Overview

Complexity of CTL and LTL model checking

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LTL model checking problem:

PSPACE-complete and solvable in time

$$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(|\varphi|))$$

CTL model checking problem:

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Recall: LTL fairness assumptions

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are conjunctions of **LTL** formulas of the form

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\psi \rightarrow \Box\Diamond\phi$
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note: CTL fairness assumptions

- are not CTL (state or path) formulas
- just a syntactic formalism to specify fairness assumptions

CTL fairness assumptions

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e.g., a strong CTL fairness assumption has the form:

$$\mathit{fair} = \bigwedge_{1 \le j \le k} (\Box\Diamond\Psi_j \rightarrow \Box\Diamond\Phi_j)$$

where Ψ_j, Φ_j are CTL state formulas

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$s \models_{fair} \Phi_1 \wedge \Phi_2$ iff $s \models_{fair} \Phi_1$ and $s \models_{fair} \Phi_2$

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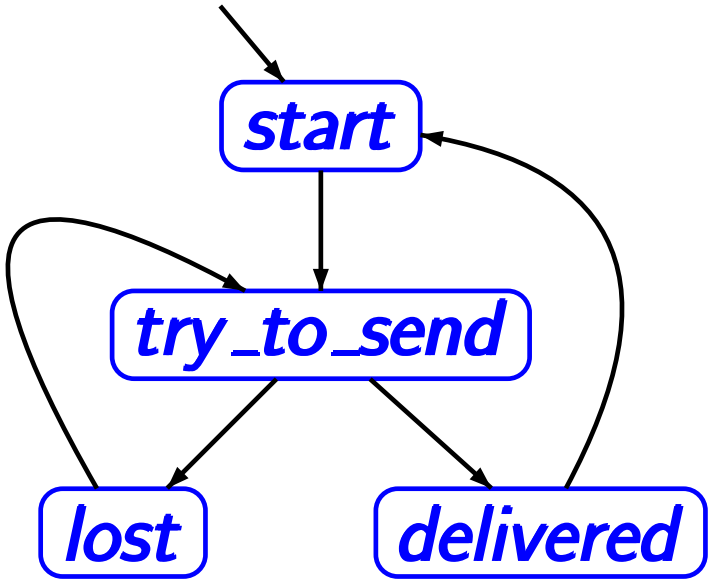
e.g., $s_0\, s_1\, s_2 \ldots \models \Box\Diamond\Phi$ iff $\overset{\infty}{\exists}\, i \geq 0$ s.t. $s_i \models \Phi$

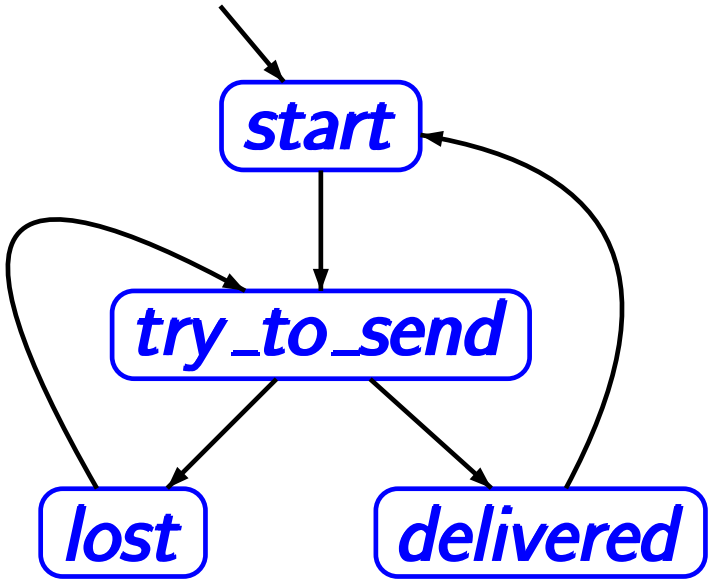
Simple communication protocol



CTL formula

$\Phi = \forall\Box\forall\Diamond start$

Simple communication protocol



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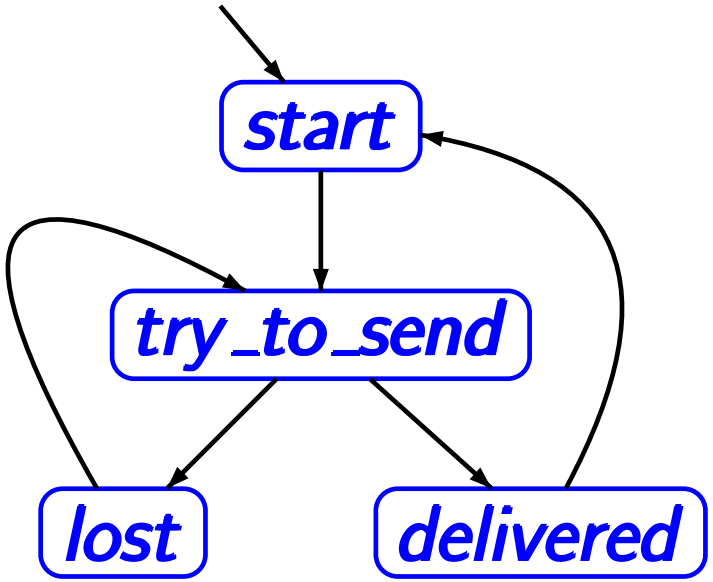

CTL formula

$\Phi = \forall\Box\forall\Diamond \mathit{start}$

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unconditional CTL fairness assumption:

$\mathit{ufair} = \Box\Diamond \mathit{delivered}$

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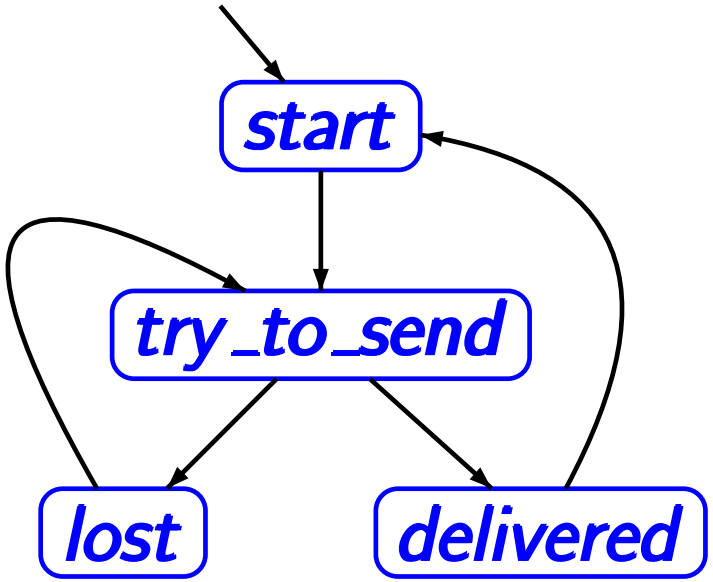

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unconditional CTL fairness assumption:

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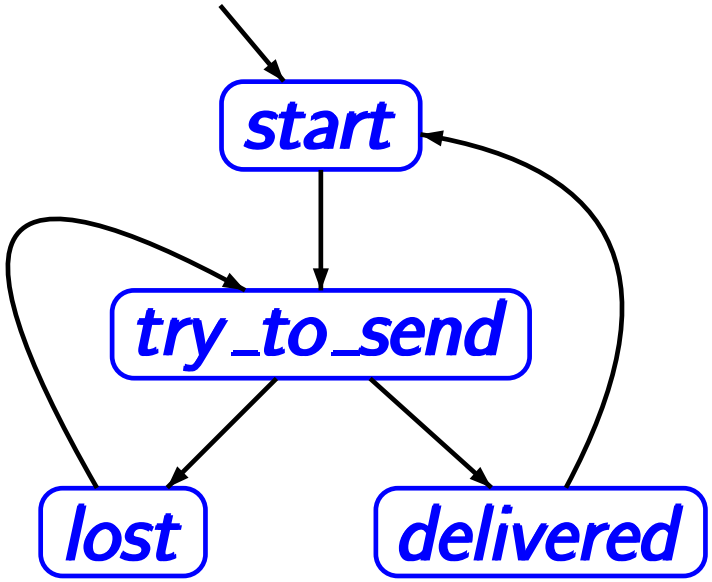
strong CTL fairness assumption:

$$\mathit{sfair} = \Box\Diamond \mathit{try\_to\_send} \rightarrow \Box\Diamond \mathit{delivered}$$

Simple communication protocol



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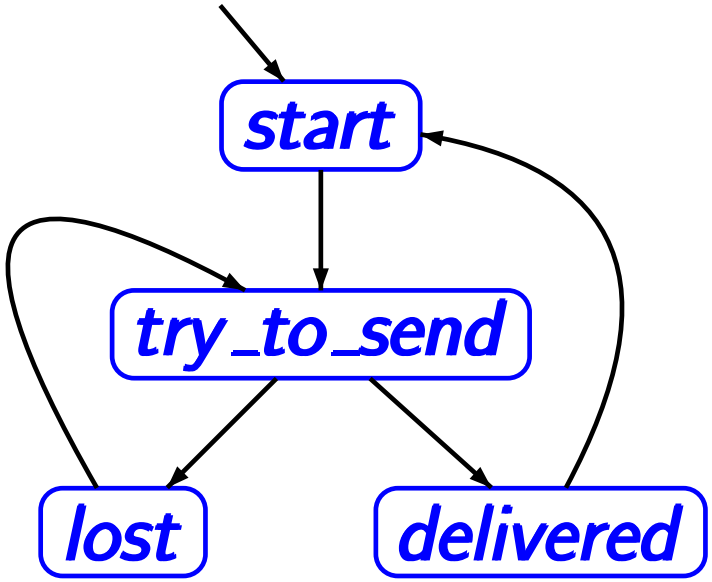
unconditional fairness: $ufair = \Box\Diamond\exists\bigcirc start$

Simple communication protocol



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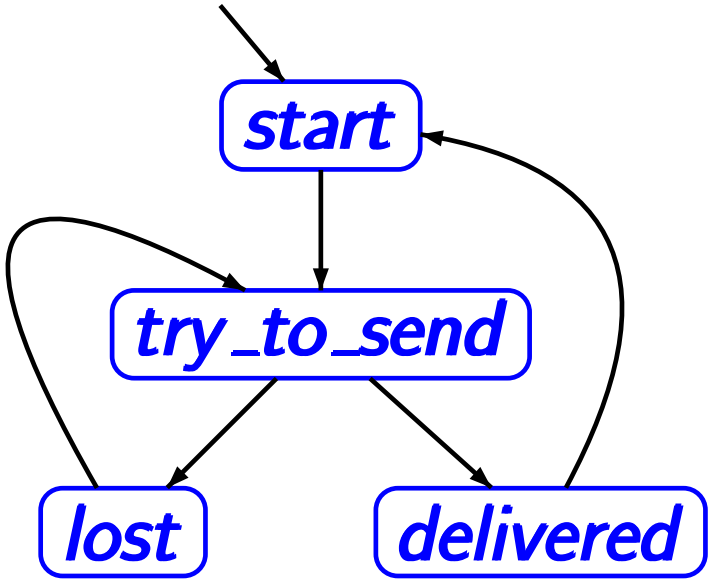

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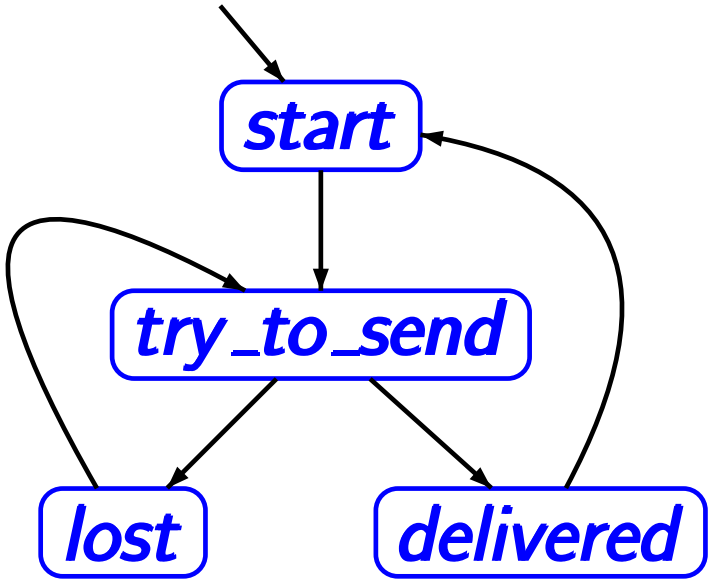

