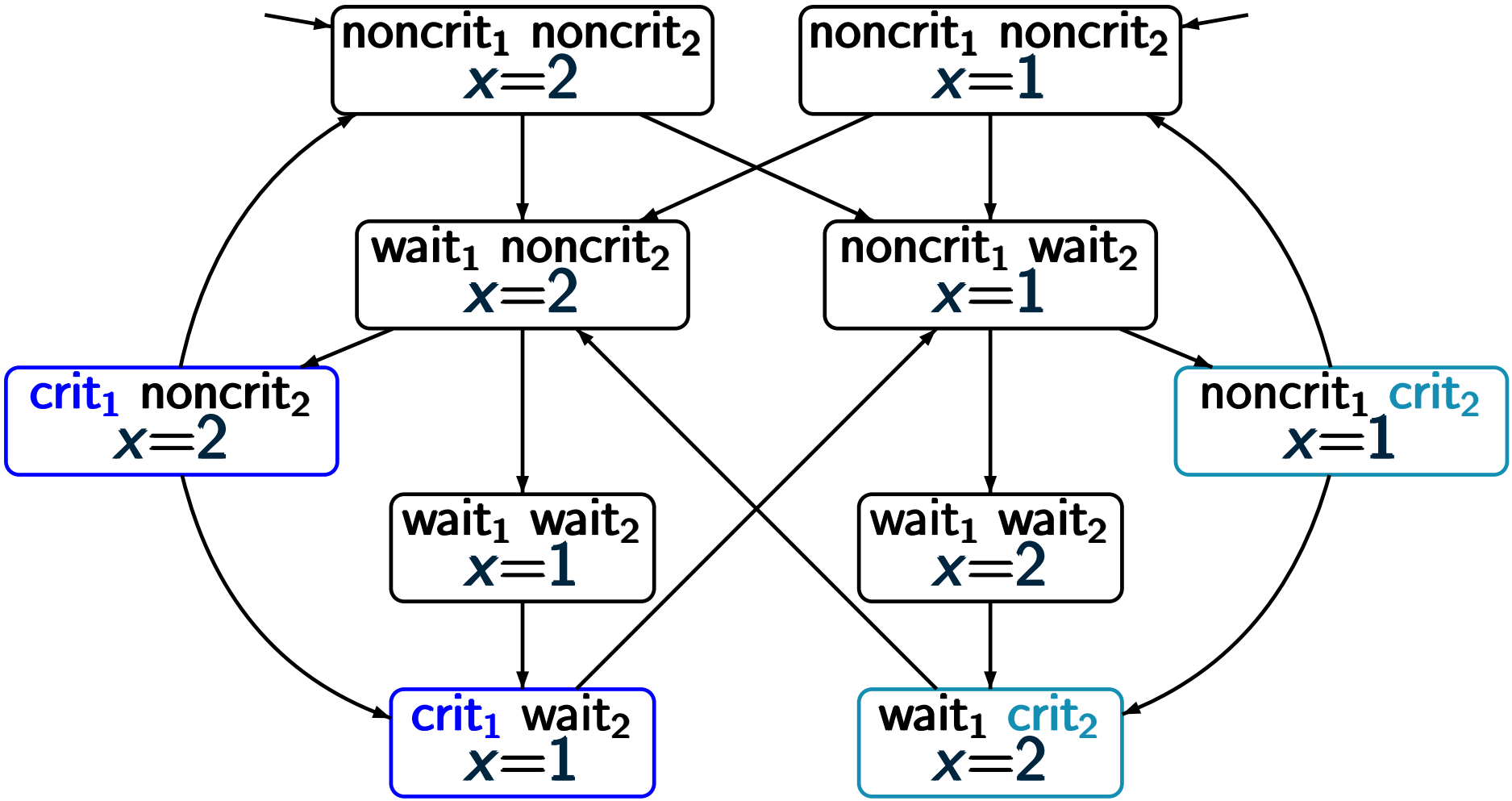
Program graphs for Peterson algorithm



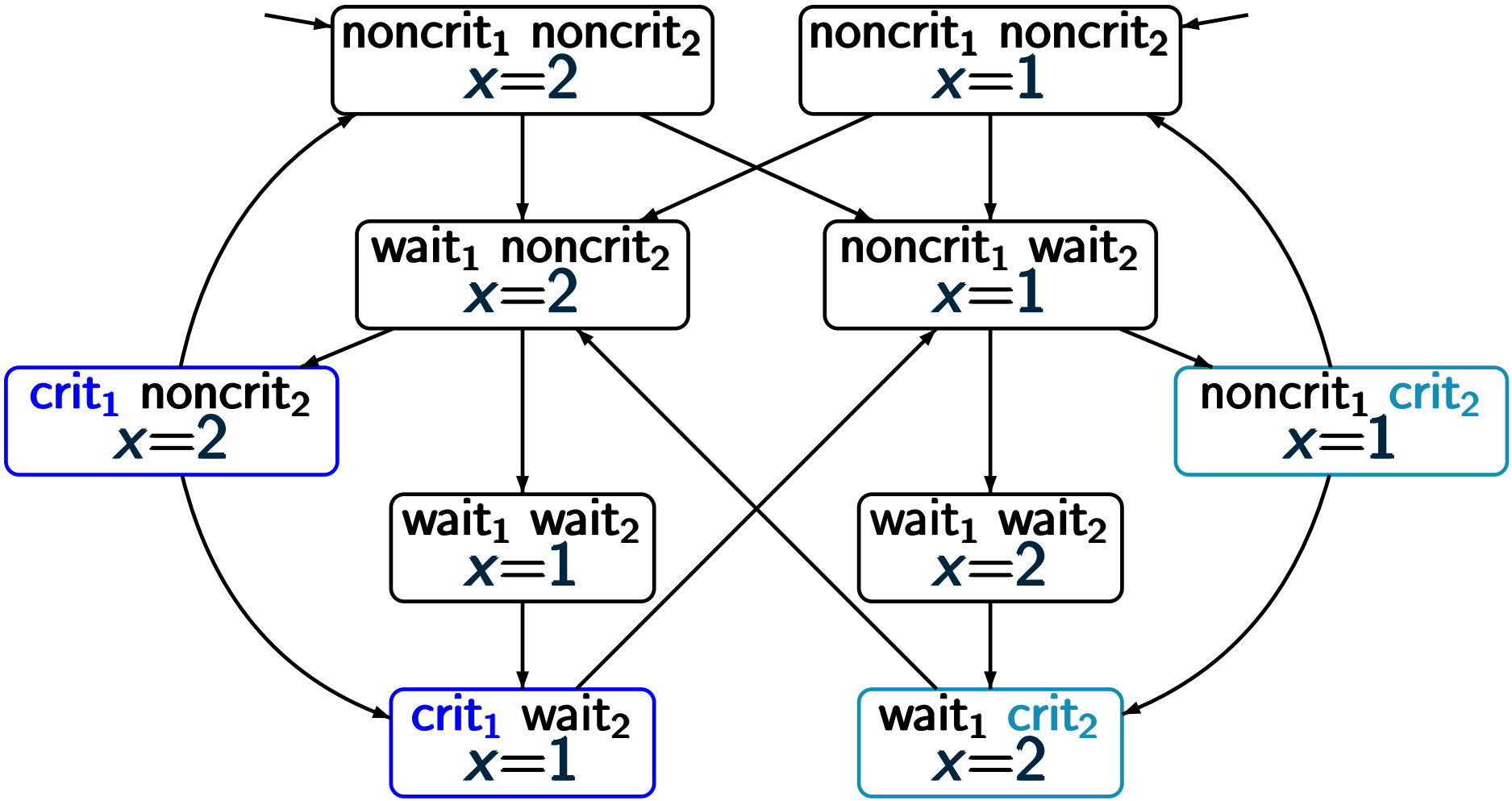
Program graphs for Peterson algorithm



TS for the Peterson algorithm



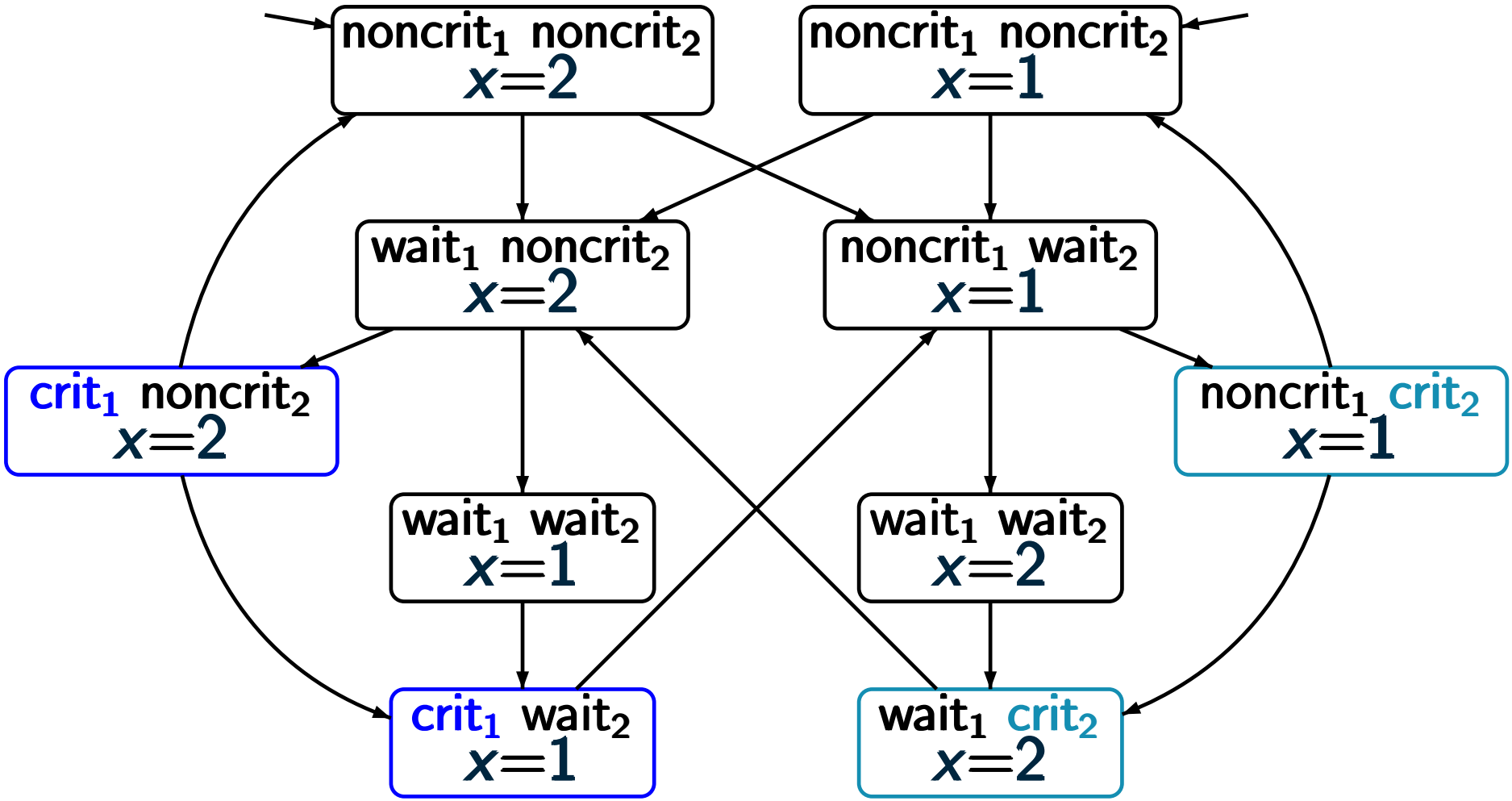
TS for the Peterson algorithm



value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

TS for the Peterson algorithm

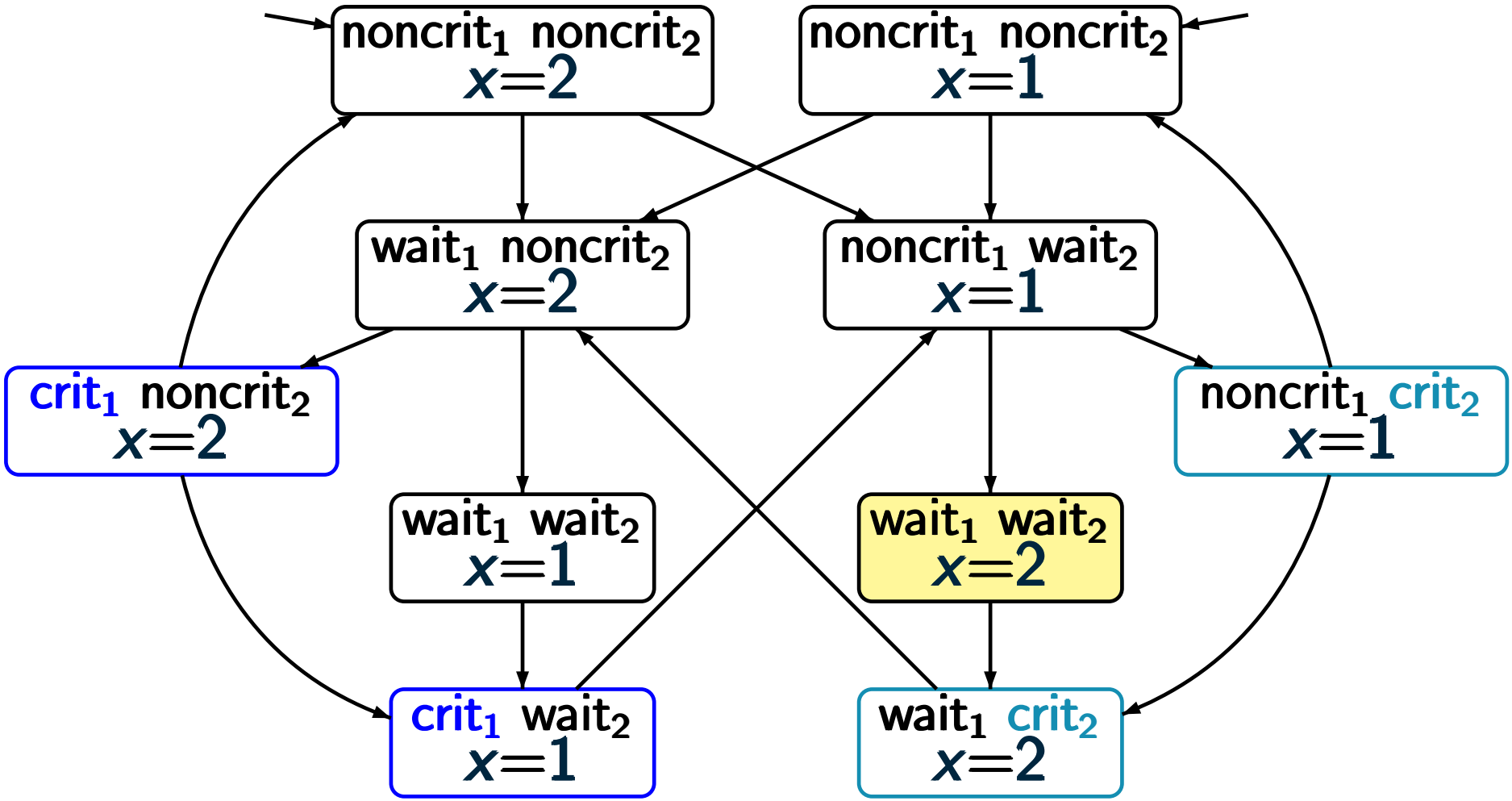


value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

+ unreachable states

TS for the Peterson algorithm

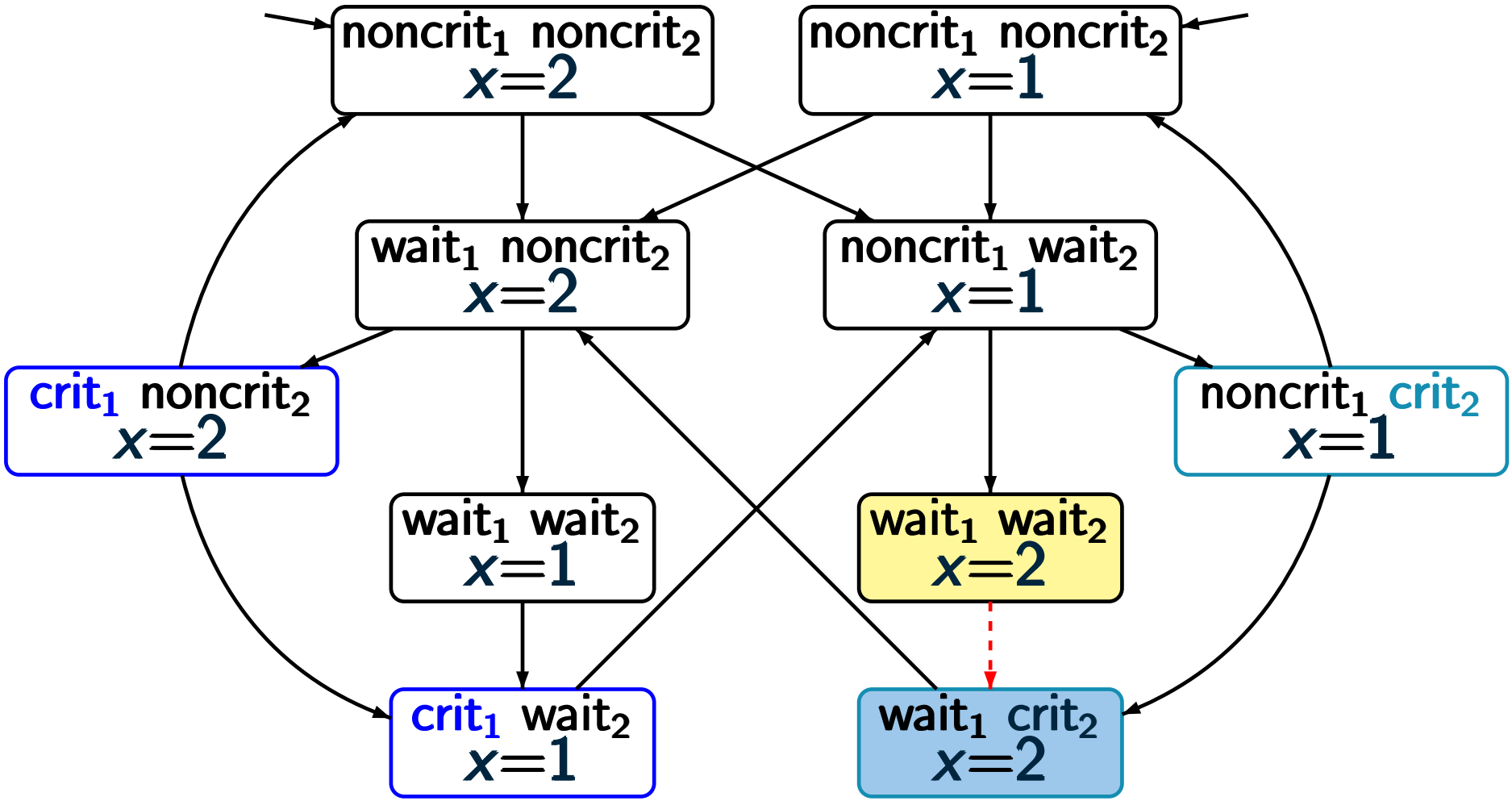


value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

+ unreachable states

