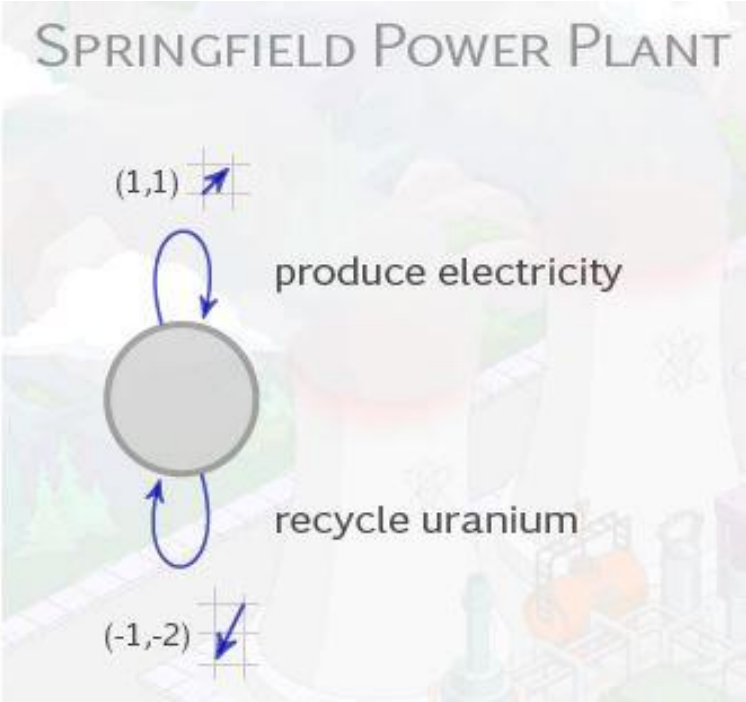
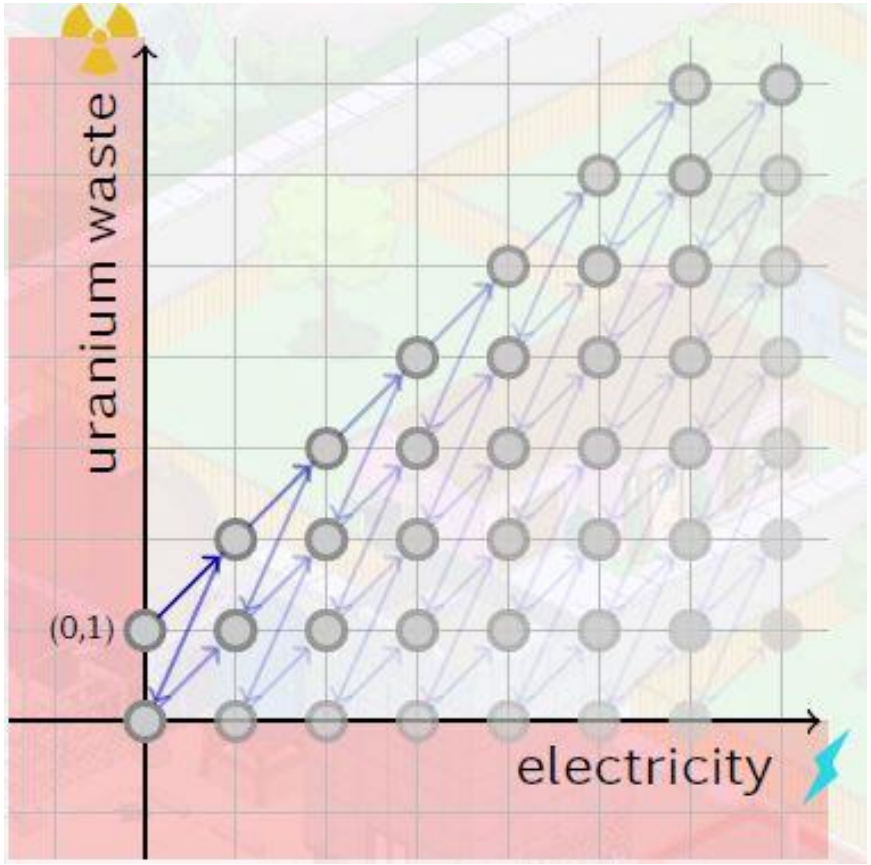
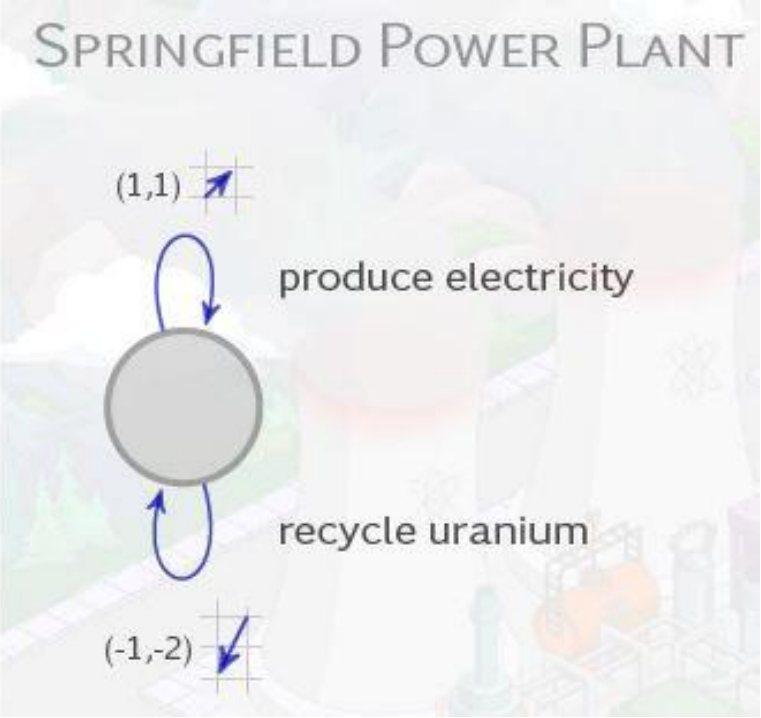
Reachability in 1-VASS with tests

