TD3 MVFA: Finite and Büchi automata

Exercise 1

Let $n \geq 1$. Consider the language $\mathcal{L}_n \subseteq \Sigma^*$ over the alphabet $\Sigma = \{a, b\}$ that consists of all finite words where the symbol b is on position n from the right, *i.e.*, $\mathcal{L}_n$ contains exactly the words $\alpha_1\alpha_2 \ldots \alpha_k \in \{a, b\}^*$ where $k \geq n$ and $\alpha_{k-n+1} = b$. For instance, the word *abbaabab* is in $\mathcal{L}_3$.

1. Construct a NFA $\mathcal{A}_n$ with at most $n + 1$ states such that $\mathcal{L}(\mathcal{A}_n) = \mathcal{L}_n$.
2. Determinize this NFA $\mathcal{A}_n$ using the powerset construction algorithm.

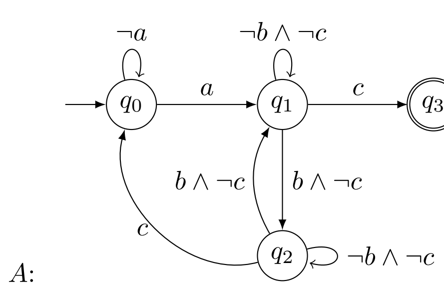
Exercise 2

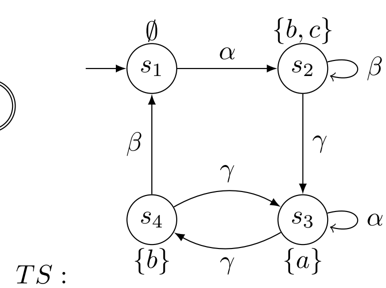
Let C a circuit with one input bit x, one output y and one register r. C is defined by the law $f_y(x, r) = \delta_r(x, r) = x \bigoplus r$.

1. Give a transition system for C. (Use $AP = \{y\}$)
2. We want to verify the property "the circuit will never output two ones after each other". Give a DFA recognizing the bad prefixes.
3. Use the product between a transition system and a DFA to check whether or not C verify the property.

Exercise 3

Let $AP = \{a, b, c\}$. Consider the following NFA $\mathcal{A}$ (over the alphabet 2^{AP}) and the following transition system TS:





Construct the product $TS \otimes \mathcal{A}$ of the transition system and the NFA.

Exercise 4

Let $\Sigma = \{a, b\}$. Construct an NBA $\mathcal{A}$ that accepts the set of infinite words σ over Σ such that a occurs infinitely many times in σ and between two successive a's an odd number of b's occur.

Exercise 5

We look at the emptiness problem for NBA, i.e. whether or not $\mathcal{L}(\mathcal{A}) = \emptyset$ for a NBA$\mathcal{A}$.

1. Show that this problem can be reduced to a graph problem.
2. Show that this graph problem can be reduced to the emptiness problem for NBA.
3. Give an algorithm for this problem on graphs.