Computing the price of anarchy in atomic network congestion games

— CONFEST 2023 —

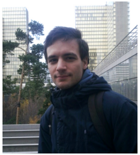
Nicolas Markey
CNRS – IRISA (Univ. Rennes, France)

based on joint works with

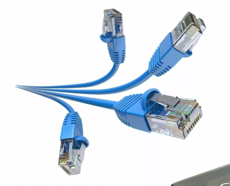

Nathalie Bertrand

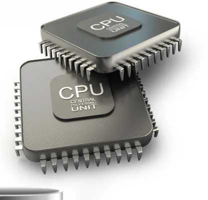

Aline Goeminne


Suman Sadhukhan


Ocan Sankur

Resource allocation problems





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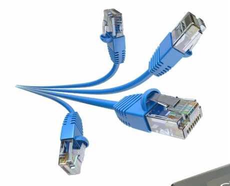

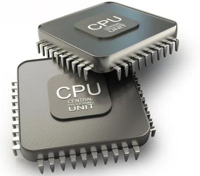

cost-sharing

vs.

congestion

Congestion in a communication network

In this talk, we consider network congestion problems.

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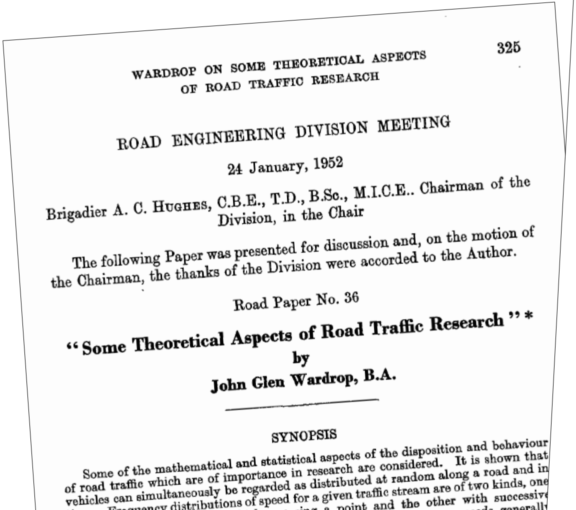

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WARDROP ON SOME THEORETICAL ASPECTS
OF ROAD TRAFFIC RESEARCH

ROAD ENGINEERING DIVISION MEETING

24 January, 1952

Brigadier A. C. HUGHES, C.B.E., T.D., B.Sc., M.I.C.E.. Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Road Paper No. 36

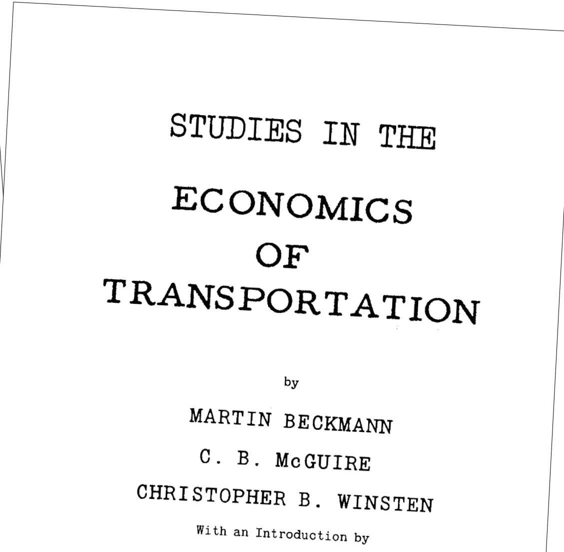
"Some Theoretical Aspects of Road Traffic Research"*

by

John Glen Wardrop, B.A.

SYNOPSIS

Some of the mathematical and statistical aspects of the disposition and behaviour of road traffic which are of importance in research are considered. It is shown that vehicles can simultaneously be regarded as distributed at random along a road and in ... distributions of speed for a given traffic stream are of two kinds, one ... a point and the other with successive ...



STUDIES IN THE

ECONOMICS

OF

TRANSPORTATION

by

MARTIN BECKMANN

C. B. McGUIRE

CHRISTOPHER B. WINSTEN

With an Introduction by

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DISTRIBUTION OF TRAFFIC OVER ALTERNATIVE ROUTES

When the effect of some future improvement of a road system is to be judged, some estimate must be made of the distribution of traffic on the various roads affected, including not only new roads but all existing roads from which traffic may be diverted. This is usually done by making some rather arbitrary assumption about speeds on the new roads, and, given the results of an Origin and Destination survey, by assuming that every vehicle will travel by the quickest route. However, it has been seen that speed is a function of flow, so that redistribution of traffic upsets the pattern of speeds. The problem is to discover how traffic may be expected to distribute itself over alternative routes, and whether the distribution adopted is the most efficient one. Although there has not been a sufficiently detailed investigation of a road network to allow this to be done in practice, it seems worth while to consider the theoretical aspects of this problem.

C. B. MCGUIRE

CHRISTOPHER B. WINSTEN

With an Introduction by

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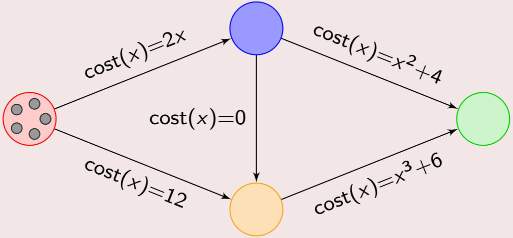
In this talk, we consider network congestion problems.

Definition (Atomic network congestion game (aka. routing games) [Ros73])

An ANCG is a pair $(\mathcal{G}, n)$ where

- $\mathcal{G}$ is a graph decorated with cost (or latency) functions on edges
- n is the number of players.



- resources are paths from source to destination;
- strategy profiles assign such a path to each player;
- the cost of an edge is $\text{cost}(x)$ where x is the number of players using that edge;
- the cost of a path is the sum of costs of all its edges.

[Ros73] Rosenthal. The network equilibrium problem in integers. Networks 3(1):53-59. John Wiley & Sons, 1973.

Example of an ANCG

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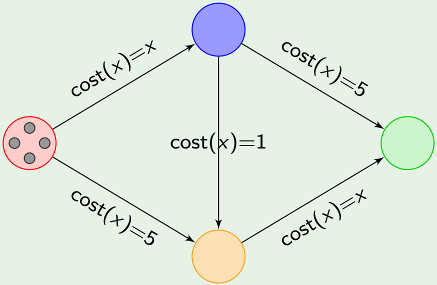

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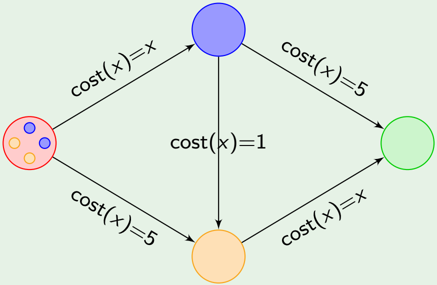

- cost for 🔵 players: 7 (2+5)
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- total cost for all four players: 28

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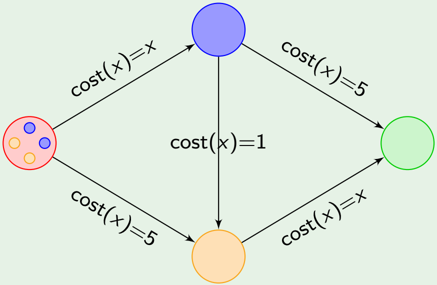

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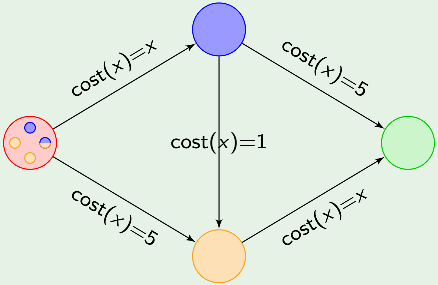

- cost for  player: 7 (2+5)
- cost for  player: 6 (2+1+3)
- cost for  players: 8 (5+3)
- total cost for all four players: 29

Centralized vs. selfish behaviours

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Centralized vs. selfish behaviours

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"Some Theoretical

Consider two alternative criteria based on these journey times which can be used to determine the distribution on the routes, as follows:

(1) The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.

