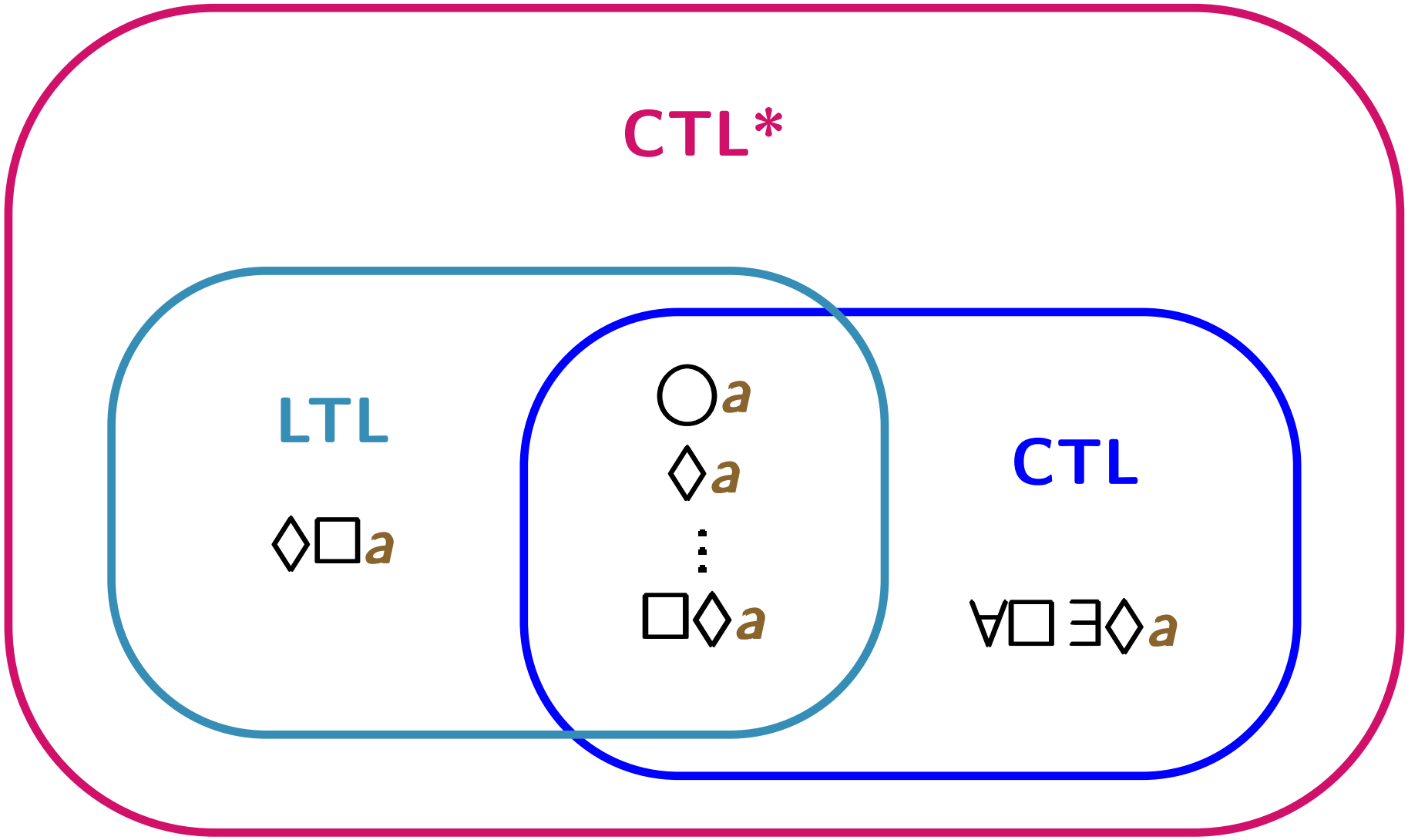
Overview

Combining LTL and CTL $\rightsquigarrow$ CTL*



Syntax of CTL*

state formulas:

$$\Phi \;::=\; \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

path formulas:

$$\varphi \;::=\; \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

derived operators:

- $\vee$, $\rightarrow$, etc.
- eventually, always as in **LTL**:

  $$\Diamond\varphi = \mathit{true}\,\mathsf{U}\,\varphi, \quad \Box\varphi = \neg\Diamond\neg\varphi$$

- universal quantification: $\forall\varphi = \neg\exists\neg\varphi$

Semantics of CTL*

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system without terminal states.