TS for the Peterson algorithm

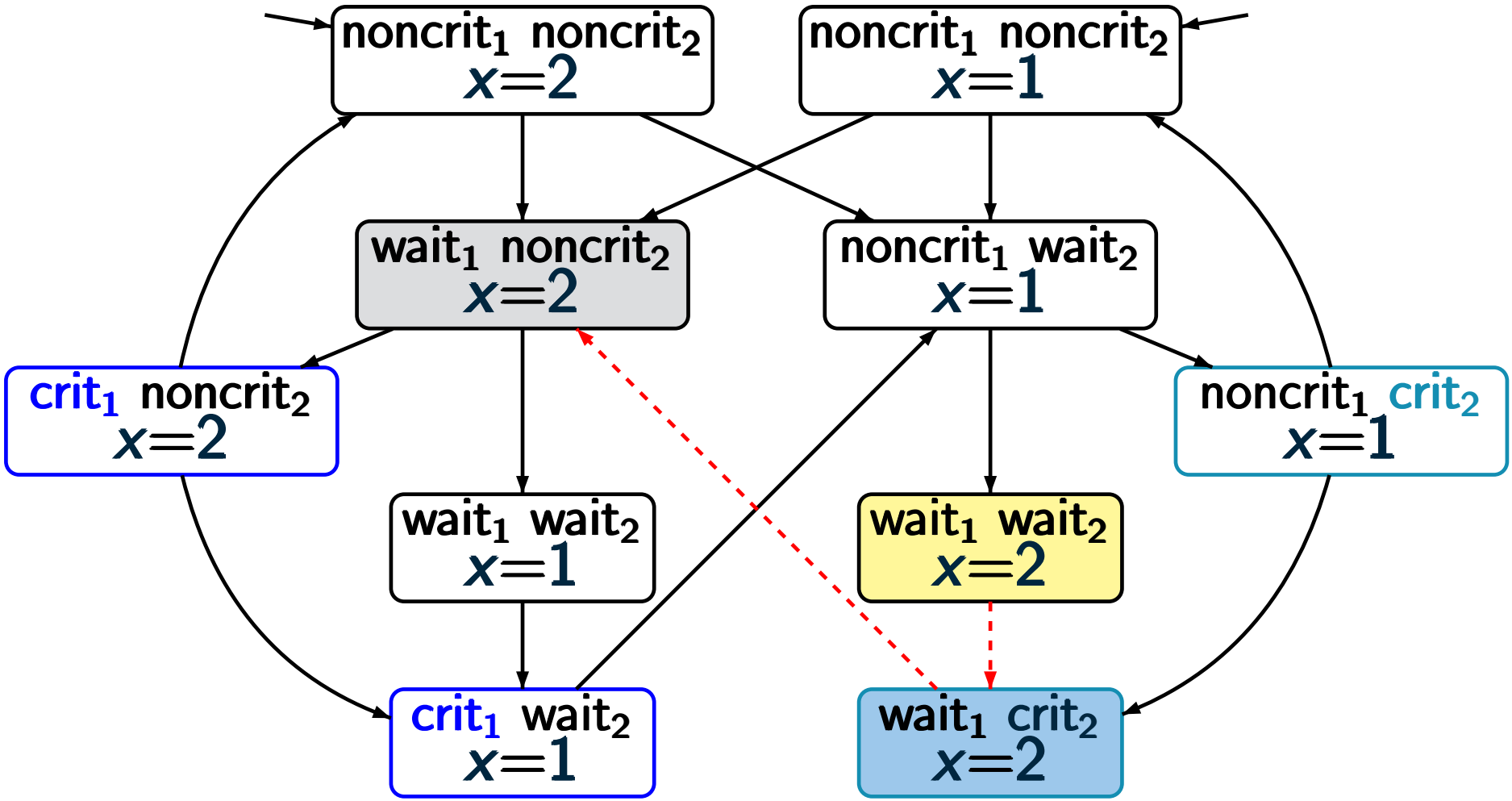


value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

+ unreachable states

TS for the Peterson algorithm

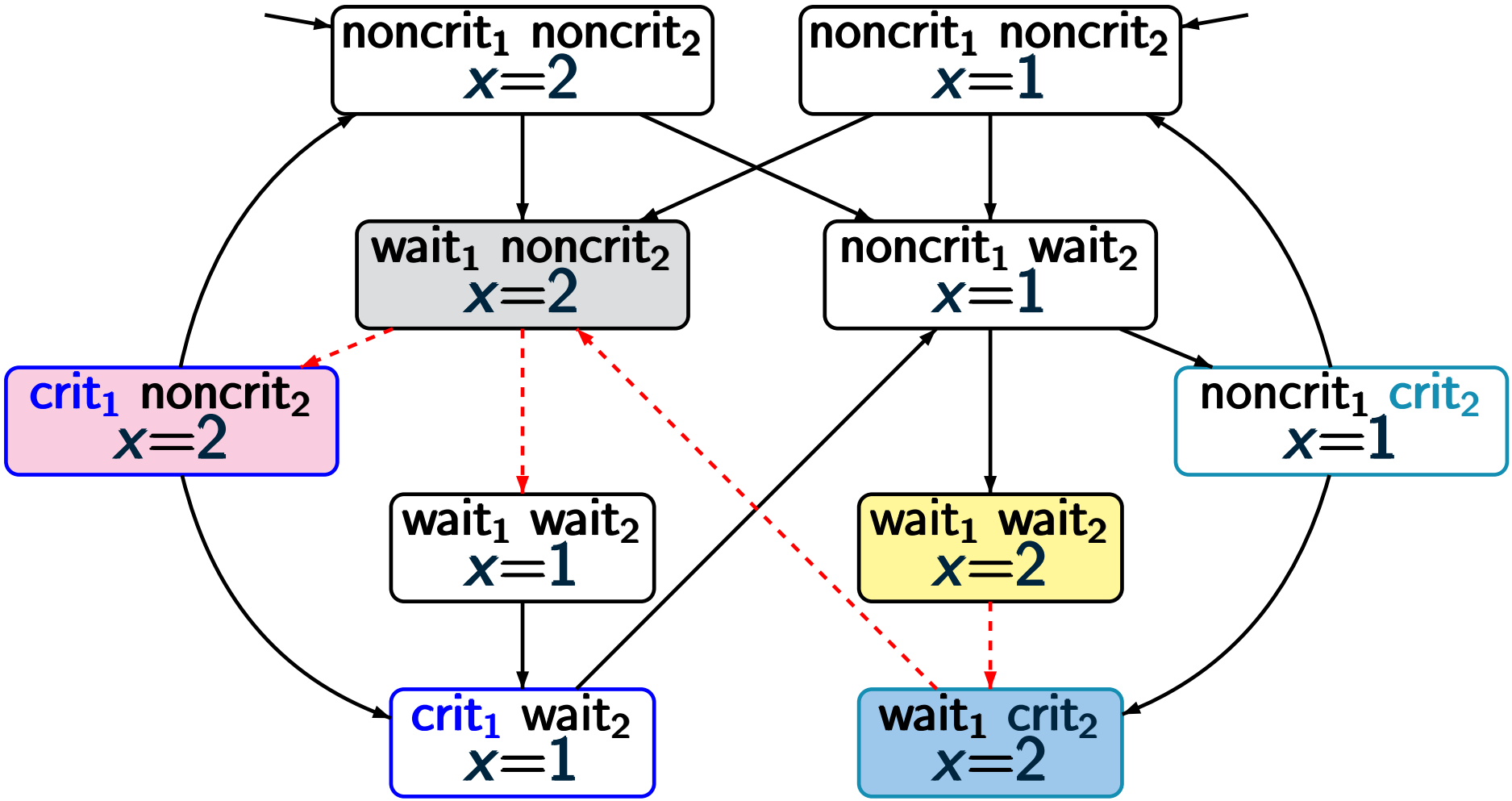


value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

+ unreachable states

TS for the Peterson algorithm

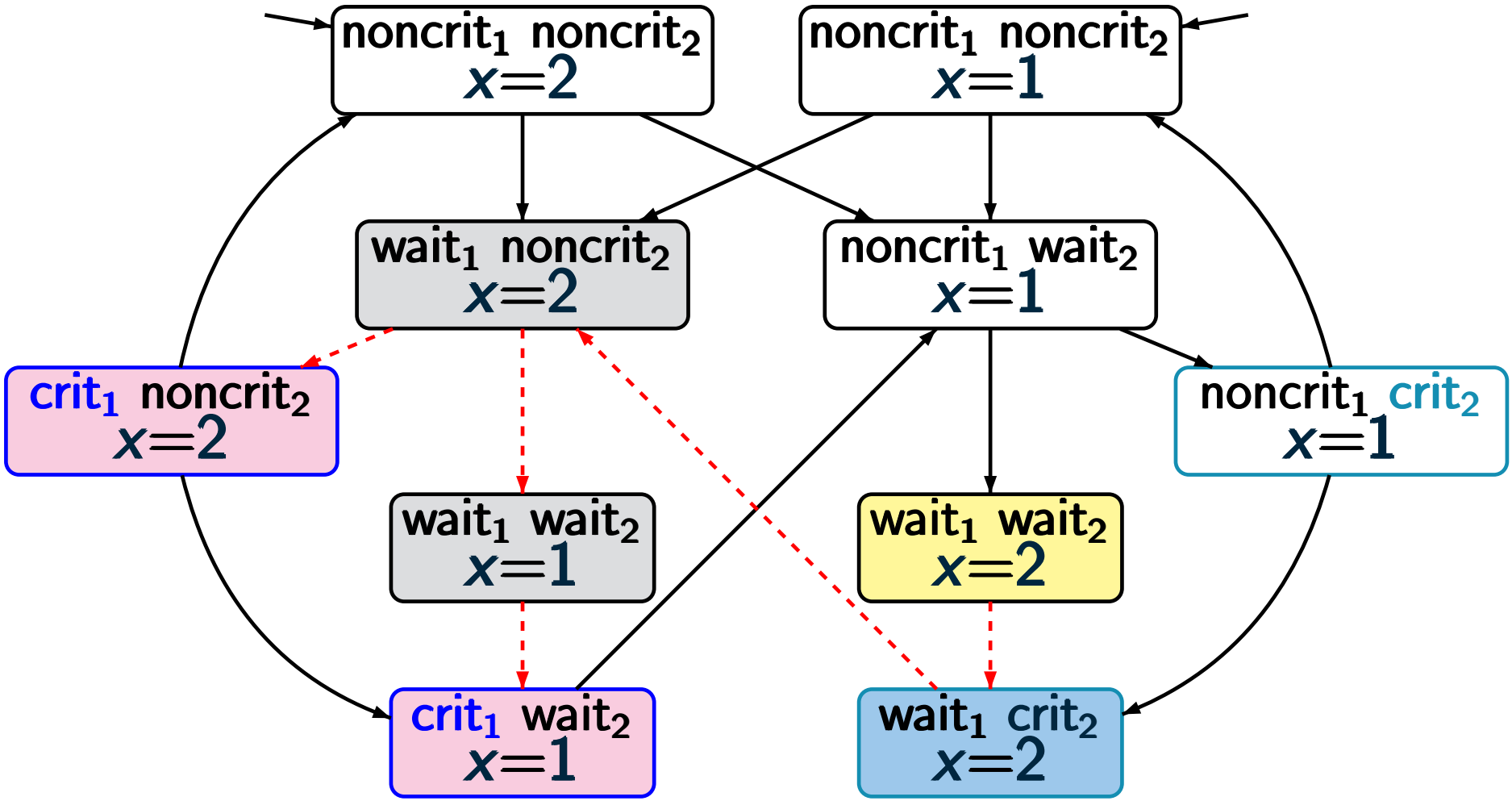


value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

+ unreachable states

TS for the Peterson algorithm

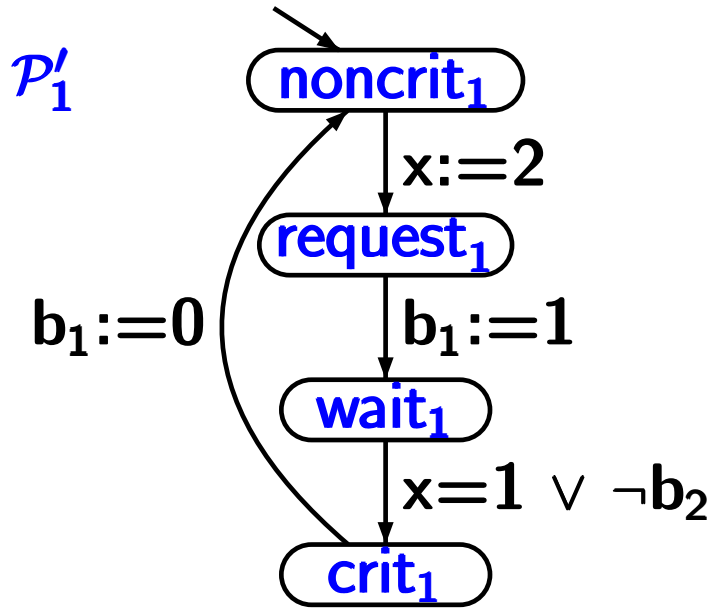


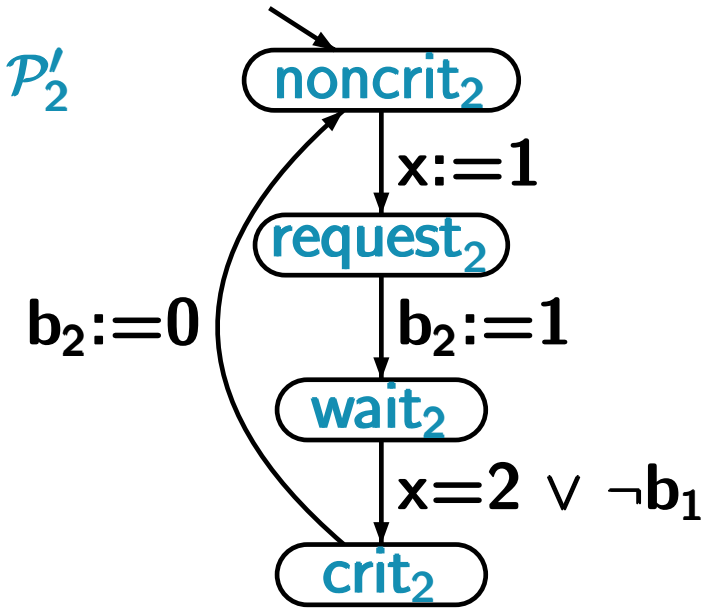
value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

