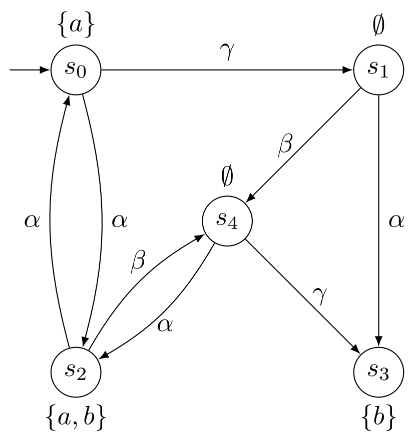
TD1 MVFA: modeling concurrent systems

Exercise 1

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system.

- TS is called *action-deterministic* if $|I| \leq 1$ and $|Post(s, \alpha)| \leq 1$ for all states s and actions α.
- TS is called *AP-deterministic* if $|I| \leq 1$ and $|Post(s) \cap \{s' \in S \mid L(s') = A\}| \leq 1$ for all states $s \in S$ and $A \in 2^{AP}$.

Consider the following transition system TS_1.



1. Give the formal definition of TS_1.
2. Specify a finite and an infinite execution of TS_1.
3. Show whether TS_1 is an AP-deterministic or an action-deterministic transition system.

Exercise 2

We consider the handshaking operator $||_H$ with H a set of actions. The definition is the same than the interleaving operator, except for the transition function for actions $\alpha \in H$ in $TS_1||_H TS_2$: instead of just taking one of the transitions in TS_1 or TS_2 by α, both transition systems must take a transition by α simultaneously. For actions $\alpha \notin H$, the definition of the transition function is the same than in the interleaving operator.

1. Formally define $TS_1||_H TS_2$ for two transition systems TS_1 and TS_2.
2. Show that $||_H$ is associative, that is, for any transition systems TS_1, TS_2, TS_3:

$$(TS_1||_H TS_2)||_H TS_3 = TS_1||_H(TS_2||_H TS_3)$$

Exercise 3

We are given two processes P_1, P_2 with shared integer variable x. The program of process P_i is as follows:

for $k_i = 1, \ldots, N_i$ **do**
  LOAD(x);
  INC(x);
  STORE(x);
end

Algorithm 1: Process P_i

with $N_i \geq 1$. That is, P_i executes N_i times the assignment $x := x + 1$. The assignment $x := x + 1$ is realized using the three actions LOAD(x), INC(x) and STORE(x). Consider the parallel program:

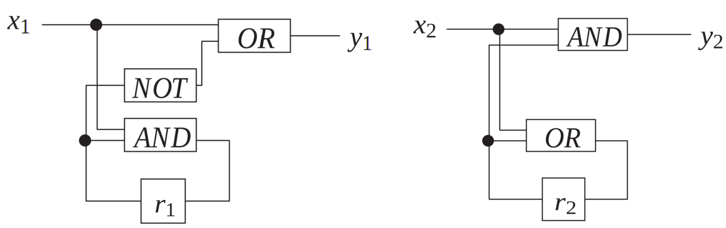
$x := 0$
$P_1 || P_2$

Algorithm 2: Parallel program P

1. Sketch the program graph for P with $N_1 = N_2 = 2$. Does P have an execution that halts with the terminal value $x = 2$?
2. What is the size of the program graph for $N_1 = N_2 = 100$? For $N_1 = N_2 = 10000$? Same question for the associated transition system.

Exercise 4

We consider the following two sequential hardware circuits:



Circuit 1 Circuit 2

For each $i \in \{1, 2\}$, x_i is the input of Circuit i, y_i the output and r_i is a register.

The values of r_i and y_i depend on the current value of x_i and the previous value of r_i. For instance, the new value of r_2 is determined by the function $\delta_{r_2}(r_2, x_2) = r_2 \vee x_2$, and The value of the output y_i depends on the values of x_i and r_i, for instance the value of y_2 corresponds to the value of the function $f_{y_2}(x_2, r_2) = x_2 \wedge r_2$.

1. Write the functions f_{y_1} and δ_{r_1}.
2. Represent Circuit 1 and Circuit 2 as transition systems.
3. Determine the reachable part of the transition systems of the synchronous product of these transition systems. Assume that the initial values of the registers are $r_1 = 0$ and $r_2 = 1$.