Multivariate and Supervised Approaches for Mathematical Morphology in Remote Sensing

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Mathematical Morphology is a powerful toolbox for image analysis but . . .

How to extend it to multivariate data such as colour/multispectral/hyperspectral images ?

How to involve some domain knowledge into the morphological processes ?

Outline

Basics of Mathematical morphology

Definition

A framework for **analysis of spatial structures in images**.
A set of operators applied on an image f with a pattern (SE) B.
Two fundamentals operators (erosion ε and dilation δ) ...
... from which are built more complex ones.

Theoretical framework

For binary images, MM may be formalized with set theory ...
but not for more complex images (grayscale, multispectral).
Thus MM is rather defined using **complete lattice theory**.

Basics of Mathematical morphology

Complete lattice theory

A complete lattice is defined from :

- a partially **ordered set** $(L, \geq)$ *(e.g. the natural order of scalars)*
- an **infimum** or greatest lower bound $\wedge$ *(e.g. minimum)*
- a **supremum** or least upper bound $\vee$ *(e.g. maximum)*

Main operators

Erosion : $\varepsilon_B(f)(x,y) = \bigwedge_{(r,s)\in B} f(x+r, y+s), \ (x,y) \in E$

Dilation : $\delta_B(f)(x,y) = \bigvee_{(r,s)\in B} f(x-r, y-s), \ (x,y) \in E$

Opening : $\gamma_B(f) = \delta_B(\varepsilon_B(f))$

Closing : $\phi_B(f) = \varepsilon_B(\delta_B(f))$

Orderings : Definitions and Properties

A binary relation $\mathcal{R}$ on a set $\mathcal{S}$ is

- **reflexive** if $x \mathcal{R} x, \ \forall x \in \mathcal{S}$
- **anti-symmetric** if $x \mathcal{R} y$ and $y \mathcal{R} x \Rightarrow x = y, \ \forall x, y \in \mathcal{S}$
- **transitive** if $x \mathcal{R} y$ and $y \mathcal{R} w \Rightarrow x \mathcal{R} w, \ \forall x, y, w \in \mathcal{S}$
- **total** if $x \mathcal{R} y$ or $y \mathcal{R} x, \ \forall x, y \in \mathcal{S}$

An ordering is a reflexive, anti-symmetric and transitive relation.

It is :

- a **pre-ordering** if the anti-symmetry statement does not hold
- a **partial** ordering if the totality statement does not hold
- a **total** ordering if the totality statement holds

Outline

Principles

MM for multivariate data

- multivalued images : $f : \mathbb{Z}^2 \rightarrow \mathbb{R}^n, \quad n > 1$
- spectral signature : $f(x, y) = [f_1(x, y), \ldots, f_n(x, y)]$

Main operators

Opening : $\gamma_B^v(f) = \delta_B^v(\varepsilon_B^v(f))$
Closing : $\phi_B^v(f) = \varepsilon_B^v(\delta_B^v(f))$

Erosion : $\varepsilon_B^v(f)(x, y) = \inf^v_{(r,s) \in B} \{f(x + r, y + s)\}$
Dilation : $\delta_B^v(f)(x, y) = \sup^v_{(r,s) \in B} \{f(x - r, y - s)\}$

... but there is no universal ordering relation between vectors !
e.g. min or max of $((3, 1), (5, 5), (4, 2), (2, 4), (1, 1)) = ?$

Barnett taxonomy of vectorial orderings (1976)

4 classes of orderings

Marginal : data are ordered independently inside each band
$\forall \mathbf{v}, \mathbf{v}' \in \mathbb{R}^n, \;\; \mathbf{v} \leq \mathbf{v}' \Leftrightarrow \forall\, i \in \{1, \ldots, n\}, \;\; v_i \leq v'_i$

Conditional : data are ordered according to an ordered (sub)set of available bands (e.g. lexicographical ordering).

Partial : data are split into groups, and then ordered inside each group but not between different groups.

Reduced : data are reduced to scalar values and then ordered according to these values :
$\forall \mathbf{v}, \mathbf{v}' \in \mathbb{R}^n, \;\; \mathbf{v} \leq \mathbf{v}' \Leftrightarrow h(\mathbf{v}) \leq h(\mathbf{v}'), \;\; h : \mathbb{R}^n \rightarrow \mathbb{R}$

Processing strategies

Marginal processing

Pixel = scalar value, processed independently in each band.