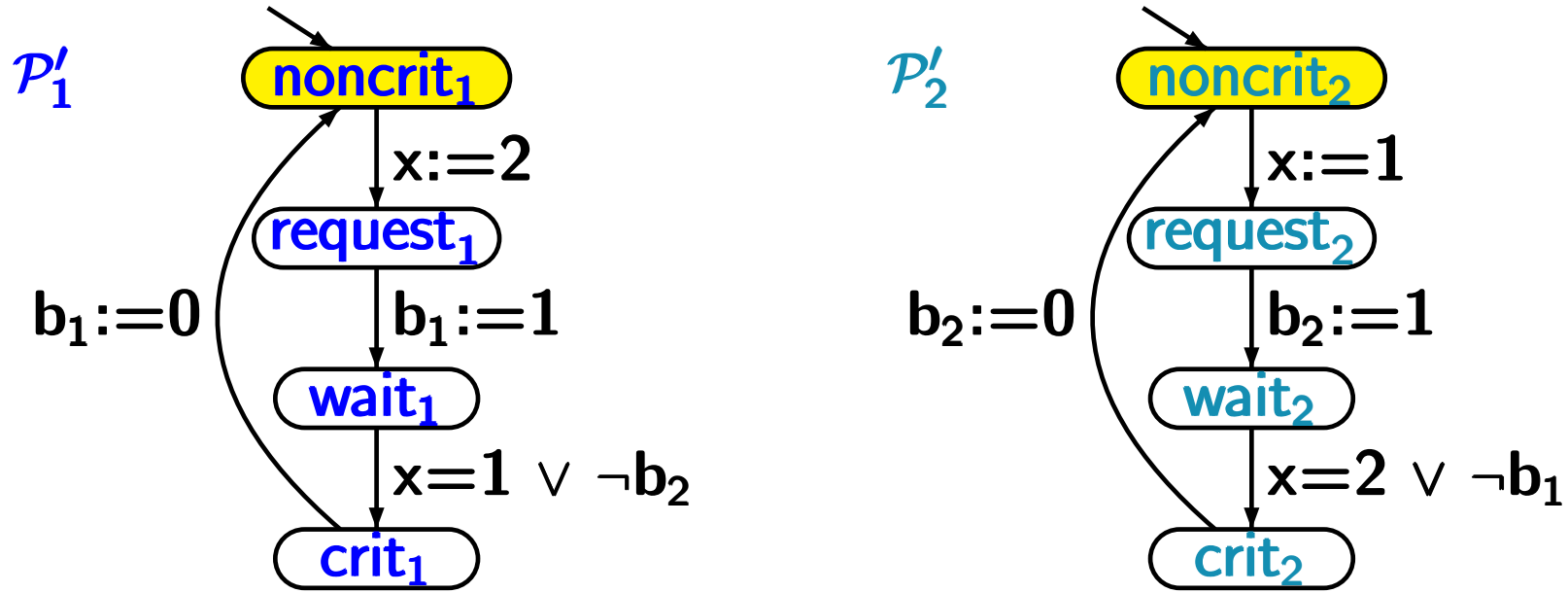
+ unreachable states

Variant of Peterson algorithm





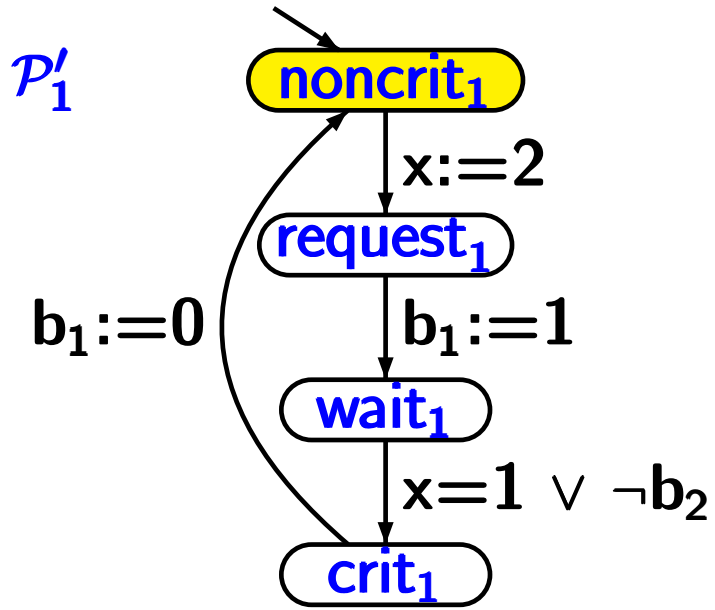
Variant of Peterson algorithm

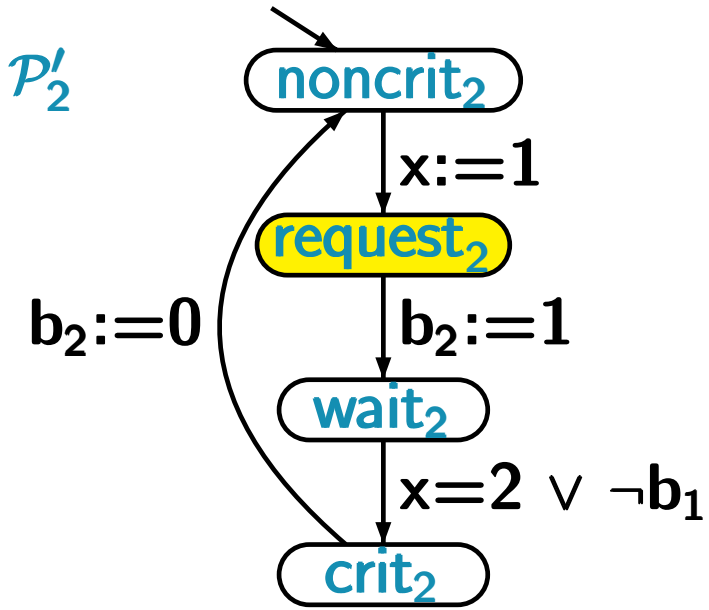


possible executions

noncrit$_1$ noncrit$_2$ x=1 ¬b$_1$ ¬b$_2$

Variant of Peterson algorithm

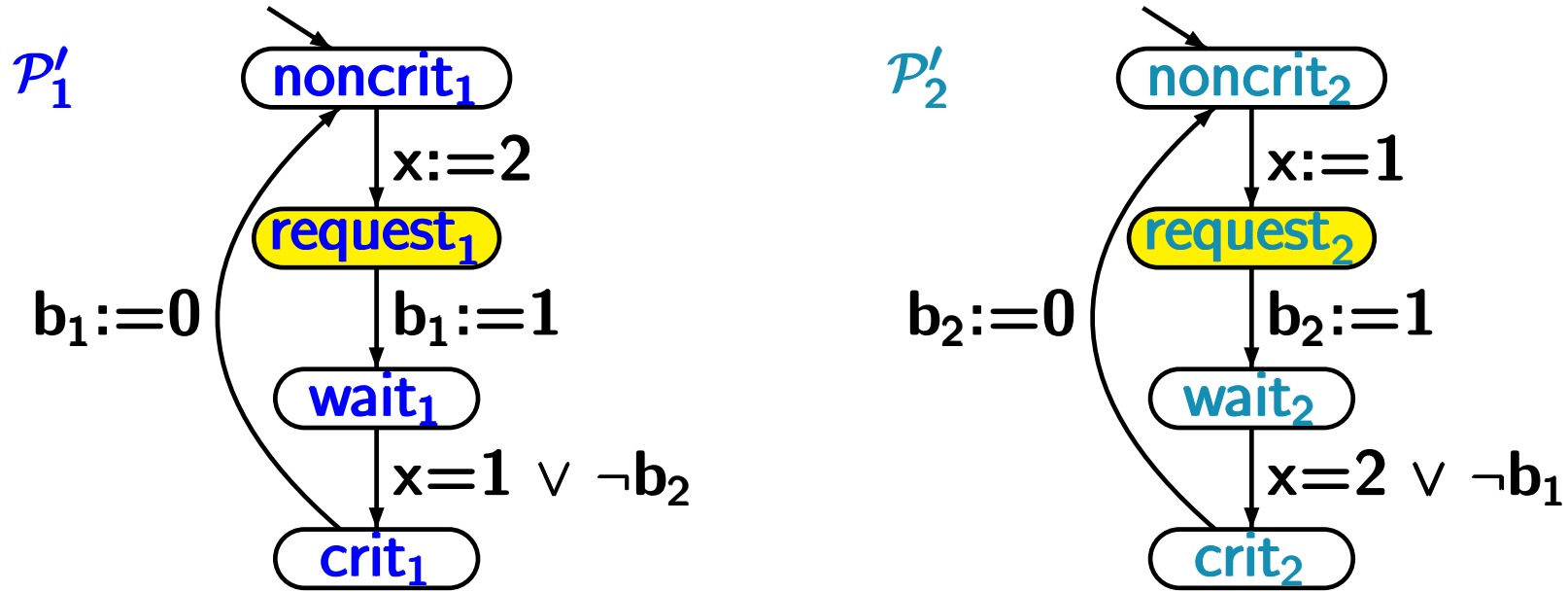




possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$

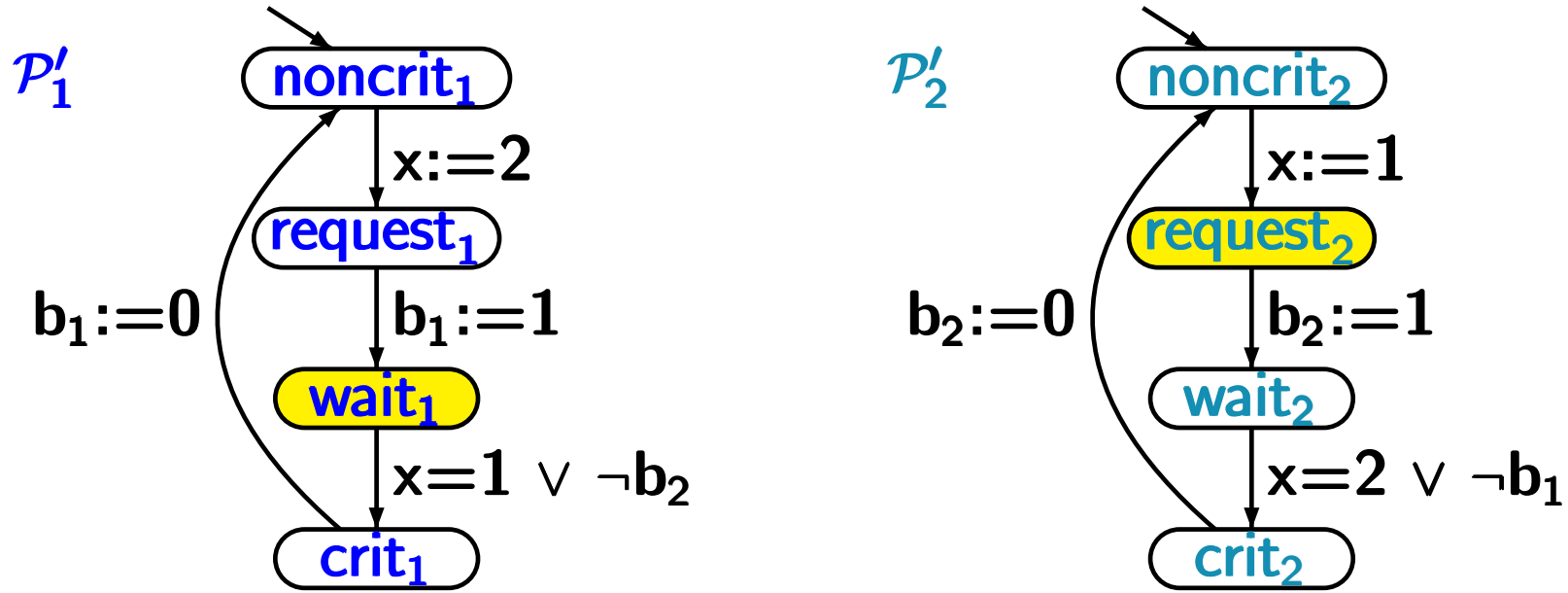
Variant of Peterson algorithm



possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$

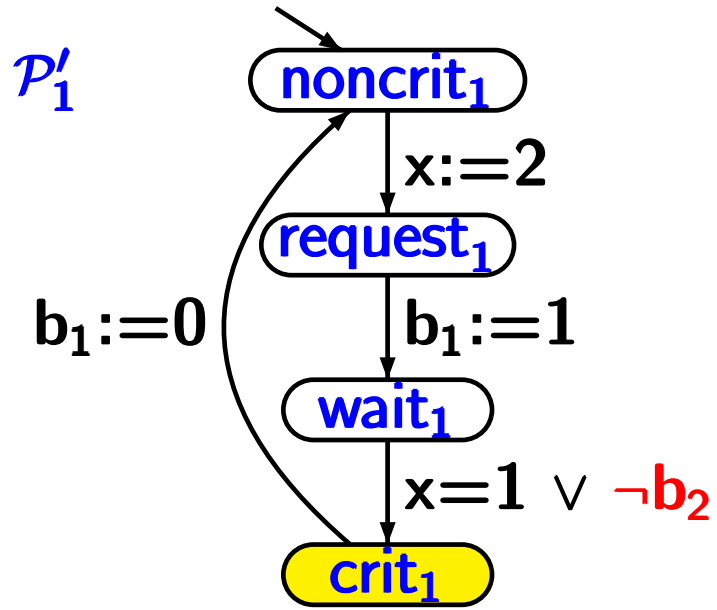
Variant of Peterson algorithm

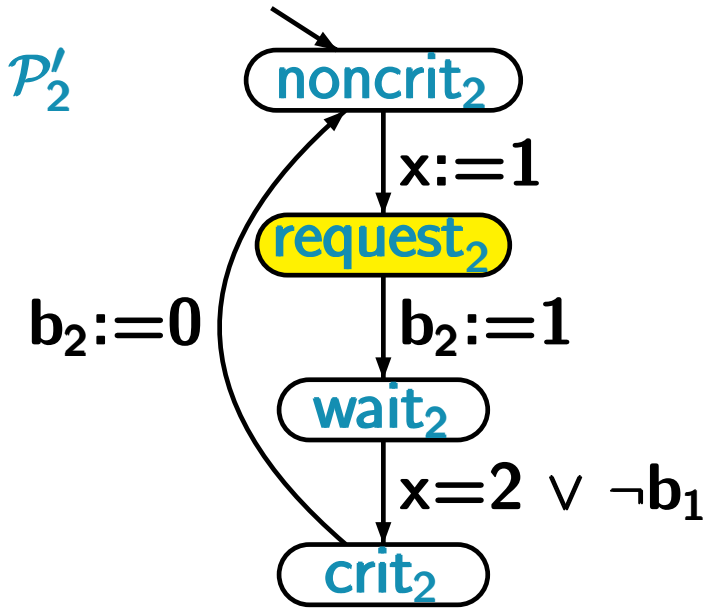


possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$

Variant of Peterson algorithm

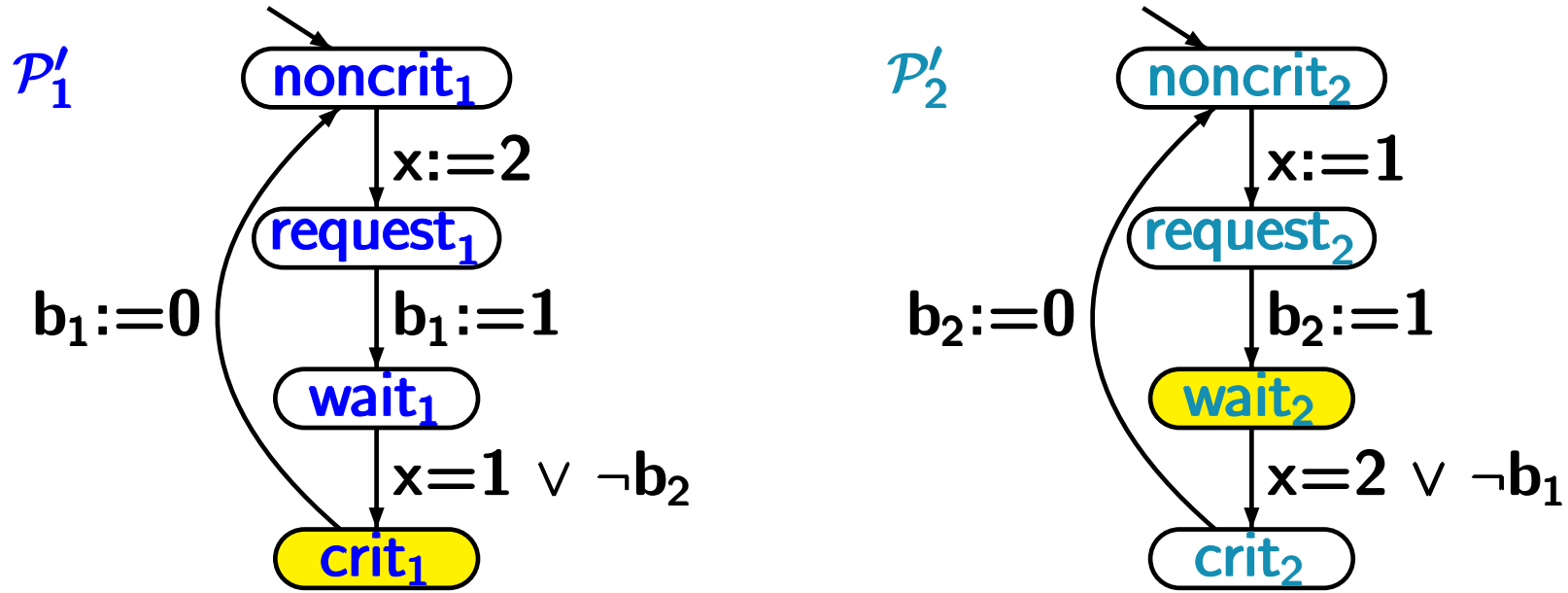




possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$

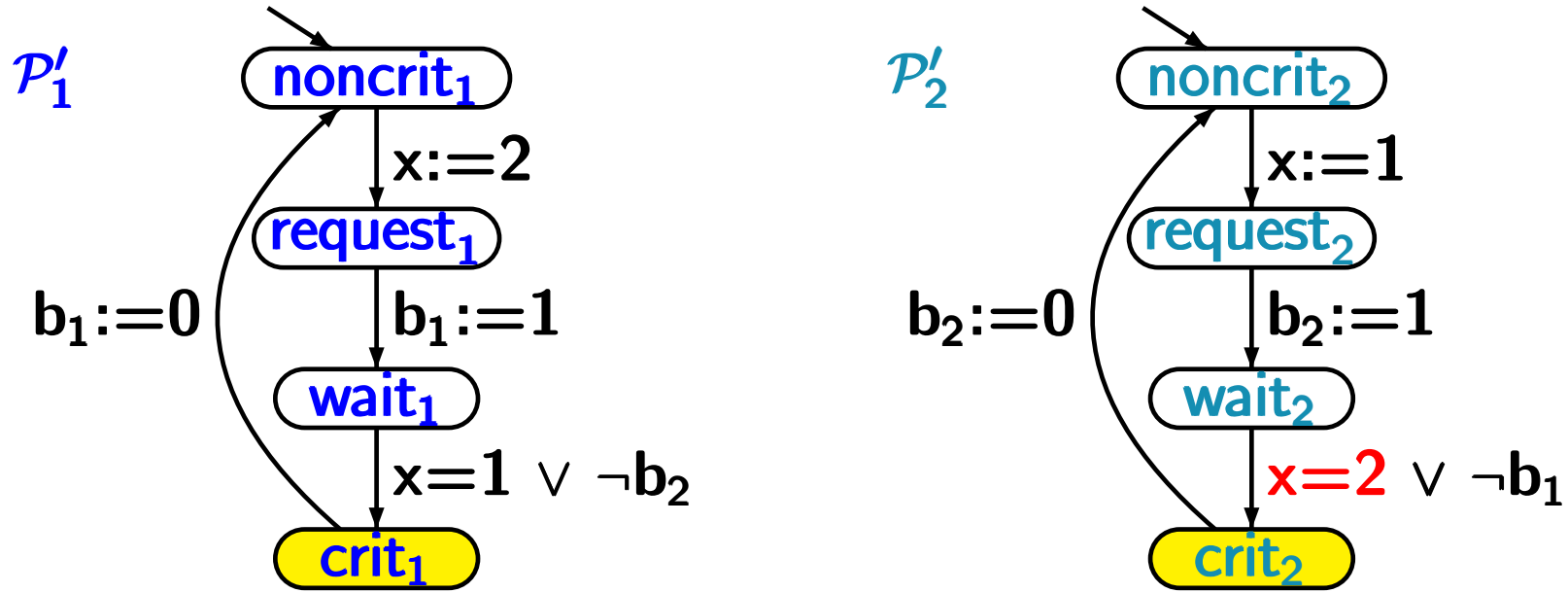
Variant of Peterson algorithm



possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	wait$_2$	x=2	b$_1$	b$_2$