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define by structural induction:

- a satisfaction relation $\models$ for states $s \in S$ and **CTL*** state formulas
- a satisfaction relation $\models$ for infinite path fragments π in $\mathcal{T}$ and **CTL*** path formulas

Semantics of CTL* state formulas

CTLST4.6-2A

Semantics of CTL* state formulas

$s \models true$

$s \models a$ iff $a \in L(s)$

$s \models \neg\Phi$ iff $s \not\models \Phi$

$s \models \Phi_1 \wedge \Phi_2$ iff $s \models \Phi_1$ and $s \models \Phi_2$

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satisfaction relation $\models$ for **CTL*** path formulas

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let $\pi = s_0\, s_1\, s_2 \ldots$ be an infinite path fragment in $\mathcal{T}$

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$\pi \models \bigcirc\varphi$ iff $\mathit{suffix}(\pi, 1) \models \varphi$

$\pi \models \varphi_1 \,\mathsf{U}\, \varphi_2$ iff there exists $j \geq 0$ such that

$\mathit{suffix}(\pi, j) \models \varphi_2$

$\mathit{suffix}(\pi, i) \models \varphi_1$ for $0 \leq i < j$

Semantics of CTL* path formulas

let $\pi = s_0\, s_1\, s_2 \ldots$ be an infinite path fragment in $\mathcal{T}$

$$
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Examples of CTL*-formulas

mutual exclusion:

safety $\forall\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

liveness $\forall\Box\Diamond \mathit{crit}_1 \wedge \forall\Box\Diamond \mathit{crit}_2$

progress property, e.g., $\forall\Box(\mathit{request} \rightarrow \Diamond \mathit{response})$