+ : no need to adapt existing algorithms.

- : correlation between band is ignored.

- : input vectors are not preserved.

Vectorial processing

Pixel = vector, processed simultaneously in all bands.

+ : correlation between band is taken into account.

- : need to adapt existing algorithms.

- : may be computationally prohibitive

Partial orderings : marginal approach

Marginal approach

It is a variant of vectorial approach using a partial ordering.

Erosion

$$\varepsilon_b^v(\mathbf{f})(\mathbf{x}) = [\varepsilon_b(f_1)(\mathbf{x}), \ldots, \varepsilon_b(f_n)(\mathbf{x})]^T$$

Dilation

$$\delta_b^v(\mathbf{f})(\mathbf{x}) = [\delta_b(f_1)(\mathbf{x}), \ldots, \delta_b(f_n)(\mathbf{x})]^T$$

Partial orderings : matrix approach

Generalisation of the marginal approach

Each image $\mathbf{F}$ is processed with a matrix of structuring elements $\mathbf{B} = \{B_{ij},\ \ i,j \in \{1,\ldots,n\}\}$.
Equivalent to marginal approach if :
$\mathbf{B} = \{B_{ij} \mid (B_{ij} = \emptyset,\ \ i \neq j) \wedge B_{ii} = b,\ \ i,j \in \{1,\ldots,n\}\}$

Erosion

$\varepsilon_{m,\mathbf{B}}(\mathbf{F}) = (E_1,\ldots E_n)$ with $E_i = \bigwedge_{j=1}^{n} \varepsilon_{B_{ji}}(F_j),\ \ i \in \{1,\ldots,n\}$

Dilation

$\delta_{m,\mathbf{B}}(\mathbf{F}) = (D_1,\ldots D_n)$ with $D_i = \bigvee_{j=1}^{n} \delta_{B_{ij}}(F_j),\ \ i \in \{1,\ldots,n\}$

Partial orderings : comments

Drawbacks of marginal approaches

Correlation between band is ignored.
Spectral composition of images may be altered.

Space transformation (decorrelation)

- Principal Component Analysis
- Discrete Cosine Transform
- Perceptual colour space

Corrected marginal approach

Computed values can be replaced by the closest existing values in the input image, thus input vectors are preserved.

Total pre-orderings

Reduced approaches

- ordering based on a cumulative distance
  $\forall\, \mathbf{v}_k, \mathbf{v}_l \in \{\mathbf{v}_j\}\,,\ \ \mathbf{v}_k \leq \mathbf{v}_l \Leftrightarrow \sum_j d(\mathbf{v}_k, \mathbf{v}_j) \leq \sum_j d(\mathbf{v}_l, \mathbf{v}_j)$
- ordering based on a distance to a reference vector
  $\forall\, \mathbf{v}_k, \mathbf{v}_l \in \{\mathbf{v}_j\}\,,\ \ \mathbf{v}_k \leq \mathbf{v}_l \Leftrightarrow d(\mathbf{v}_k, \mathbf{v}_{ref}) \leq d(\mathbf{v}_l, \mathbf{v}_{ref})$
- ordering based on a double distance
  $h(\mathbf{v}) = w(\mathbf{v}, \mathbf{v}_c) \cdot a(\mathbf{v}, \mathbf{v}_c) + (1 - w(\mathbf{v}, \mathbf{v}_c)) \cdot d_E(\mathbf{v}, \mathbf{v}_c)$

Conditional approaches

- ordering on one component
  $\mathbf{v} \leq \mathbf{v}' \Leftrightarrow v_1 \leq v_1'$
- restricted lexicographical ordering (with $k < n$)
  $\mathbf{v} <_L \mathbf{v}' \Leftrightarrow \exists\, i \in \{1, \ldots, k\}\,,\ \ (\forall\, j < i,\ v_j = v_j') \wedge (v_i < v_i')$

Total pre-orderings : comments

Advantages

Vectors are preserved

Possible symmetry between bands (if desired).

Drawbacks

Several different vectors may be considered as same extrema.

Possible instability of the dilation operator.