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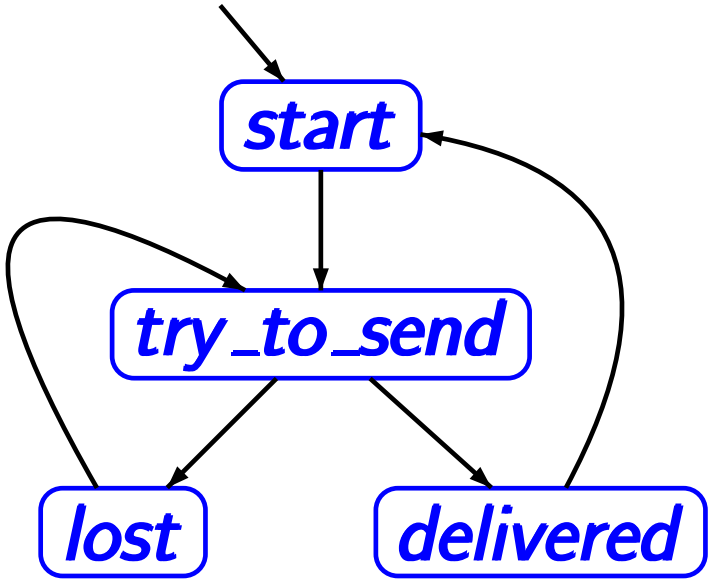

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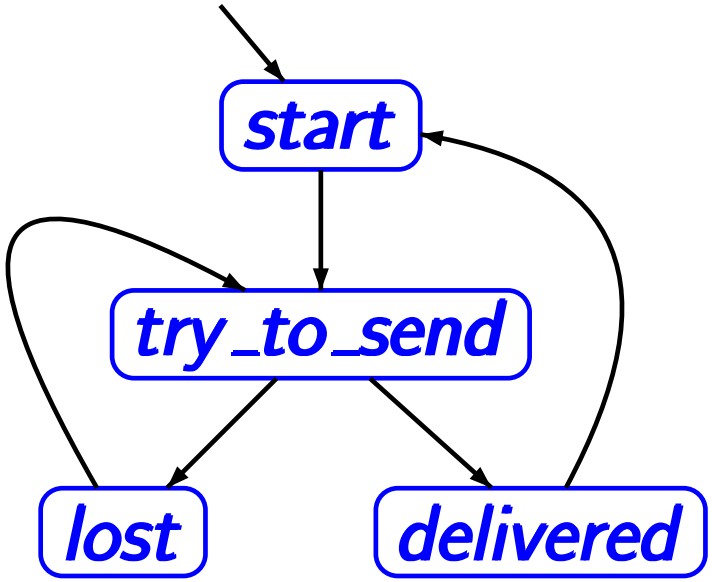

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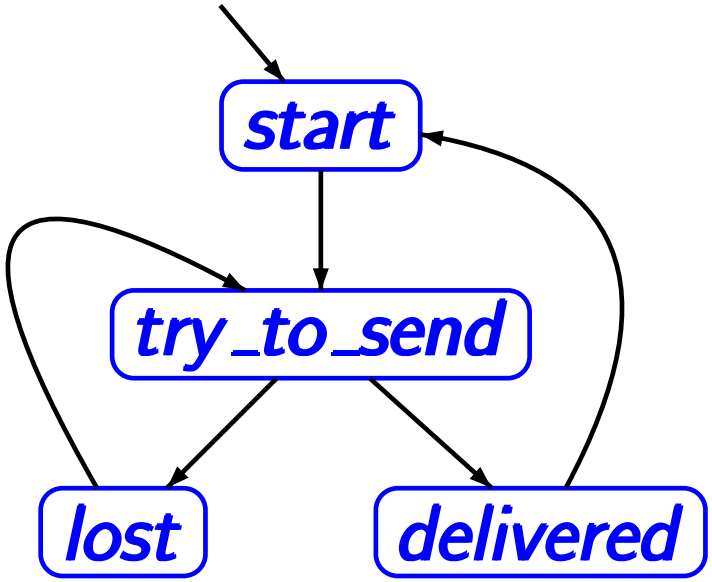

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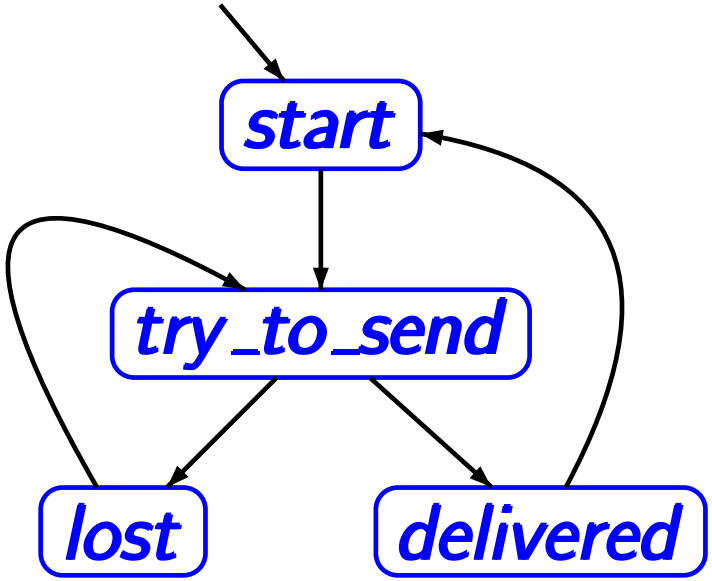

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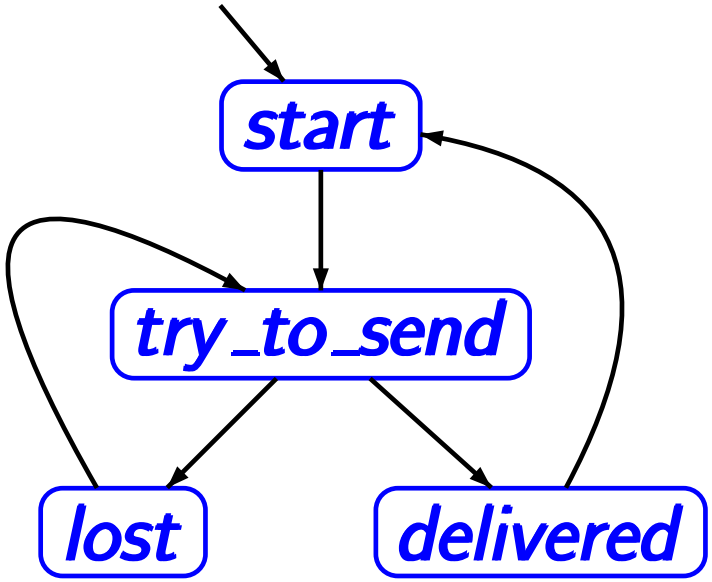

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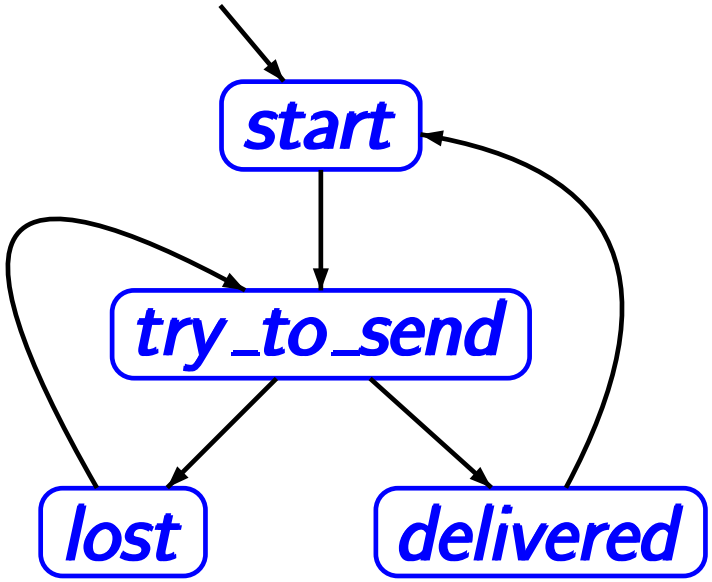

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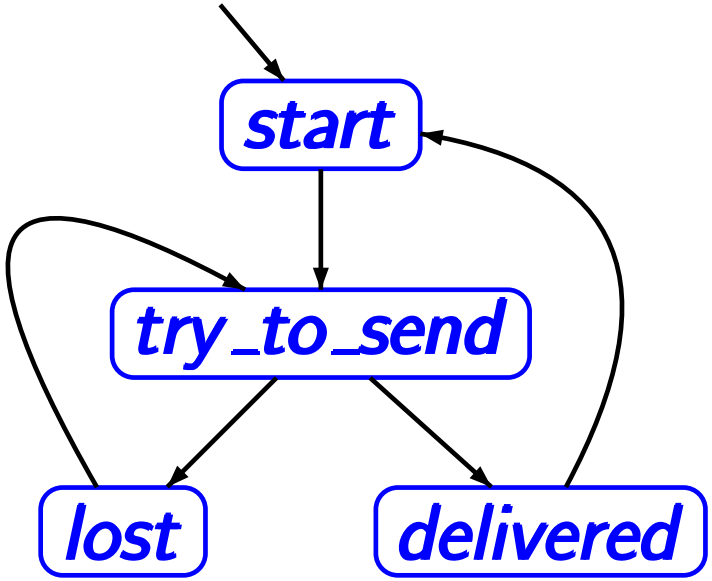

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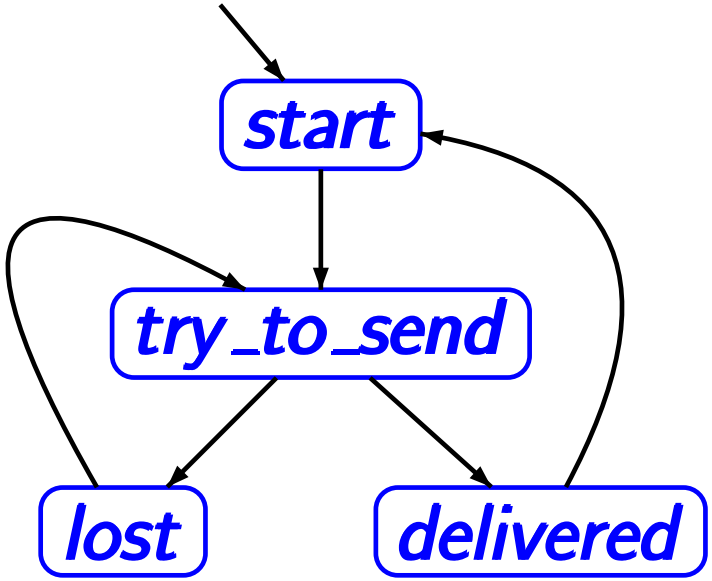

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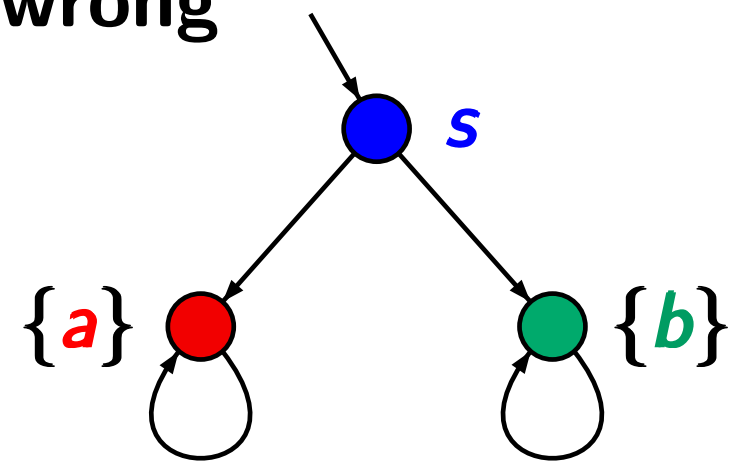

$fair = \Box\Diamond b$

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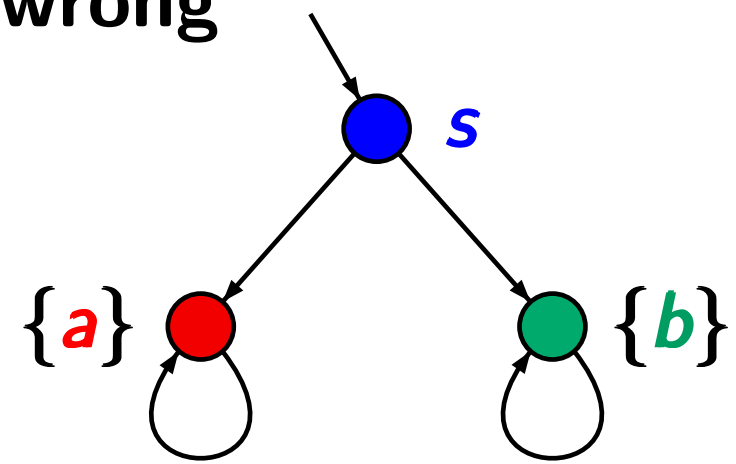

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just one fair path ● ● ● ● . . .