persistence property, e.g., $\forall\Diamond\Box a$

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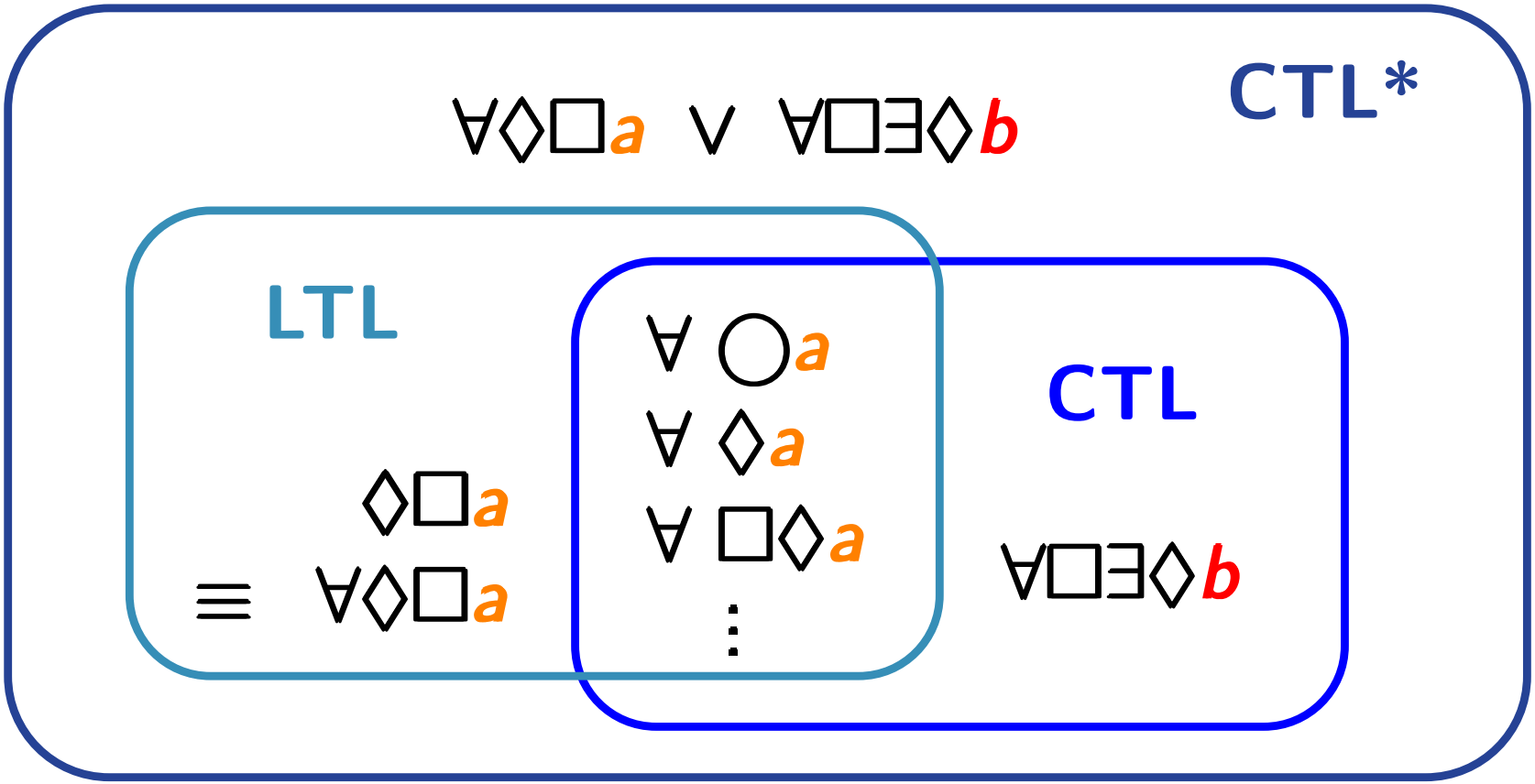
CTL* formulas with existential quantification, e.g.,
Hamilton path problem (for fixed initial state)

$$\exists\Big(\bigwedge_{v\in V}\big(\Diamond v \wedge \Box(v \rightarrow \bigcirc\Box\neg v)\big)\Big)$$

Expressiveness of CTL, LTL and CTL*

- **CTL** is a sublogic of **CTL***
- **LTL** is a sublogic of **CTL***
- **CTL*** is more expressive than **LTL** and **CTL**



Equivalence of CTL*-formulas

$\Phi_1 \equiv \Phi_2$ iff for all transition systems $\mathcal{T}$:

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Examples:

$$\neg\exists\Box\Diamond a \equiv \forall\Diamond\Box\neg a$$

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correct. Both formulas assert that an a-state is reachable from the current state within one or more steps.

Combinations of □ and ◊ in CTL*

Combinations of $\Box$ and $\Diamond$ in CTL*

we already saw:

$$\forall\Box\forall\Diamond a \equiv \forall\Box\Diamond a$$

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answer: **no**

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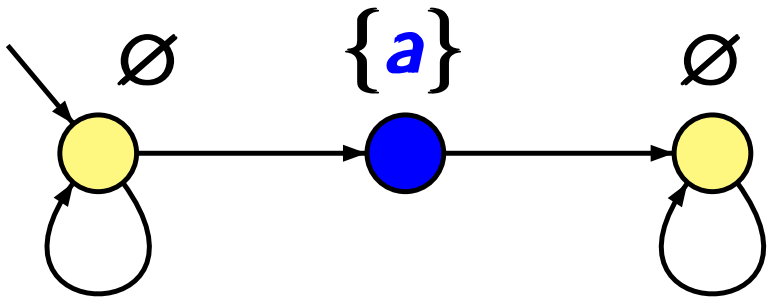