Total orderings : some examples

Lexicographical ordering

$$\mathbf{v} <_L \mathbf{v}' \Leftrightarrow \exists\, i \in \{1, \ldots, n\}\,,\ \ (\forall\, j < i,\ v_j = v'_j) \wedge (v_i < v'_i)$$

Lexicographical ordering with additional component

$$\mathbf{v} \leq \mathbf{v}' \Leftrightarrow [\|\mathbf{v}\|, v_1, \ldots, v_n]^T \leq_L [\|\mathbf{v}'\|, v'_1, \ldots, v'_n]^T$$

Lexicographical ordering modulo α

$$\mathbf{v} \leq \mathbf{v}' \Leftrightarrow [\lceil v_1/\alpha \rceil, v_2, \ldots, v_n]^T \leq_L [\lceil v'_1/\alpha \rceil, v'_2, \ldots, v'_n]^T$$

Ordering based on bit mixing

Scalar representation of a vector by mixing the different bits :
$h(\mathbf{v}) = \sum_{m=1}^{k} \left\{ 2^{n\cdot(k-m)} \cdot \sum_{i=1}^{n} 2^{n-i} \cdot v_{i,m} \right\}$

Total orderings : comments

Advantages

Vectors are preserved.
Extrema are always unique.

Drawback

Dissymmetry since a high importance is given to first components.

Solution

Dissymmetry can be attenuated using a space transformation.
Advanced orderings to ensure a better weight between bands.

Total orderings : some balanced approaches

α-trimmed lexicograpical extrema

Keep iteratively only the α% greatest/lowest values of each band.
Consider normalized distance to the extrema to avoid edge effects.
For a finer tuning, specific α_i values can be used for each band.

Quantisation-based α-lexicographical ordering

Avoid uniform quantisation and model desired priority distribution :
$\mathbf{v} \leq \mathbf{v}' \Leftrightarrow [w_1, v_2, \ldots, v_n]^T \leq_L [w'_1, v'_2, \ldots, v'_n]^T$
where w_1 and w'_1 represents the equivalence group of v_1 and v'_1.

Marker-based lexicographical ordering

Priority distribution varies for each pixel, $\forall x, y, s, t \in \mathbb{Z}$:
$\mathbf{g}(x, y) < \mathbf{g}(s, t) \Leftrightarrow$
$[m(x, y), g_2(x, y), \ldots, g_n(x, y)]^T \leq_L [m(s, t), g_2(s, t), \ldots, g_n(s, t)]^T$

Outline

Domain knowledge and MM

Existing methods for knowledge integration

- Structuring elements
  It is possible to define the shape and size of the structuring element depending on the problem under consideration.
  But definition of the SE may be irrelevant for some objects !
- Choice of morphological operators
  To solve a problem, a morphological expert will select the appropriate operators.
  But the expert is not always here !

Two kinds of knowledge

- **Expert knowledge** represented as a database, a set of rules, an ontology
- **Learning samples** or labelled visual examples

Domain knowledge and MM

Various knowledge may be available for the object(s)

- shape(s)
- size(s)
- spectral composition(s)
- textural description(s)
- number and model of classes
- ...

They can be either :

- given and formalized by the expert
- estimated from the learning data

Using expert knowledge

An object is characterized by / a SE can be defined by

- spatial information
- spectral information
- combination of both

Thus several predefined functional SE (shape and intensity profile) can be used, one for each band of interest.

The expert knowledge may be less precise

- only the number of classes of interest is known
- only an approximate size and shape are known

In both cases, several single results are computed (for each cluster, each size, each shape, etc).
There is finally a need to merge the results (e.g. with union).

Using learning samples

Learning dataset and supervised classification process

Spectral/textural information is given by the learning dataset.
This dataset is used by a fuzzy supervised classification process.

A knowledge-based representation space

A pixel is associated with membership values to each class.
Vectors contain class membership values instead of spectral values.
But both can also be combined (e.g. average, product).

Using learning samples

And spatial information ?

Remark : the spatial location of learning samples may also be relevant and could be used.