Vector Addition Systems with States

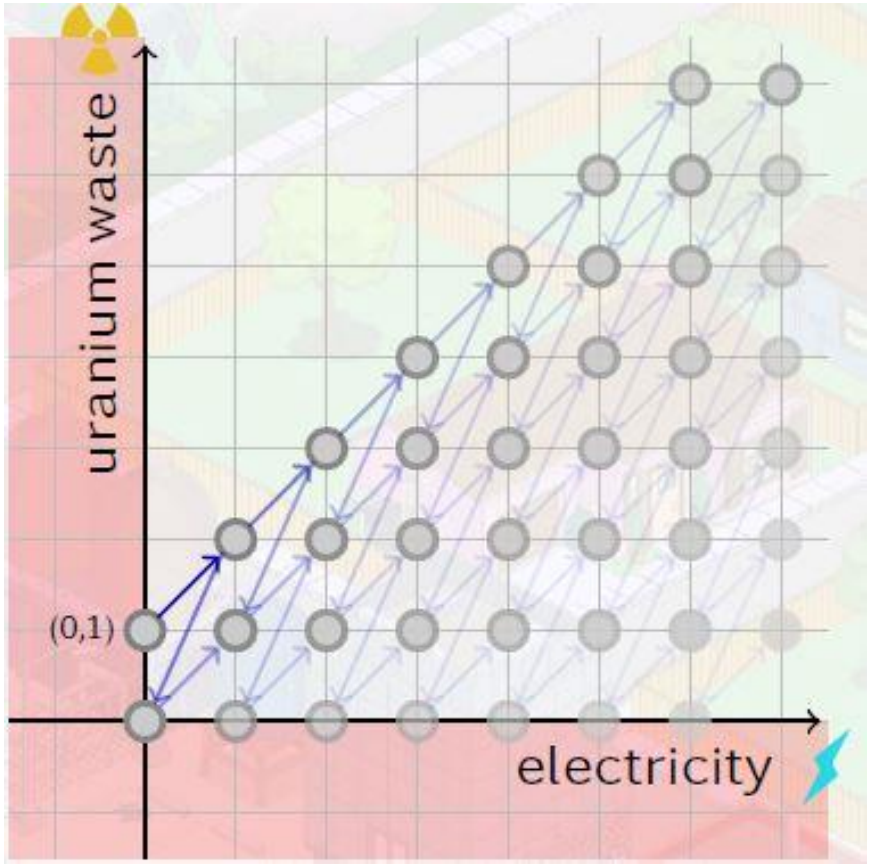


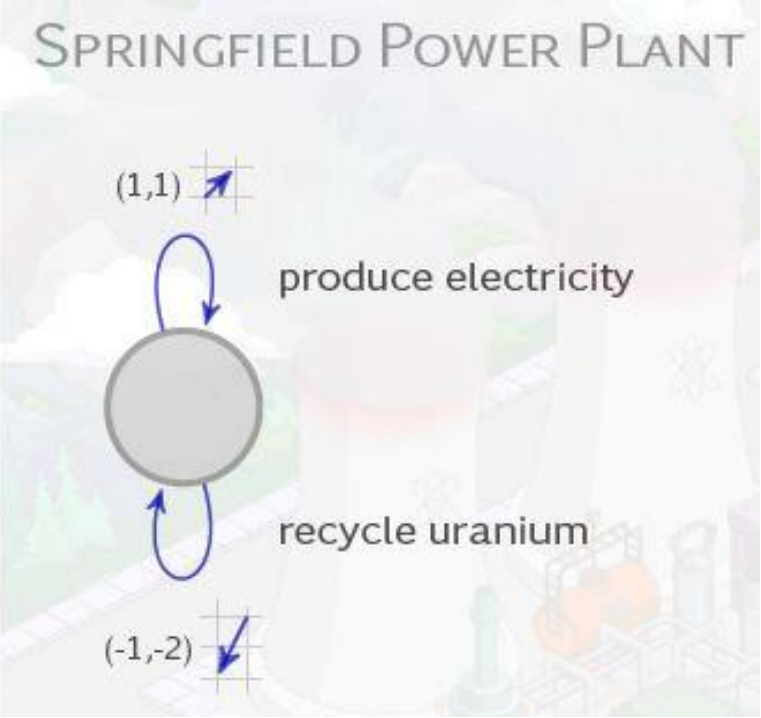
VASS


uranium waste
(0,1)
electricity


SPRINGFIELD POWER PLANT
(1,1)
produce electricity
recycle uranium
(-1,-2)

VASS





Example from *Reachability in Vector Addition Systems is Primitive-Recursive in Fixed Dimension* by Sylvain Schmitz, Highlights 2019

Reachability in VASS

Lower bound: Tower-hard[1]

$$2^{2^{2^{2^{2^{\overbrace{2}^{\text{Input size}}}}}}}$$

Upper bound: Ackermann[2]

$A(4, 2) \sim 20{,}000$ decimal digits

Reachability with $=$ already in 2-VASS is undecidable
(Minsky machine)

1, Wojciech Czerwiński, Sławomir Lasota, Ranko Lazić, Jérôme Leroux, and Filip Mazowiecki. The reachability problem for Petri nets is not elementary, 2019
2, Jérôme Leroux and Sylvain Schmitz. Reachability in vector addition systems is primitive-recursive in fixed dimension, 2019

Reachability in VASS

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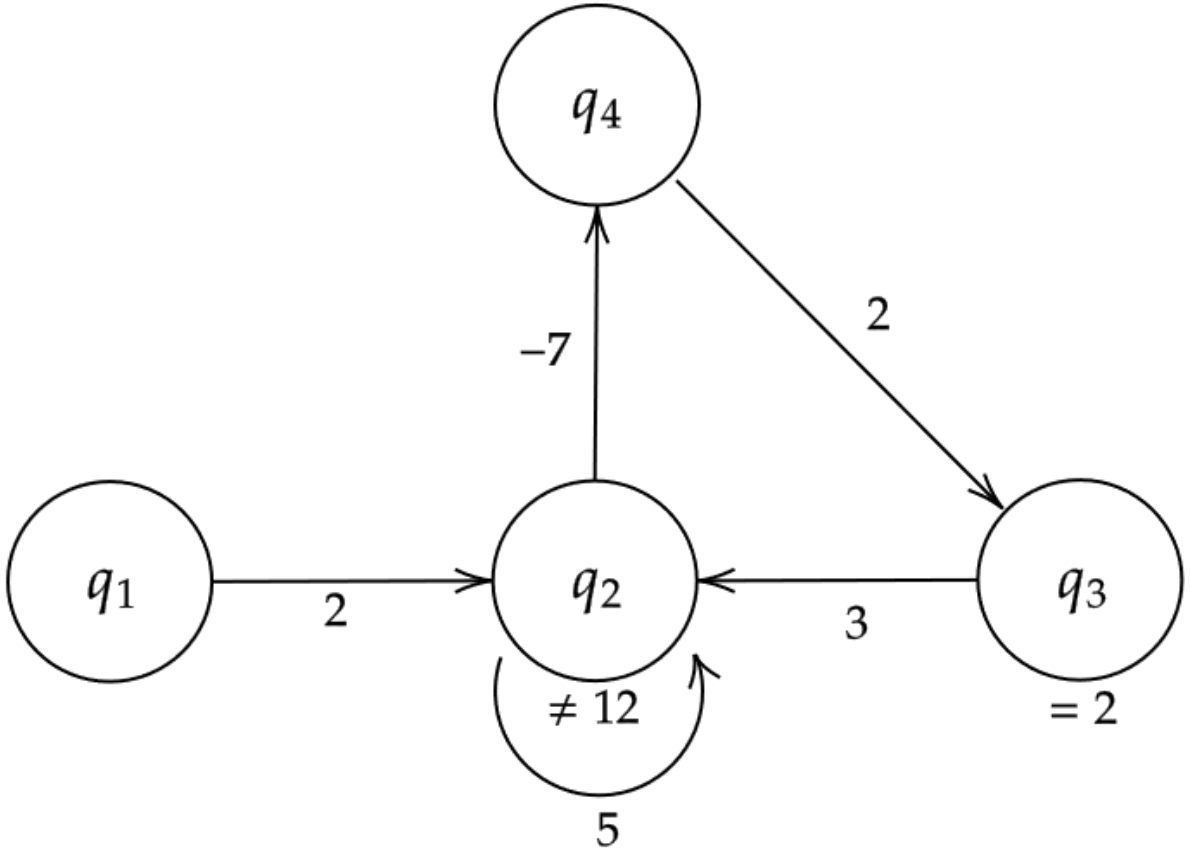
Recent results!

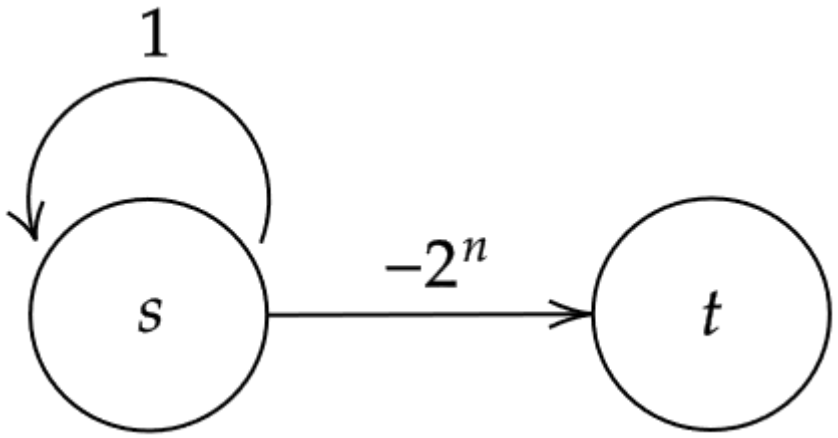
1, Wojciech Czerwiński, Sławomir Lasota, Ranko Lazić, Jérôme Leroux, and Filip Mazowiecki. The reachability problem for Petri nets is not elementary, 2019

2, Jérôme Leroux and Sylvain Schmitz. Reachability in vector addition systems is primitive-recursive in fixed dimension, 2019