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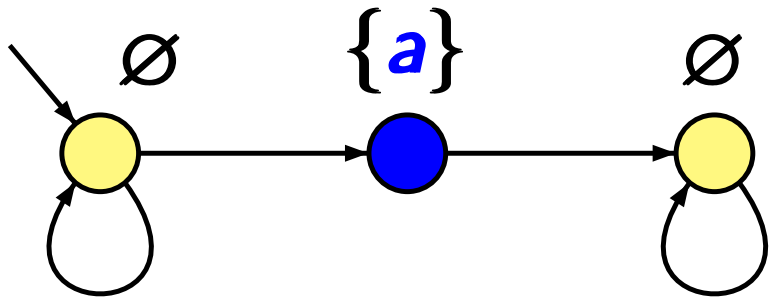
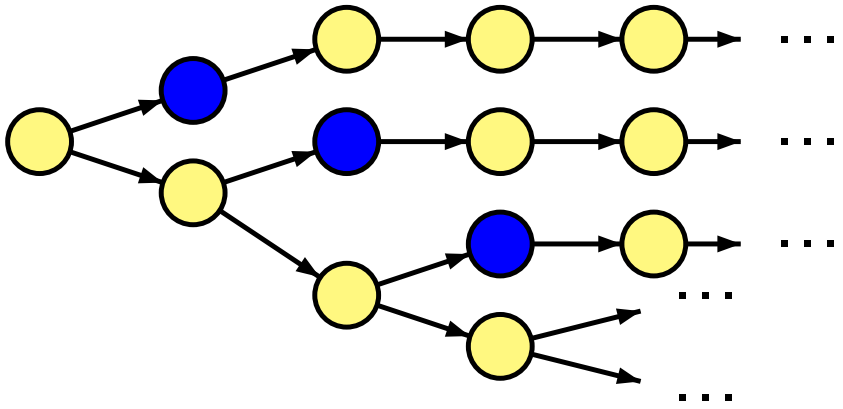


computation tree:



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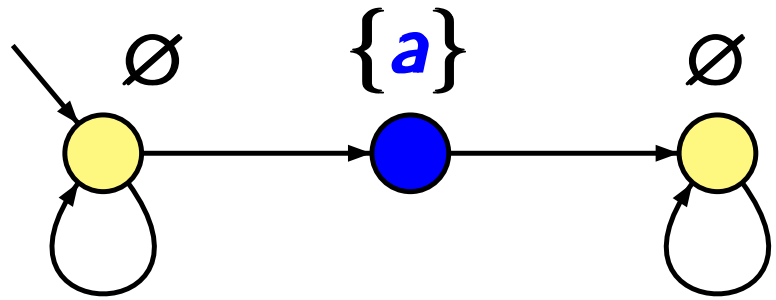

$\mathcal{T} \not\models \exists\Box\Diamond a$

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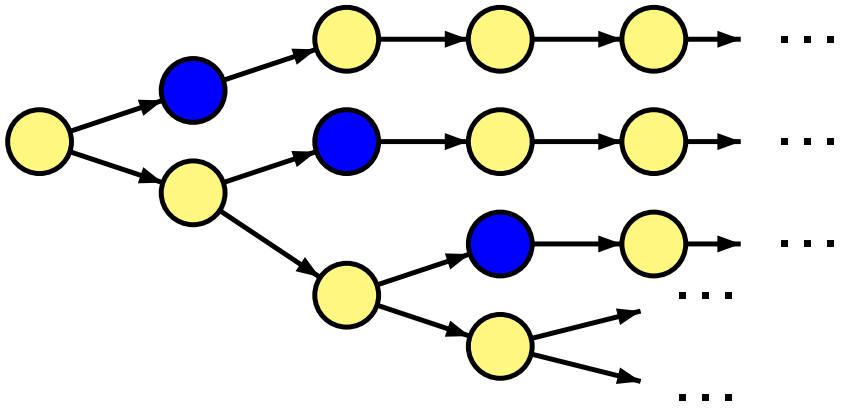