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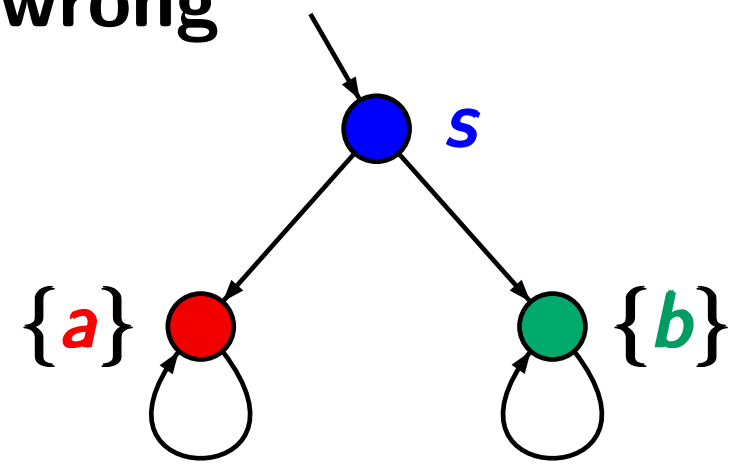

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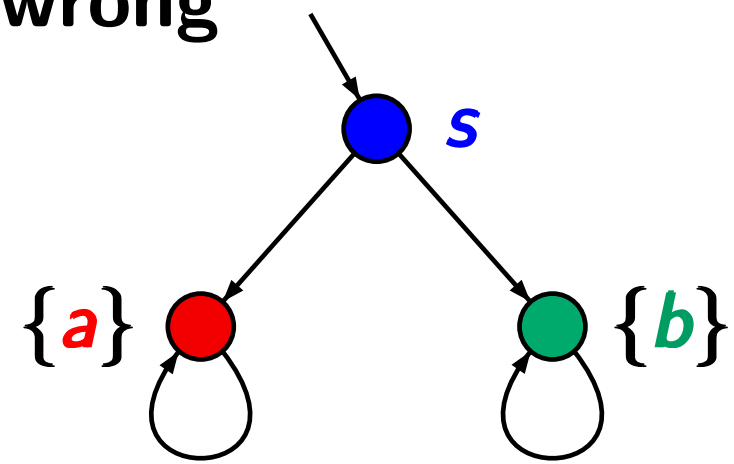

$fair = \Box\Diamond b$

$s \not\models_{fair} \exists\Diamond a$

$s \models \exists\Diamond a$

just one fair path ● ● ● ● . . .

Correct or wrong?

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct.

Does the same condition hold if a is replaced with an arbitrary state formula **?**

Correct or wrong?

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

Correct or wrong?

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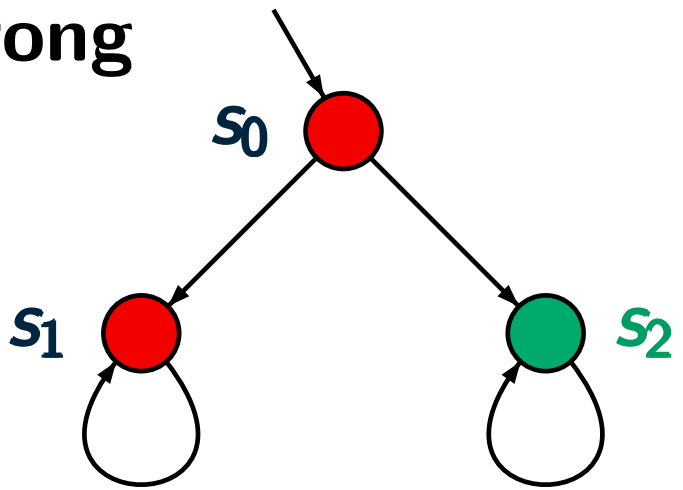

● $= \{b\}$

● $= \{a\}$

Correct or wrong?

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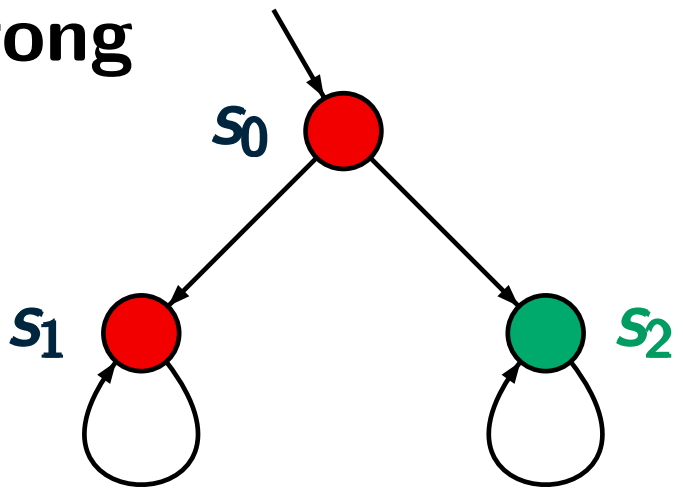

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$Sat(\exists\Box a) = \{s_0, s_1\}$

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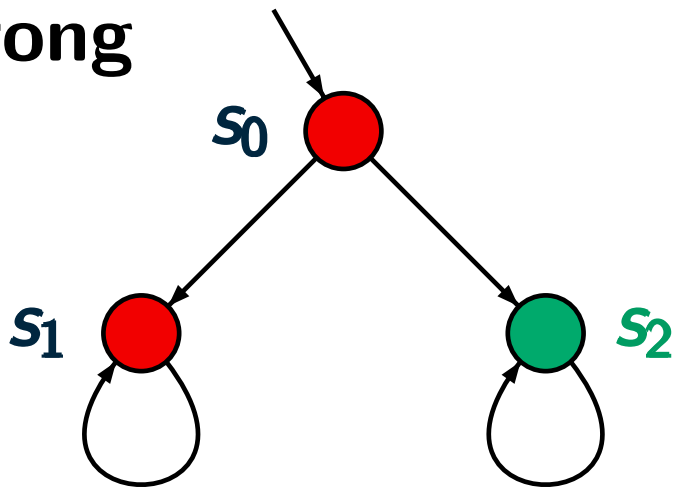

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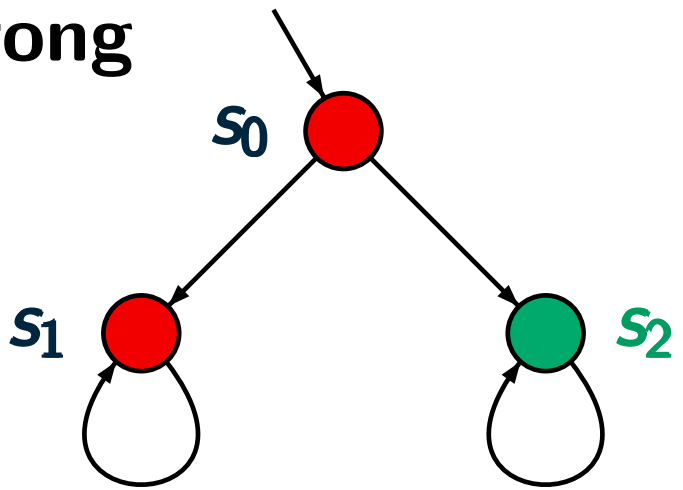

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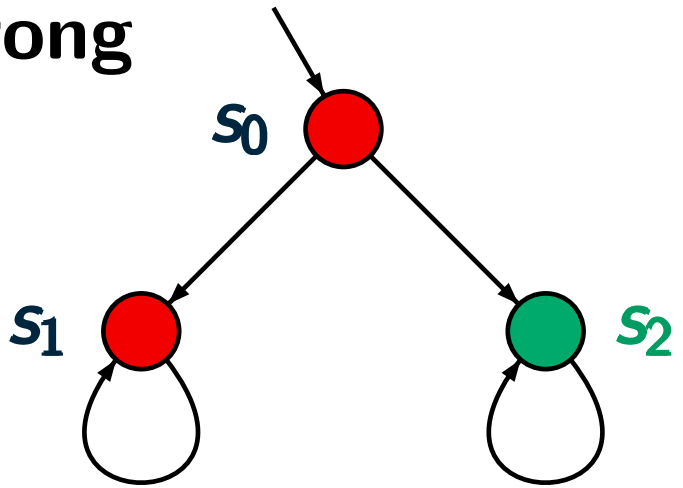

$\bigcirc = \{b\}$

$\bigcirc = \{a\}$

$fair = \Box\Diamond b$

$Sat(\exists\Box a) = \{s_0, s_1\}$

$Sat_{fair}(\exists\Box a) = \varnothing$

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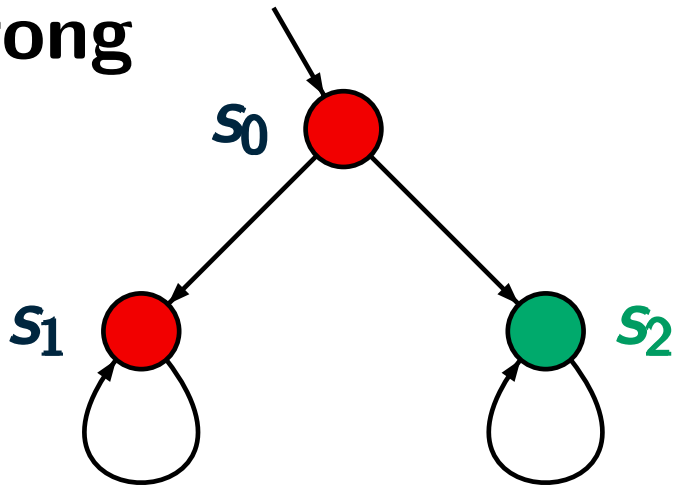

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$Sat_{fair}(\exists\Box true) = ?$

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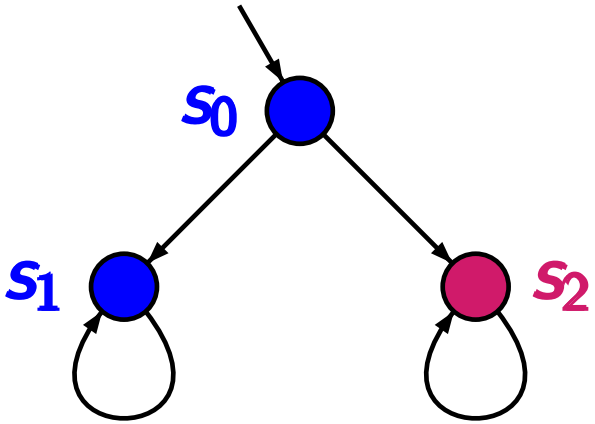

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● $= \varnothing$

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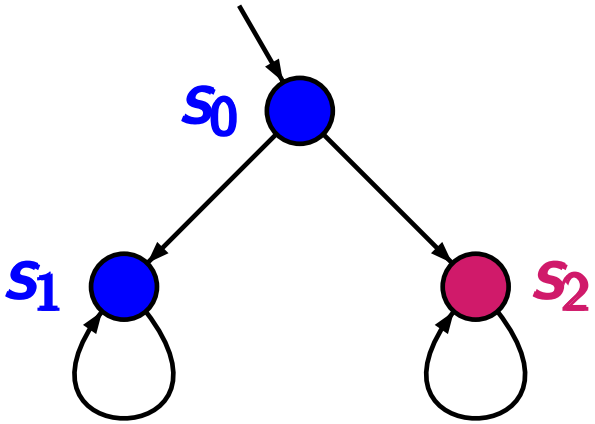

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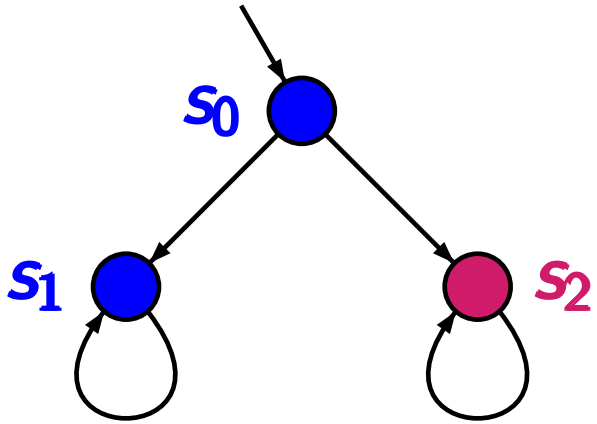