Towards an iterative process

If posterior evaluation is possible, optimal parameters for the morphological process can be estimated :

- shape
- size
- spectral band
- other parameters (e.g. for oversegmentation reduction)

Outline

Pixel-based Classification using DMP : Experimental setup

Methodology

- Computation of a DMP (using both openings and closings) : $\Delta(f)(x, y)$ (with $\lambda = 5$ and $\omega = 3$)
- Bayes supervised classification with 5 classes : buildings, water, shadow, roads, vegetation
- Learning dataset contains 1% of the full dataset, 10-fold cross-validation

Data

- VHR Image of Strasbourg (1100×900 pixels)
- SPOT 5 simulation, spatial resolution of 1.3m and 3 bands (G, R, NIR)

Multispectral image classification

Visual result with 5 classes :

Results

Precision

Class	Ordering				
	Marginal	Lex.	Norm	Cumul. dist.	Red. Lex.
Roads	83.3	82.4	85.5	85.5	**85.6**
Buildings	69.3	**72.3**	68.6	68.0	68.4
Water	91.3	**96.8**	89.5	89.7	89.8
Vegetation	94.0	**94.4**	93.1	93.0	93.4
Shadow	94.2	92.9	95.1	94.7	**95.2**
Overall	87.0	**88.2**	86.9	86.8	87.0

Comments

- Global quality is rather close for the different orderings
- Disymmetric approaches may be relevant for some classes
- Marginal approach does not preserve spectral signatures !

Object Detection using HMT : Building Detection

Assumptions

A building has a regular shape (e.g. rectangle) but it may contains several materials, thus its content may not be homogeneous but rather composed of homogeneous parts.

Convert the input image to a set of binary images through clustering and addition

Algorithm and results

Algorithm

- Image clustering and generation of binary images (clusters)
- Optimal opening of each cluster (using 2-D granulometry)
- Adaptive Hit or Miss Transform and geodesic reconstruction
- Unification of the results (and additional HMT)

Object Detection using HMT : Coastline Extraction

Extended SEs for Multispectral Hit or Miss Transform

- Size and shift from the origin
- Spectral band
- Class (foreground, background), i.e. operator to be used
- Threshold

HMT fitting

$$\mathrm{Fitting}_{\Omega}(I)(p) = \bigwedge_{s \in \Omega} \mathrm{Fitting}_{s}(I)(p)$$

$$\mathrm{Fitting}_{s}(I)(p) = \begin{cases} \varepsilon_{s_{\text{shape}}}(I_{s_b})(p) \geq s_t & \text{if } s_{\text{type}} = \varepsilon \\ \delta_{s_{\text{shape}}}(I_{s_b})(p) \leq s_t & \text{if } s_{\text{type}} = \delta \end{cases}$$

Coastline Extraction

HMT Valuation

HMT returns a value to each fitted pixel :

$$\mathrm{Valuation}_{\Omega}(I)(p) = \frac{\sum_{s \in \Omega} \mathrm{Valuation}_s(I)(p)}{|\Omega|}$$