(2) The average journey time is a minimum.

The first criterion is quite a likely one in practice, since it might be assumed that traffic will tend to settle down into an equilibrium situation in which no driver can reduce his journey time by choosing a new route. On the other hand, the second criterion is the most efficient in the sense that it minimizes the vehicle-hours spent on the journey. In practice, of course, drivers will be influenced by other factors, such as the state of the roads, and the comfort or discomfort of driving in general. However, it is clearly difficult to allow for these psychological factors.

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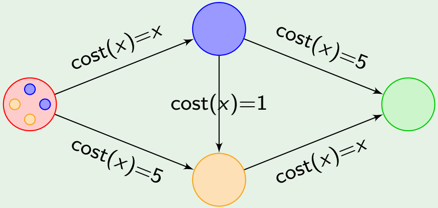

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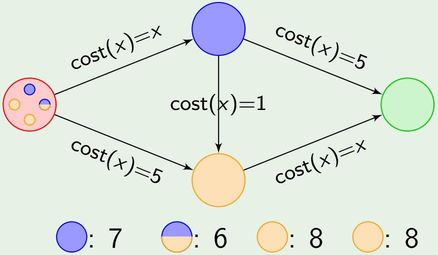

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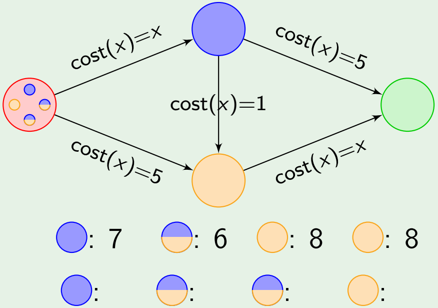

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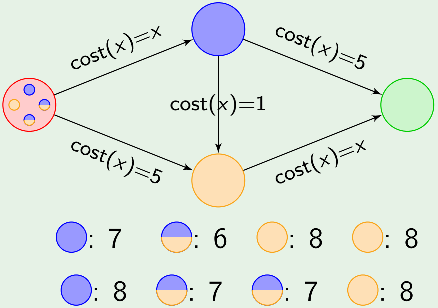

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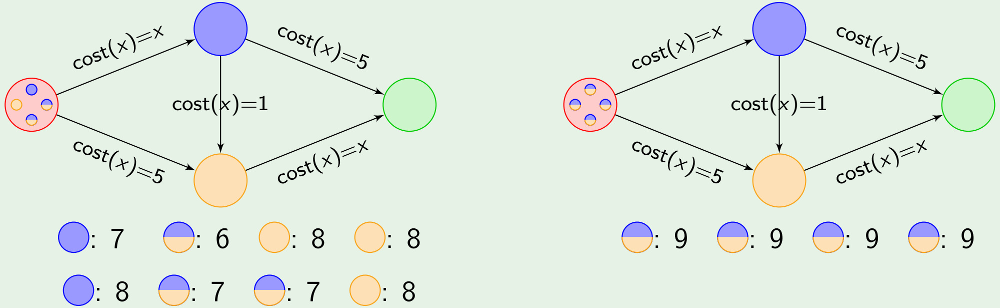

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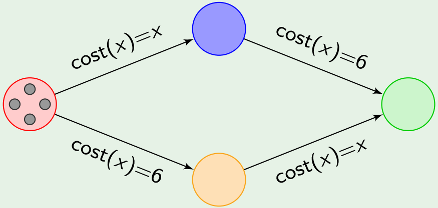

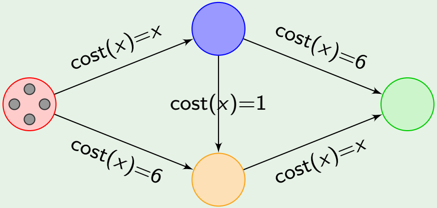

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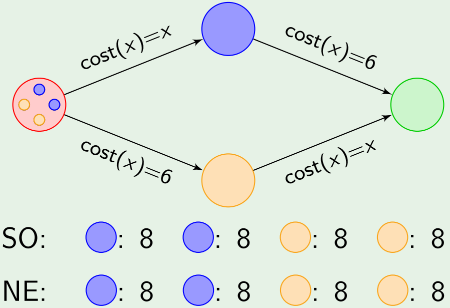

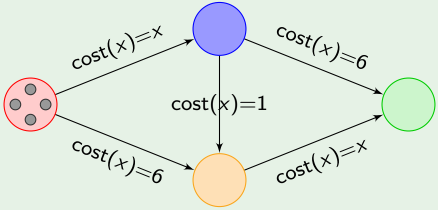

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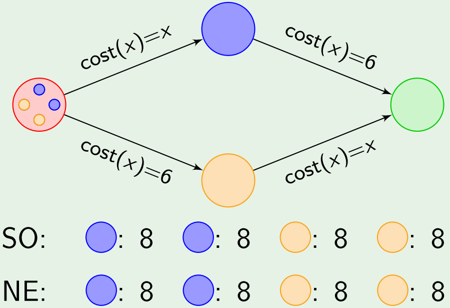

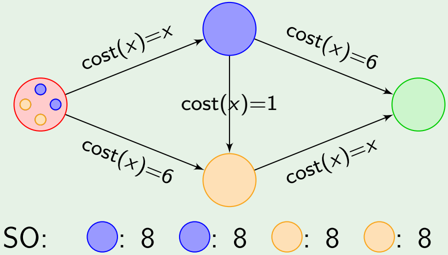

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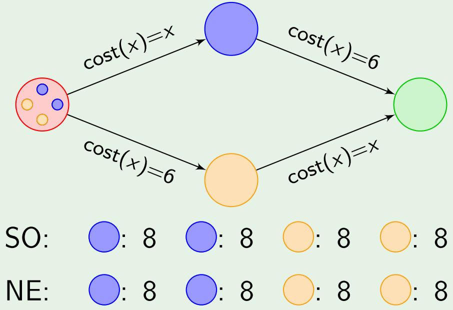

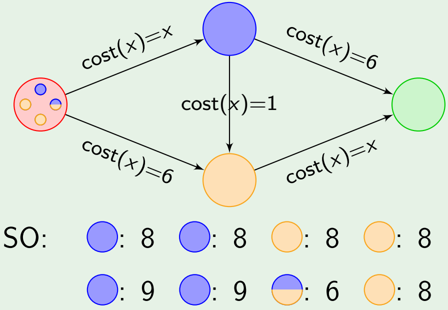

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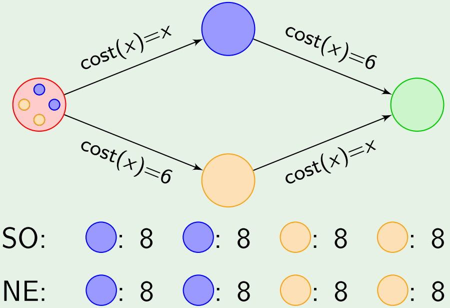

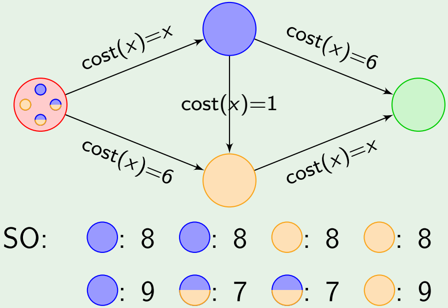

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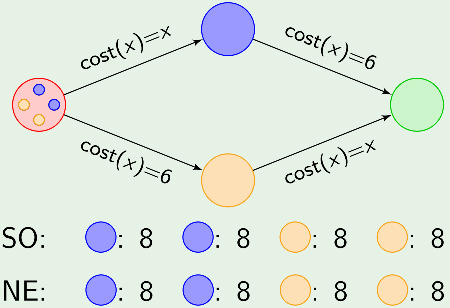