1-VASS with tests



Path length can be exponential



Reachability in 1-VASS with tests

Without tests				
NP-complete				

Uses flow = Parikh image of the run

Reachability in 1-VASS with tests

Without tests				In 2-VASS without tests
NP-complete				PSPACE-complete

Reachability in 1-VASS with tests

Without tests	With = tests			In 2-VASS without tests
NP-complete	NP-complete			PSPACE-complete

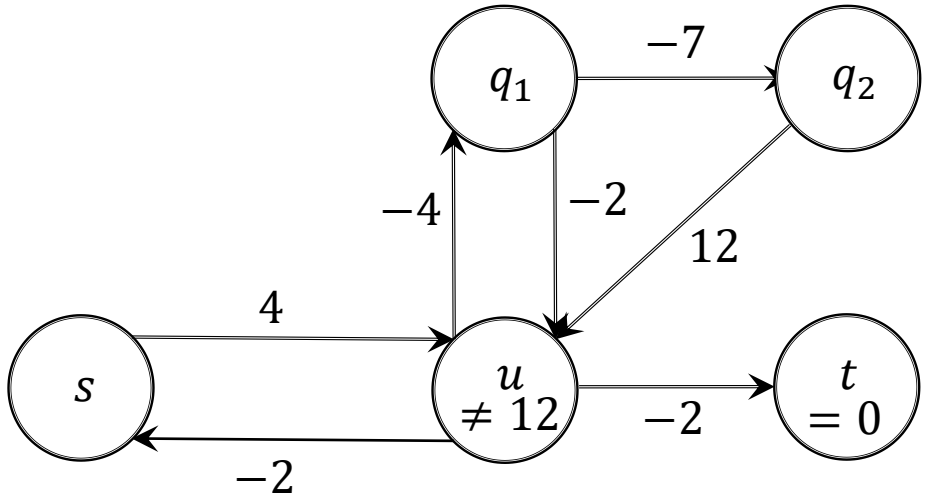
Reachability in 1-VASS with tests

Without tests	With $=$ tests		with $\leq$ & $\geq$	In 2-VASS without tests
NP-complete	NP-complete		PSPACE-complete	PSPACE-complete

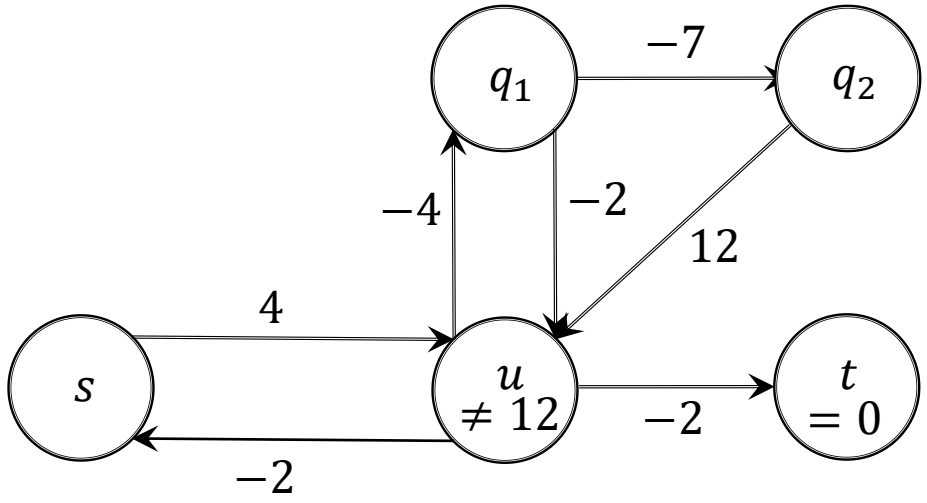
Reachability in 1-VASS with tests

Without tests	With $=$ tests	With $=$ & $\neq$	with $\leq$ & $\geq$	In 2-VASS without tests
NP-complete	NP-complete	Open	PSPACE-complete	PSPACE-complete

1-VASS with $=$ and $\neq$:

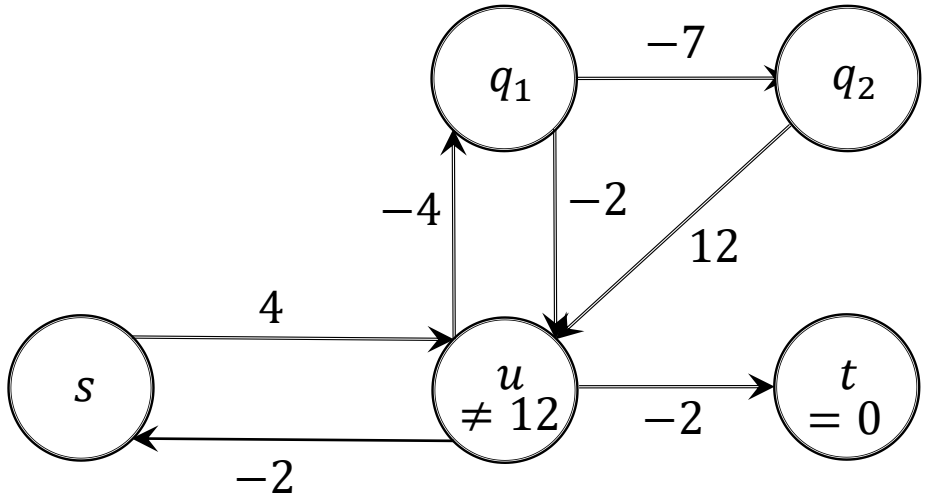


1-VASS with $=$ and $\neq$:



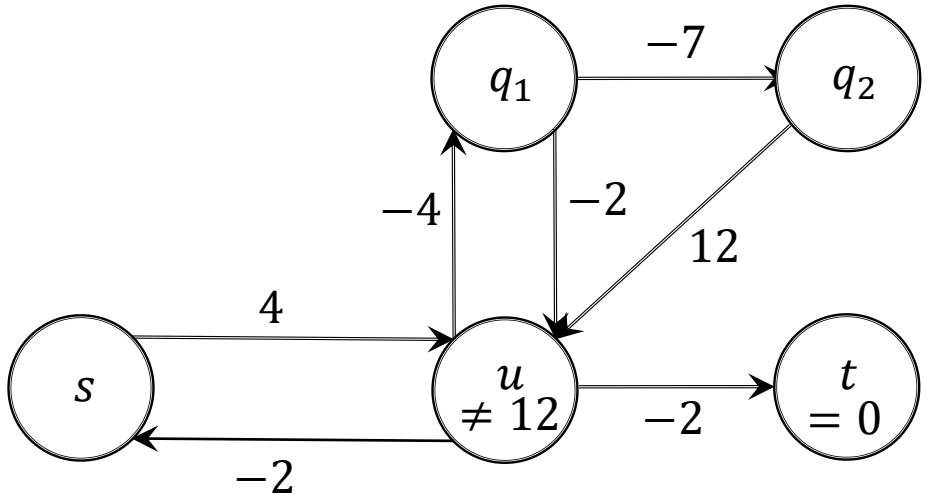
Reachability: PSPACE, NP-hard

1-VASS with $=$ and $\neq$:



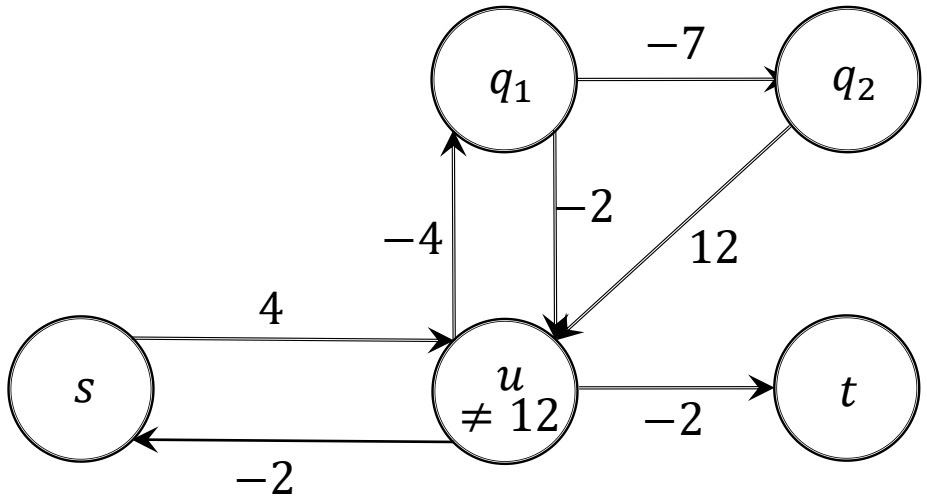
My conjecture = NP-complete

1-VASS with $=$ and $\neq$:



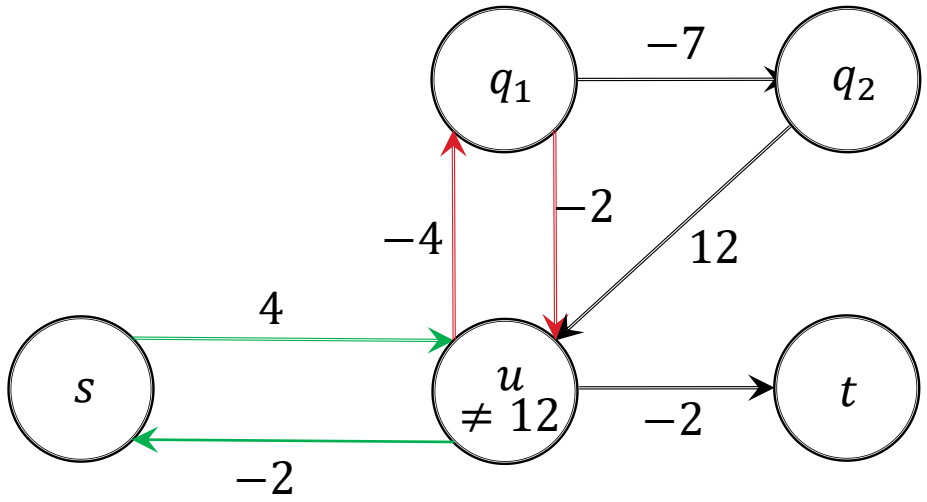
My result: NP-membership when all $\neq$ are on the same state

Proving NP-Membership



Valid run: ρ = (s,0), (u,4), (s,2), (u,6), (s,4), (u,8), (q_1,4), (u,2), (t,0)

u-simple cycles

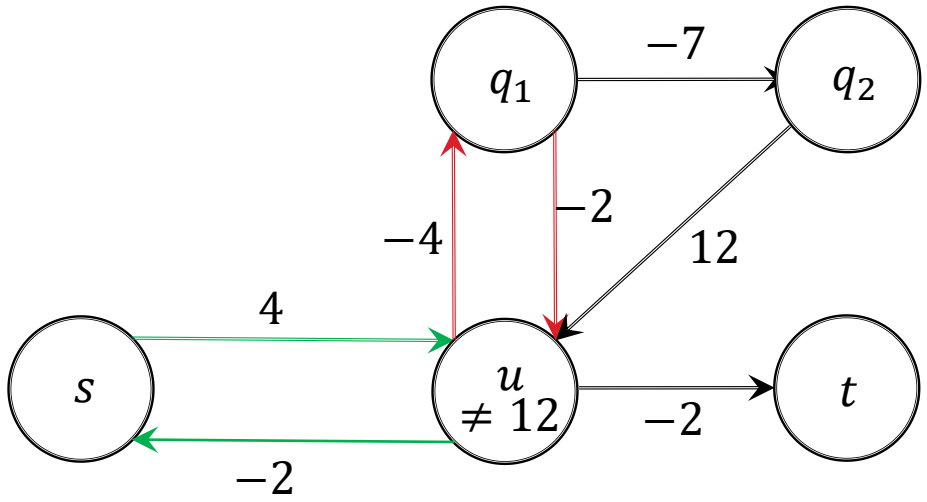


Valid run: ρ = (s,0), (u,4), (s,2), (u,6), (s,4), (u,8), (q_1,4), (u,2), (t,0)

c_1 c_1 c_2

z_i New ρ z_f

u-simple cycles



$weight(c_1) = 2$
$weight(c_2) = -6$

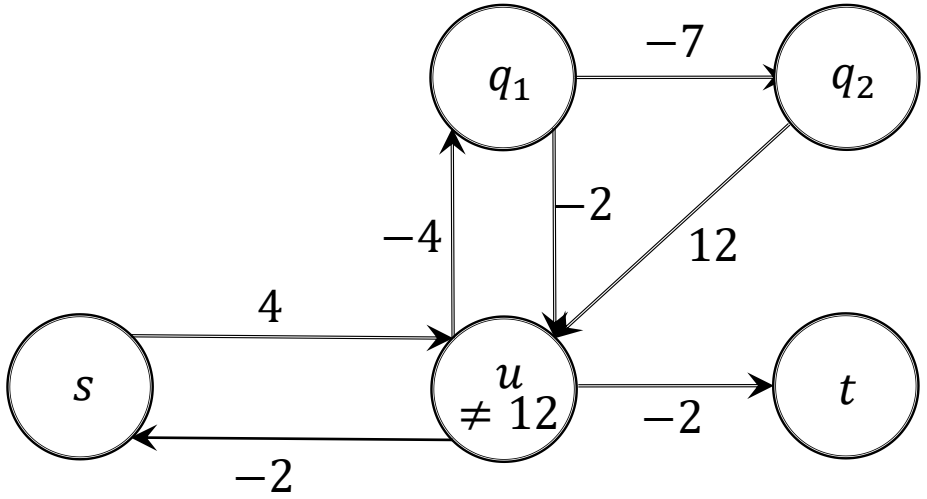
$drop(c_1) = 2$
$drop(c_2) = 6$

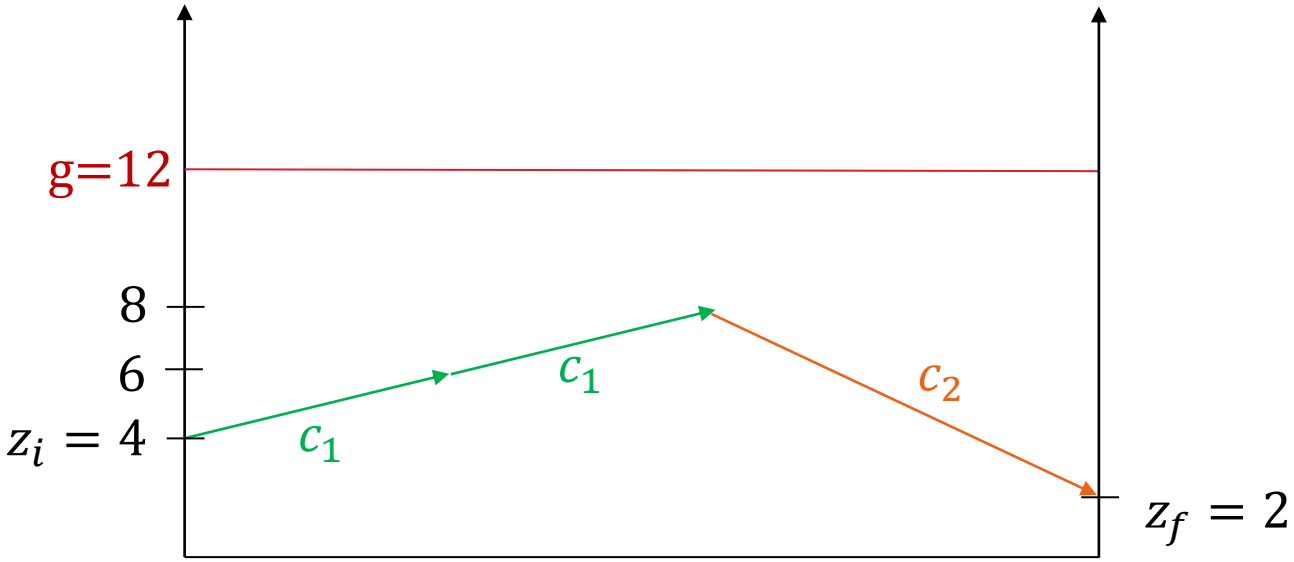
Valid run: ρ = (s,0), (u,4), (s,2), (u,6), (s,4), (u,8), (q_1,4), (u,2), (t,0)