with

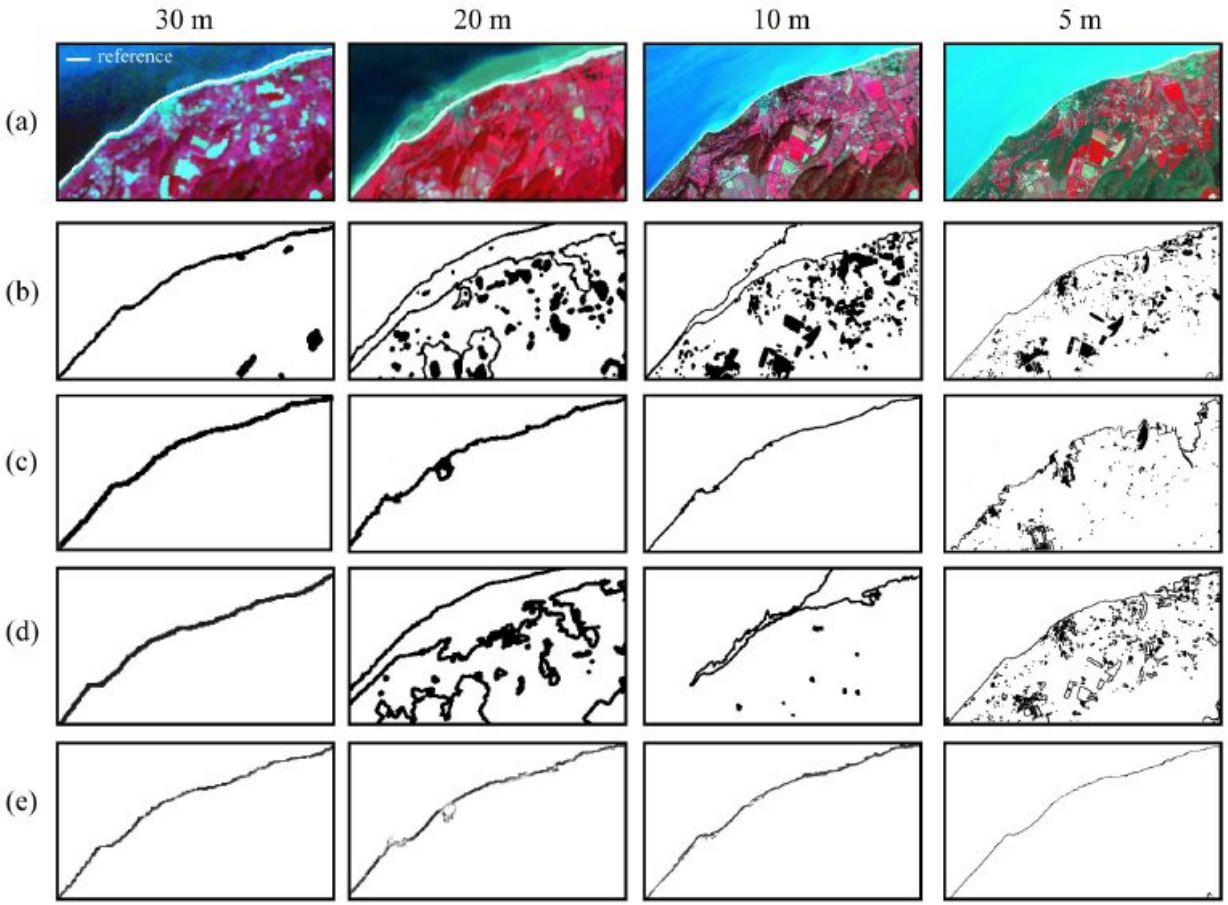
$$\mathrm{Valuation}_s(I)(p) = \begin{cases} \frac{\varepsilon_{s_{\text{shape}}}(I_{s_b})(p) - s_t}{I_{s_b}^{+} - s_t} & \text{if } s_{\text{type}} = \varepsilon \\ \frac{s_t - \gamma_{s_{\text{shape}}}(I_{s_b})(p)}{s_t - I_{s_b}^{-}} & \text{if } s_{\text{type}} = \delta \end{cases}$$

where $[I_k^-, I_k^+]$ is the range of pixel values for the band I_k.

Result on a VHR Image

$$\{s_{\text{shape}} = 180 + 0, \quad s_b = \text{NDVI}, \quad s_t = 0, \quad s_{\text{type}} = \delta\},$$
$$\{s_{\text{shape}} = 800 + 300, \quad s_b = \text{NIR}, \quad s_t = 0,125, \quad s_{\text{type}} = \delta\},$$
$$\{s_{\text{shape}} = -180 + 0, \quad s_b = \text{NDVI}, \quad s_t = 0,004, \quad s_{\text{type}} = \varepsilon\}$$