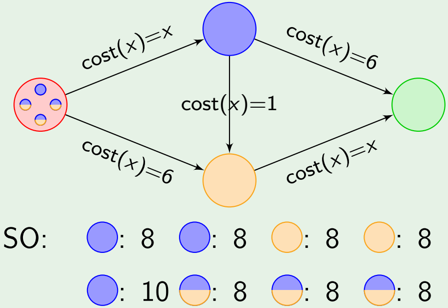

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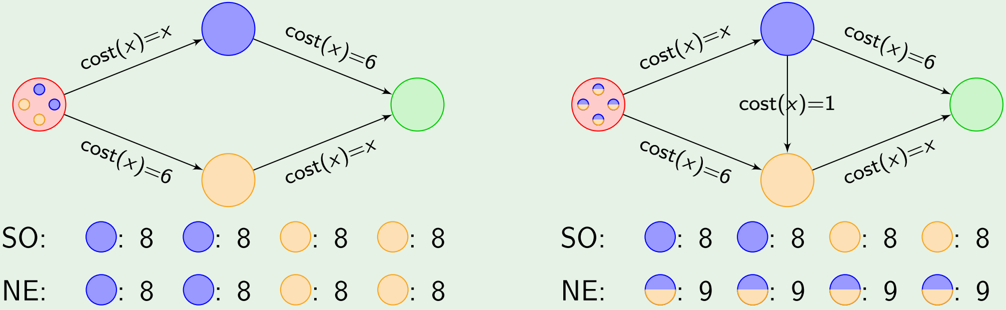

Any ANCG admits Nash equilibria

Theorem ([Ros73])

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Theorem ([Ros73])

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Proof

For any strategy profile σ, define the potential function:

$$\Phi(\sigma) = \sum_{e \in E} \sum_{k=1}^{\mathsf{load}_\sigma(e)} \mathsf{cost}_e(k).$$

Then for any player i and any path ρ, letting $\sigma' = \sigma[i \mapsto \rho]$, it holds

$$\Phi(\sigma') - \Phi(\sigma) = \mathsf{cost}_i(\sigma') - \mathsf{cost}_i(\sigma).$$

Any σ minimizing Φ (over the finitely-many possible strategy profiles) is a Nash equilibrium.

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Price of anarchy, price of stability

Definition ([KP99])

The price of anarchy is the ratio between the cost of the worst Nash equilibrium and the social optimum.

$\rightsquigarrow$ measures how much can be lost when agents act selfishly.

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[ADK+04] Anshelevich *et al.* The Price of Stability for Network Design with Fair Cost Allocation. FOCS'04, p. 295-304. IEEE, 2004.

Classical results about ANCG in algorithmic game theory

Theorem ([CK05,CJKU19])

The price of anarchy of any ANCG with affine cost functions is at most $5/2$.

[CK05] Christodoulou, Koutsoupias. The price of anarchy of finite congestion games. STOC'05, p. 67-73. ACM Press, 2005.
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Let σ_N be a Nash equilibrium, and σ_S be a strategy profile. The total cost of σ_N is

$$\begin{aligned}
\mathrm{cost}(\sigma_N) &= \sum_{1 \leq i \leq k} \sum_{e \in \sigma_N(i)} a_e \cdot \mathrm{load}_{\sigma_N}(e) + b_e \\
&\leq \sum_{1 \leq i \leq k} \sum_{e \in \sigma_S(i)} a_e \cdot (\mathrm{load}_{\sigma_N}(e) + 1) + b_e \qquad \text{(because } \sigma_N \text{ Nash eq.)} \\
&= \sum_{e \in E} \mathrm{load}_{\sigma_S}(e) \cdot (a_e \cdot (\mathrm{load}_{\sigma_N}(e) + 1) + b_e)
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(because $\alpha \cdot (\beta + 1) \leq \frac{5}{3}\beta^2 + \frac{1}{3}\alpha^2$)

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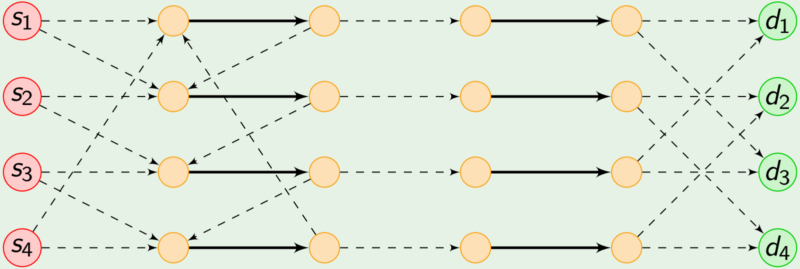

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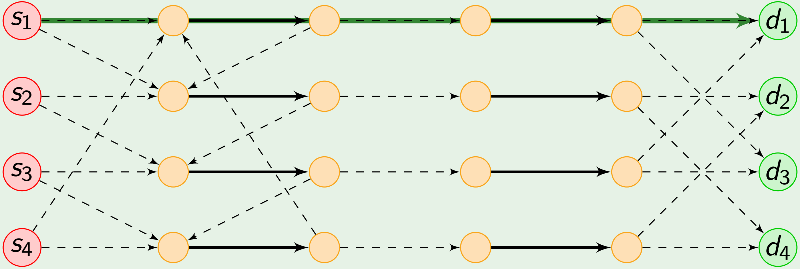

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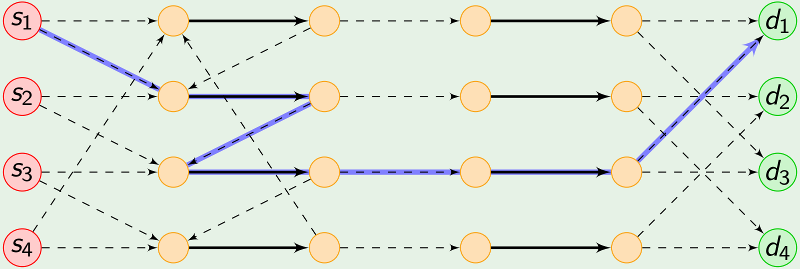

[CK05] Christodoulou, Koutsoupias. The price of anarchy of finite congestion games. STOC'05, p. 67-73. ACM Press, 2005.

[CJKU19] J.R. Correa *et al.* The inefficiency of Nash and subgame-perfect equilibria [...] Math. Op. Res. 44(4):1286-1303. Informs, 2019.

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Computing a Nash equilibrium in a symmetric ANCG can be performed in polynomial time; it is PLS-complete in the non-symmetric case.

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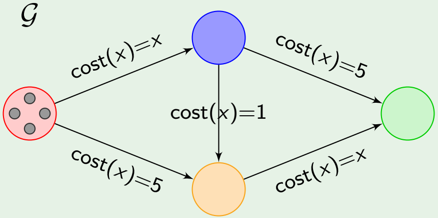

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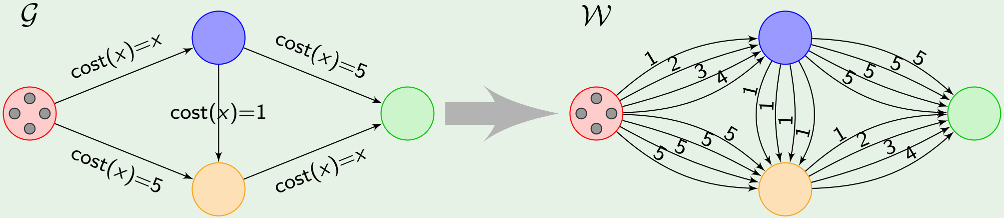

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Computing the price of anarchy

Part I: price of anarchy for arbitrarily many players

- we establish a semi-linear representation of Nash equilibria and local social optima;
- we show that they extend in a single direction in series-parallel networks.