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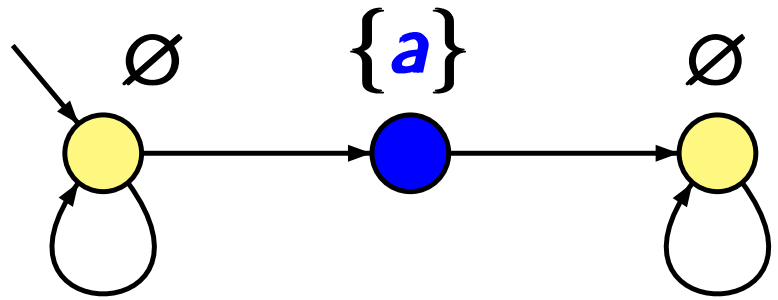

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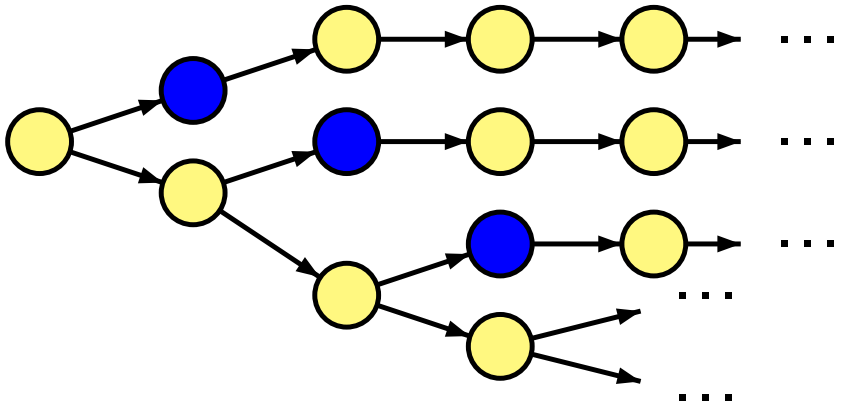

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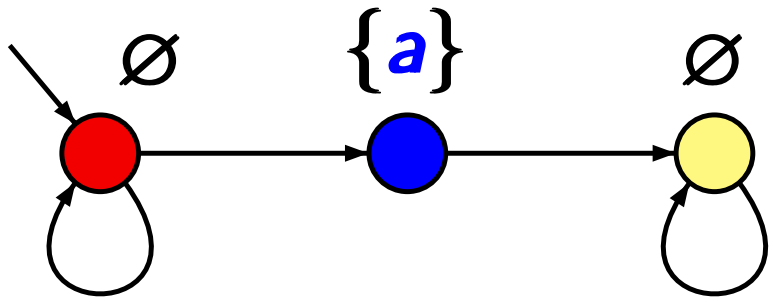

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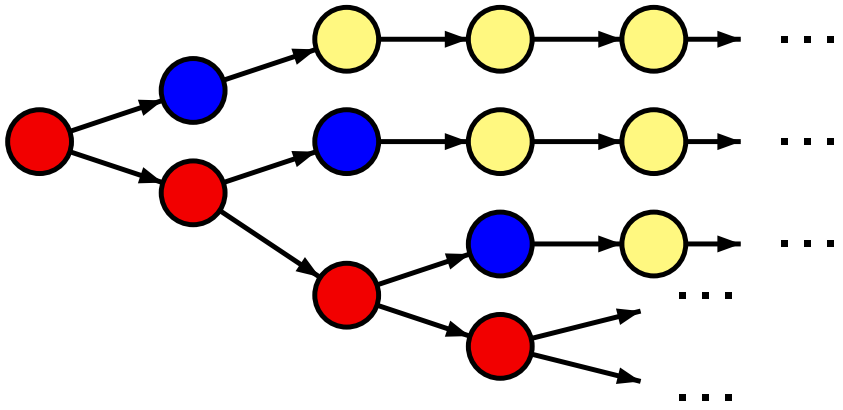