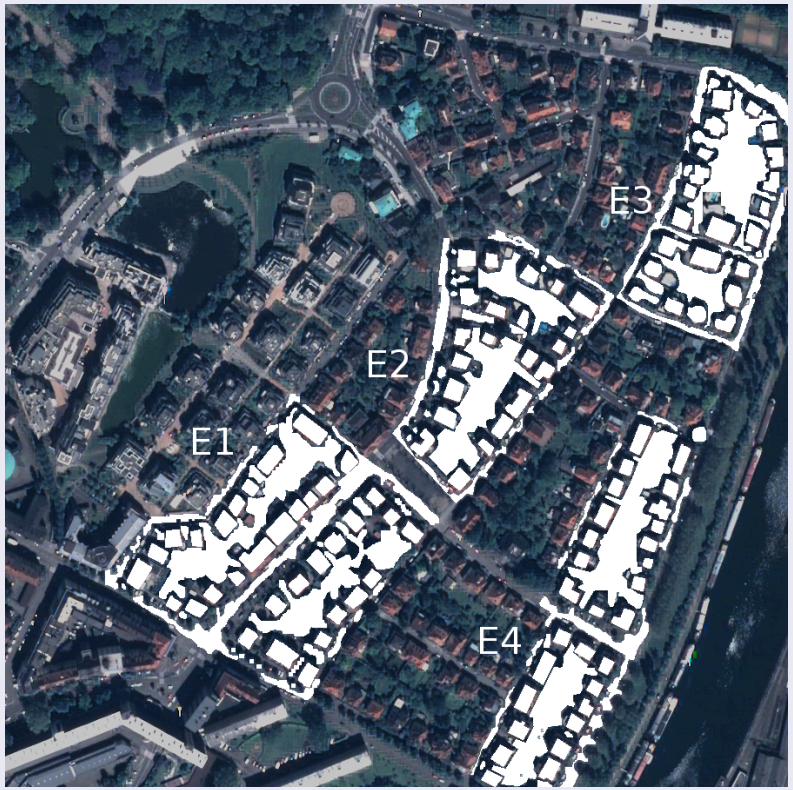
Comparison with other approaches



Watershed Segmentation of urban areas

Context

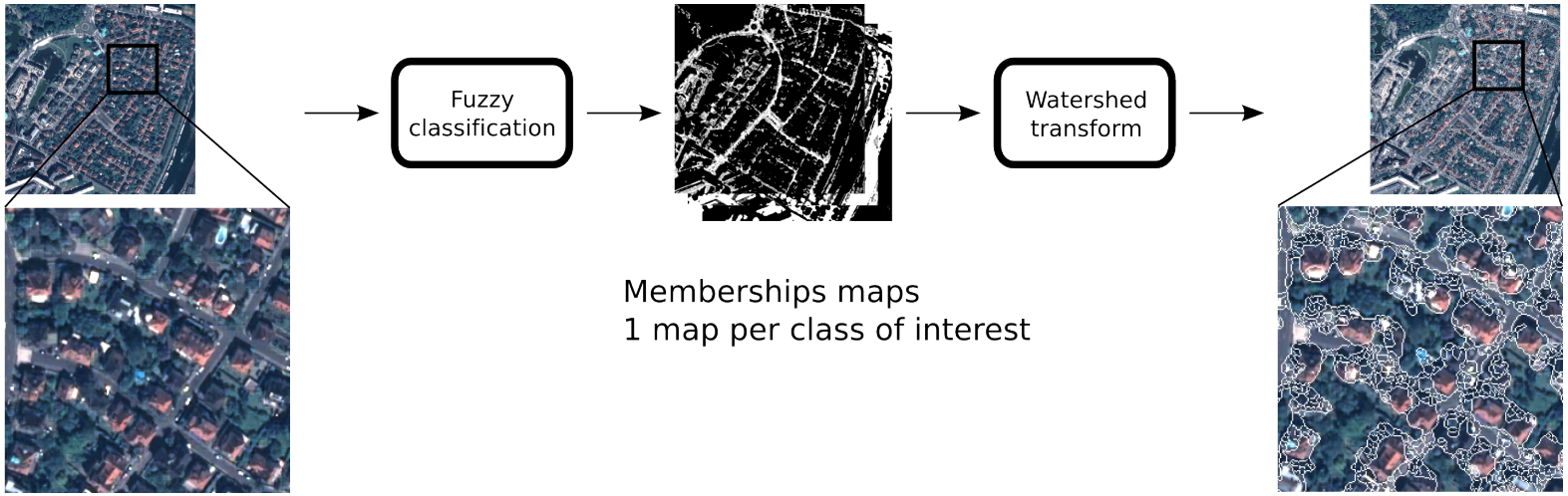
A set of labelled samples is given by the expert.



Segmentation of urban areas

Algorithm

- Conversion from the input space to the membership space
- Compute watershed relief from gradient of membership values
- Perform watershed algorithm
- Evaluate results through classification to optimize oversegmentation reduction parameters.