c_1 c_1 c_2

z_i New ρ z_f

u-simple cycles



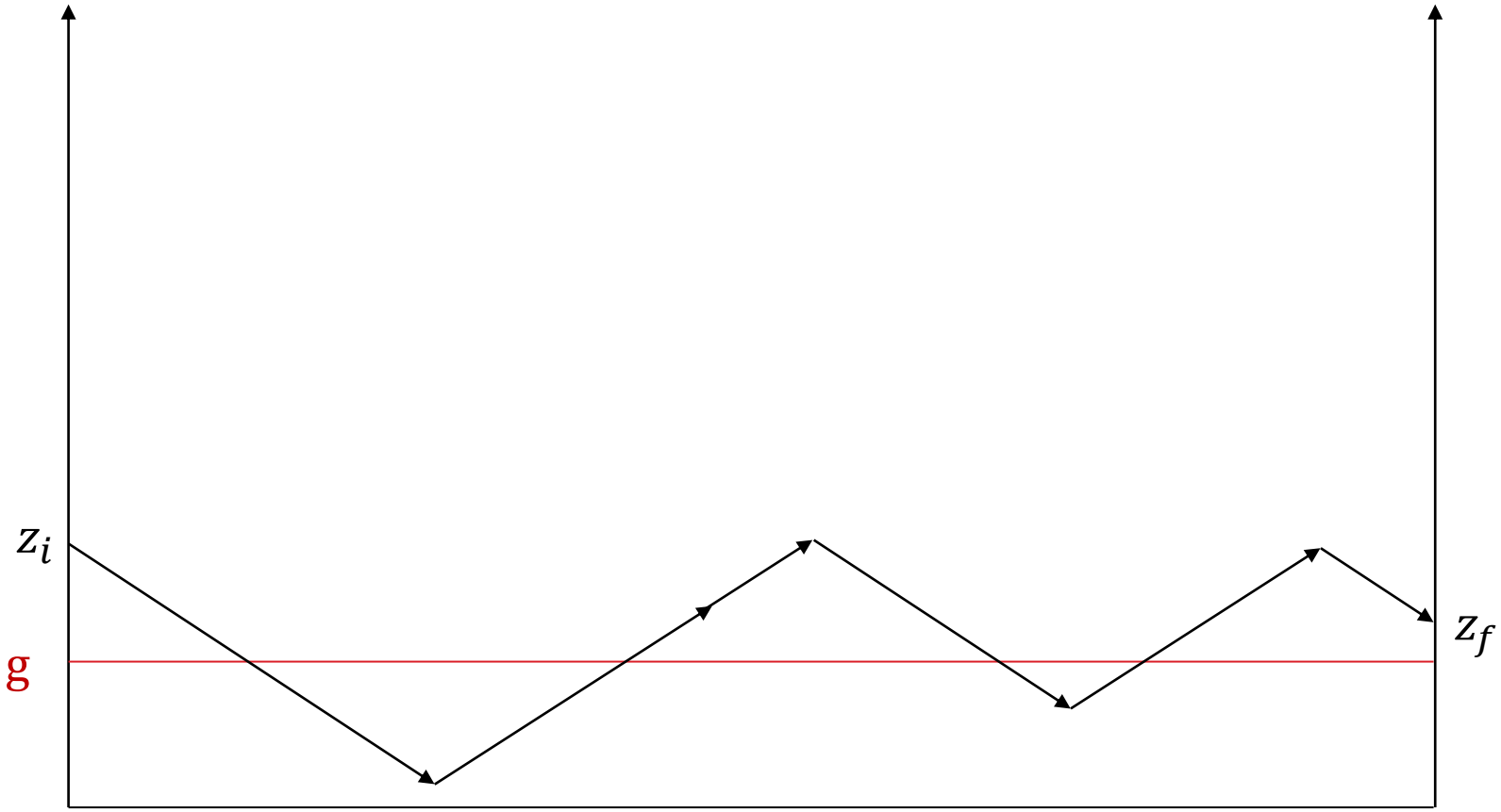


Valid run: ρ = (s,0), (u,4), (s,2), (u,6), (s,4), (u,8), (q_1,4), (u,2), (t,0)