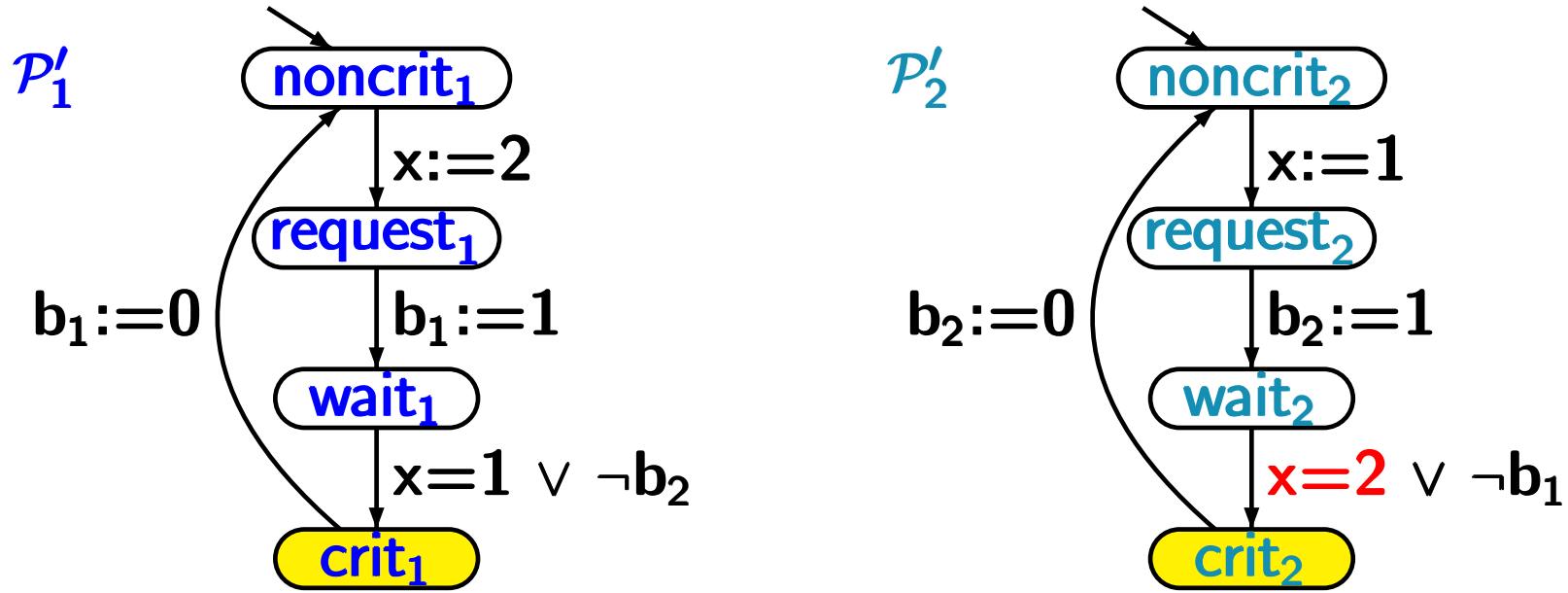
Variant of Peterson algorithm



possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	wait$_2$	x=2	b$_1$	b$_2$
crit$_1$	crit$_2$	x=2	b$_1$	b$_2$

Variant of Peterson algorithm incorrect!



possible executions

noncrit$_1$	noncrit$_2$	x=1	¬b$_1$	¬b$_2$
noncrit$_1$	request$_2$	x=1	¬ b$_1$	¬b$_2$
request$_1$	request$_2$	x=2	¬b$_1$	¬b$_2$
wait$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	request$_2$	x=2	b$_1$	¬b$_2$
crit$_1$	wait$_2$	x=2	b$_1$	b$_2$
crit$_1$	crit$_2$	x=2	b$_1$	b$_2$

Operators for parallelism and communication

Operators for parallelism and communication

- true concurrency: interleaving operator $|||$ for TS (no communication, no dependencies)

Operators for parallelism and communication

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- channel systems
  communication via shared variables + via channels
- synchronous product

Synchronous message passing

Synchronous message passing

$\mathcal{T}_1 = (S_1, Act_1, \rightarrow_1, \ldots)$, $\mathcal{T}_2 = (S_2, Act_2, \rightarrow_2, \ldots)$ TS

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composite transition system:

$$\mathcal{T}_1 \,\|_{Syn}\, \mathcal{T}_2 = (S_1 \times S_2, Act_1 \cup Act_2, \rightarrow, \ldots)$$

for modeling the concurrent execution of $\mathcal{T}_1$ and $\mathcal{T}_2$ with synchronization over all actions in Syn

