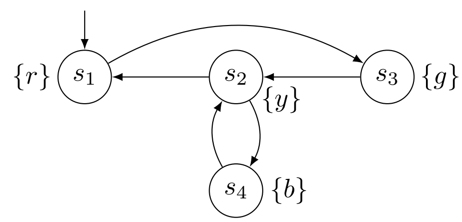
TD5 MVFA: CTL

Exercise 1

Consider the following transition system over $AP = \{b, g, r, y\}$:

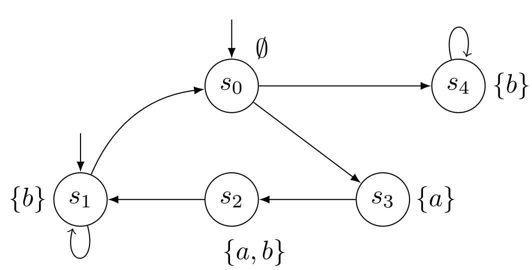


The following atomic propositions are used: r (red), y (yellow), g (green), and b (black). The model is intended to describe a traffic light that is able to blink yellow. You are requested to indicate for each of the following CTL formulas the set of states for which these formulas hold:

1. $\forall\Diamond y$
2. $\forall\Box y$
3. $\forall\Box\forall\Diamond y$
4. $\forall\Diamond g$
5. $\exists\Diamond g$
6. $\exists\Box g$
7. $\exists\Box\neg g$
8. $\forall(b \text{ U } \neg b)$
9. $\exists(b \text{ U } \neg b)$
10. $\forall(\neg b \text{ U } \exists\Diamond b)$
11. $\forall(g \text{ U } \forall(y \text{ U } r))$
12. $\forall(\neg b \text{ U } b)$

Exercise 2

Consider the following transition system over $AP = \{a, b\}$:



You are requested to indicate for each of the following CTL formulas the set of states for which these formulas hold and whether the transition system verify the property:

- $\Phi_1 = \forall(a \text{ U } b) \vee \exists \bigcirc (\forall\Box b)$
- $\Phi_2 = \forall\Box\forall(a \text{ U } b)$
- $\Phi_3 = (a \wedge b) \rightarrow \exists\Box\exists \bigcirc \forall(a \text{ W } b)$
- $\Phi_4 = \forall\Box\exists\Diamond\Phi_3$

Exercise 3

Which of the following assertions are correct? Provide a proof or a counterexample.

1. If $s \models \exists\Box a$, then $s \models \forall\Box a$.
2. If $s \models \forall\Box a$, then $s \models \exists\Box a$.
3. If $s \models \forall\Diamond a \vee \forall\Diamond b$, then $s \models \forall\Diamond(a \vee b)$.
4. If $s \models \forall\Diamond(a \vee b)$, then $s \models \forall\Diamond a \vee \forall\Diamond b$.
5. If $s \models \forall(a \text{ U } b)$, then $s \models \neg(\exists(\neg b \text{ U } (\neg a \wedge \neg b)) \vee \exists\Box\neg b)$.

Exercise 4

Prove that there exists a formula φ_{CTL} of CTL, two transition systems TS_1 and TS_2 such that:

- $TS_1 \models \varphi_{CTL}$ and $TS_2 \not\models \varphi_{CTL}$.
- for all formulas φ_{LTL} of LTL, $TS_1 \models \varphi_{LTL}$ if and only if $TS_2 \models \varphi_{LTL}$.

Exercise 5

We define a function f such that for all LTL formula ϕ, ψ:

- $f(\neg\phi) = \neg f(\phi)$
- $f(\psi \wedge \phi) = f(\psi) \wedge f(\phi)$
- $f(\psi \vee \phi) = f(\psi) \vee f(\phi)$
- $f(\psi U \phi) = \forall(f(\psi) U f(\phi))$
- $f(\Diamond\phi) = \forall\Diamond f(\phi)$
- $f(\Box\phi) = \forall\Box f(\phi)$

Prove that there exists a formula ϕ of LTL and a transition system TS such that $TS \models \phi$ and $TS \not\models f(\phi)$.