Related work: [CDS23]

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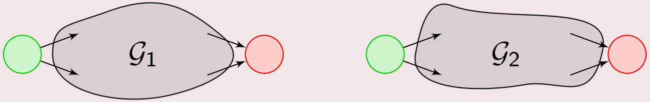
Series-parallel graphs

Definition

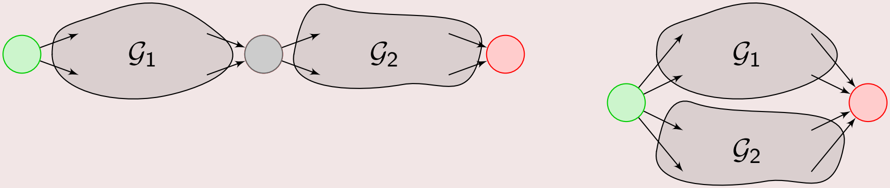
Given two graphs



we define their compositions:

series composition

parallel composition



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Theorem ([HM22])

The price of anarchy for series-parallel ANCG with affine cost functions is at most 2.

[HM22] Hao, Michini. Inefficiency of Pure Nash Equilibria in Series-Parallel Network [...]. WINE'22, p. 3-20. Springer, 2022.

Representation of strategy profiles

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Consider an ANCG $(\mathcal{G}, n)$.

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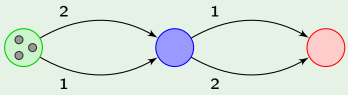
Example (flow of a strategy profile)



- a single flow may correspond to several strategy profiles;
- the total cost of a strategy profile only depends on its flow.

Semi-linear sets

Definition

A subset of $\mathbb{N}^d$ is *semi-linear* if it can be written as a finite union of sets of the form

$$L(b, \{v_i \mid 1 \leq i \leq p\}) = \left\{ b + \sum_{1 \leq i \leq p} \lambda_i \cdot v_i \mid (\lambda_i)_{1 \leq i \leq p} \in \mathbb{N}^p \right\}.$$

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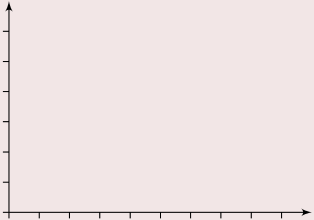

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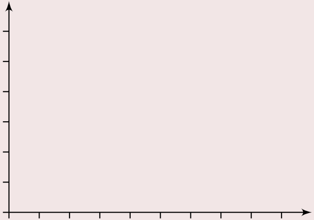

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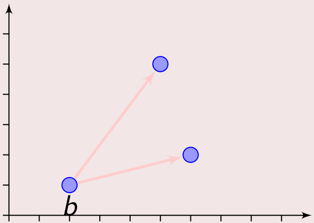

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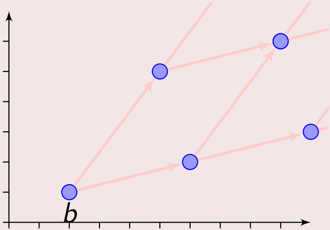

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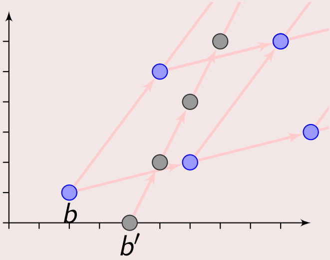

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base vector: $b' = (4, 0)$
period vector: $v_1' = (1, 2)$

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Theorem ([GS66])

A set is semi-linear if, and only if, it is definable in Presburger arithmetic.

[GS66] Ginsburg, Spanier. Semigroups, Presburger formulas, and languages. Pacific Journal Math. 16(2):285-296. 1966.

Expressing Nash equilibria

Lemma

A strategy profile σ is a Nash equilibrium if, and only if,

$$\forall \pi, \pi' \in Paths(\mathcal{G}).\ \sigma^{-1}(\pi) \neq \varnothing \ \Rightarrow \sum_{e \in \pi \setminus \pi'} cost_e(flow_\sigma(e)) \leq \sum_{e \in \pi' \setminus \pi} cost_e(flow_\sigma(e) + 1).$$

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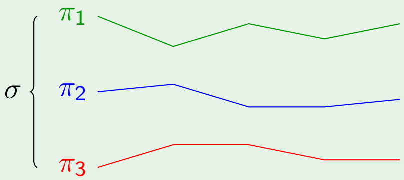

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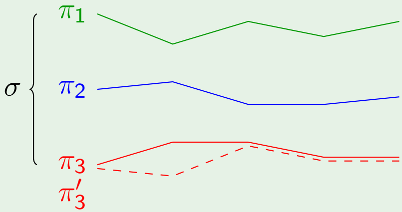

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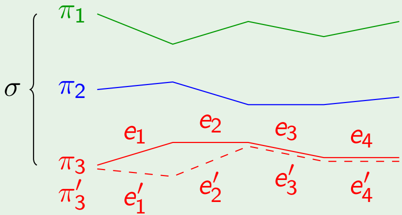

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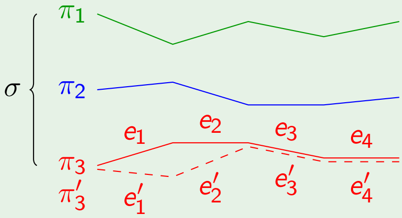

$$\sum_{e_i \in \pi_3} \mathrm{cost}_{e_i}(\mathrm{flow}_\sigma(e_i)) \leq$$

$$\sum_{e'_i \in \pi'_3} \mathrm{cost}_{e'_i}(\mathrm{flow}_{\sigma - \pi_3 + \pi'_3}(e'_i))$$

Expressing Nash equilibria

Corollary

If $\mathcal{G}$ is a network with linear cost functions, then the set of (flows of) Nash equilibria $NE(\mathcal{G})$ is semi-linear.

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Proof

Membership of $(p_\pi)_{\pi \in \mathsf{Paths}(\mathcal{G})}$ in $\mathsf{NE}(\mathcal{G})$ can be expressed as

$$\exists (q_e)_{e \in E} . \bigwedge_{\pi, \pi' \in \mathsf{Paths}(\mathcal{G})} \left(p_\pi > 0 \Rightarrow \sum_{e \in \pi \setminus \pi'} w_e \cdot q_e \leq \sum_{e' \in \pi' \setminus \pi} w_{e'} \cdot (q_{e'} + 1) \right) \wedge \bigwedge_{e \in E} \left(q_e = \sum_{\pi \ni e} p_\pi \right) .$$

Period vectors of flows of Nash equilibria

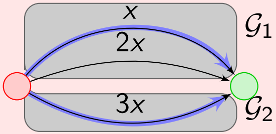
Lemma

Let $\mathcal{S}$ and $\mathcal{P}$ be the series- and parallel compositions of $\mathcal{G}_!$ and $\mathcal{G}_2$.

- *A strategy profile σ is a Nash equilibrium in $\mathcal{S}$ if, and only if, its projections in $\mathcal{G}_1$ and $\mathcal{G}_2$ are.*
- *If a strategy profile σ is a Nash equilibrium in $\mathcal{P}$, then its projections in $\mathcal{G}_1$ and $\mathcal{G}_2$ are.*

Remark

The converse direction fails for parallel composition, as can be seen on the small example opposite.



Period vectors of flows of Nash equilibria

Theorem

In any series-parallel ANCG $(\mathcal{G}, n)$ with linear cost functions, all period vectors of $flow(NE(\mathcal{G}))$ have the same cost along all paths:

$$\text{for all period vector } q \text{ of } flow(NE(\mathcal{G})).\ \exists \kappa.\ \forall \pi \in Paths(\mathcal{G}).\ \sum_{e \in \pi} cost_e(q_e) = \kappa.$$

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Proof

By induction:

- for single-edge graphs: trivial;

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Proof

By induction:

- for series compositions $\mathcal{G} = \mathcal{G}_1 \odot \mathcal{G}_2$:
  - if v is a period vector of $\mathrm{NE}(\mathcal{G})$, then $v_{\mathcal{G}_i}$ is a period vector of $\mathrm{NE}(\mathcal{G}_i)$;
  - by induction, we get constants κ_i for each $\mathcal{G}_i$;
  - $\kappa = \kappa_1 + \kappa_2$.