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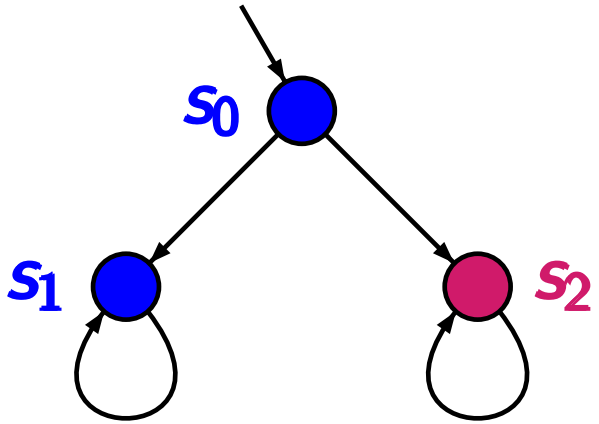

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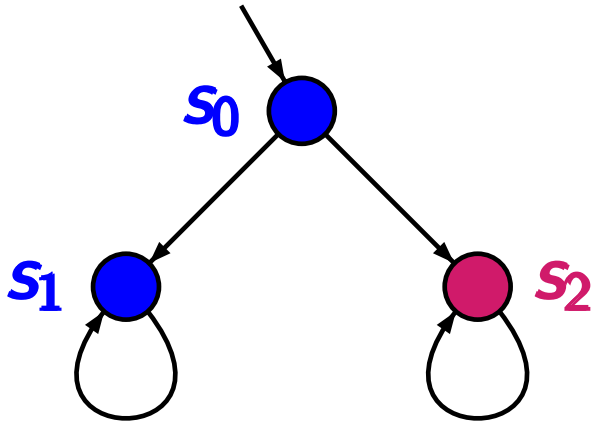

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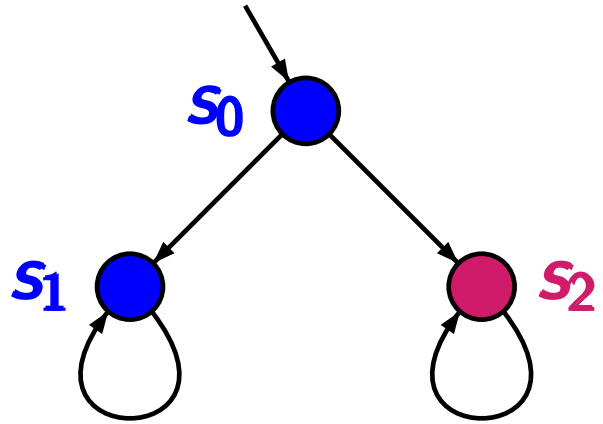

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fair is realizable iff

$Sat_{fair}(\exists\Box true) \supseteq$ set of all reachable states

Model checking problem for FairCTL

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given: finite transition system $\mathcal{T}$
CTL formula Φ
CTL fairness assumption *fair*

question: does $\mathcal{T} \models_{fair} \Phi$ hold **?**

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CTL fairness assumption $fair$, e.g.,

$$fair = \bigwedge_{1 \leq i \leq k} \Box\Diamond\Psi_{i,1} \rightarrow \Box\Diamond\Psi_{i,2}$$

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for simplicity:

we suppose that Φ is in existential normal form,
i.e., a $\forall$-free CTL formula with temporal modalities

$$\exists\bigcirc,\ \exists\mathbf{U},\ \exists\Box$$

Preprocessing of FairCTL model checking

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   - $\exists\bigcirc$, $\exists\mathsf{U}$: via standard **CTL** model checking
   - $\exists\Box$: via analysis of **SCCs**

recursive computation of the fair satisfaction sets:

$$Sat_{fair}(\Psi) = \{s \in S : s \models_{fair} \Psi\}$$

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simple cases: $\Psi = true$ or $\Psi = a \in AP$ or the outer most operator of Ψ is a negation or conjunction:

$$\begin{aligned}
Sat_{fair}(true) &= S \\
Sat_{fair}(a) &= \{s \in S : a \in L(s)\} \\
Sat_{fair}(\neg\Psi) &= S \setminus Sat_{fair}(\Psi) \\
Sat_{fair}(\Psi_1 \wedge \Psi_2) &= Sat_{fair}(\Psi_1) \cap Sat_{fair}(\Psi_2)
\end{aligned}$$

Idea of FairCTL model checking

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CTL formula Φ in $\exists$-normal form
CTL fairness assumption ***fair***

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FairCTL model checking: treatment of $\exists\bigcirc$

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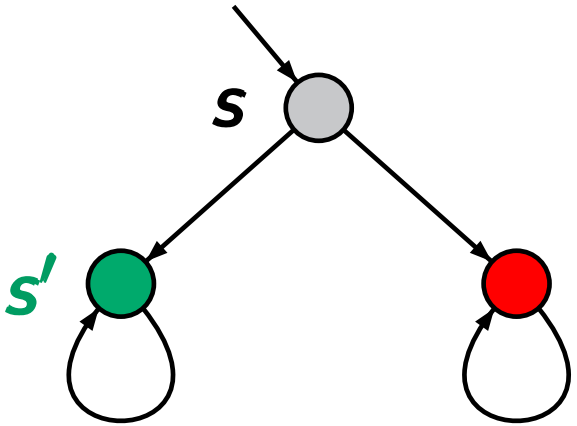
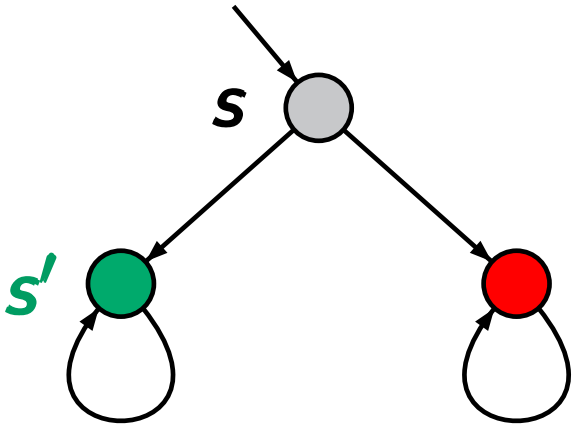


$fair = \Box\Diamond red$

FairCTL model checking: treatment of $\exists\bigcirc$



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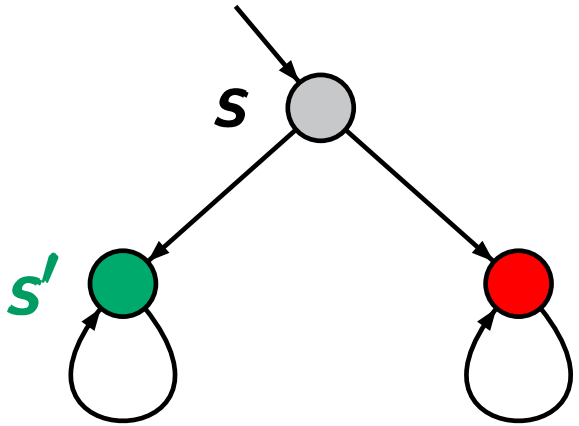
$s \not\models_{fair} \exists\bigcirc green$

FairCTL model checking: treatment of $\exists\bigcirc$



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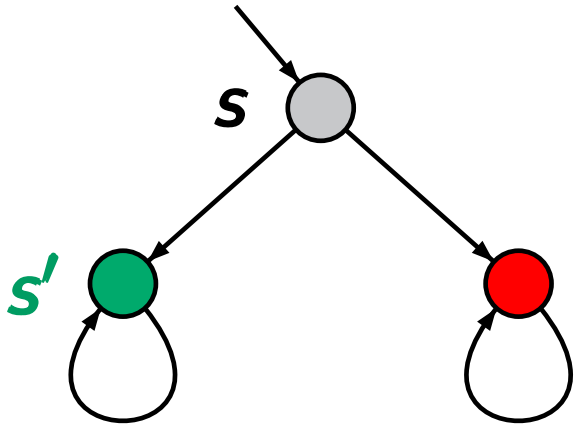
as $s' \not\models_{fair} \exists\Box true$

FairCTL model checking: treatment of $\exists\bigcirc$



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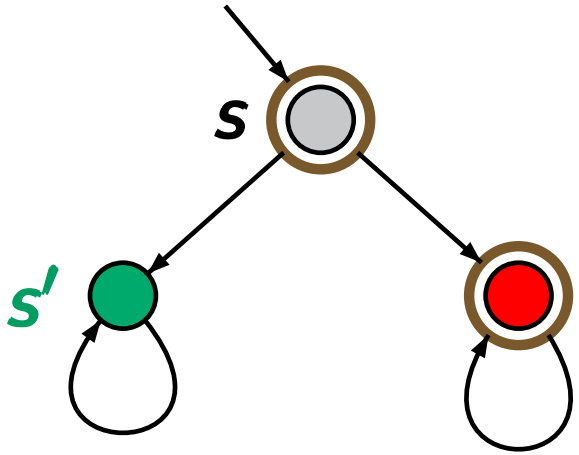
as $s' \not\models_{fair} \exists\Box true$

introduce an additional atomic proposition a_{fair} s.t. for all states s:

$$a_{fair} \in L(s) \text{ iff } s \models_{fair} \exists\Box true$$

FairCTL model checking: treatment of $\exists\bigcirc$



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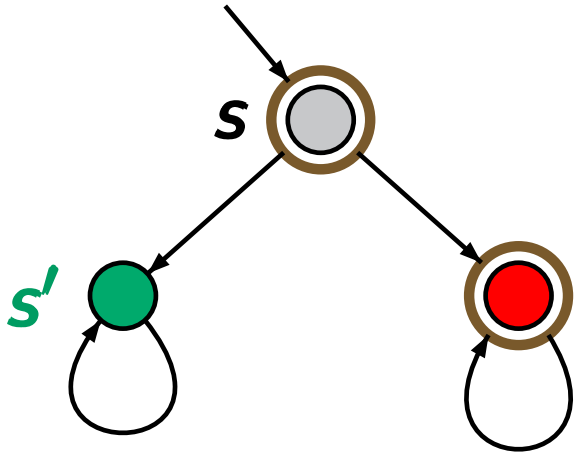

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This yields that for all $b \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \text{ iff } s \models \exists\bigcirc(b \wedge a_{fair})$$

FairCTL model checking: $\exists\bigcirc$ and $\exists\mathsf{U}$

introduce an additional atomic proposition a_{fair} s.t.