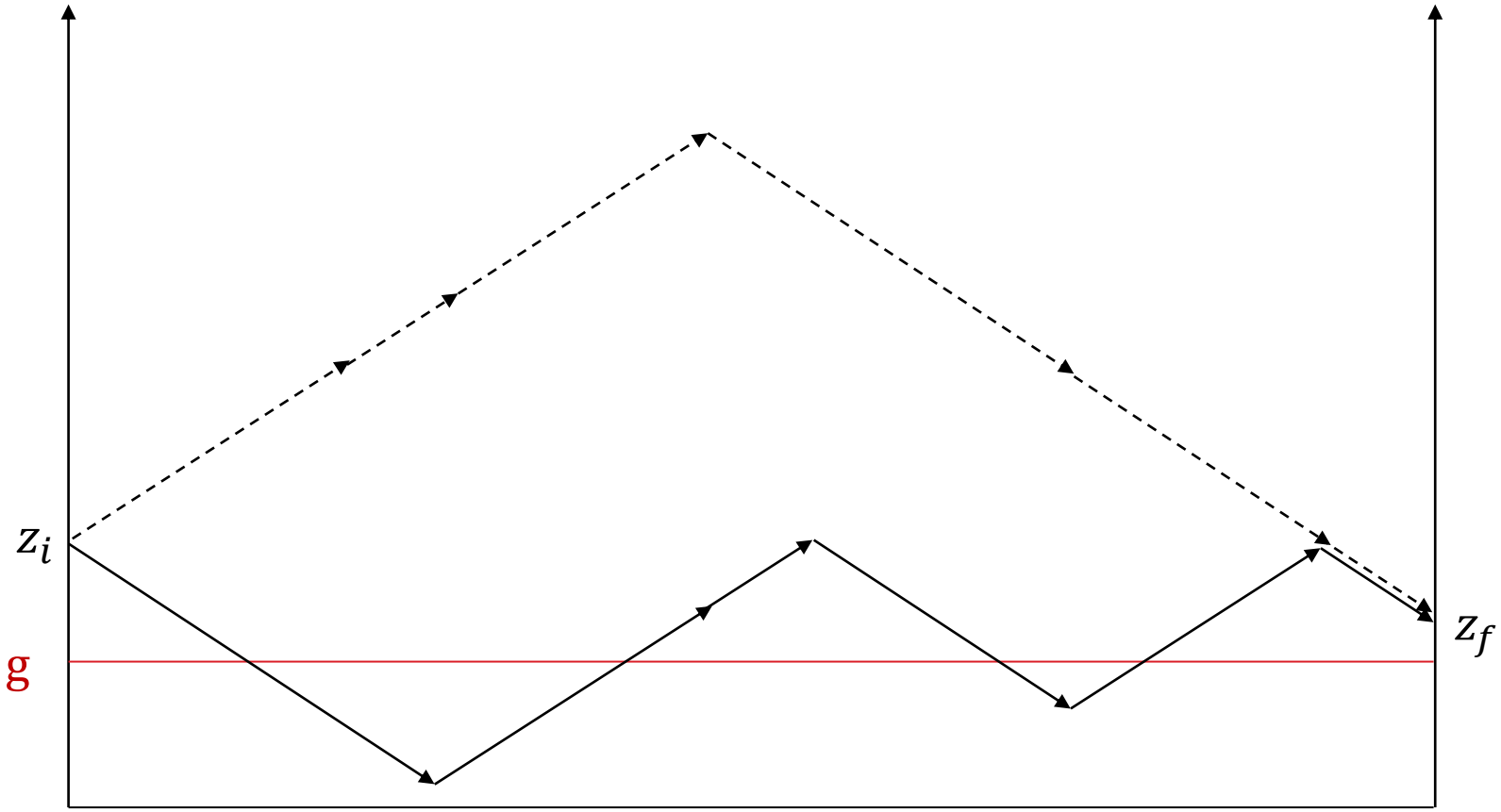
c_1 c_1 c_2

z_i New ρ z_f

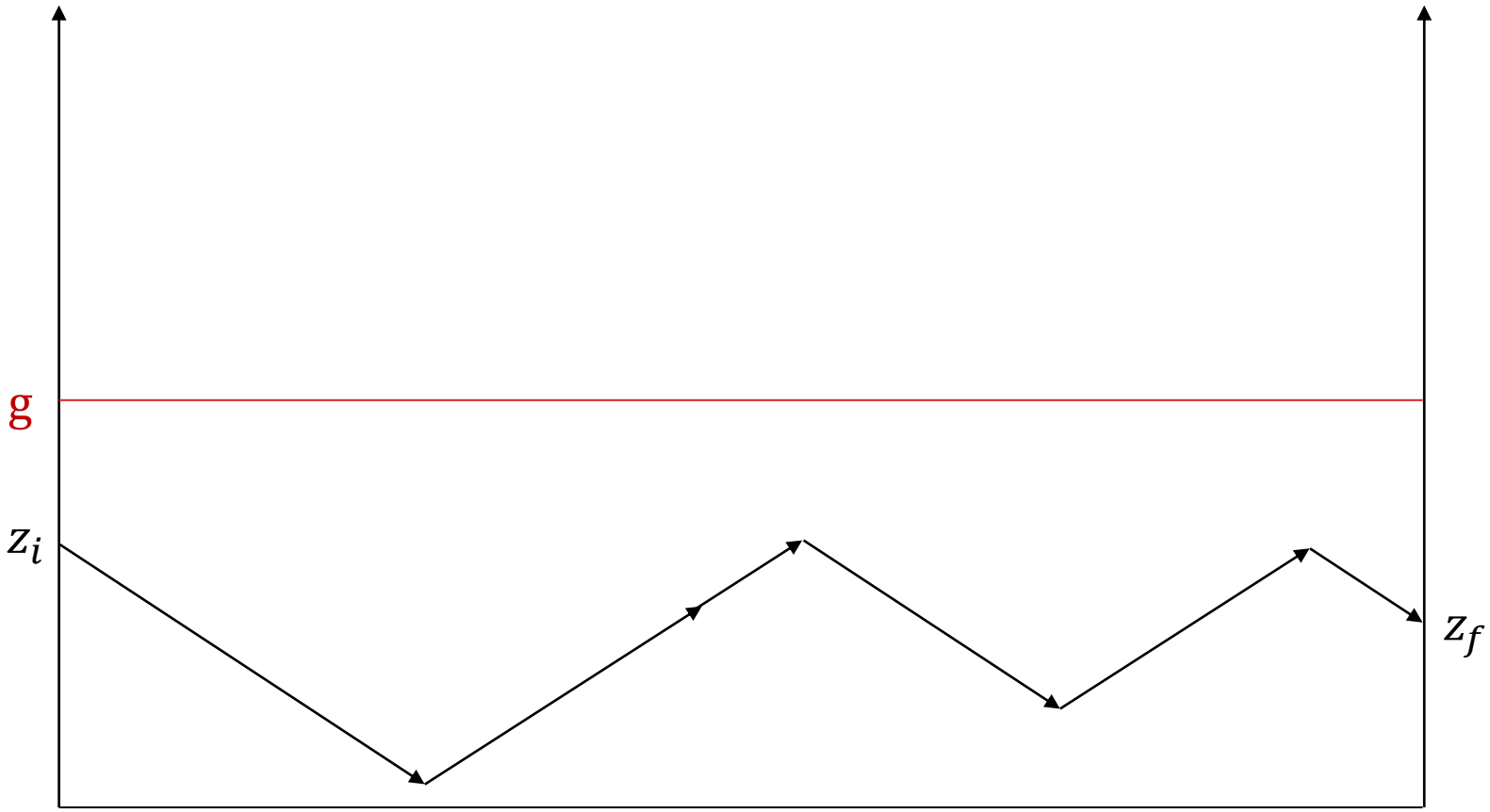
Above the guard



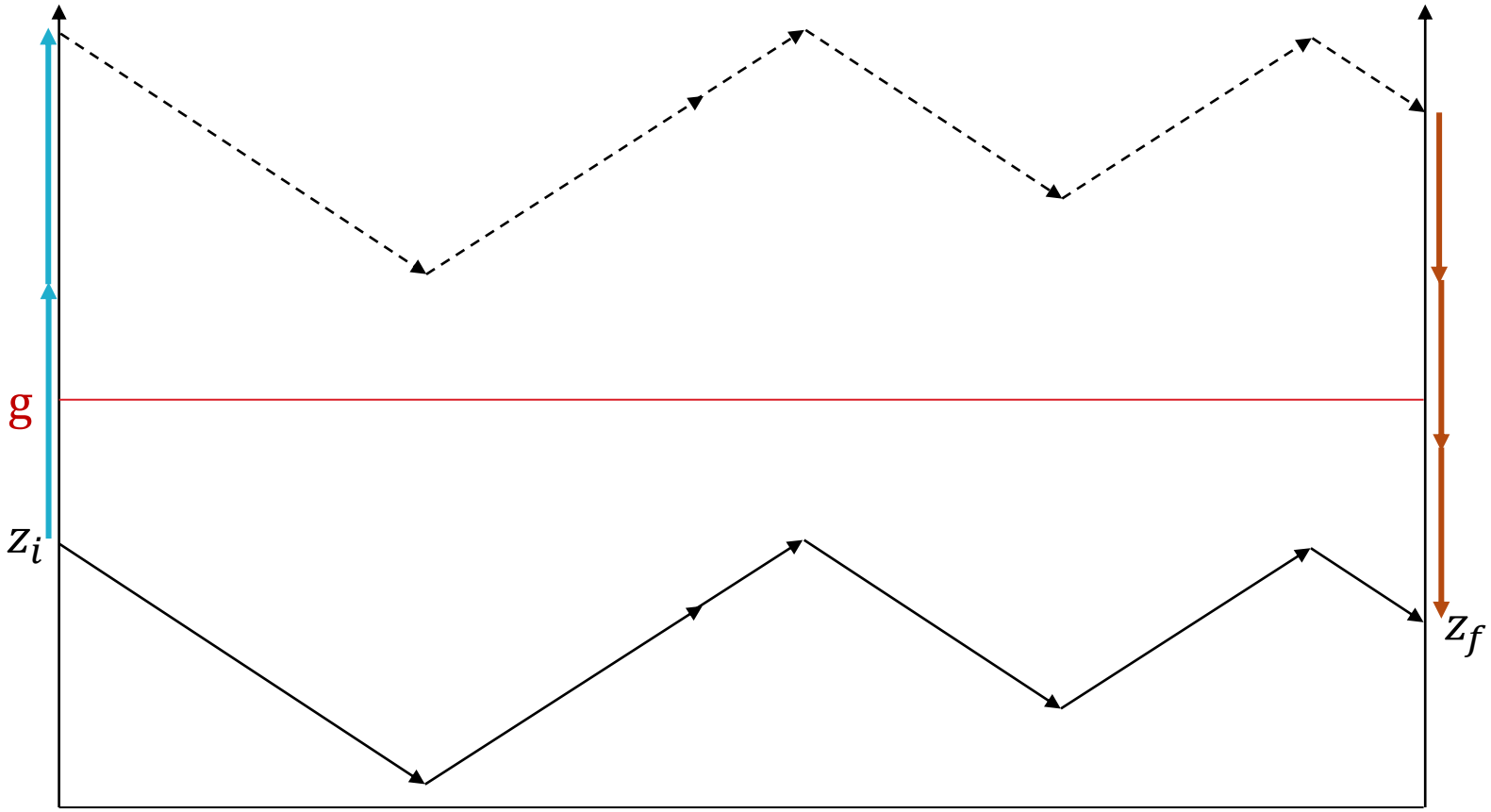
Above the guard



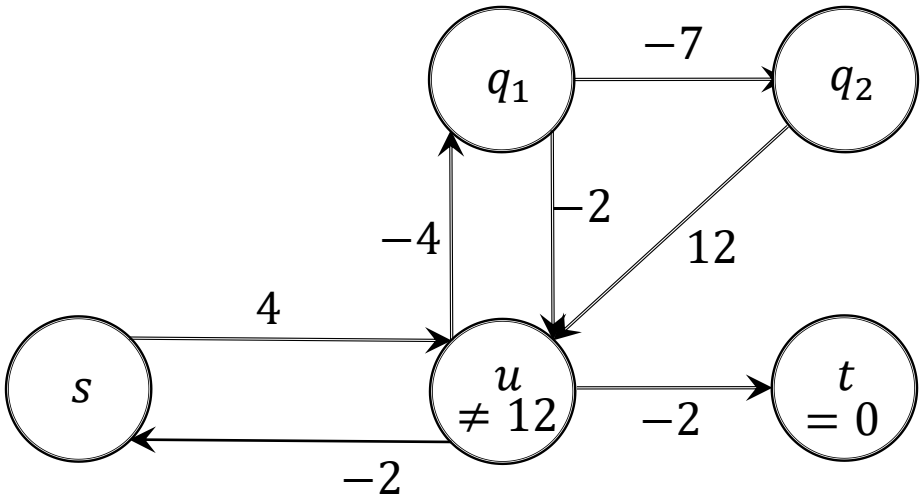
Below the guard



Below the guard but unblocked



Blocked under the guard



$c_1 = u, s, u$	$w = 2$	$drop = 2$
$c_2 = u, q_1, u$	$w = -6$	$drop = 6$
$c_3 = u, q_1, q_2, u$	$w = 1$	$drop = 11$

Valid run: $\rho =$ (u,4), (s,2), (u,6), (s,4), (u,8), (q_1,4), (u,2)