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Proof

By induction:

- for parallel compositions $\mathcal{G} = \mathcal{G}_1||\mathcal{G}_2$:
  - we again get constants κ_i for each $\mathcal{G}_i$;
  - we prove that $\kappa_1 = \kappa_2$;
  - we let $\kappa = \kappa_1$.

Expressing Nash equilibria

Proposition (see also [CDS23])

If $\mathcal{G}$ is a series-parallel network, the following system of equations $(\mathcal{E}_\kappa)$ has a unique solution:

$$\begin{cases} \forall \pi \in \mathit{Paths}(\mathcal{G}).\ \sum_{e \in \pi} w_e \cdot q_e = \kappa \\ \forall v \in V \setminus \{\mathit{src}, \mathit{tgt}\}.\ \sum_{e \in \mathit{In}(v)} q_e - \sum_{e' \in \mathit{Out}(v)} q_e = 0 \end{cases} \qquad (\mathcal{E}_\kappa)$$

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Corollary

If $\mathcal{G}$ is a series-parallel network with linear cost functions, then the period vectors of $flow(NE(\mathcal{G}))$ are multiples of a single vector $\delta_{\mathcal{G}}$.

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Expressing (local) social optimality

Expressing social optimality for linear cost functions gives rise to a quadratic expression with universal quantification over profiles:

$$\forall (q'_e)_{e \in E}.\ \sum_{e \in E} w_e \cdot ({q'_e}^2 - {q_e}^2) \geq 0.$$

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$\rightsquigarrow$ we relax optimality to *local optimality*:

Definition

A strategy profile σ is a local social optimum if, and only if, no player alone can improve the total cost of σ.

Expressing (local) social optimality

Theorem

A strategy profile σ is a local social optimum if, and only if,

$$\forall \pi, \pi' \in Paths(\mathcal{G}).\ (\forall e \in \pi.\ flow_\sigma(e) > 0) \Rightarrow \sum_{e \in \pi \setminus \pi'} w_e \cdot (2q_e - 1) \leq \sum_{e \in \pi' \setminus \pi} w_e \cdot (2q_e + 1).$$

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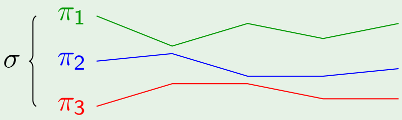

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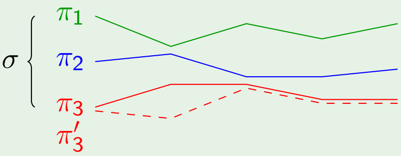

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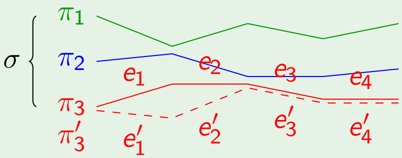

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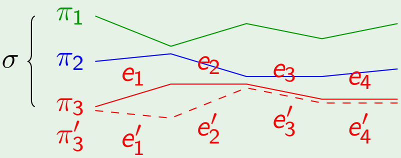

$$\sum_{e \in \pi_3 \cup \pi_3'} w_e q_e^2 \leq \sum_{e \in \pi_3 \cap \pi_3'} w_e \cdot q_e^2 + \sum_{e' \in \pi_3' \setminus \pi_3} w_{e'} \cdot (q_{e'} + 1)^2 + \sum_{e \in \pi_3 \setminus \pi_3'} w_e \cdot (q_e - 1)^2$$

Expressing (local) social optimality

Theorem

A strategy profile σ *is a local social optimum if, and only if,*

$$\forall \pi, \pi' \in \mathit{Paths}(\mathcal{G}).\ (\forall e \in \pi.\ \mathit{flow}_\sigma(e) > 0) \Rightarrow \sum_{e \in \pi \setminus \pi'} w_e \cdot (2q_e - 1) \leq \sum_{e \in \pi' \setminus \pi} w_e \cdot (2q_e + 1).$$

Corollary

If $\mathcal{G}$ *is a network with linear cost functions, then the set of (flows of) local social optima is semi-linear.*

Expressing (local) social optimality

Proposition

In series-parallel networks, with sufficiently many players, any local social optimum involves all edges.

Expressing (local) social optimality

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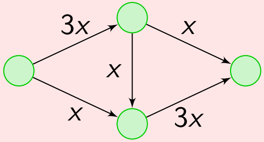
Remark

This fails to hold in non-series-parallel networks, such as



Expressing (local) social optimality

Theorem

In any series-parallel ANCG $(\mathcal{G}, n)$ with linear cost functions, all period vectors of $\mathit{flow}(LSO_{\geq n_0}(\mathcal{G}))$ have the same cost along all paths:

$$\textit{for all period vector } q \textit{ of } \mathit{flow}(LSO_{\geq n_0}(\mathcal{G})).\ \exists \kappa.\ \forall \pi \in \mathit{Paths}(\mathcal{G}).\ \sum_{e \in \pi} \mathit{cost}_e(q_e) = \kappa.$$

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In any series-parallel ANCG $(\mathcal{G}, n)$ *with linear cost functions, all period vectors of* $flow(LSO_{\geq n_0}(\mathcal{G}))$ *have the same cost along all paths:*

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Proof

Assume $L(b, v) \subseteq \mathrm{flow}(\mathrm{LSO}_{\geq n_0}(\mathcal{G}))$, and take two paths π and π'. For any $k \geq 0$:

$$\sum_{e \in \pi \setminus \pi'} w_e \cdot (2(b_e + k \cdot v_e) - 1) \leq \sum_{e' \in \pi' \setminus \pi} w_e \cdot (2(b_{e'} + k \cdot v_{e'}) + 1).$$

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In any series-parallel ANCG $(\mathcal{G}, n)$ *with linear cost functions, all period vectors of* $\mathit{flow}(LSO_{\geq n_0}(\mathcal{G}))$ *have the same cost along all paths:*

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Corollary

If $\mathcal{G}$ *is a series-parallel network with linear cost functions, then the semi-linear set* $\mathit{flow}(LSO(\mathcal{G}))$ *admits a single period vector* $\delta_{\mathcal{G}}$.

Expressing (local) social optimality

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Corollary (see also [WMRX23])

In series-parallel networks, PoA and PoS tend to 1 when the number of players grows.

[WMRX23] Wu *et al.* A convergence analysis of the price of anarchy in atomic congestion games. Math. Prog 199(1):937-993. 2023.

Computing the price of anarchy in series-parallel networks

Python prototype using `sympy` and Z3

- compute (symbolic representation of) period vector $\delta_{\mathcal{G}}$;
- compute base points for flows of LSO($\mathcal{G}$) and NE($\mathcal{G}$).
- for each n, compute optimal cost and costs of worst and best Nash equilibria.

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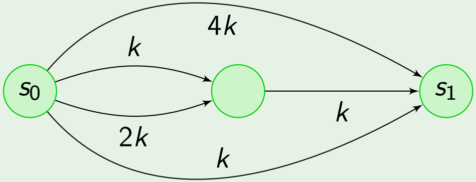
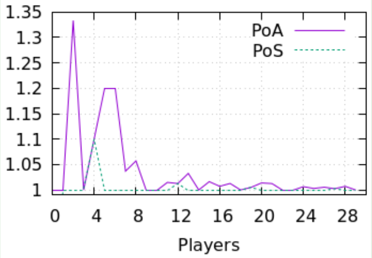
Example





Computing the price of anarchy

Part I: price of anarchy for arbitrarily many players

- we establish a semi-linear representation of Nash equilibria and local social optima;
- we show that they extend in a single direction in series-parallel networks.

Related work: [CDS23]

Part II: adding time in network congestion games

- we adapt the semantics to better model the congestion effect: synchronized costs, non-blind strategies;
- we develop algorithms to compute Nash equilibria and social optima in this setting;
- we extend this approach to timed network congestion games.

Related works: [CJKU19, AGK17]

[CDS23] Cominetti, Dose, Scarsini. The price of anarchy in routing games as a function of the demand. Math. Prog. To appear.