$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

This yields that for all $b, c \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$

$$s \models_{fair} \exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad ?$$

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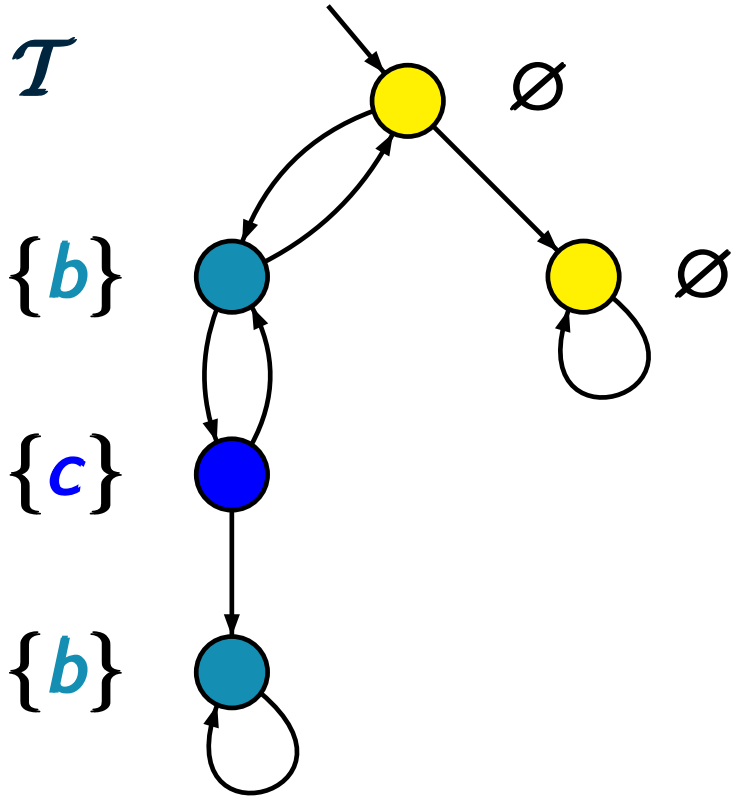
hence: treatment of $\exists\bigcirc$ and $\exists\mathbf{U}$ for FairCTL via

- special methods to compute $Sat_{fair}(\exists\Box true)$
- standard CTL model checking for $\exists\bigcirc$ and $\exists\mathbf{U}$

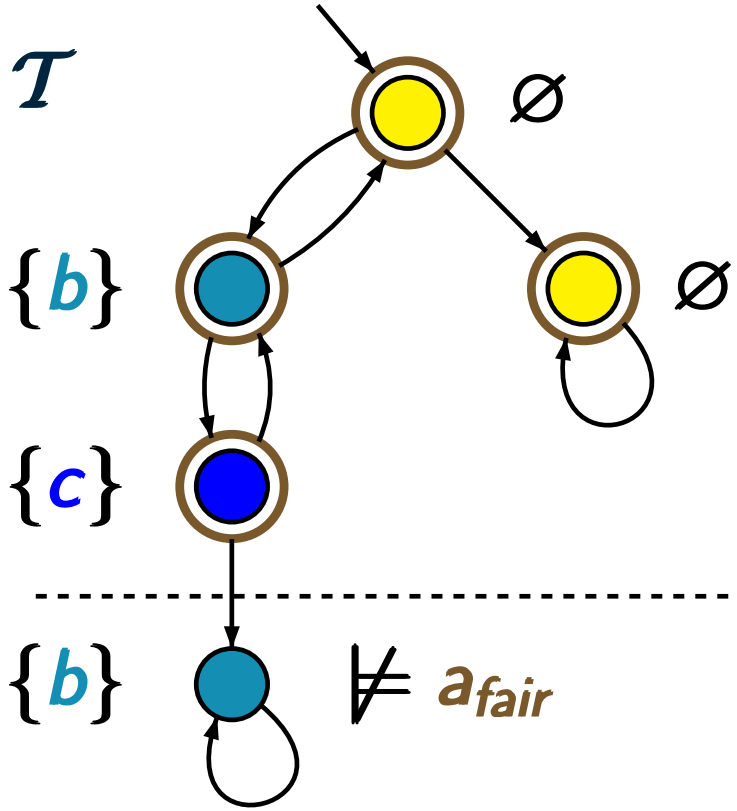
Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

strong fairness assumption: $\mathit{fair} = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$



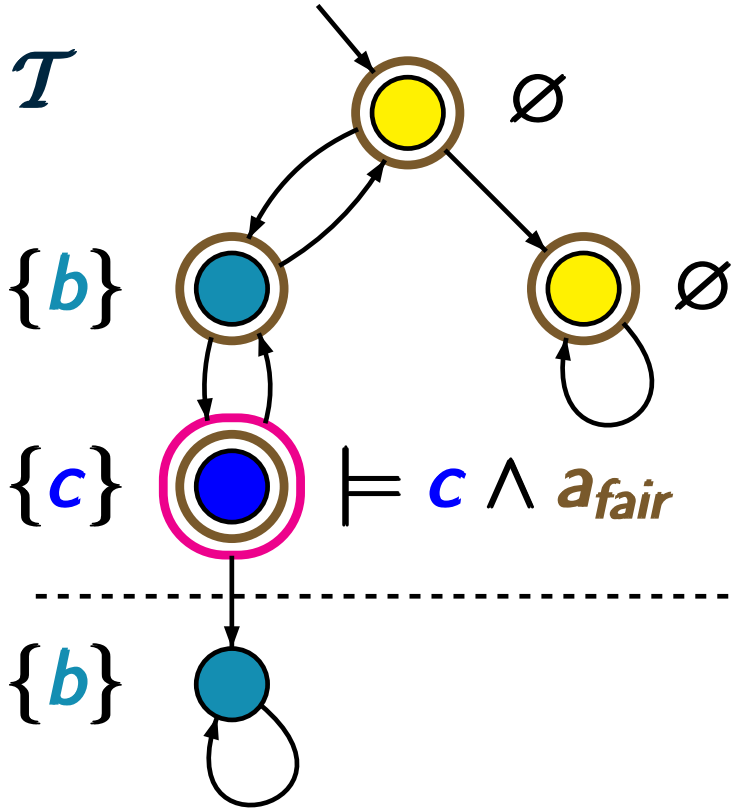
CTL formula $\exists\Diamond c$

$\downarrow$

$\exists\Diamond (c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$



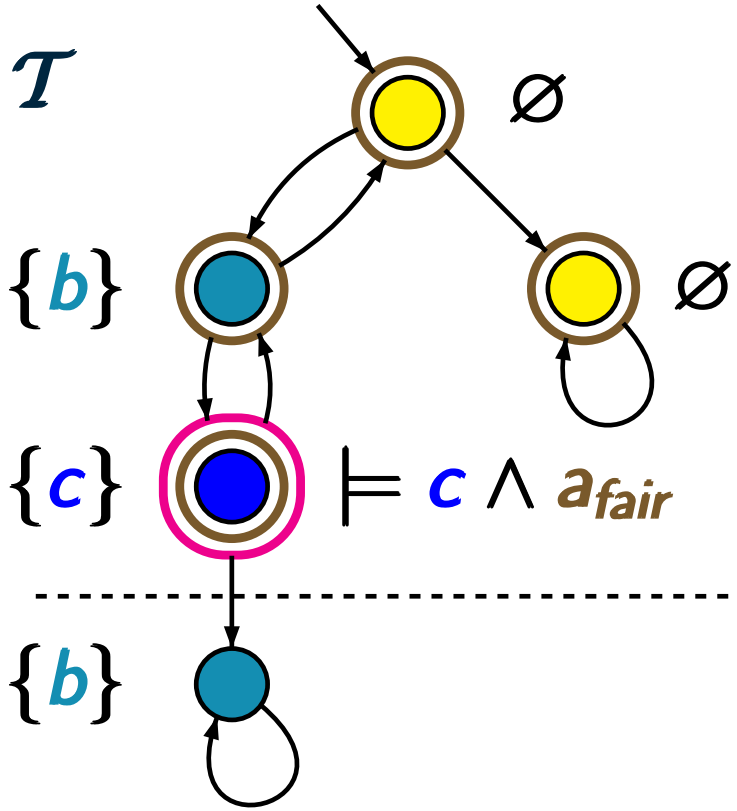
CTL formula $\exists\Diamond c$

$\downarrow$

$\exists\Diamond \boxed{(c \wedge a_{fair})}$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

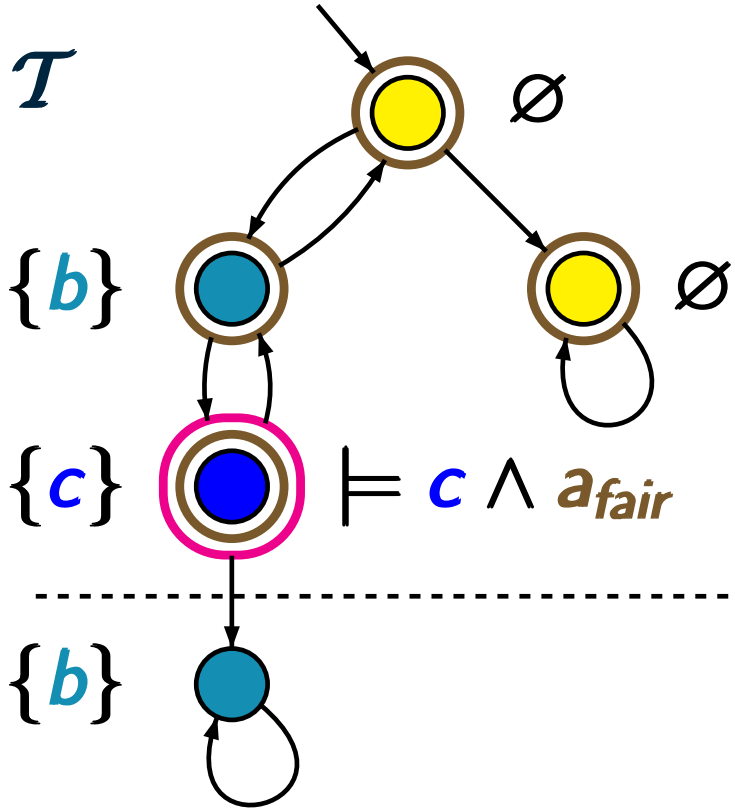
$\downarrow$

$\exists\Diamond \boxed{(c \wedge a_{fair})}$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists\Diamond(c \wedge a_{fair})$

Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

$\downarrow$

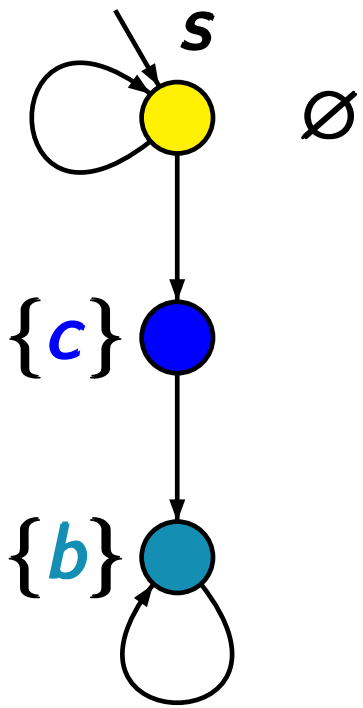
$\exists\Diamond \boxed{(c \wedge a_{fair})}$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists\Diamond(c \wedge a_{fair}) \implies \mathcal{T} \models_{fair} \exists\Diamond c$$

Example: treatment of $\exists \mathsf{U}$

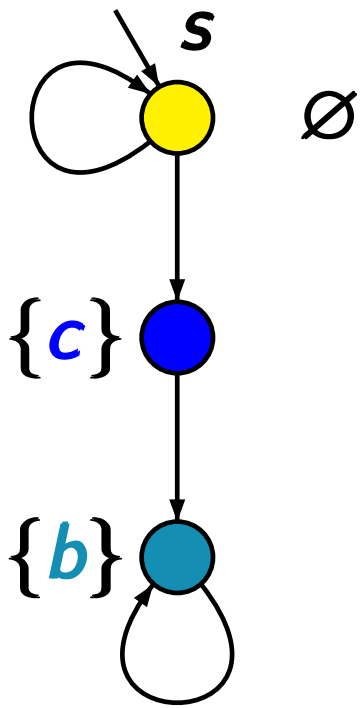
$\mathcal{T}$:



$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of ∃U

$\mathcal{T}$:

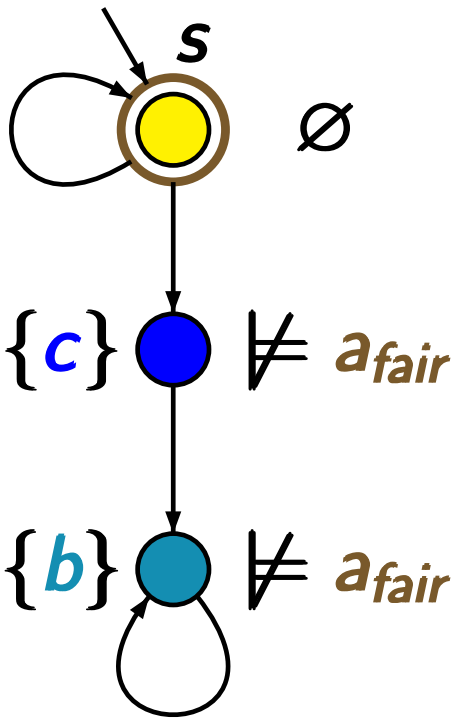


strong fairness assumption: $\mathit{fair} = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of $\exists U$

$\mathcal{T}$:

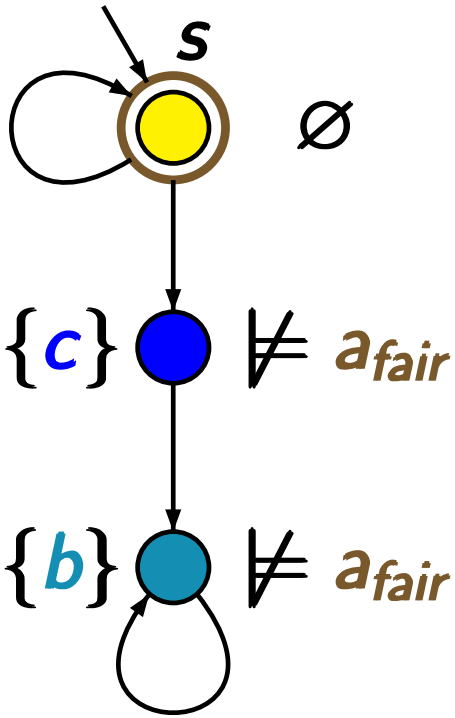


strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of $\exists U$

$\mathcal{T}$:



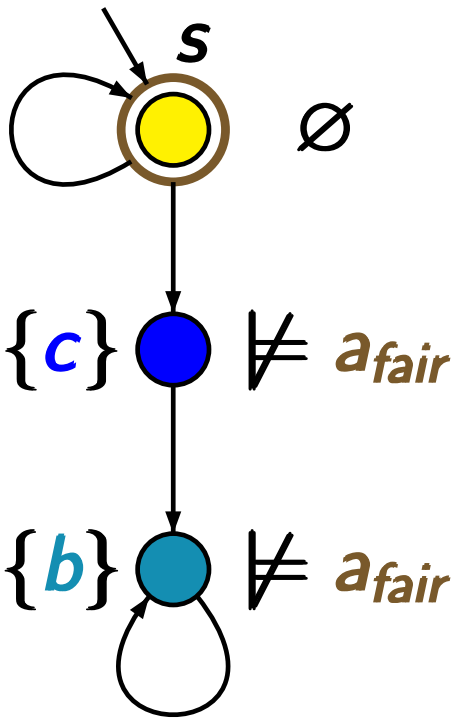
$Sat(c \wedge a_{fair}) = \emptyset$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of $\exists \mathsf{U}$

$\mathcal{T}$:



$$s \not\models \exists(\neg b \,\mathsf{U}(\, c \wedge a_{fair}))$$

$$\Uparrow$$

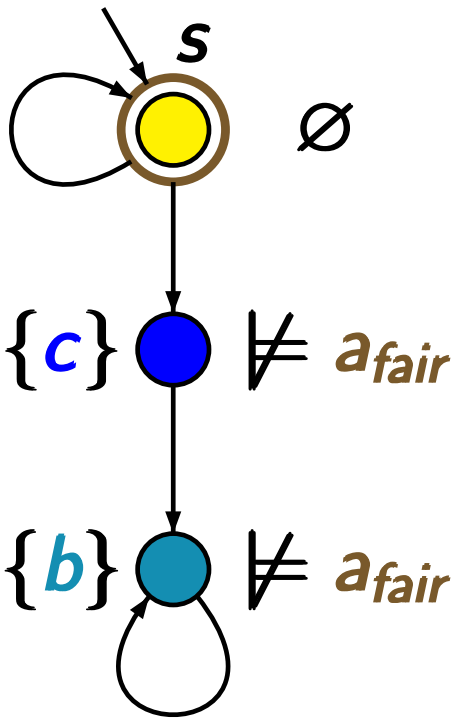
$$Sat(c \wedge a_{fair}) = \varnothing$$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of $\exists \mathsf{U}$

$\mathcal{T}$:



$$s \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$$

$$\Uparrow$$

$$s \not\models \exists(\neg b \,\mathsf{U}\, (c \wedge a_{fair}))$$

$$\Uparrow$$

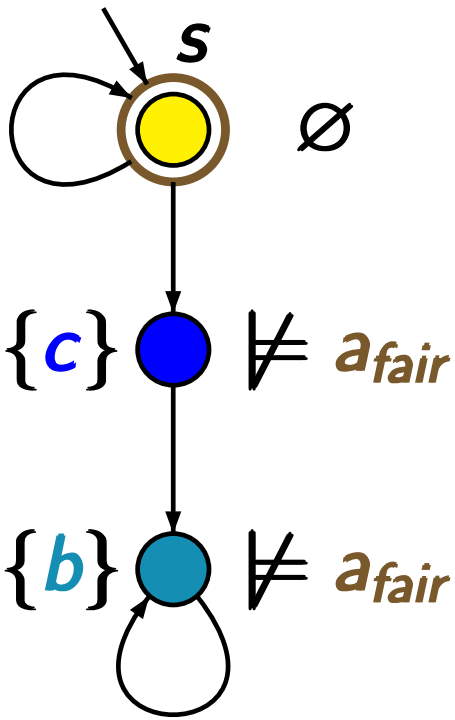
$$Sat(c \wedge a_{fair}) = \varnothing$$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of $\exists \mathsf{U}$

$\mathcal{T}$:



$$s \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$$

$$\Uparrow$$

$$s \not\models \exists(\neg b \,\mathsf{U}\, (c \wedge a_{fair}))$$

$$\Uparrow$$

$$Sat(c \wedge a_{fair}) = \varnothing$$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c), \quad \text{but } \mathcal{T} \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

correct.

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\, (b \wedge a_{fair}))$$

correct. Note that:

if $s_0\, s_1 \ldots s_{n-1} s_n$ is a path fragment from $s_0 = s$ s.t. $s_n \models a_{fair}$ then $s_0, s_1, \ldots, s_{n-1} \models a_{fair}$.

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

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$$\begin{aligned} & s \models \quad \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair})) \\ \iff \quad & s \models \quad \exists\bigcirc\exists((c \wedge a_{fair}) \,\mathsf{U}(b \wedge a_{fair})) \end{aligned}$$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

correct. Note that:

if $s_0\, s_1 \ldots s_{n-1} s_n$ is a path fragment from $s_0{=}s$ s.t. $s_n \models a_{fair}$ then $s_0, s_1, \ldots, s_{n-1} \models a_{fair}$. Hence:

$$\begin{aligned}
& s \models \quad \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair})) \\
\iff\ & s \models \quad \exists\bigcirc\exists((c \wedge a_{fair}) \,\mathsf{U}(b \wedge a_{fair})) \\
\iff\ & s \models \quad \exists\bigcirc\big(\exists(c \,\mathsf{U}(b \wedge a_{fair})) \wedge a_{fair}\big)
\end{aligned}$$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

correct. Note that:

if $s_0\, s_1 \ldots s_{n-1} s_n$ is a path fragment from $s_0{=}s$ s.t. $s_n \models a_{fair}$ then $s_0, s_1, \ldots, s_{n-1} \models a_{fair}$. Hence:

$$\begin{aligned}
& s \models \exists\bigcirc\exists(c \,\mathsf{U}(b \wedge a_{fair})) \\
\iff\ & s \models \exists\bigcirc\exists((c \wedge a_{fair}) \,\mathsf{U}(b \wedge a_{fair})) \\
\iff\ & s \models \exists\bigcirc\big(\exists(c \,\mathsf{U}(b \wedge a_{fair})) \wedge a_{fair}\big) \\
\iff\ & s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b)
\end{aligned}$$

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\,(b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

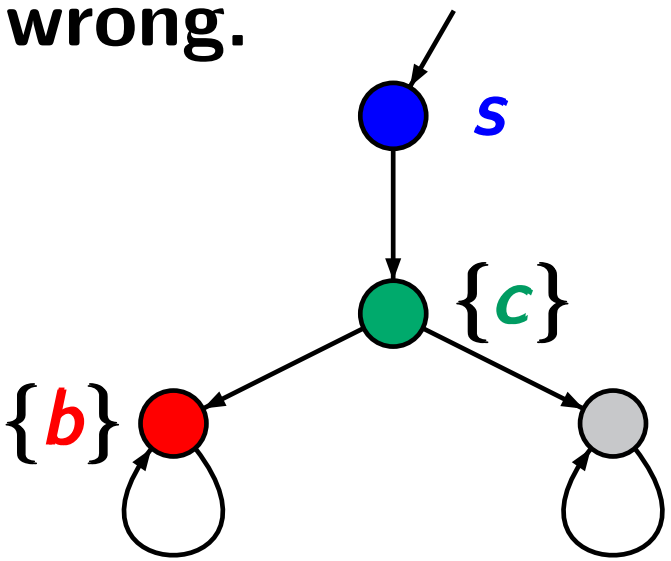
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\,(b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

wrong.



$$fair = \Box\Diamond gray$$

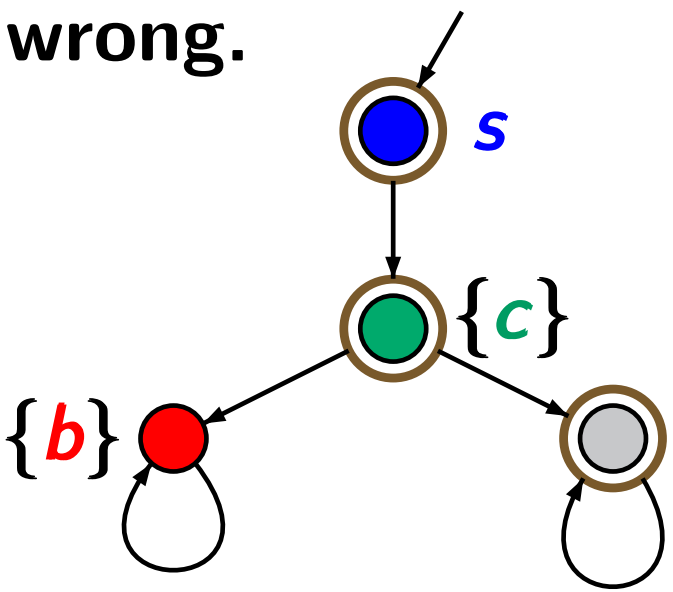
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\,(b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

wrong.



$$fair = \Box\Diamond gray$$

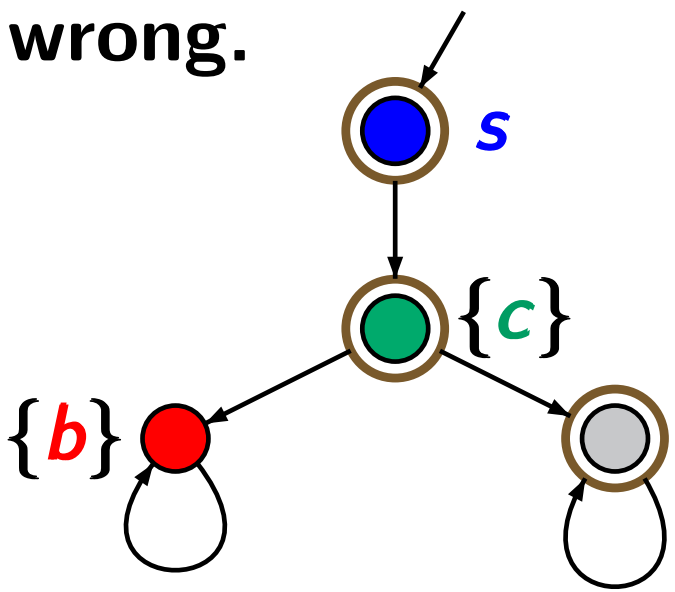
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\,(b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

wrong.



$$fair = \Box\Diamond gray$$

$$Sat_{fair}(\exists(c \,\mathsf{U}\, b)) = \varnothing$$

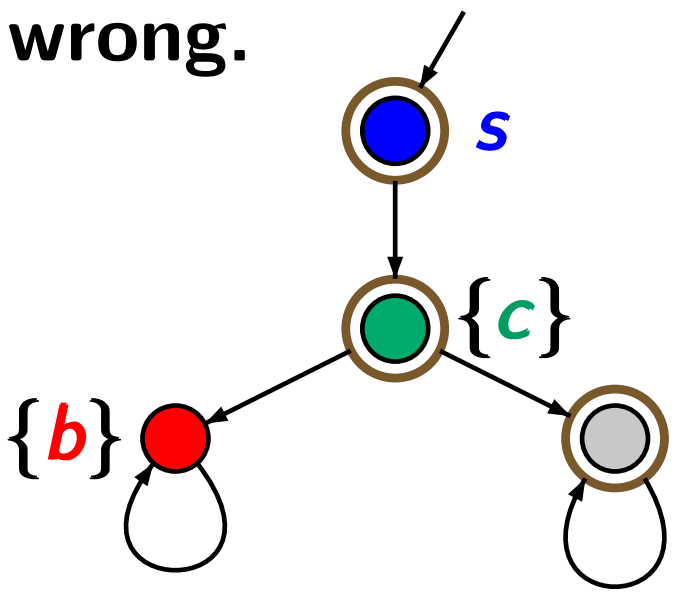
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\, (b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

wrong.