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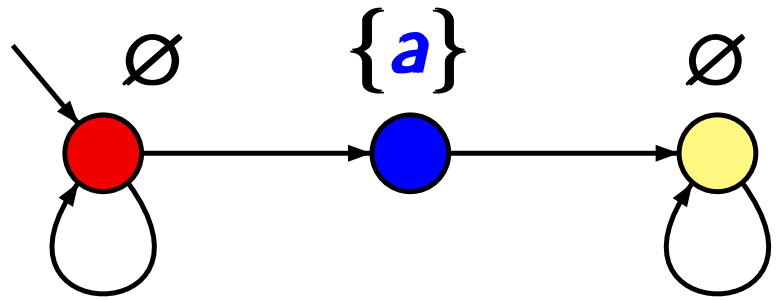

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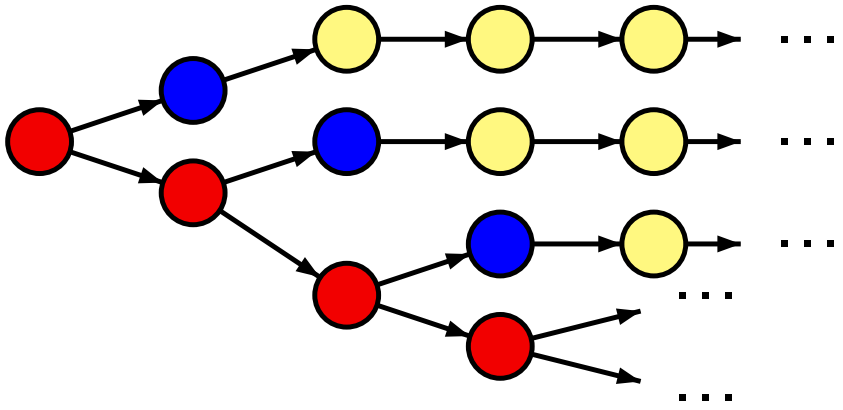

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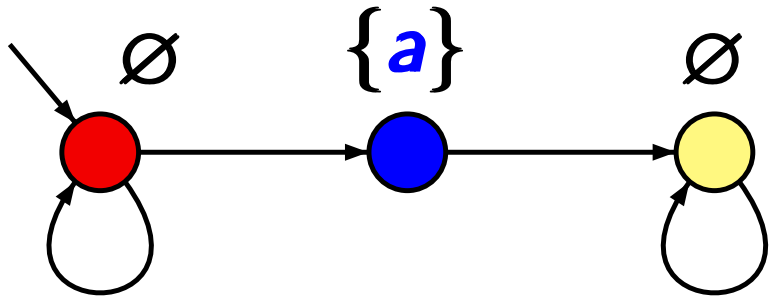

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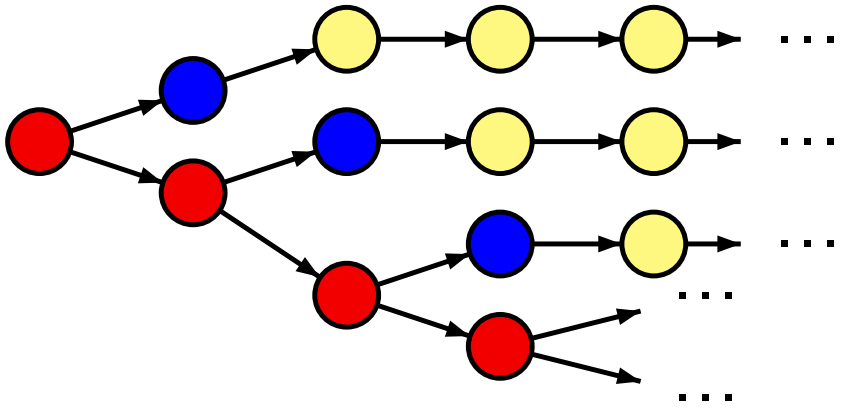

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$\forall\Box\Diamond\varphi \equiv \forall\Box\forall\Diamond\varphi$ but: $\forall\Diamond\Box\varphi \not\equiv \forall\Diamond\forall\Box\varphi$

$\exists\Diamond\Box\varphi \equiv \exists\Diamond\exists\Box\varphi$ $\exists\Box\Diamond\varphi \not\equiv \exists\Box\exists\Diamond\varphi$