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Mutual exclusion

by synchronous message passing

Mutual exclusion

by synchronous message passing using an arbiter

Mutual exclusion with an arbiter

protocol for process P_i

```
LOOP FOREVER DO
     noncritical actions
     request
     critical section
     release
     noncritical actions
OD
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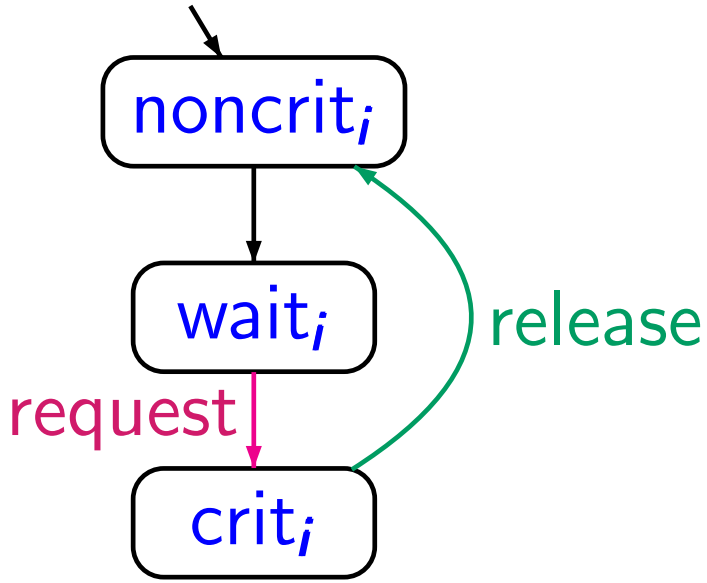
transition system $\mathcal{T}_i$



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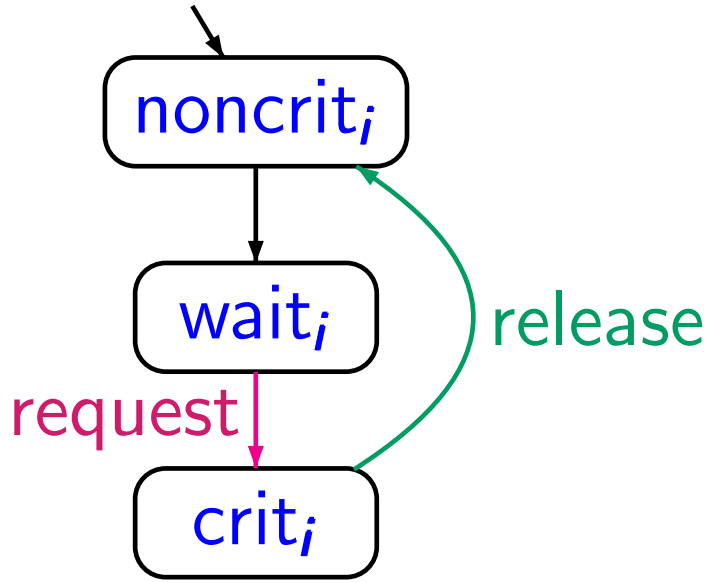
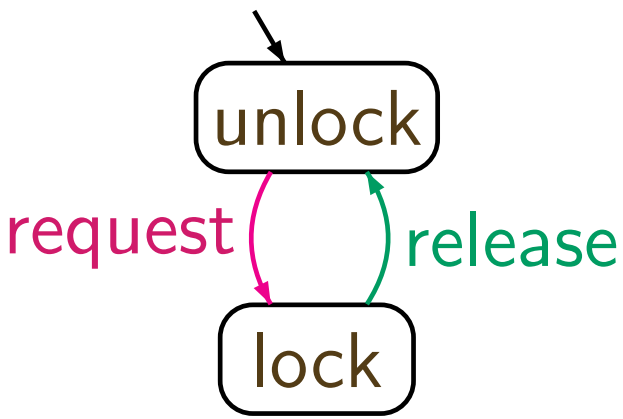


Arbiter:

selects nondeterministically
a synchronization partner
$\mathcal{T}_1$ or $\mathcal{T}_2$



Mutual exclusion with an arbiter

$$(\mathcal{T}_1 \;|||\; \mathcal{T}_2) \;\|_{Syn}\; \mathit{Arbiter} \text{ where } \mathit{Syn} = \{\text{request}, \text{release}\}$$

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"pure" interleaving for TS

handshaking for actions request and release

Mutual exclusion with an arbiter

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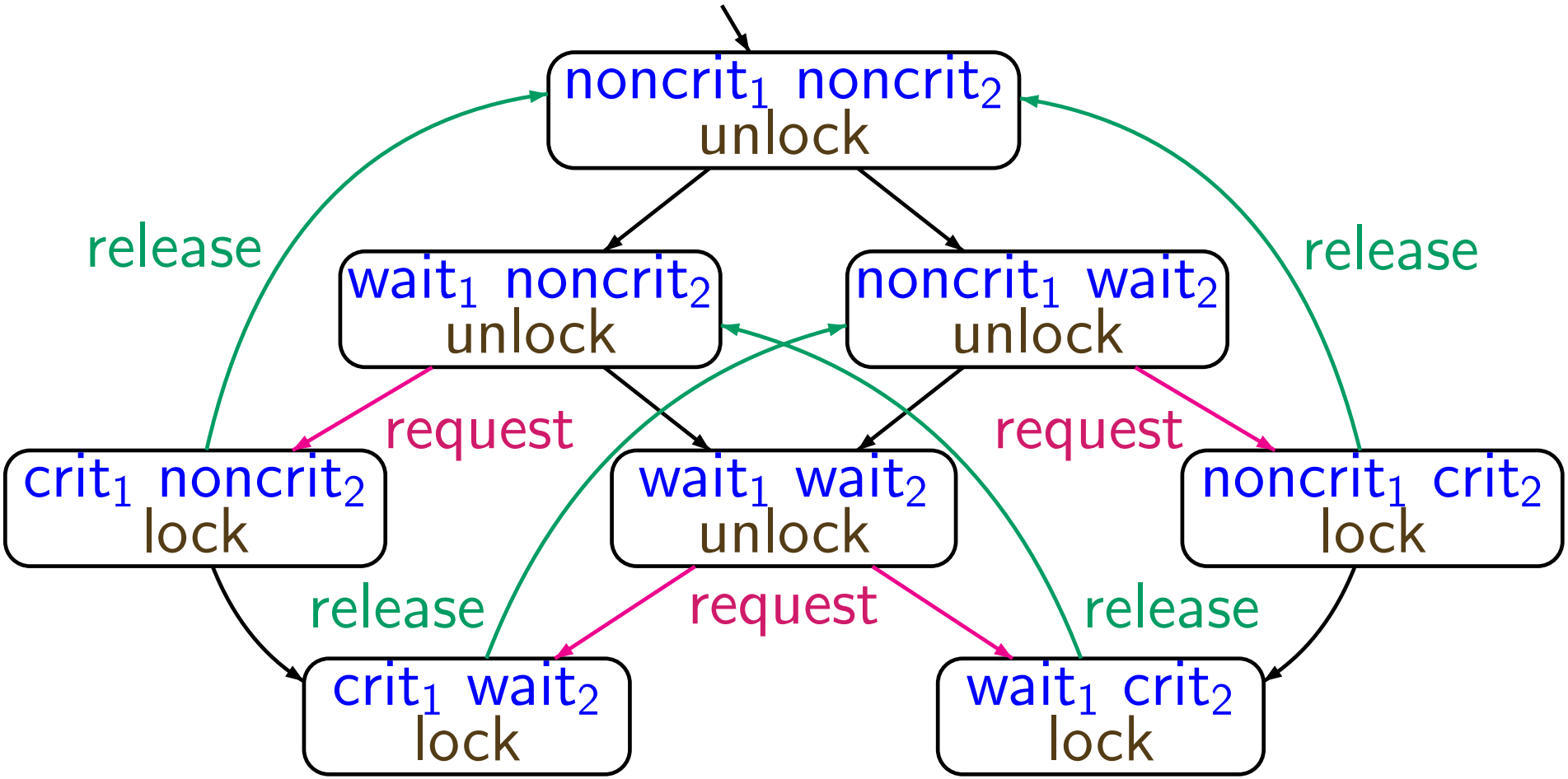

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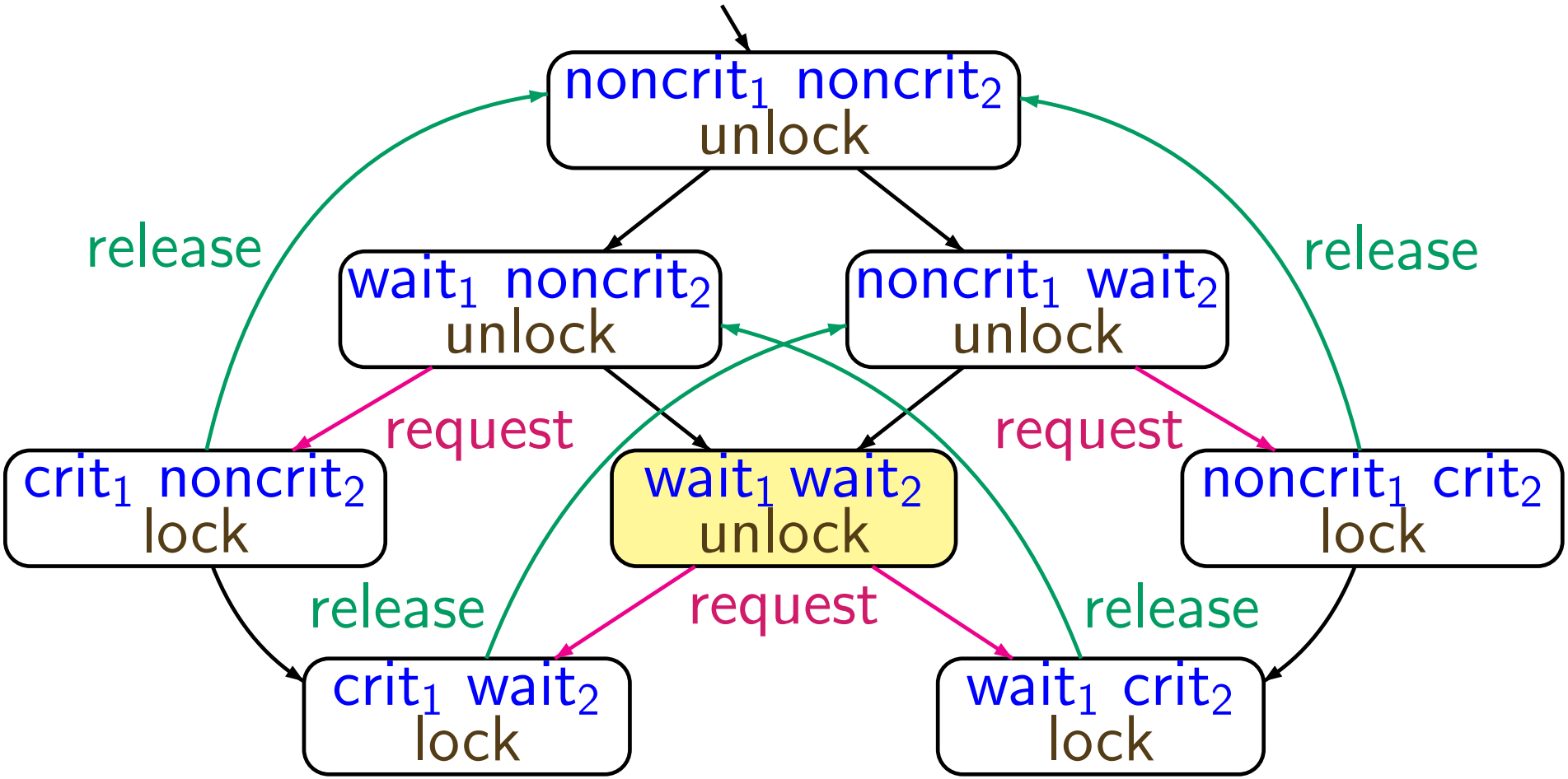