$$fair = \Box\Diamond gray$$

$$Sat_{fair}(\exists(c \,\mathsf{U}\, b)) = \varnothing$$

$$s \not\models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b)$$

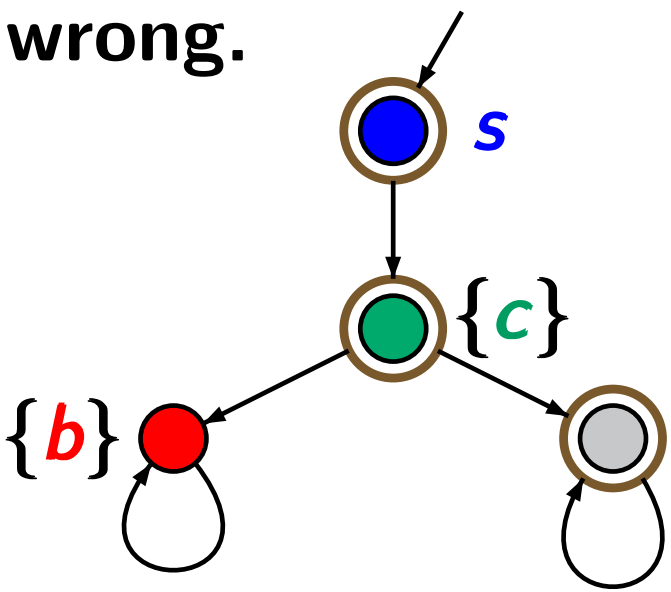
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc\exists(c \,\mathsf{U}\, (b \wedge a_{fair}))$$

correct.

$$s \models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

wrong.



$$fair = \Box\Diamond gray$$

$$Sat_{fair}(\exists(c \,\mathsf{U}\, b)) = \varnothing$$

$$s \not\models_{fair} \exists\bigcirc\exists(c \,\mathsf{U}\, b)$$

$$s \models \exists\bigcirc(\exists(c \,\mathsf{U}\, b) \wedge a_{fair})$$

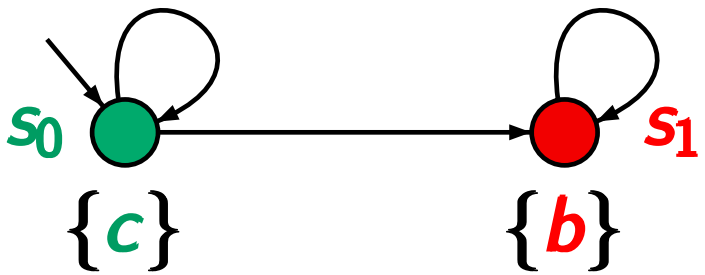
Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.

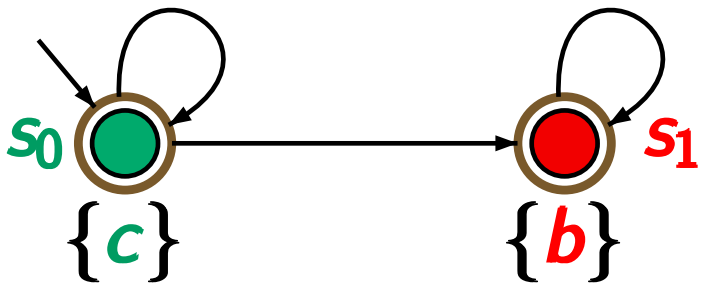


$$fair = \Box\Diamond b$$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

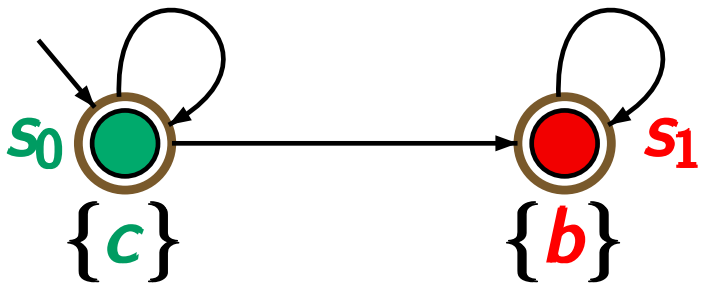
$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

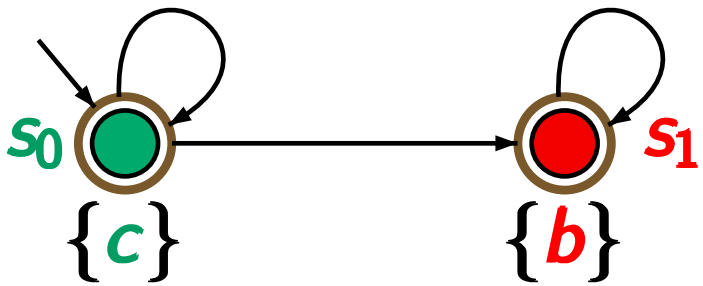
$s_1 \models a_{fair}$

regard state $s = s_0$:

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

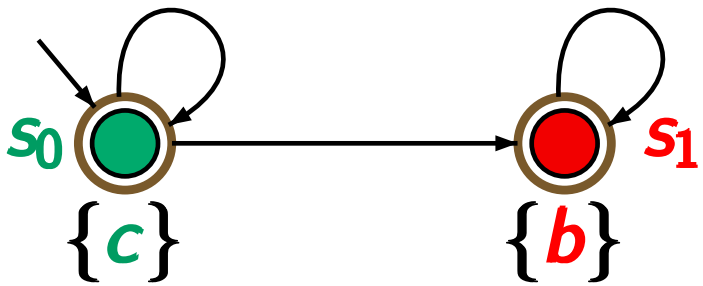
regard state $s = s_0$:

$s \models \exists\Box(c \wedge a_{fair})$,

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

regard state $s = s_0$:

$$s \models \exists\Box(c \wedge a_{fair}),$$

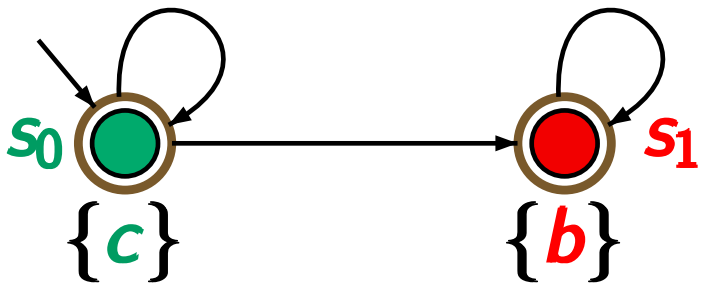
$$\uparrow$$

path $\pi = s_0\, s_0\, s_0\, s_0 \ldots \models \Box(c \wedge a_{fair})$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

regard state $s = s_0$:

$s \models \exists\Box(c \wedge a_{fair})$, but $s \not\models_{fair} \exists\Box c$

$\uparrow$

path $\pi = s_0\, s_0\, s_0\, s_0 \ldots \models \Box(c \wedge a_{fair})$

Idea of FairCTL model checking

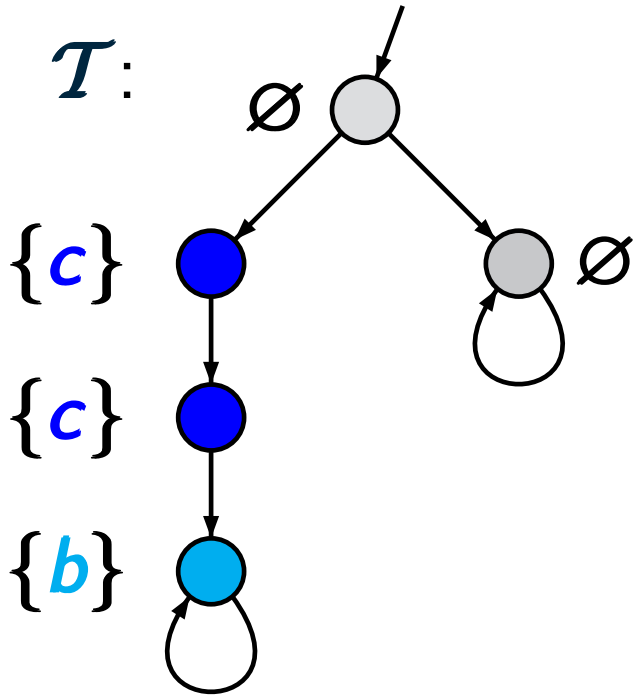
given: finite transition system $\mathcal{T}$
CTL formula Φ in $\exists$-normal form
CTL fairness assumption ***fair***

question: does $\mathcal{T} \models_{fair} \Phi$ hold **?**

1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
   - ***true***, $a \in AP$, $\wedge$, $\neg$: as for standard CTL
   - $\exists\bigcirc$, $\exists\mathsf{U}$: via standard CTL model checking
   - $\exists\Box$: via analysis of **SCCs**

$\exists\Box\Psi$ under strong fairness

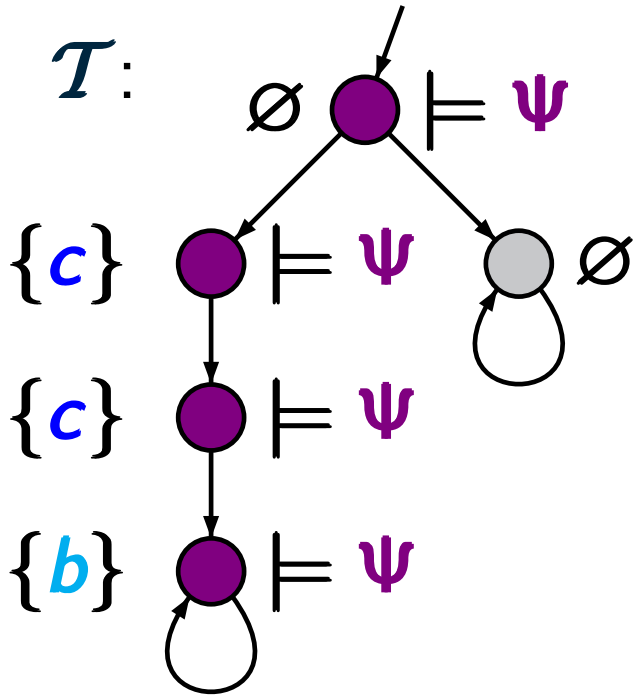
$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{fair} \exists\Box\Psi$?