[CJKU19] Correa *et al.* The inefficiency of Nash and subgame-perfect equilibria [...] Math. Op. Res. 44(4):1286-1303. Informs, 2019.

[AGK17] Avni, Guha, Kupferman. Timed Network Games. MFCS'17, p. 37:1-37:16. LZI, 2017.

Changing the semantics

Changing the semantics

Synchronizing cost computation

- more natural semantics for dealing with network congestion;
- first step towards considering timed network congestion games [AGK17].

[AGK17] Avni, Guha, Kupferman. Timed Network Games. MFCS'17, p. 37:1-37:16. LZI, 2017.

Changing the semantics

Synchronizing cost computation

- more natural semantics for dealing with network congestion;
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Allowing non-blind strategies

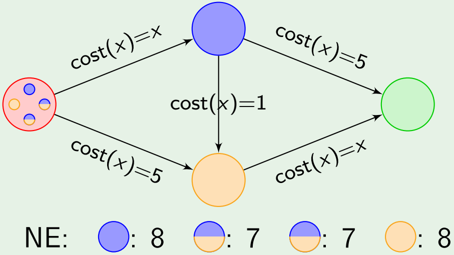
- richer setting, also more natural for dealing with congestion;
- extends recent works on sequential congestion games [RST12,CJKU19].

[AGK17] Avni, Guha, Kupferman. Timed Network Games. MFCS'17, p. 37:1-37:16. LZI, 2017.
[PST12] Paes Leme, Syrgkanis, Tardos. The curse of simultaneity. ITCS'12, p. 60-67. ACM Press, 2012.
[CJKU19] Correa *et al.* The inefficiency of Nash and subgame-perfect equilibria [...]. Math. Op. Res. 44(4):1286-1303. Informs, 2019.

Synchronizing cost computation

Example

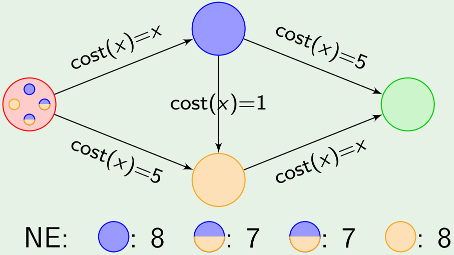
Classical semantics



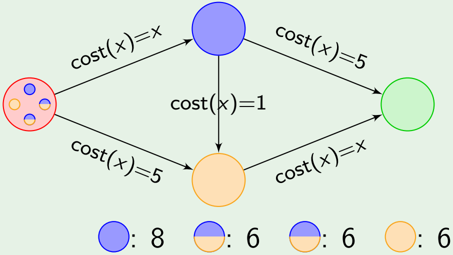
Synchronizing cost computation

Example

Classical semantics



Synchronized semantics



Synchronizing cost computation

Example

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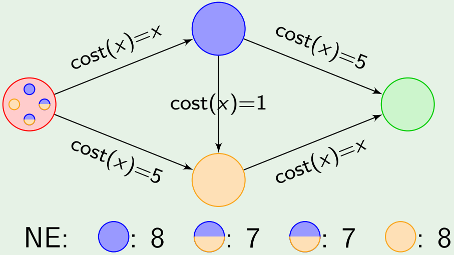

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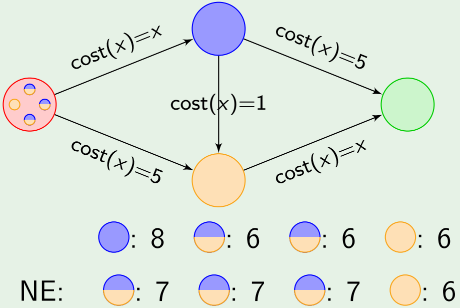
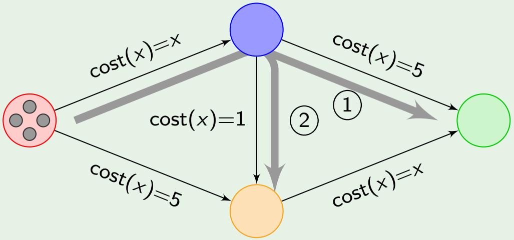


Non-blind strategies

Example



1. if at most 2 players in 
2. if more than 2 players in 

Non-blind strategies

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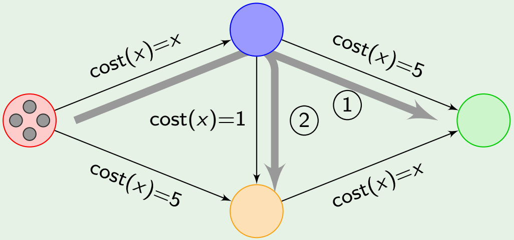


① if at most 2 players in ●
② if more than 2 players in ●

Remark

In the sequel, blind Nash equilibria are blind strategy profiles that are Nash equilibria w.r.t. blind strategies.

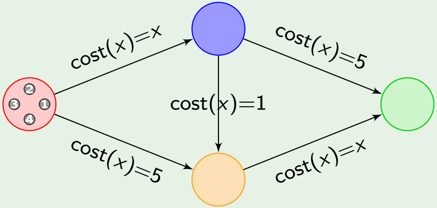
Semantics as a concurrent game

Concurrent game on a multi-weighted graph

- states are configurations (elements of V^n);
- there is a transition $(v_i)_{1\leq i\leq n} \xrightarrow{(w_i)_{1\leq i\leq n}} (v'_i)_{1\leq i\leq n}$ whenever there exist edges $(e_i)_{1\leq i\leq n}$ s.t. for all $1 \leq j \leq n$,
  - $e_j = (v_j, f_j, v'_j)$ is an edge of the network,
  - $w_j = f_j(\mathrm{load}(e_j, (e_i)_{1\leq i\leq n}))$ where $\mathrm{load}(e_j, (e_i)_{1\leq i\leq n}) = \#\{1 \leq i \leq n \mid e_i = e_j\})$.

Semantics as a concurrent game

Example



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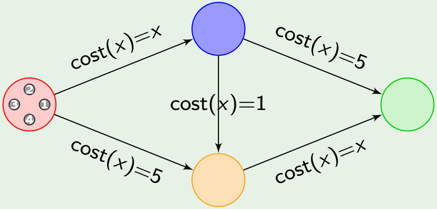

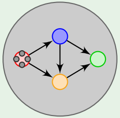

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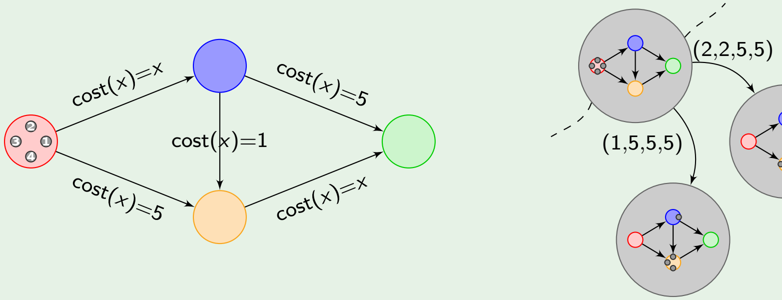

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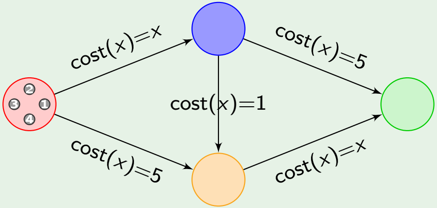

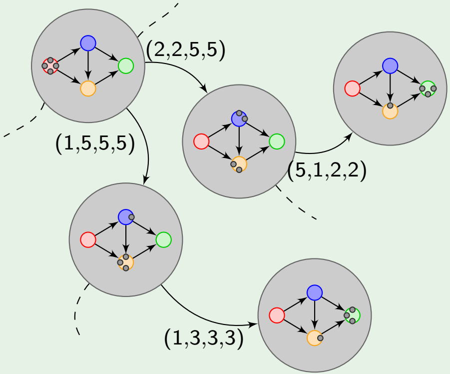

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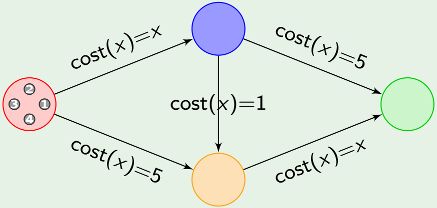