nondeterministic choice: who enters the critical section?

Synchronous message passing

synchronization operator $\|_{Syn}$ for

three or more processes

Synchronous message passing

$$\left.\begin{array}{lcl} \mathcal{T}_1 & = & (S_1, Act_1, \rightarrow_1, \ldots) \\ \mathcal{T}_2 & = & (S_2, Act_2, \rightarrow_2, \ldots) \\ \mathcal{T}_3 & = & (S_3, Act_3, \rightarrow_3, \ldots) \\ \mathcal{T}_4 & = & (S_4, Act_4, \rightarrow_4, \ldots) \\ \vdots & & \vdots \end{array}\right\} \text{transition systems}$$

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for $Syn \subseteq Act_1 \cup Act_2 \cup Act_3 \cup Act_4 \cup \ldots$

$$\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 \parallel_{Syn} \mathcal{T}_3 \parallel_{Syn} \mathcal{T}_4 \parallel_{Syn} \ldots \stackrel{\text{def}}{=}$$

$$\Big(\big((\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2) \parallel_{Syn} \mathcal{T}_3\big) \parallel_{Syn} \mathcal{T}_4\Big) \parallel_{Syn} \ldots$$

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or any other order of paranthesis

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where, e.g., $\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 \stackrel{\text{def}}{=} \mathcal{T}_1 \parallel_H \mathcal{T}_2$