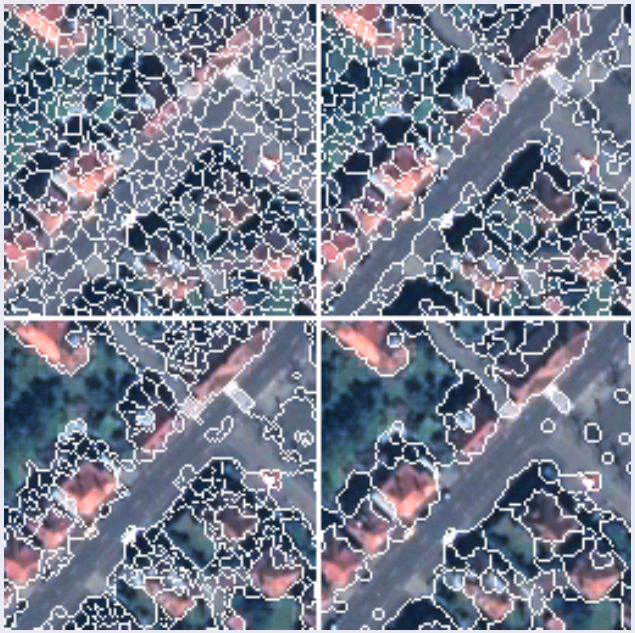


Segmentation of urban areas

Results

unsupervised watershed

optimal watershed



supervised watershed

supervised optimal watershed

Watershed Segmentation of coastal areas

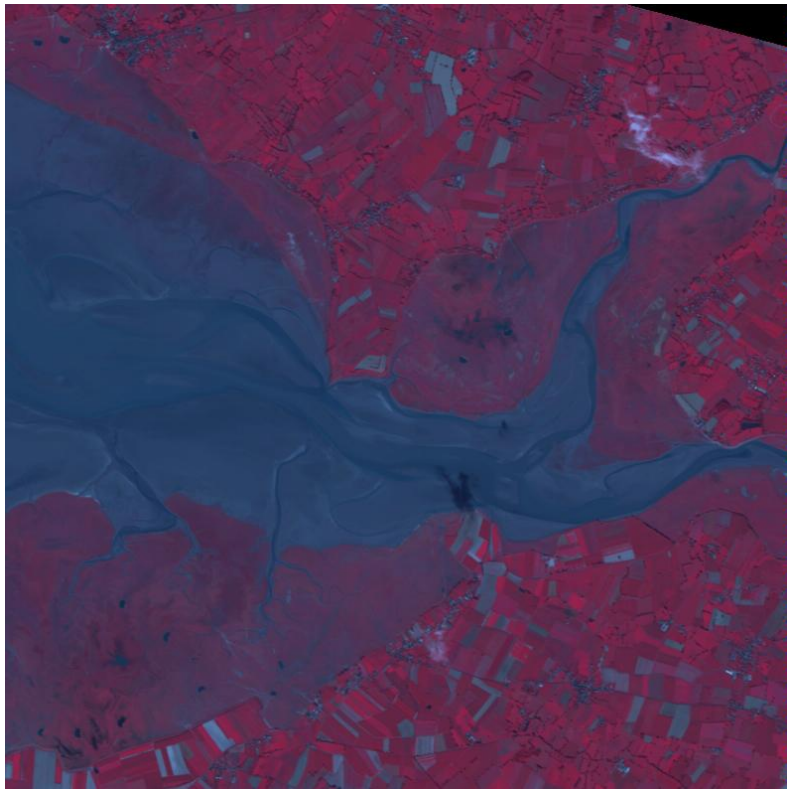
Context, coastline extraction

In such a case, a segmentation into two objects may be useful.