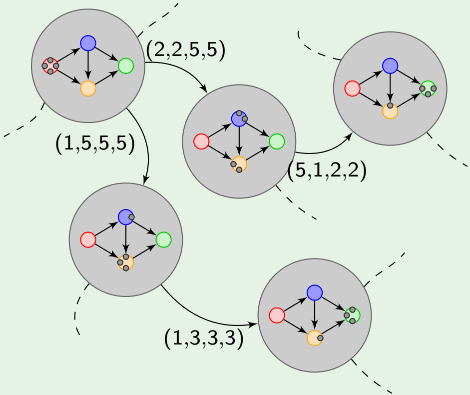

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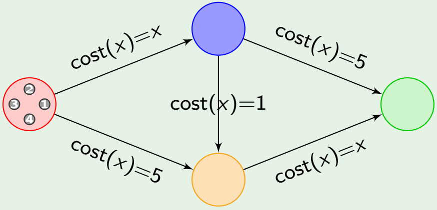

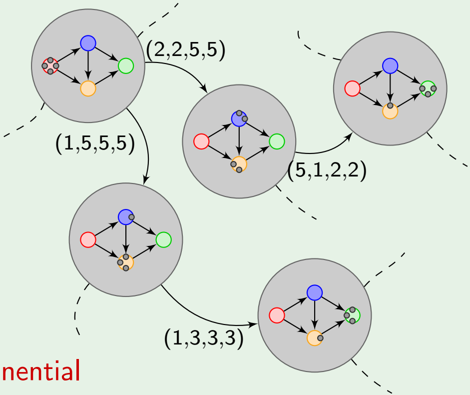

The game has size $|V|^n$, which is doubly-exponential (assuming number of players given in binary).

Computing social optima

Theorem

The social optimum can be computed in PSPACE.

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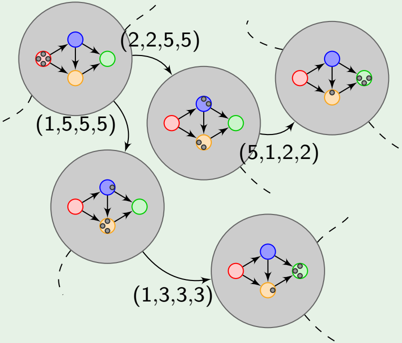

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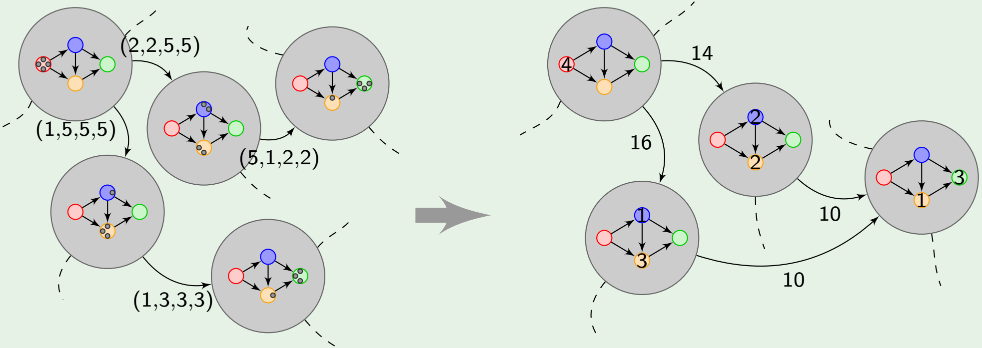

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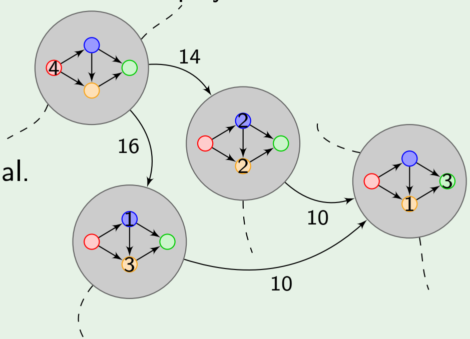
For social optimum, no need to keep track of each individual player.

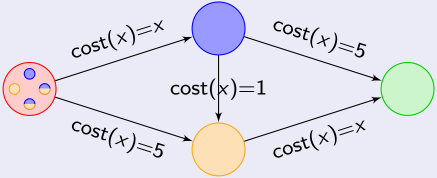
This graph has size $n^{|V|}$, which is exponential.

$\leadsto$ non-deterministically build a good path.



Pure Nash equilibria still always exist!

Modelling synchronous costs with classical ones



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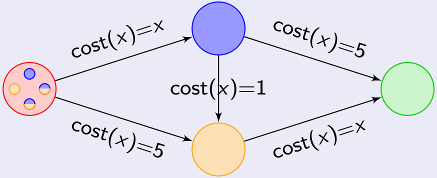

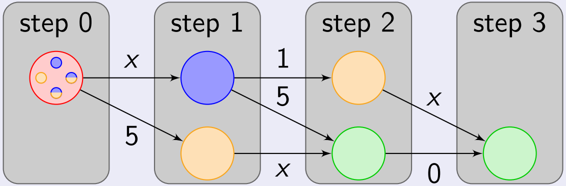

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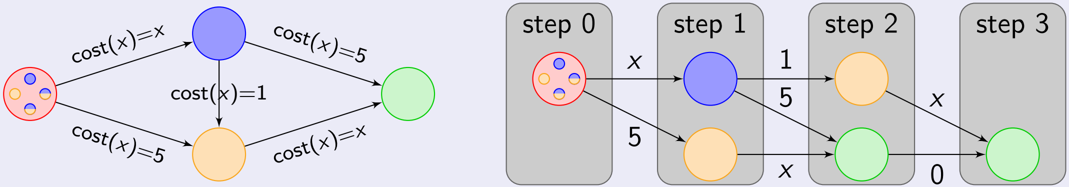
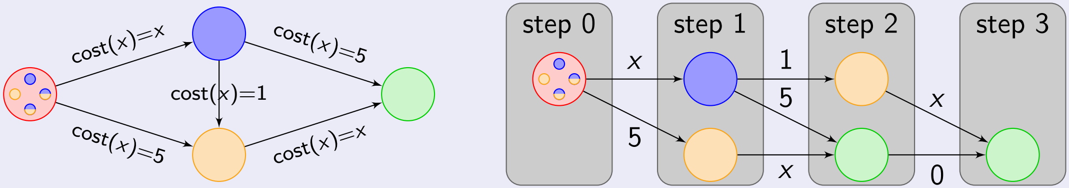


Lemma

If an ANCG with synchronous costs has a blind Nash equilibrium, then it has one whose paths have length at most $\sum_{e \in E} cost_e(n)$ *(assuming all costs are positive integers).*

Pure Nash equilibria still always exist!

Modelling synchronous costs with classical ones



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Corollary

Any ANCG with synchronous costs admits a pure blind Nash equilibrium.

Pure Nash equilibria still always exist!

Theorem

Blind Nash equilibria are Nash equilibria (w.r.t. non-blind strategies).

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If some player has a profitable deviation, they have a blind one.

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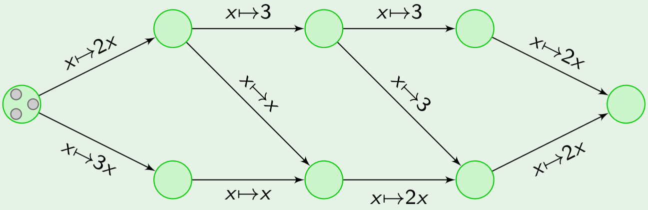

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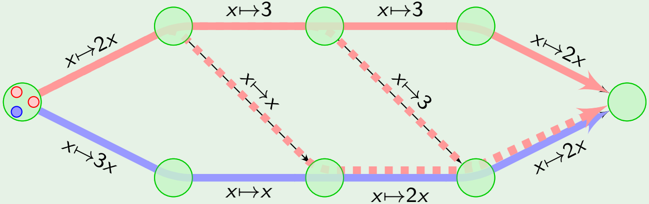

- red players take dashed edge if the other red player deviated from plain red path at previous state. cost $= 14 + 14 + 8$

Pure Nash equilibria still always exist!