with $H = Syn \cap Act_1 \cap Act_2$

Parallel operator $\|$

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transition systems s.t.

$Act_i \cap Act_j \cap Act_k = \emptyset$

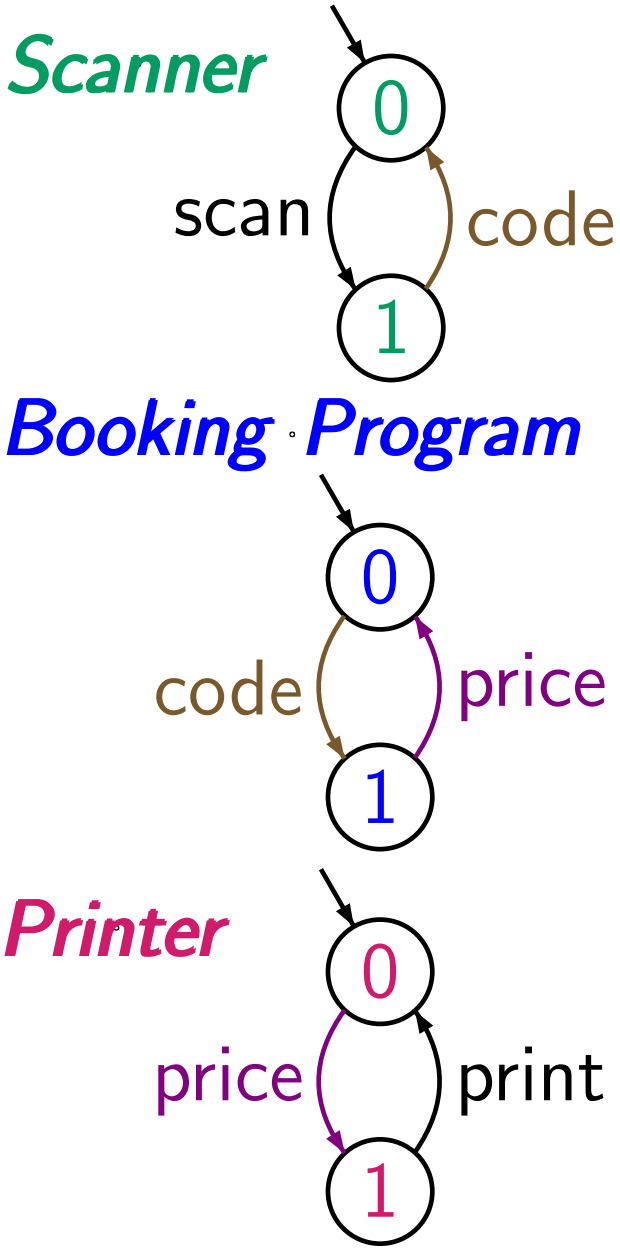
if i, j, k are pairwise distinct

$$\mathcal{T}_1 \| \mathcal{T}_2 \| \mathcal{T}_3 \| \mathcal{T}_4 \| \ldots \stackrel{\text{def}}{=} (((\mathcal{T}_1 \|_{Syn_{1,2}} \mathcal{T}_2) \|_{Syn_{1,2,3}} \mathcal{T}_3) \|_{Syn_{1,2,3,4}} \mathcal{T}_4) \ldots$$

where

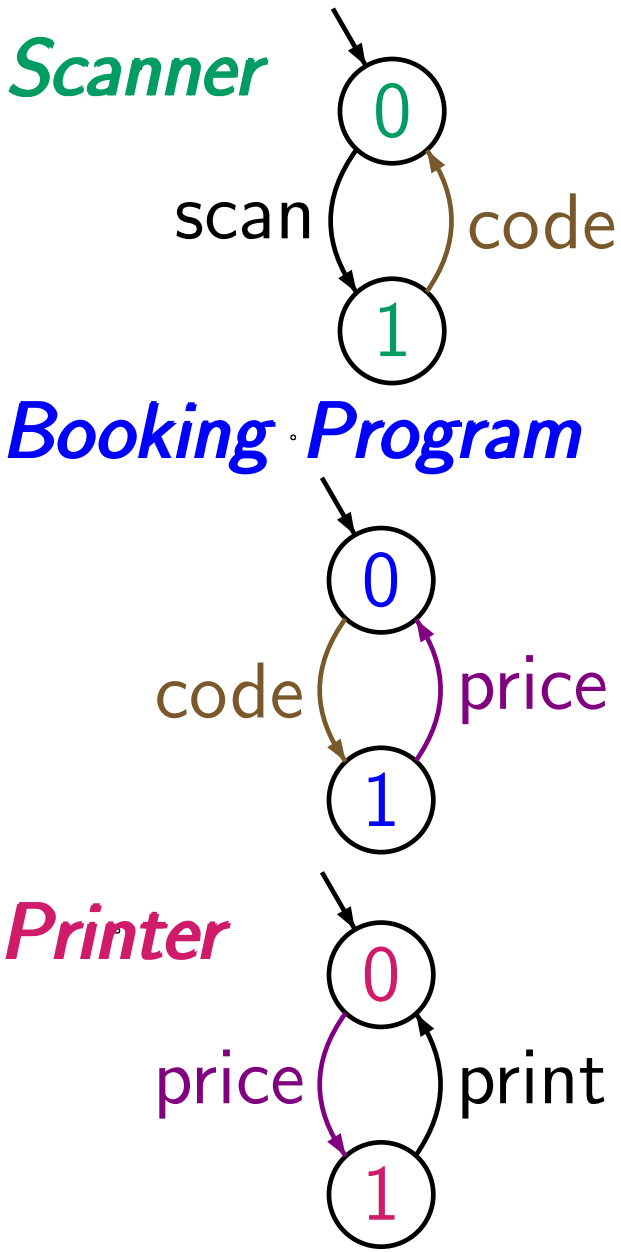
$$\begin{array}{lcl} Syn_{1,2} & = & Act_1 \cap Act_2 \\ Syn_{1,2,3} & = & (Act_1 \cup Act_2) \cap Act_3 \\ Syn_{1,2,3,4} & = & (Act_1 \cup Act_2 \cup Act_3) \cap Act_4 \\ \vdots & & \vdots \end{array}$$

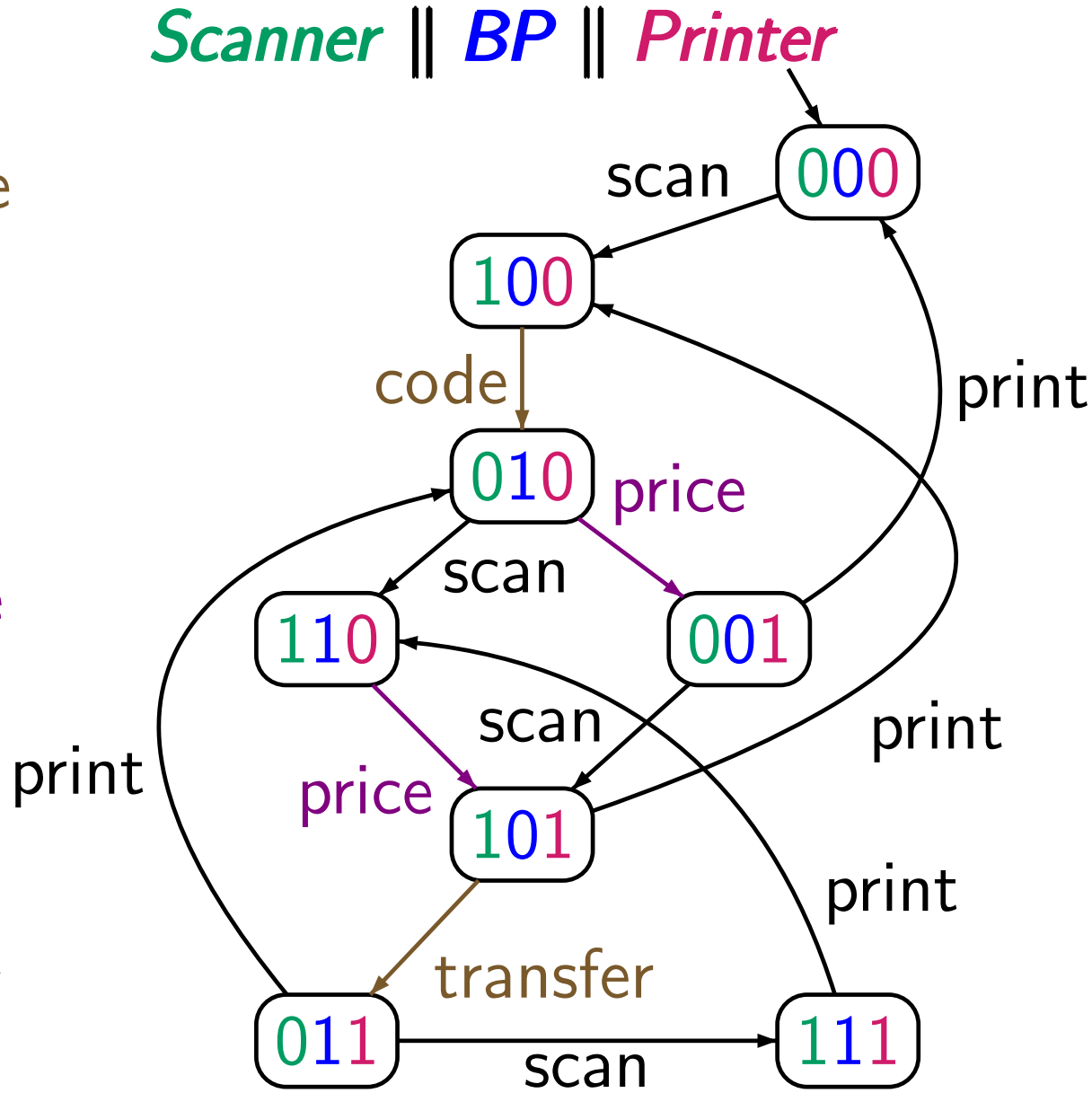
Booking system in supermarket



$Scanner \parallel BP \parallel Printer$

Booking system in supermarket





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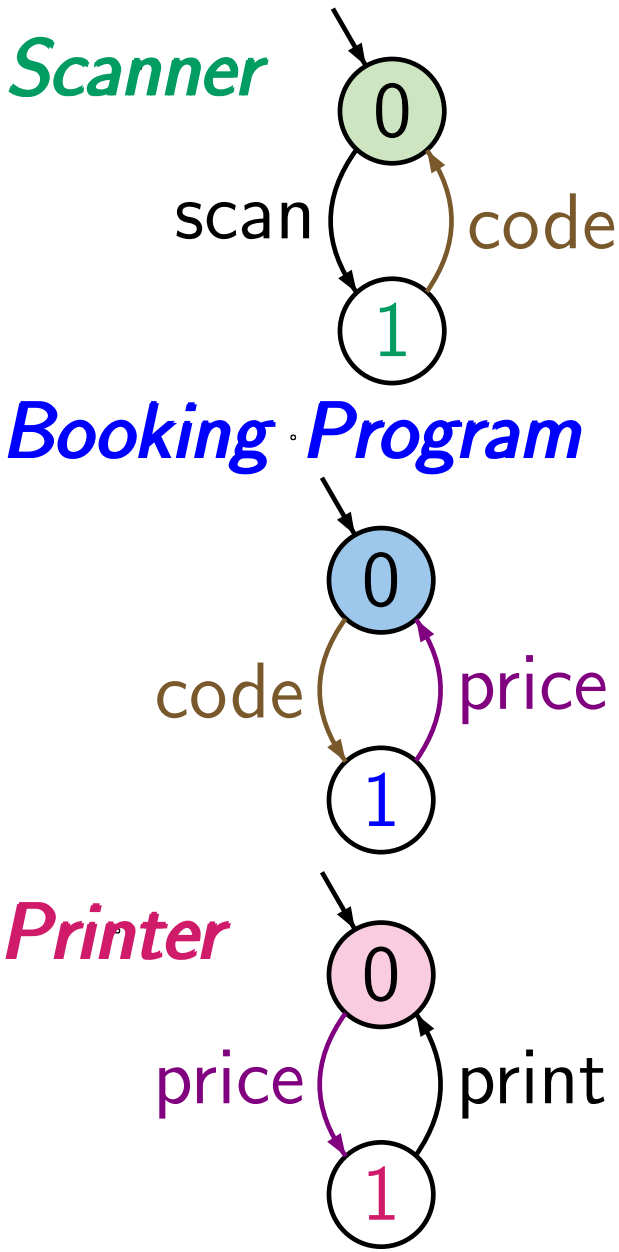

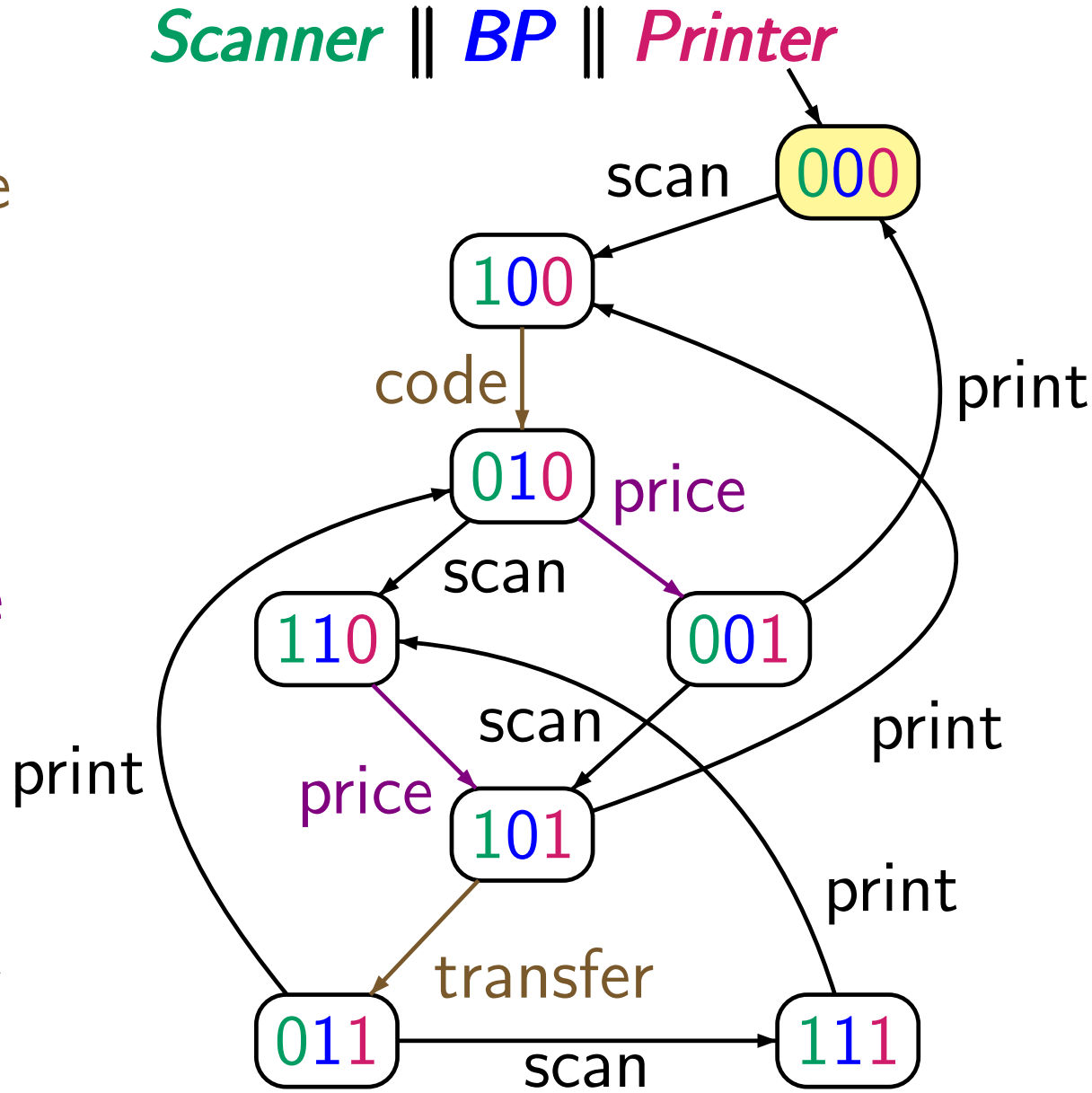

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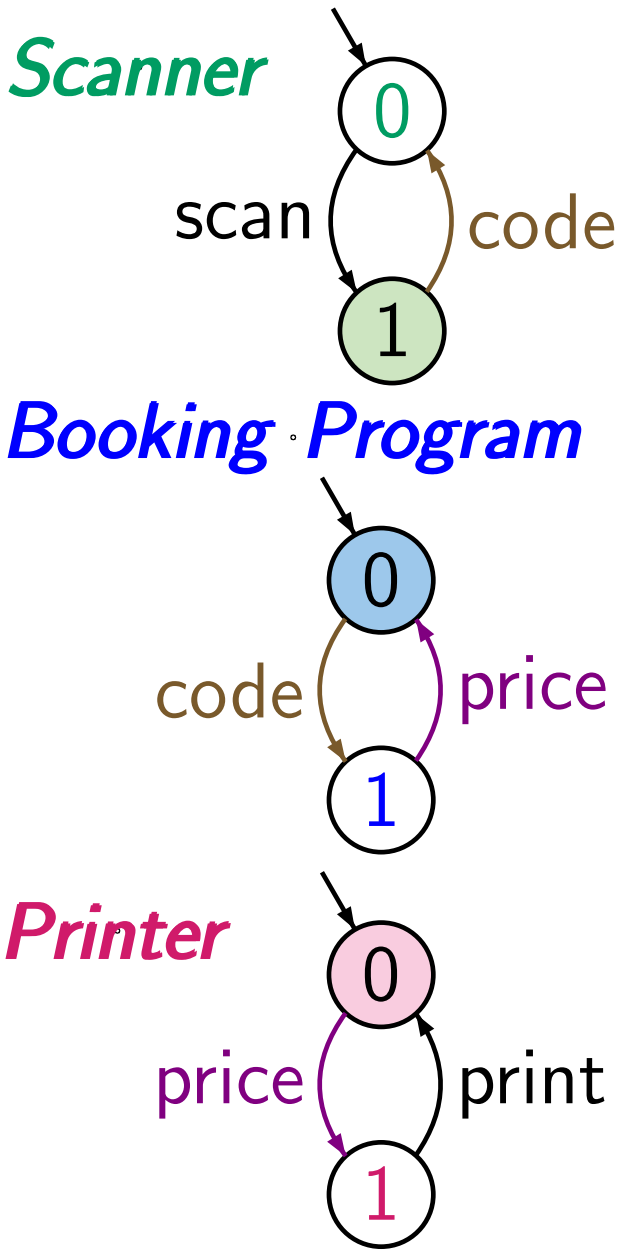

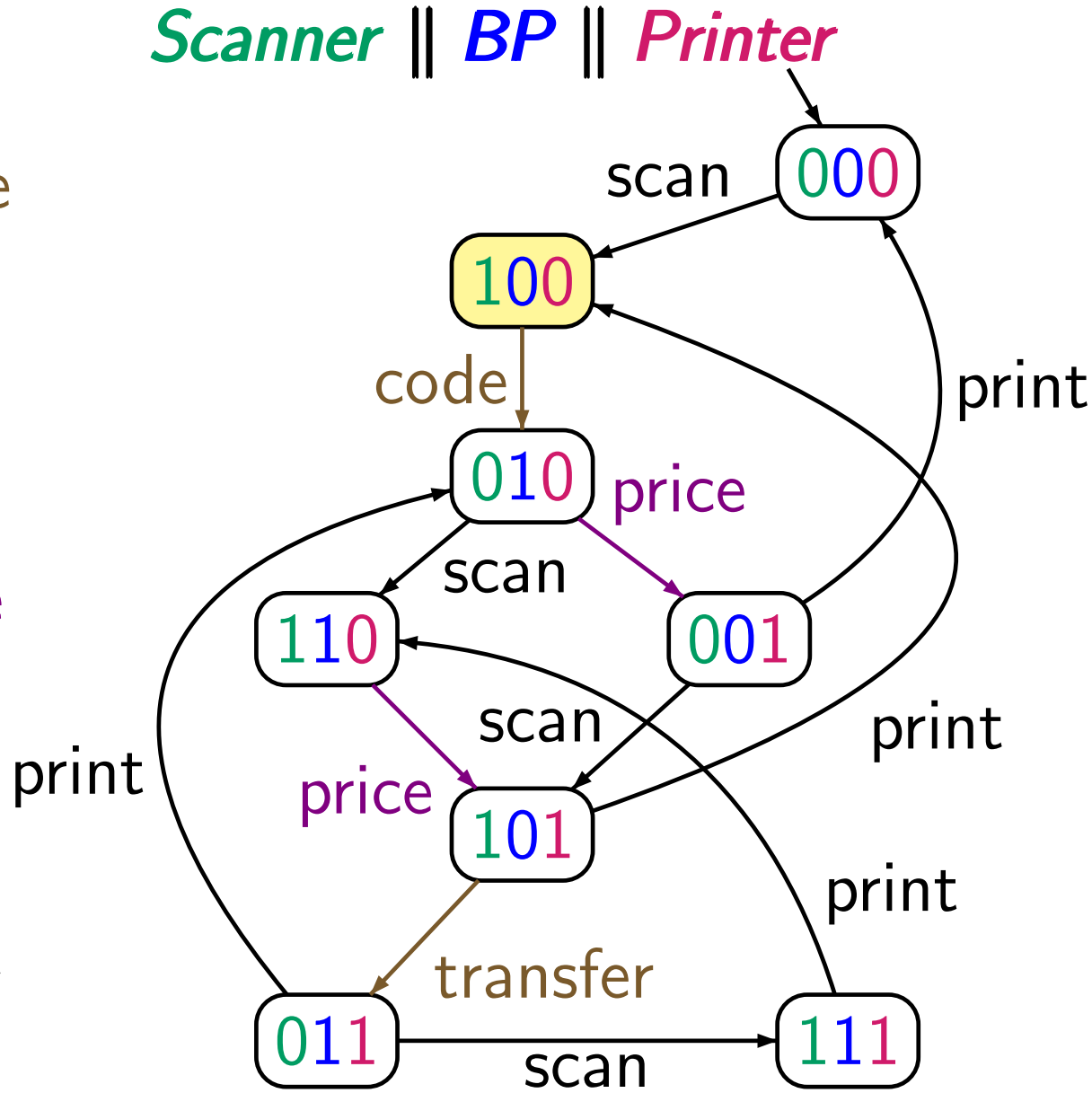

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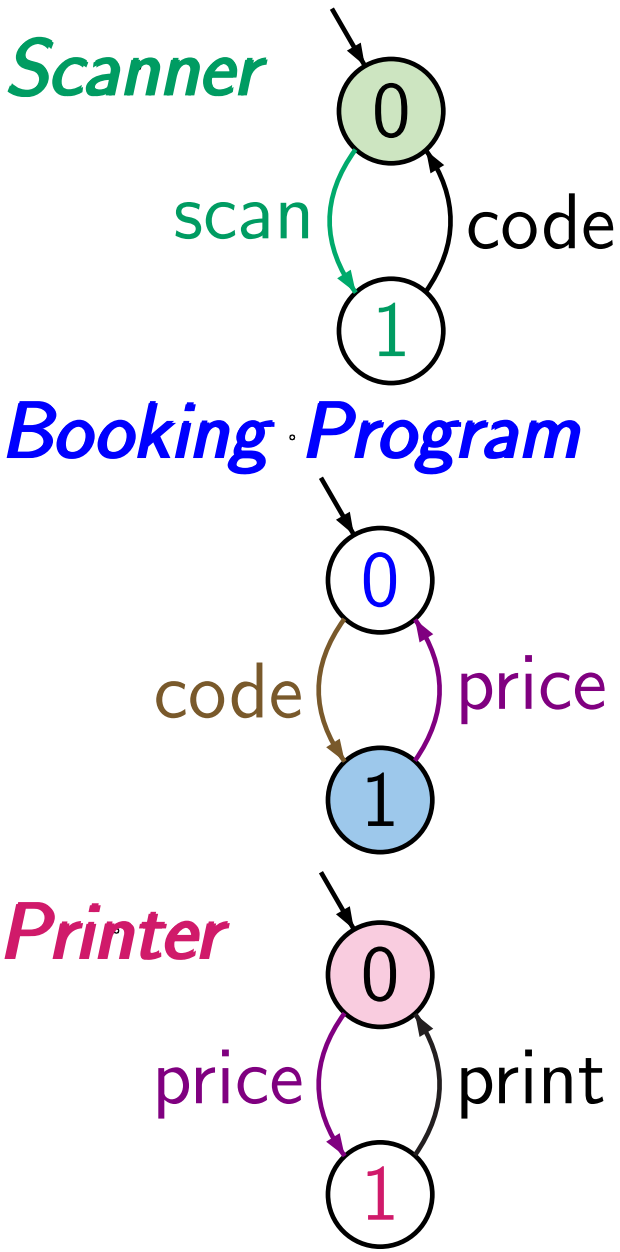