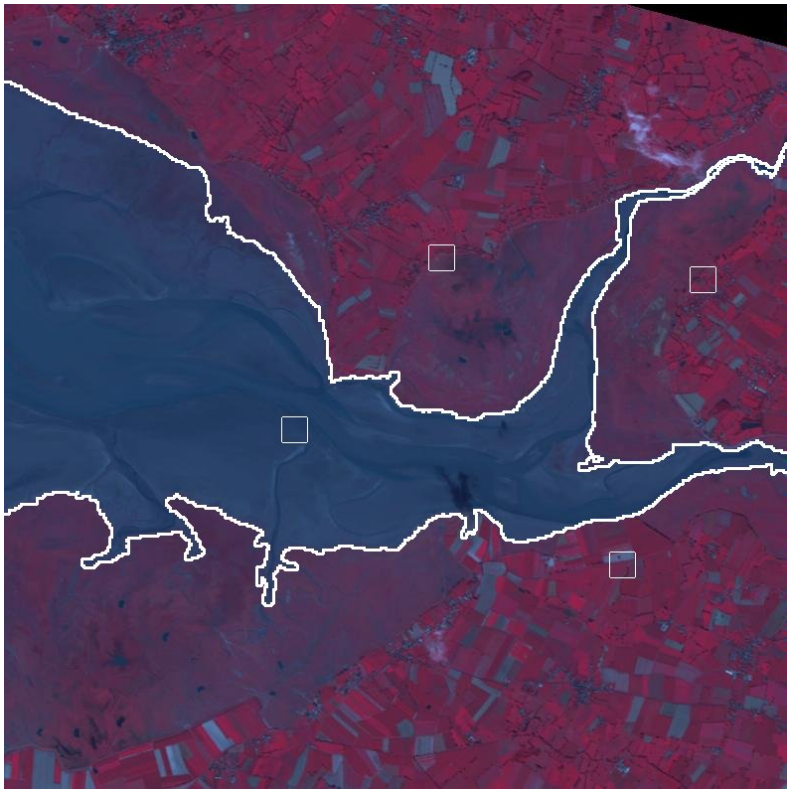
Method

- Manual definition of the markers (used in the learning step)
- Supervised fuzzy classification to build membership maps w_i
- Replace unsupervised relief f by $f_i = (1 - w_i) \cdot f$
- Flood each marker M_i using relief f_i
- p is assigned to M_i if first reached by flooding from M_i

Segmentation of coastal areas

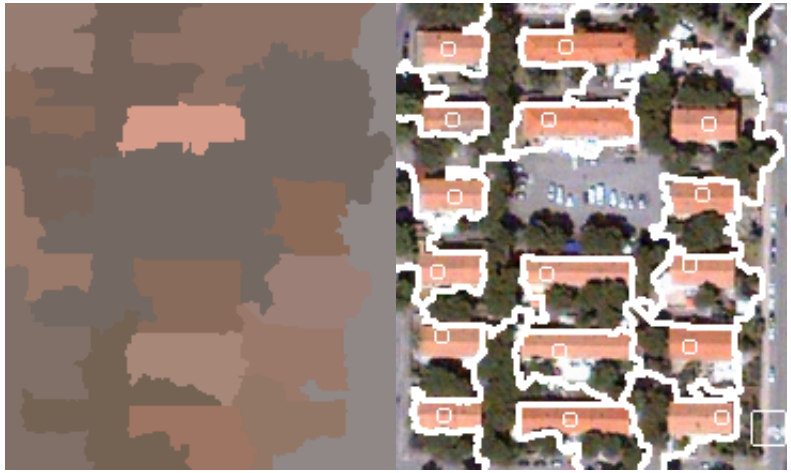


input image



result with markers

Segmentation of urban areas



marker-based watershed

using spectral markers

with one marker for each building

and a single marker
(bottom right)
for the background

Conclusions and Perspectives

Issues

- Adaptation of MM to multispectral/hyperspectral images
- Knowledge integration for MM-based image analysis

Contributions

- Survey of Multivariate MM and new vectorial orderings
- Various ways for using domain knowledge into supervised MM
- Detection, segmentation, classification in urban/coastal RS

Perspectives

- Check the relevance of proposed orderings in the RS field
- Define new physically meaningful orderings for RS
- Combine multivariate and supervised advances

Some further readings ...

Theoretical advances in multivariate MM

- Aptoula & Lefèvre (2007) *A Comparative Study on Multivariate Mathematical Morphology*, Pattern Recognition, 40(11) :2914-2929
- Aptoula & Lefèvre (2008) *On Lexicographical Ordering in Multivariate Mathematical Morphology*, Pattern Recognition Letters, 29(2) :109-118
- Aptoula & Lefèvre (2008) *α-trimmed lexicographical extrema for pseudo-morphological image analysis*, Journal of Visual Communication and Image Representation, to appear

Supervised morphological segmentation methods

- Derivaux et al (2006) *Watershed Segmentation of Remotely Sensed Images Based on a Supervised Fuzzy Pixel Classification*, IEEE IGARSS
- Derivaux et al (2007) *On machine learning in watershed segmentation*, IEEE International Workshop on Machine Learning in Signal Processing
- Lefèvre (2007), *Knowledge from markers in watershed segmentation*, IAPR CAIP

Extensions of the HMT operator for template matching

- Lefèvre et al (2007), *Automatic building extraction in VHR images using advanced morphological operators*, IEEE/ISPRS URBAN
- Weber et al (2008), *Coastline extraction in VHR imagery using mathematical morphology with spatial and spectral knowledge*, ISPRS 2008 Congress

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