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

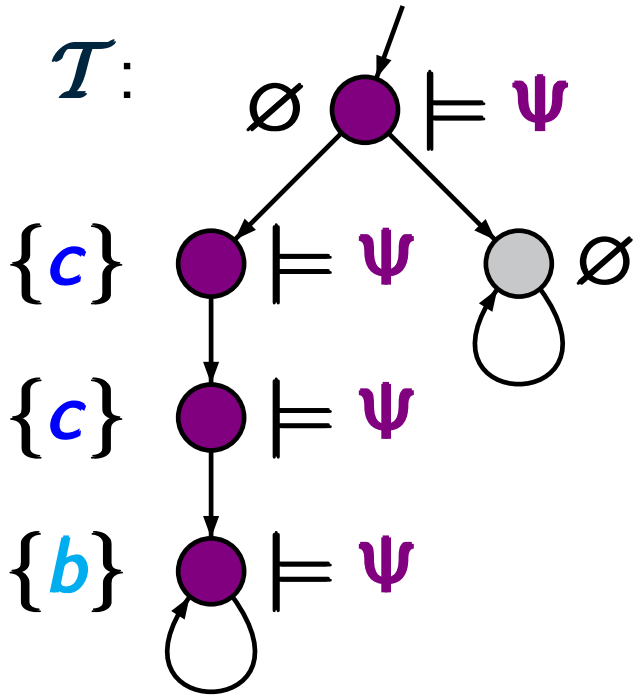


$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

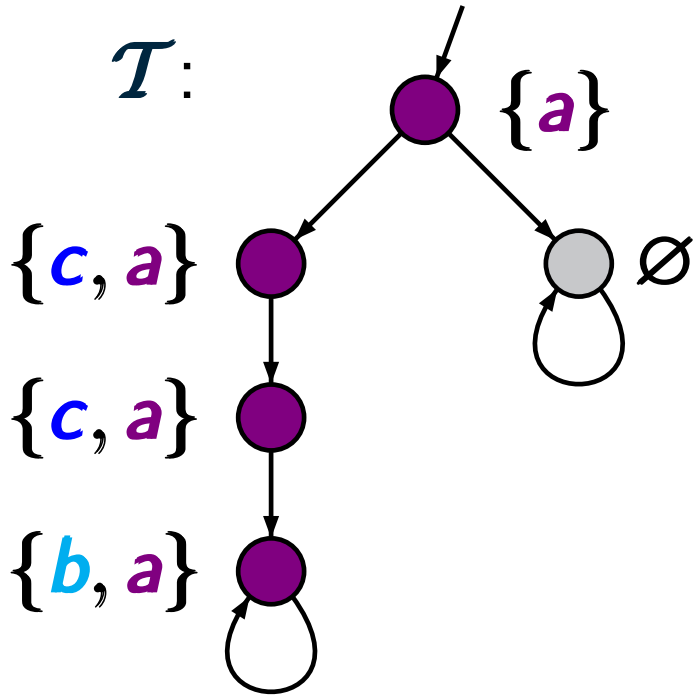


$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

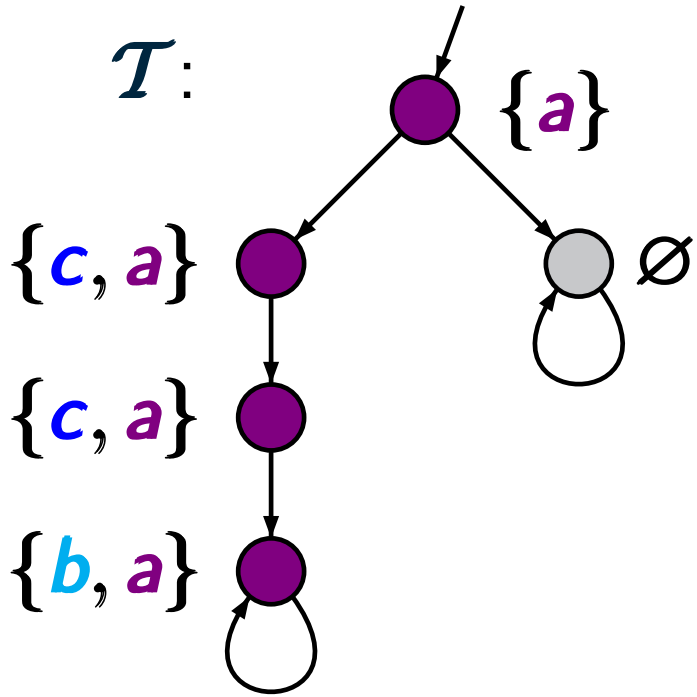


$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

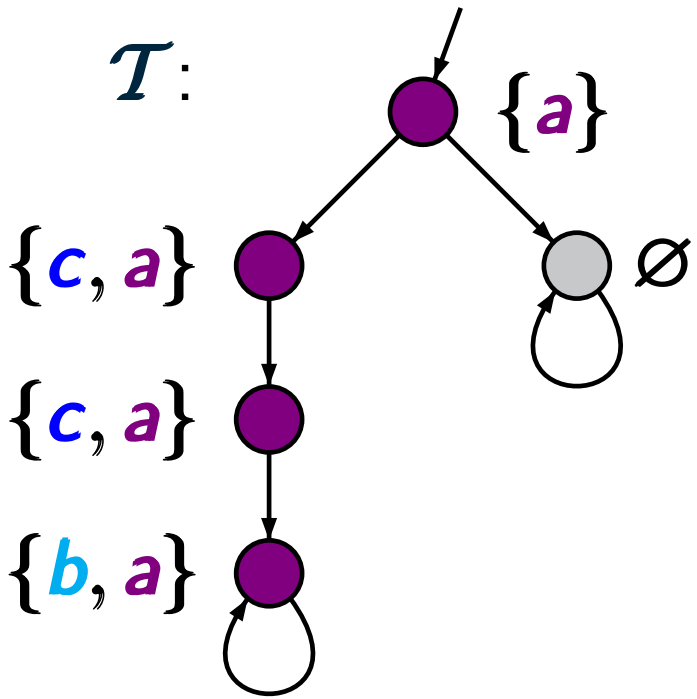


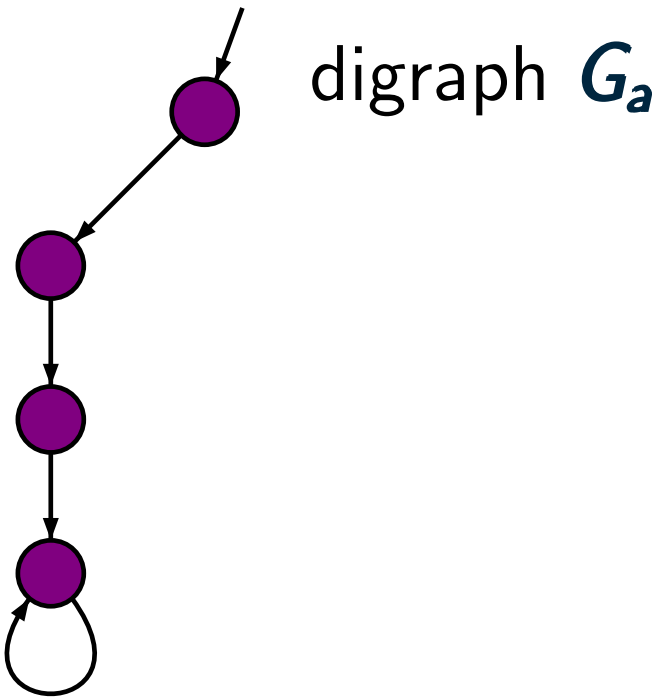
$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a)$

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

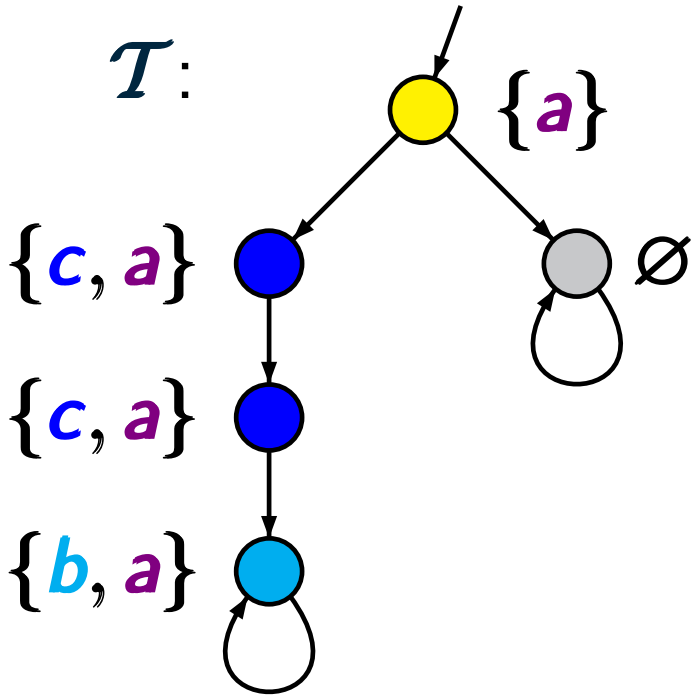


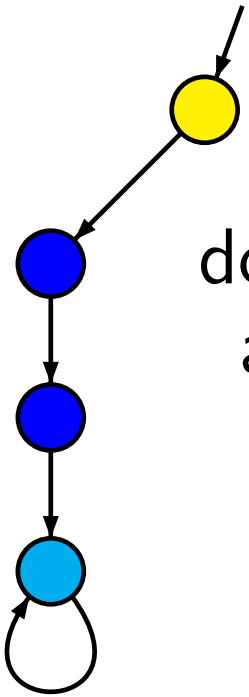


1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a)$

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ





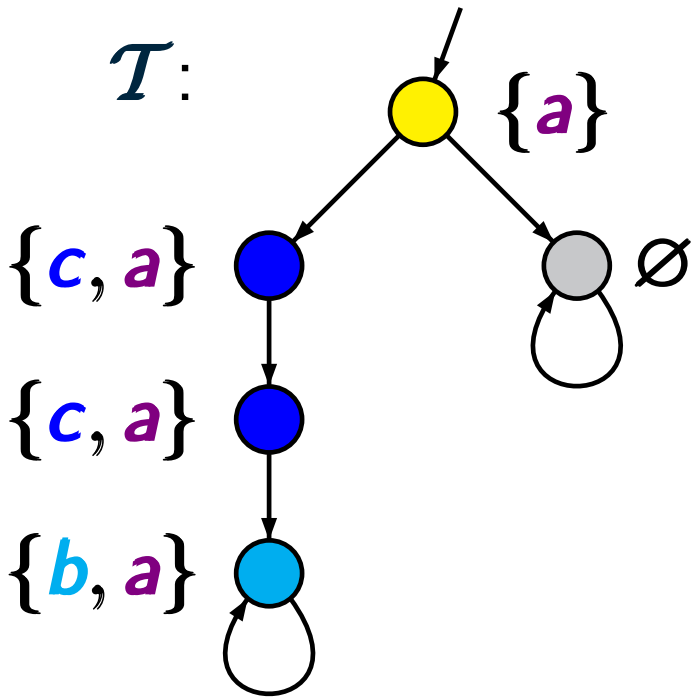
digraph G_a

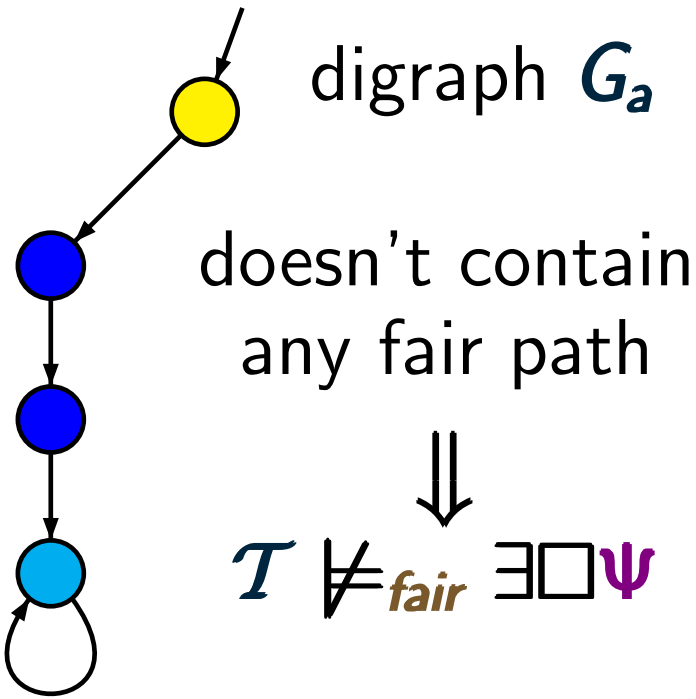
doesn't contain any fair path

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a)$

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ





1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a) = \varnothing$

Treatment of $\exists\Box a$ for FairCTL

given: finite TS $\mathcal{T}$, atomic proposition a
CTL fairness assumption $fair$

goal: compute $Sat_{fair}(\exists\Box a)$

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here: explanations only for strong fairness

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$$\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(b_i) = \varnothing$$

$$\text{or} \quad \{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(c_i) \neq \varnothing$$

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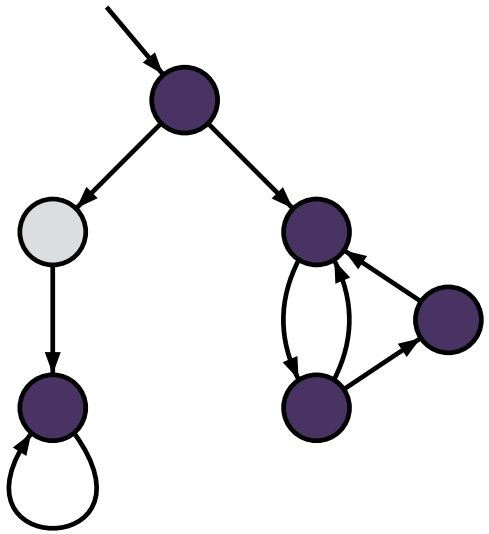
does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?



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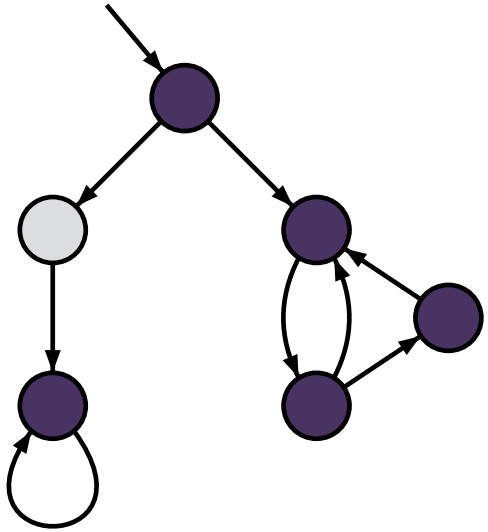

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analyze the digraph G_a that results from $\mathcal{T}$ by removing all states s with $s \not\models a$