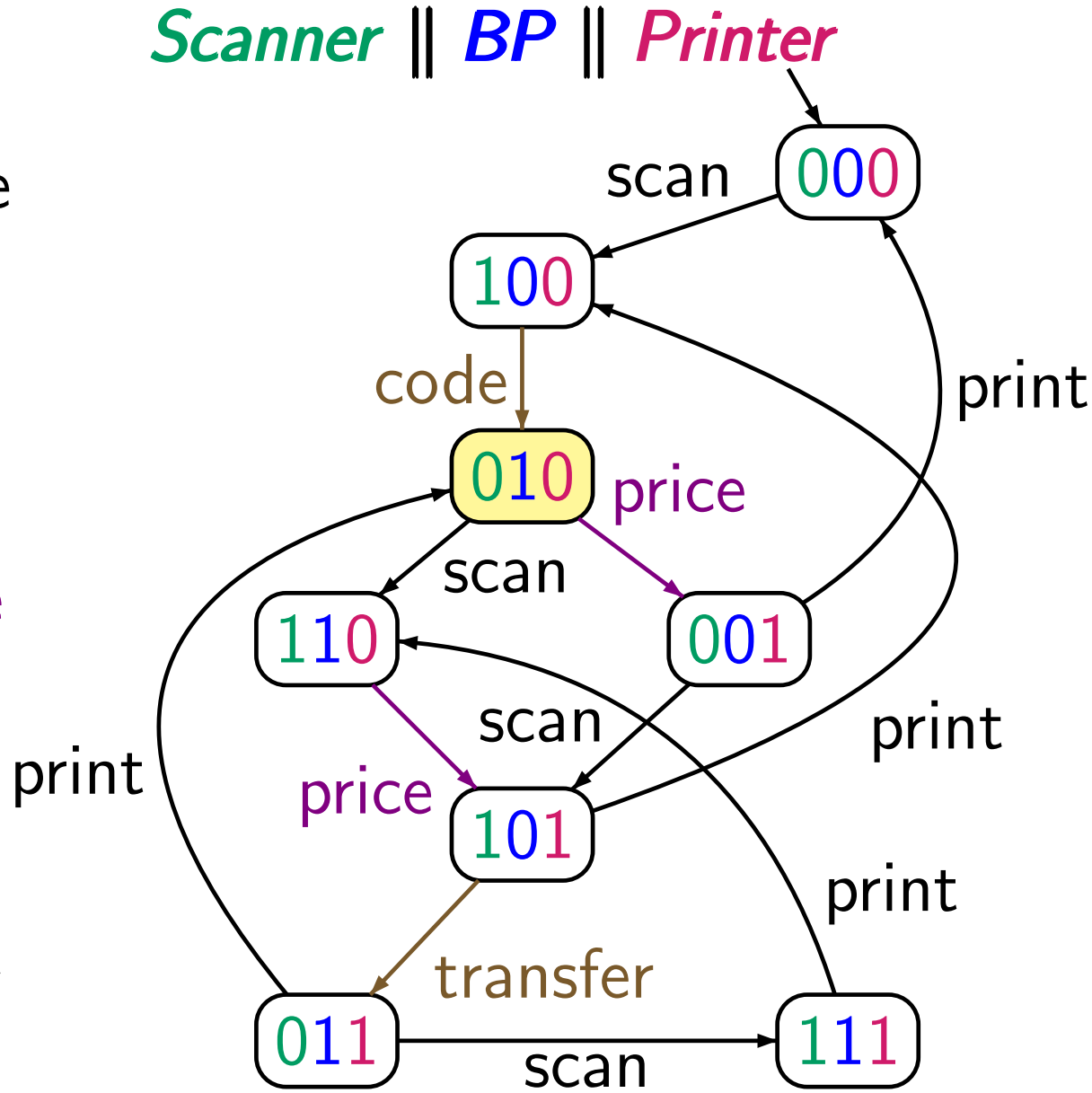
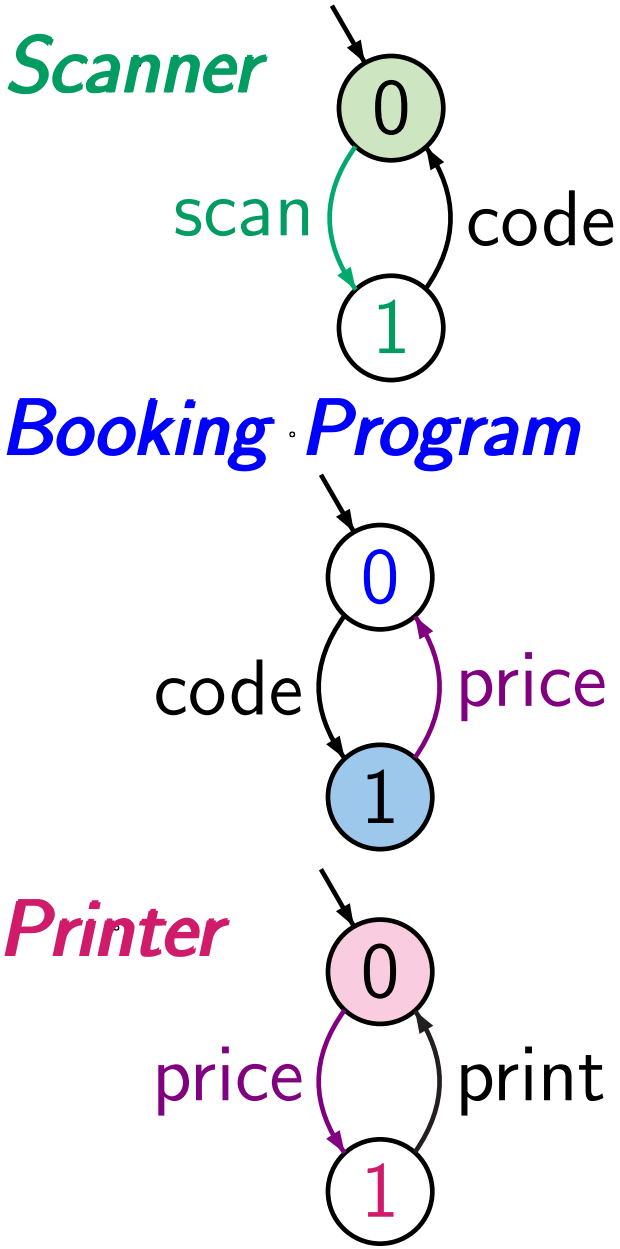
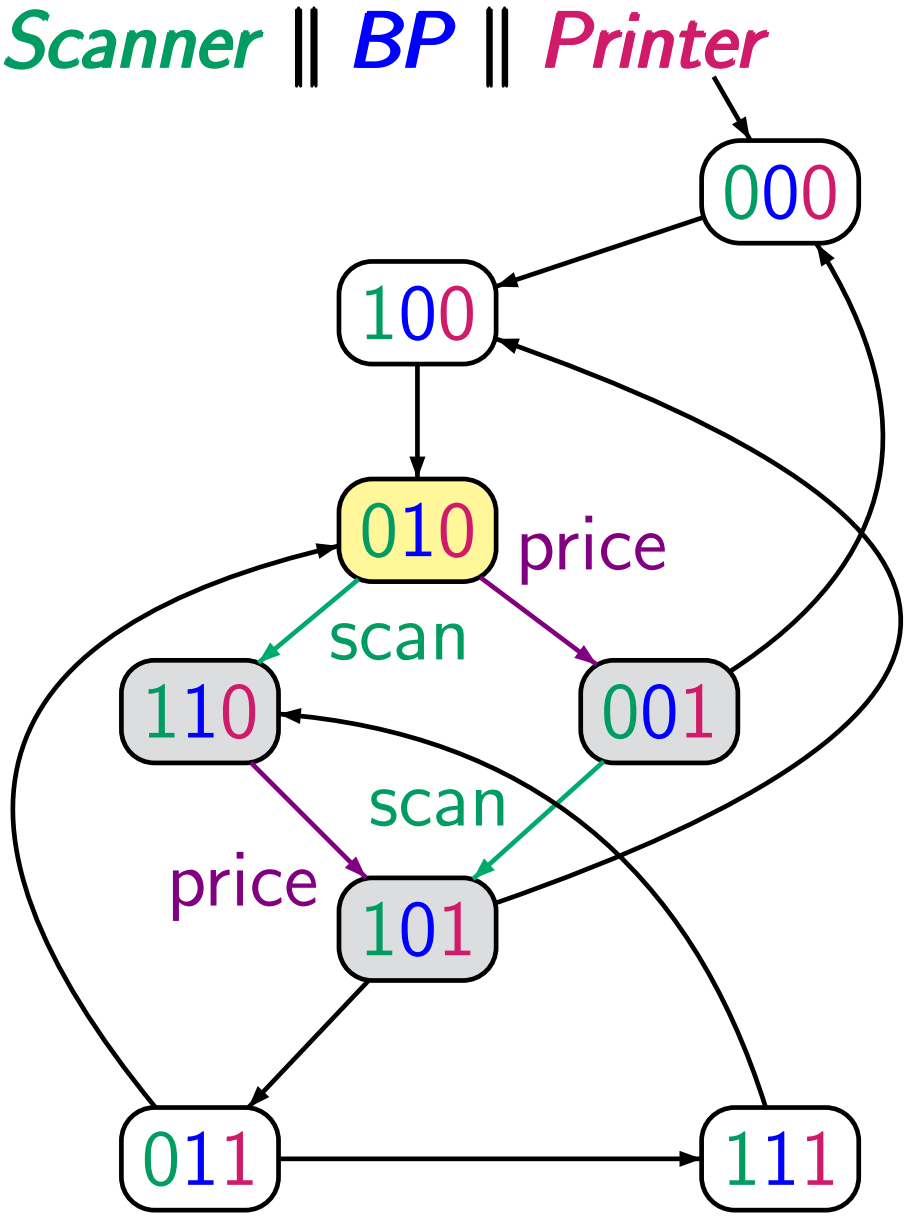




Interleaving





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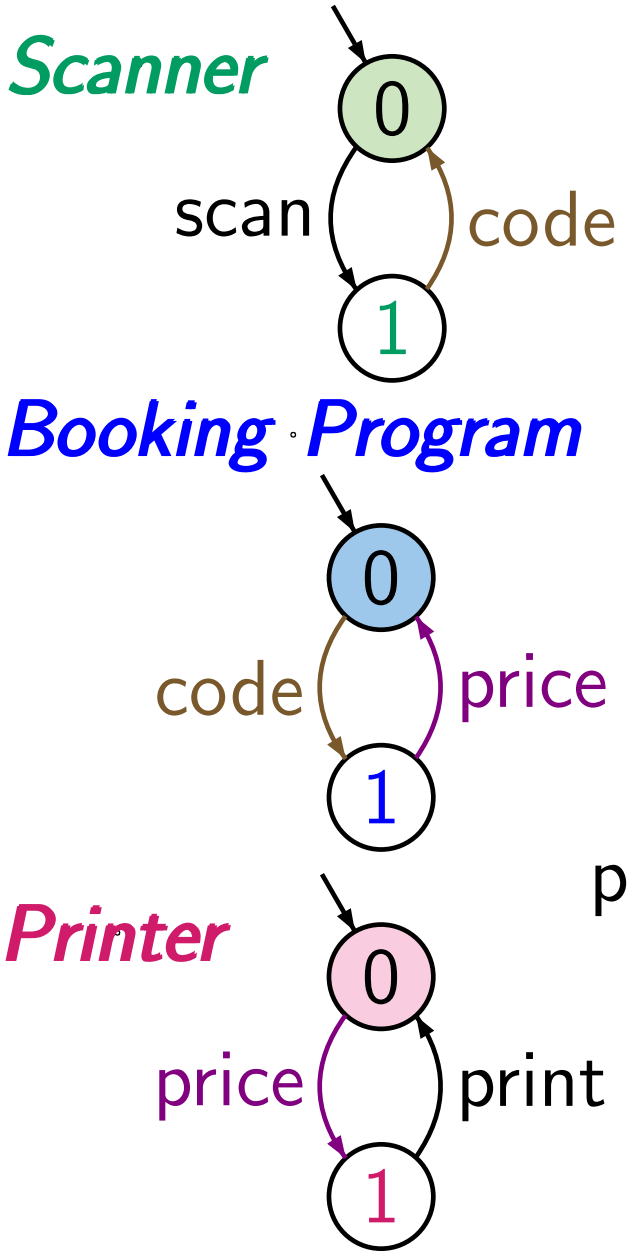

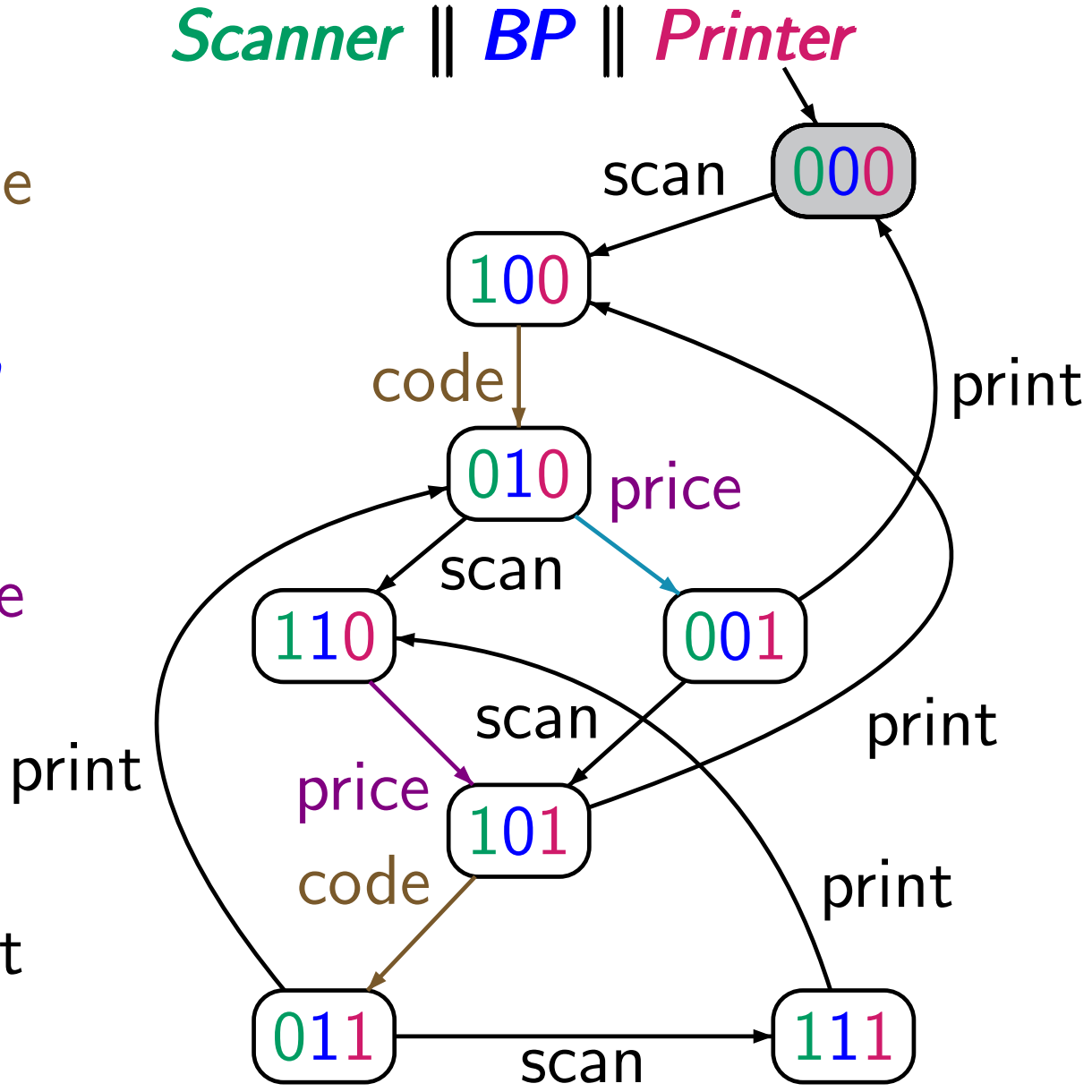

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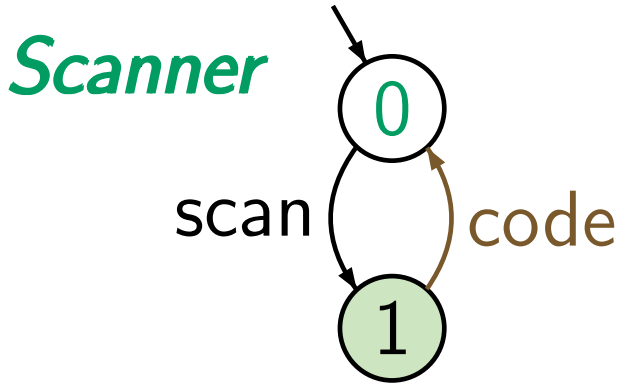

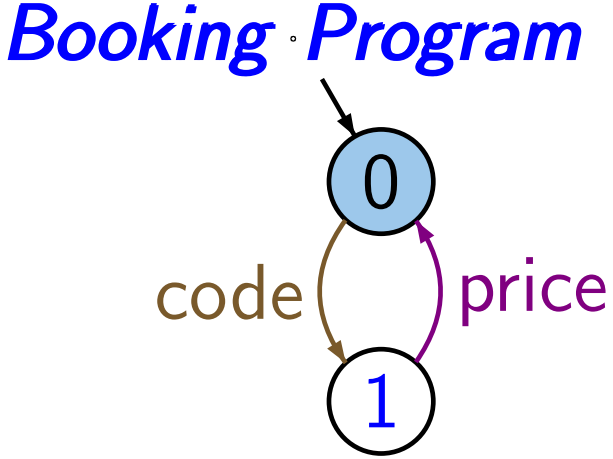

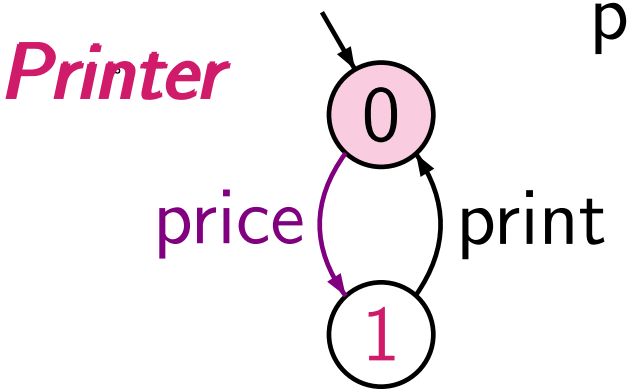

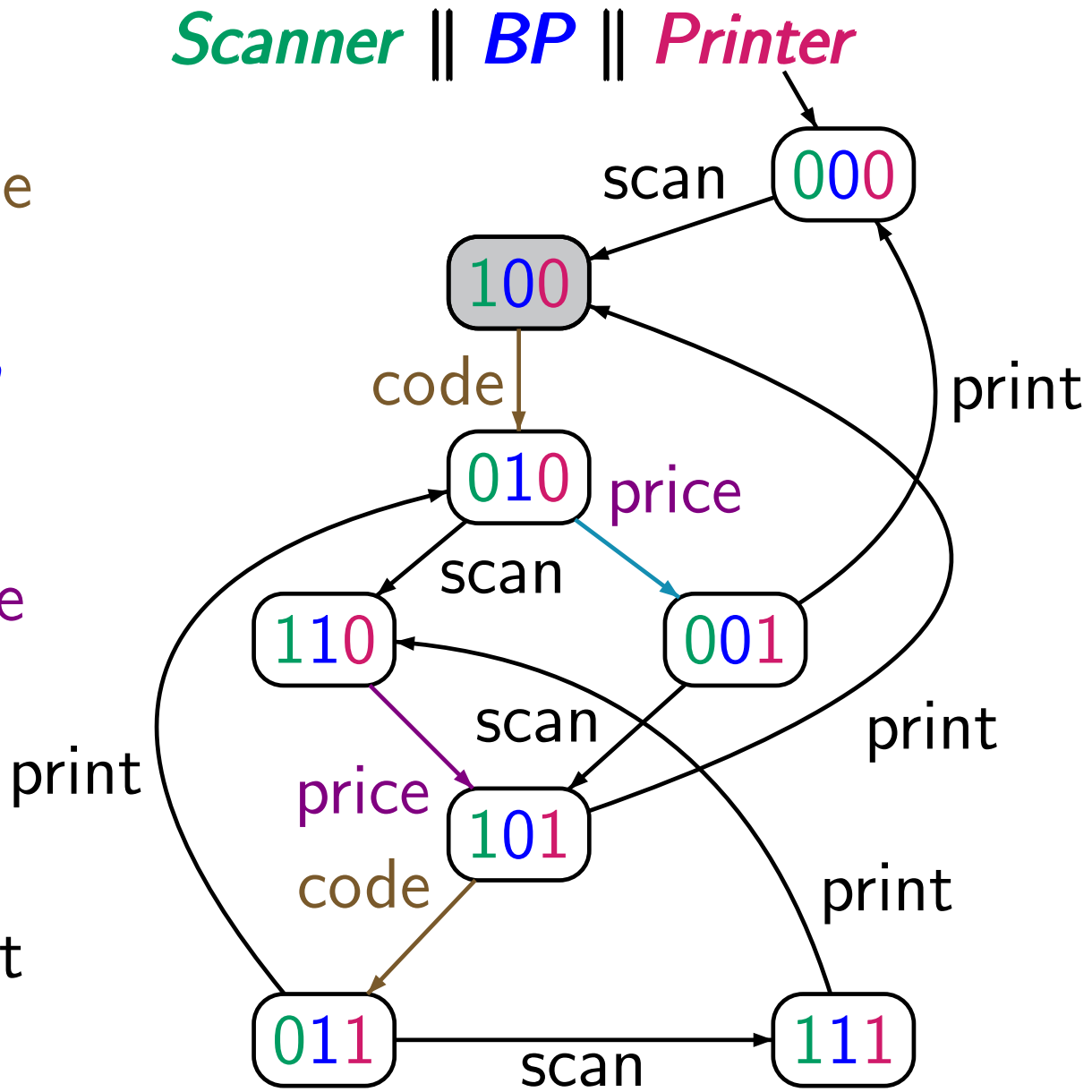

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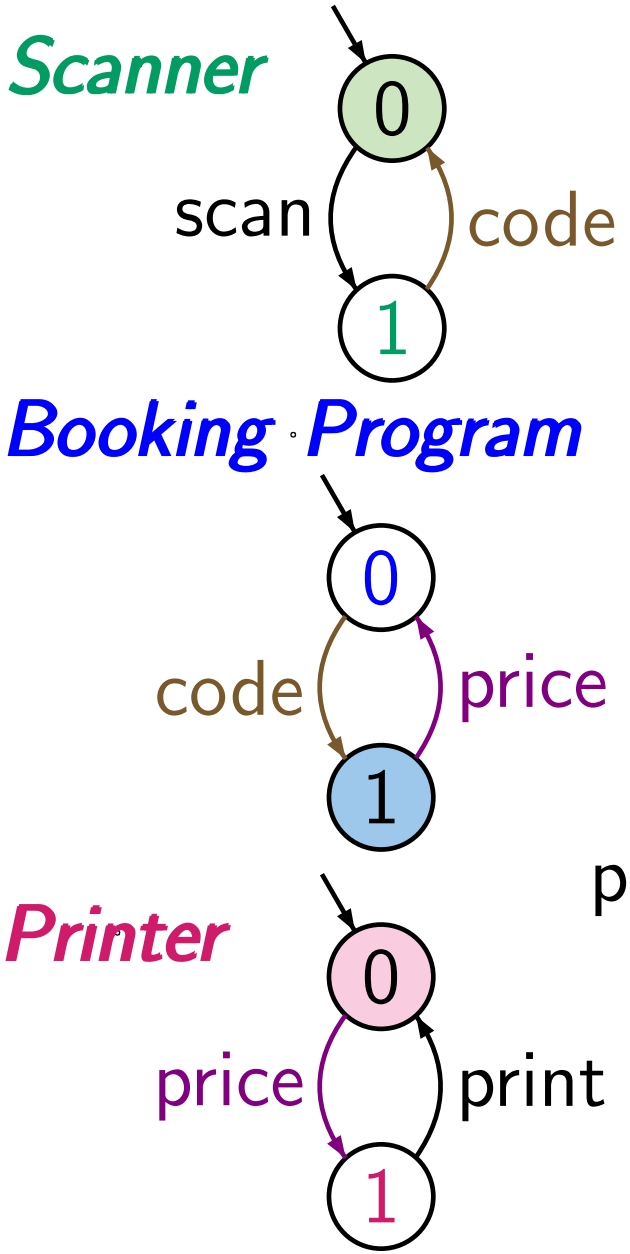

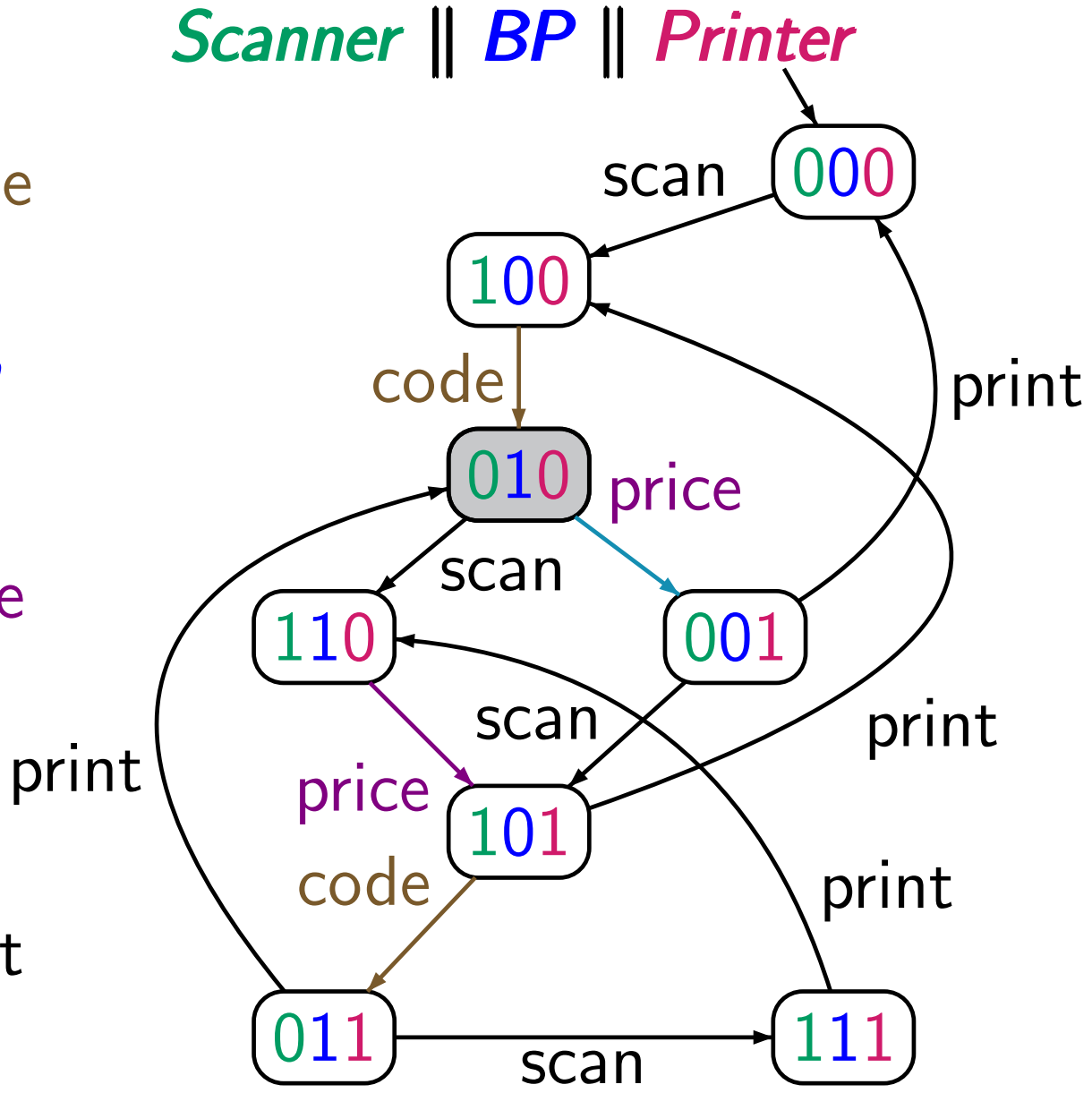

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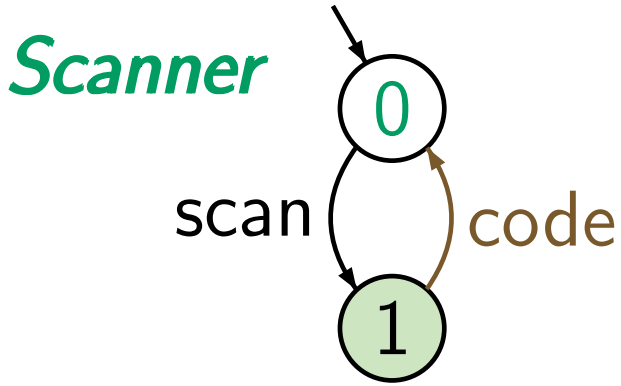

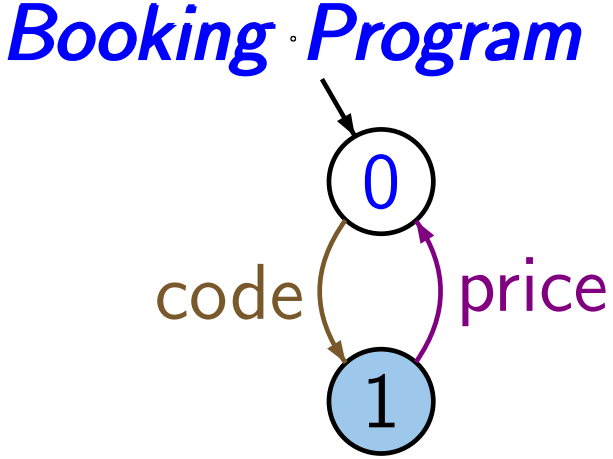

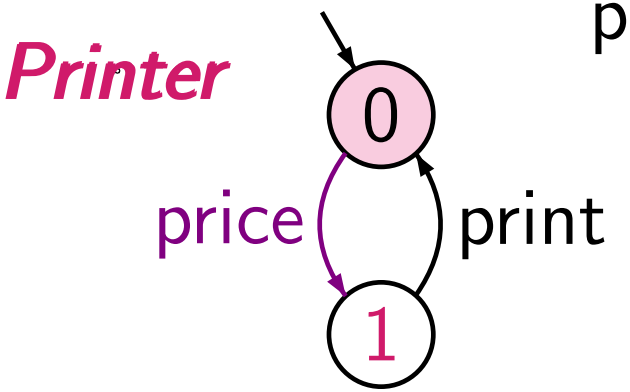

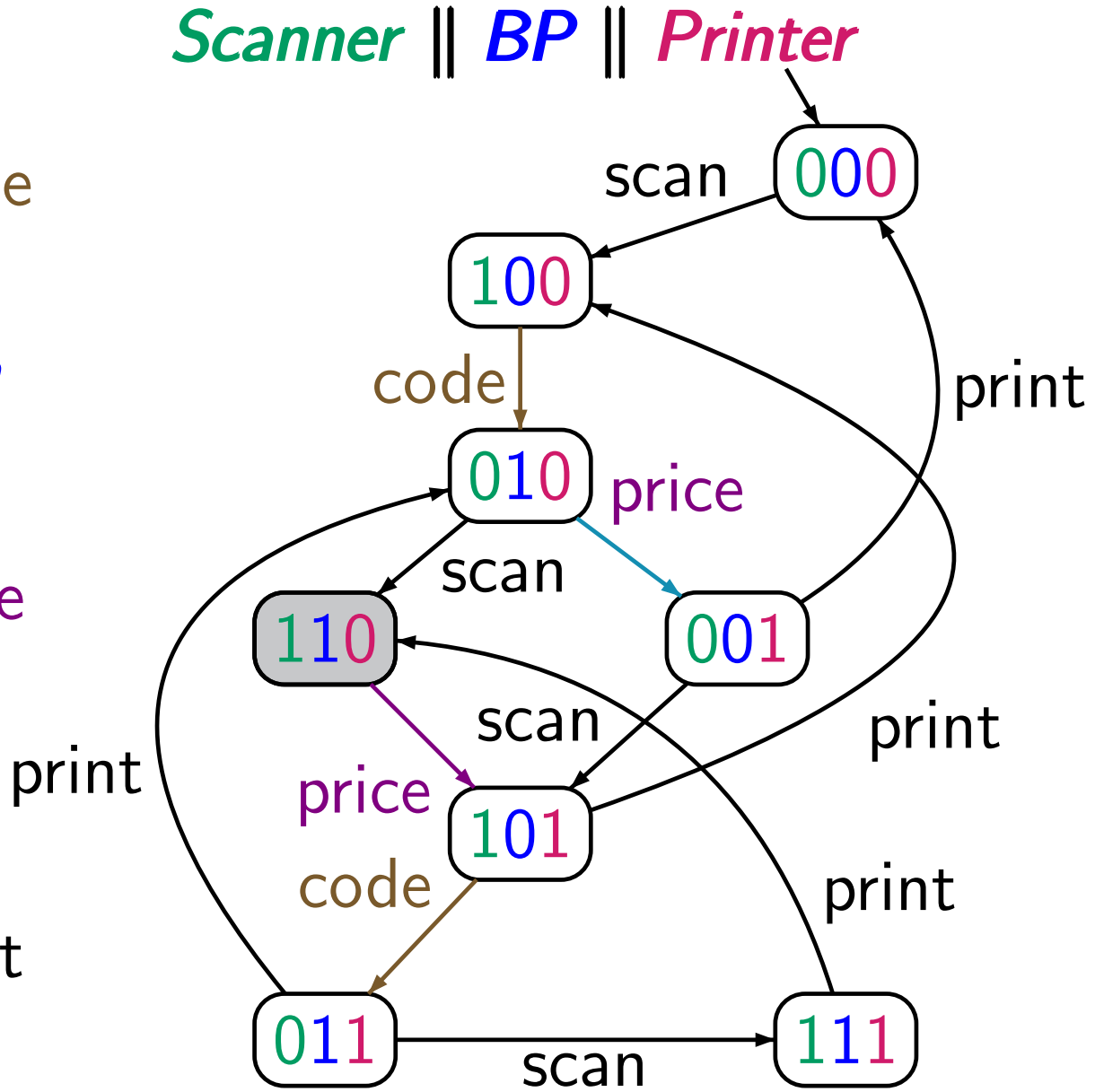

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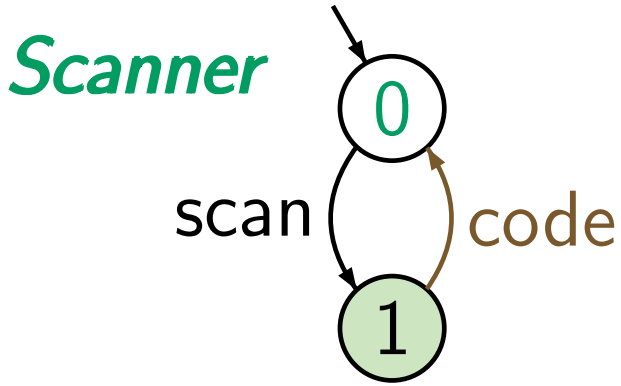

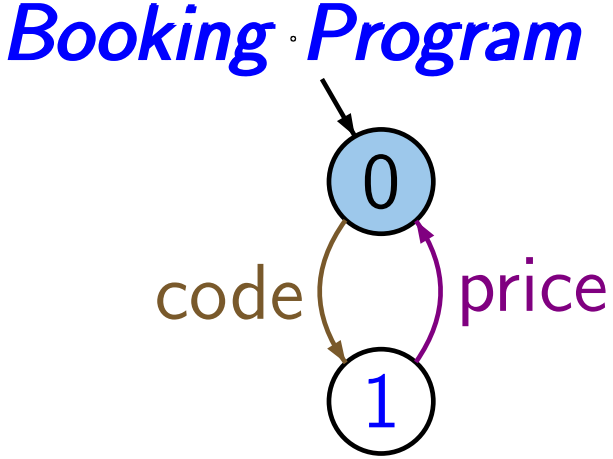

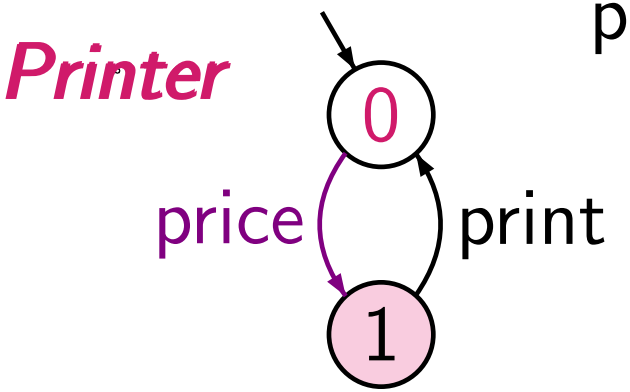

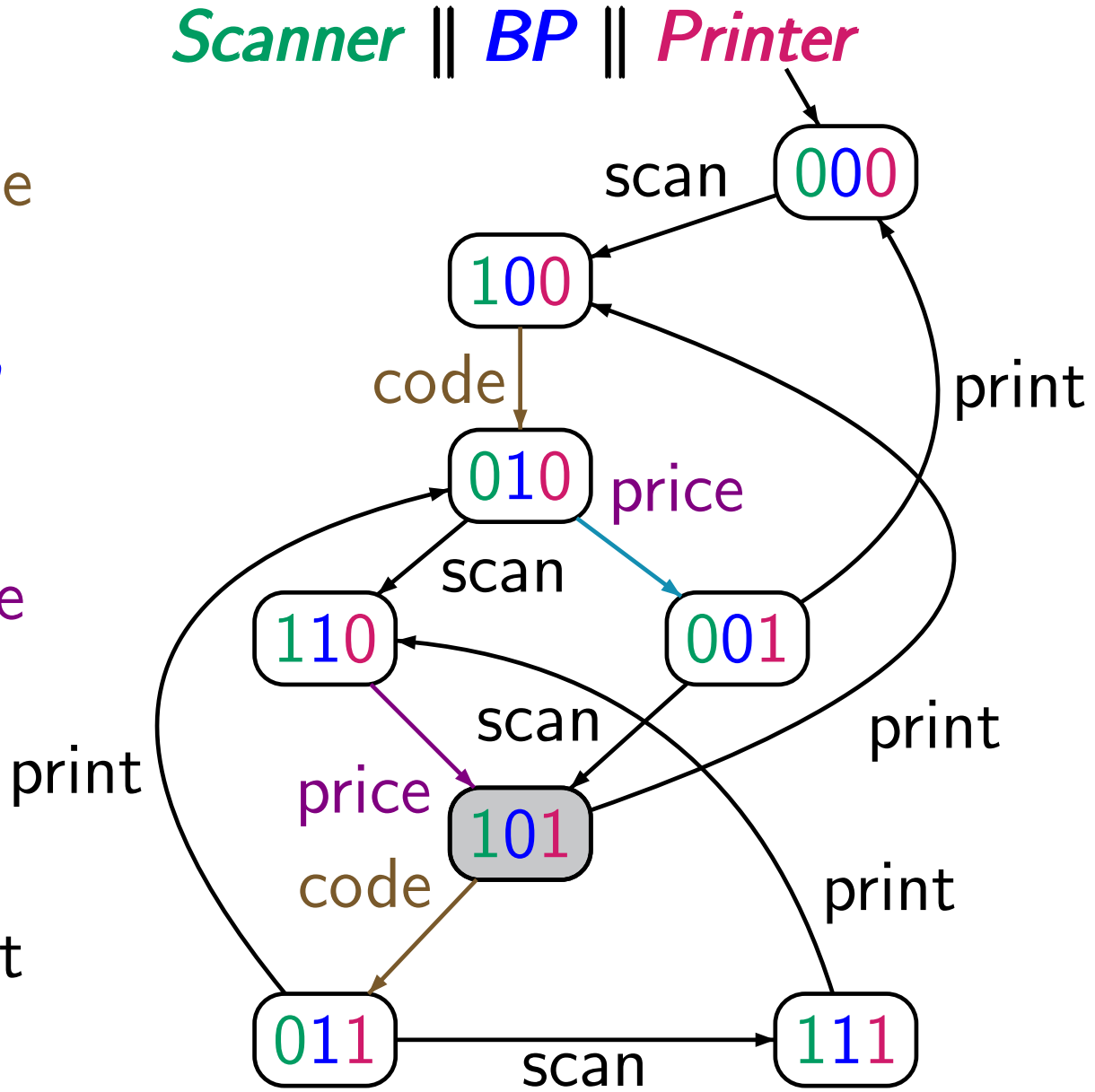

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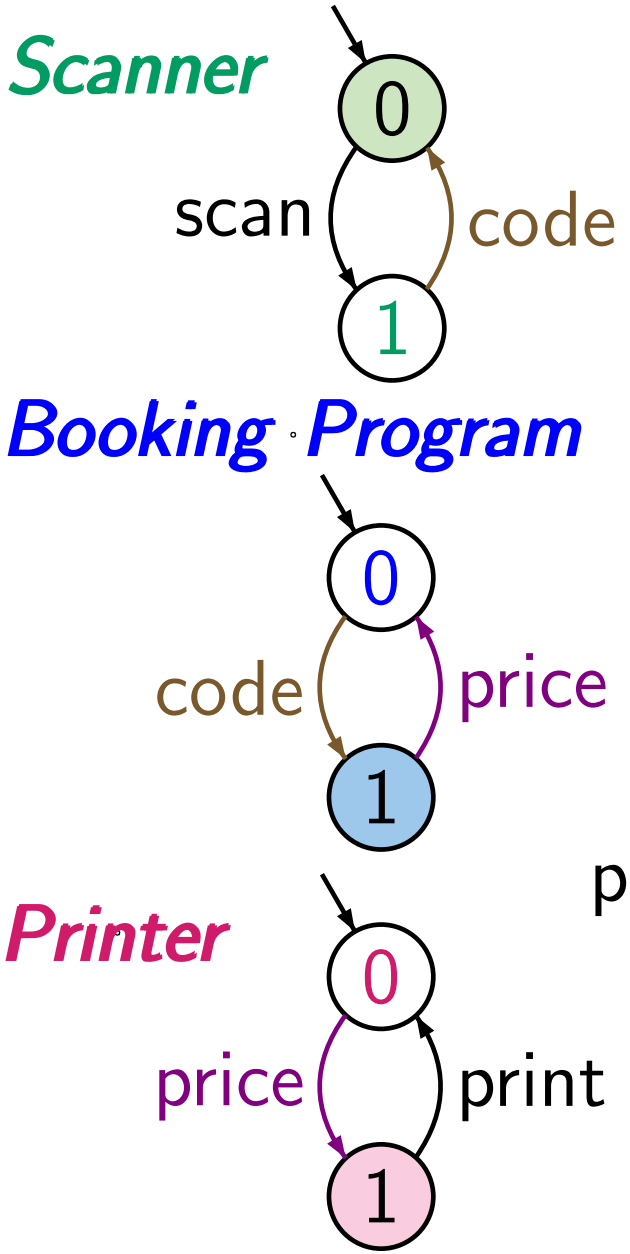

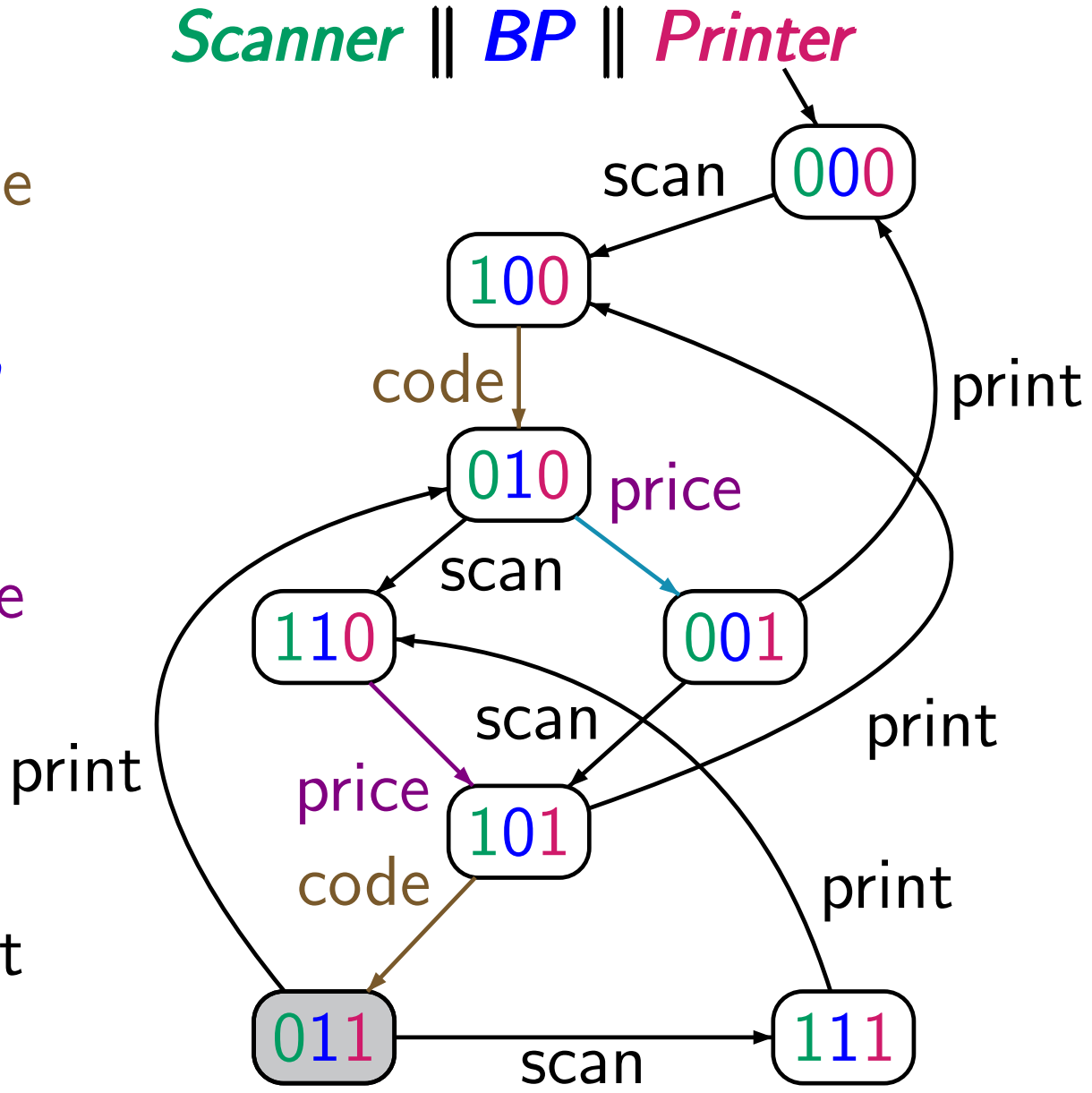

Interleaving