→ Blocked under the guard

Blocked under the guard

Blocked under the guard = very restrictive!

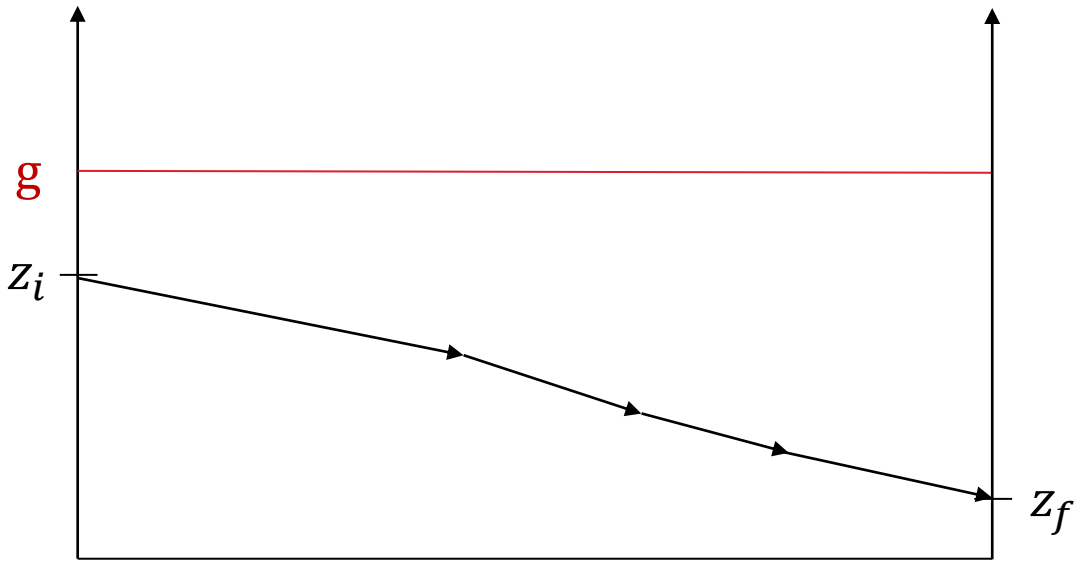
Look at f := flow of the path:

Blocked under the guard

Blocked under the guard = very restrictive!

Look at f := flow of the path:

- No positive cycle in f → f gives a certificate



Blocked under the guard

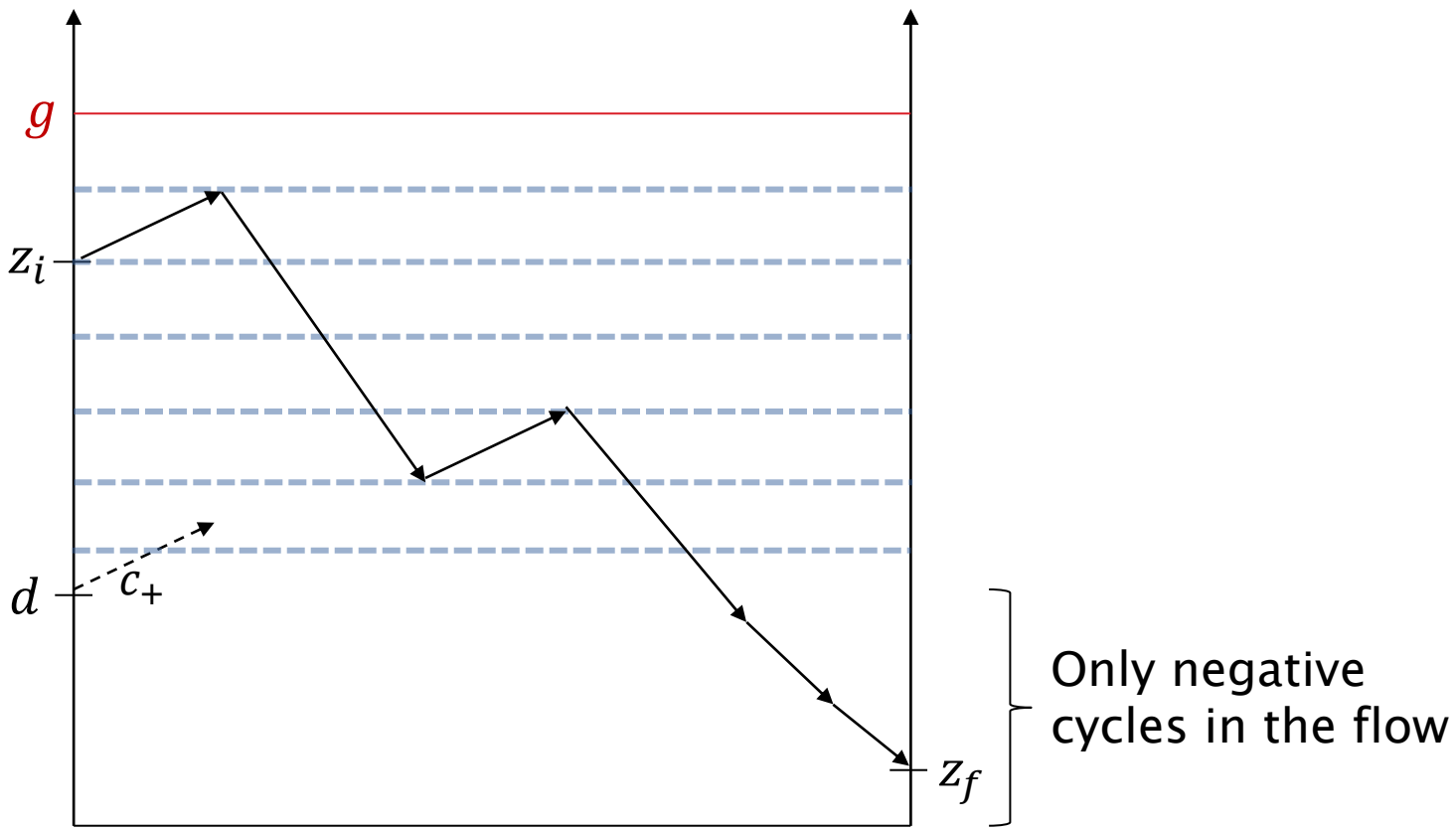
Blocked under the guard = very restrictive!

Look at f := flow of the path:

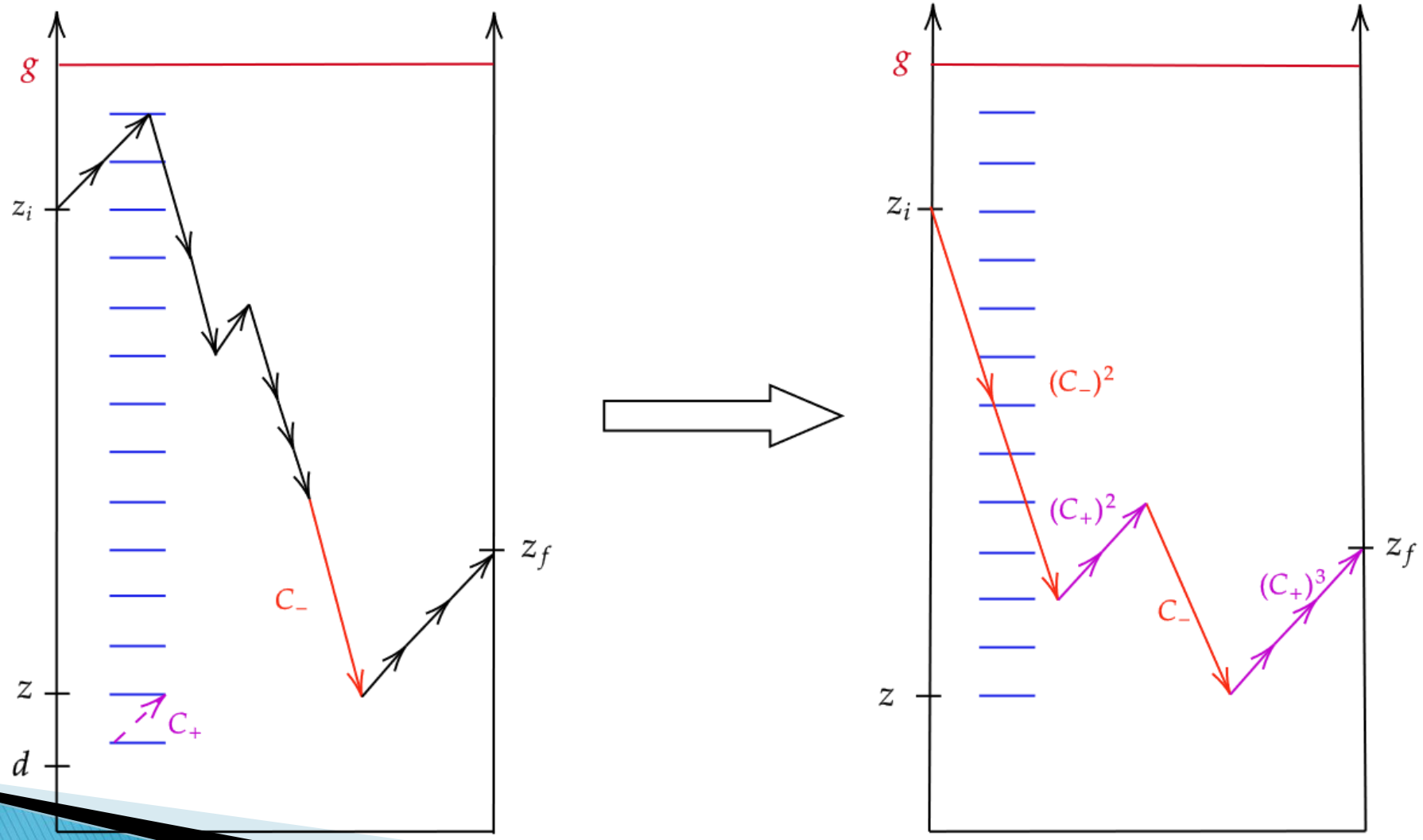
- No positive cycle in f → f gives a certificate
- Some positive cycle in f →
  All positive cycles have the same weight!
  c_+ positive u-simple cycle, d = $drop(c_+) \leq z_i$, w = $weight(c_+)$

minimal

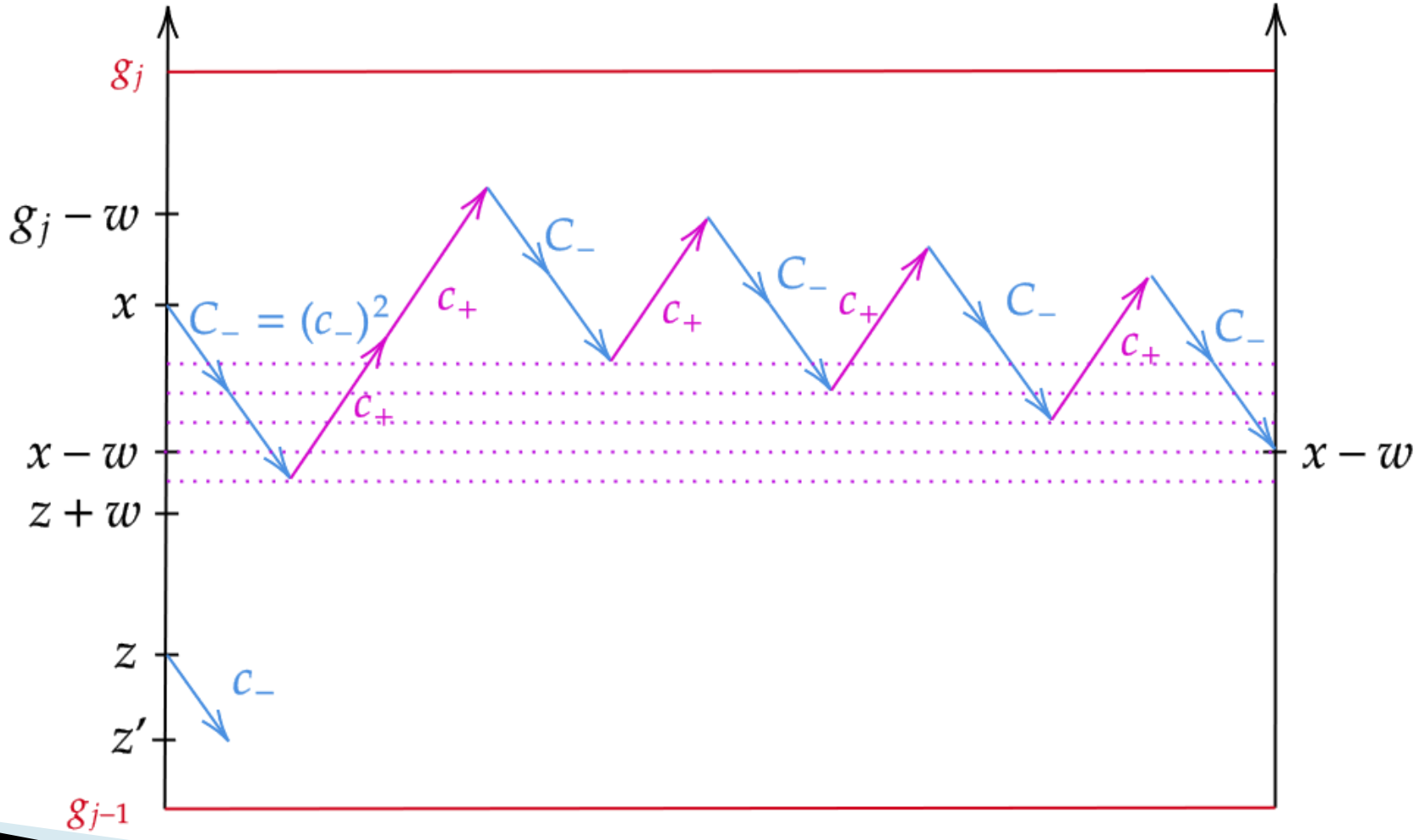
Blocked under the guard



Compressing the run



Several disequality guards on one state



counter values on u

Conclusion

Without tests	With $=$ tests	With $=$ & $\neq$ on single state	With $=$ & $\neq$	with $\leq$ & $\geq$	In 2-VASS without tests
NP-complete	NP-complete	Open	Open	PSPACE-complete	PSPACE-complete

Conclusion

Without tests	With $=$ tests	With $=$ & $\neq$ on single state	With $=$ & $\neq$	with $\leq$ & $\geq$	In 2-VASS without tests
NP-complete	NP-complete	NP-complete	Open… Not for long?	PSPACE-complete	PSPACE-complete