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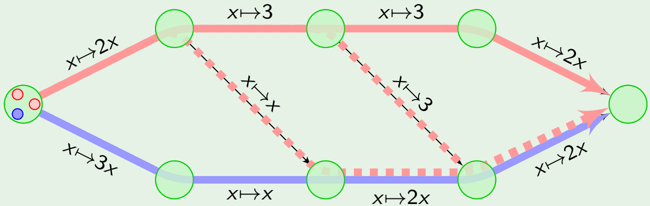


- any blind strategy profile has cost larger than 36, except one which is not a blind Nash equilibrium.

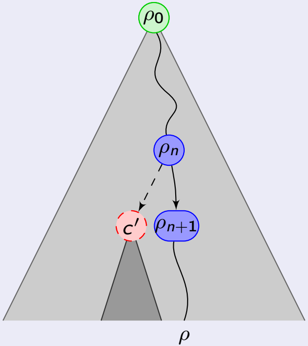
Characterizing outcomes of Nash equilibria

Theorem ([KLŠT12])

A path ρ is the outcome of a Nash equilibrium if, and only if, for any player i and any position n along ρ:

$$\forall c' \text{ deviation by player } i \text{ from } \rho_n \to \rho_{n+1},$$
$$cost_i(\rho_{\geq n}) \leq cost_i(\rho_n \to c') + val_i(c')$$

where $val_i(c') = \sup_{\sigma_{-i}} \inf_{\sigma_i} cost_i(c', \sigma)$ is the minimum cost for Player i from c' against other players.



[KLŠT12] Klimoš *et al.* Nash Equilibria in Concurrent Priced Games. LATA'12, p. 363-376. Springer, 2012.

Computing Nash equilibria

Algorithm

Build tree of outcomes, propagating constraints on the cost of the rest of the path:

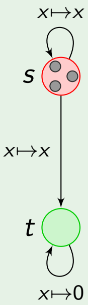
$$m_i' = \min\left\{m_i - \mathsf{cost}_i(\rho_n \to \rho_{n+1}), \min_{c'}(\mathsf{val}_i(c') + \mathsf{cost}_i(\rho_n \to c') - \mathsf{cost}_i(\rho_n \to \rho_{n+1}))\right\}.$$

Computing Nash equilibria

Algorithm

$$m_i' = \min\{m_i - \mathrm{cost}_i(\rho_n \to \rho_{n+1}), \min_{c'}(\mathrm{val}_i(c') + \mathrm{cost}_i(\rho_n \to c') - \mathrm{cost}_i(\rho_n \to \rho_{n+1}))\}.$$

Example

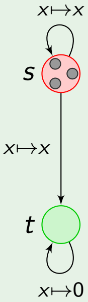


Computing Nash equilibria

Algorithm

$$m_i' = \min\{m_i - \mathrm{cost}_i(\rho_n \to \rho_{n+1}), \min_{c'}(\mathrm{val}_i(c') + \mathrm{cost}_i(\rho_n \to c') - \mathrm{cost}_i(\rho_n \to \rho_{n+1}))\}.$$

Example



$(s_{\leq +\infty}), (s_{\leq +\infty}), (s_{\leq +\infty})$

Computing Nash equilibria

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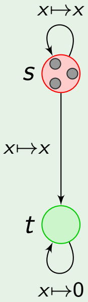

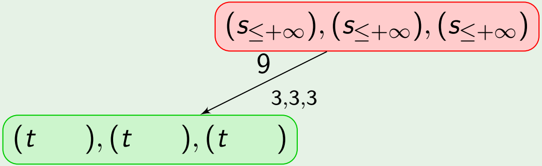

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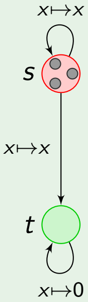

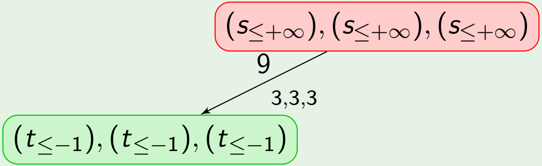

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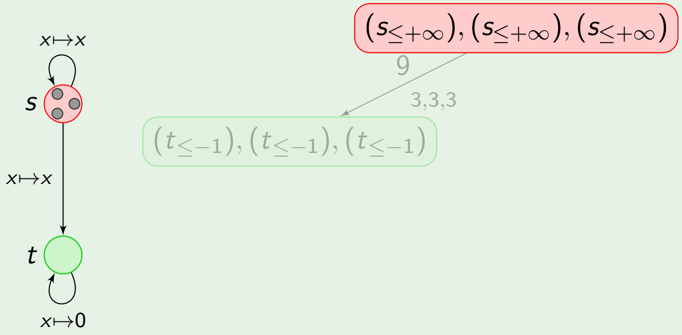

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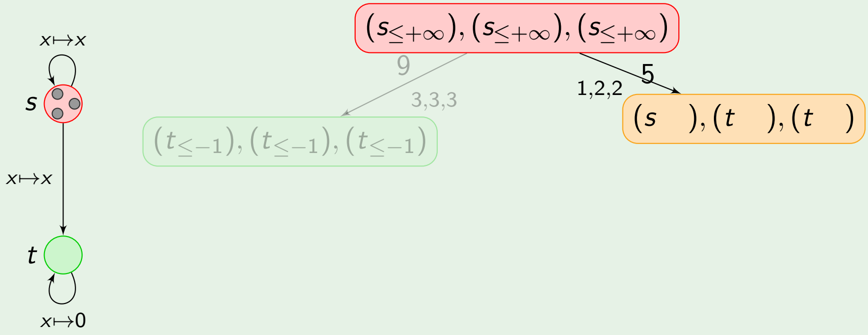

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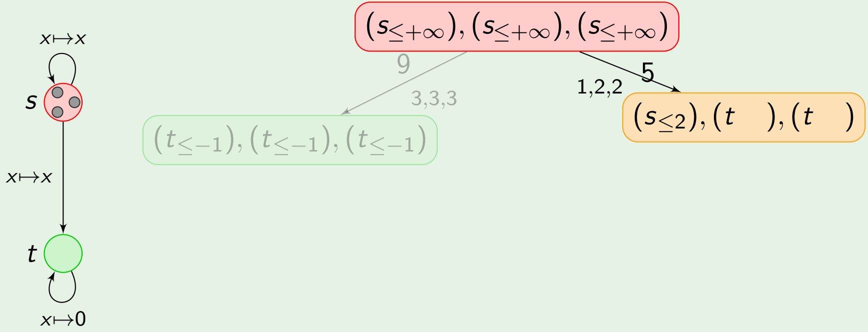

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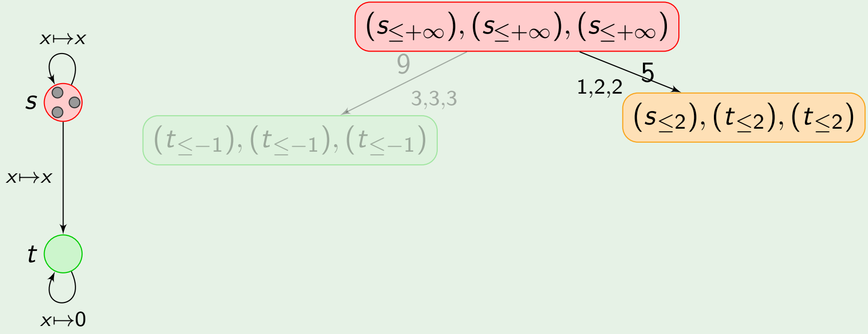

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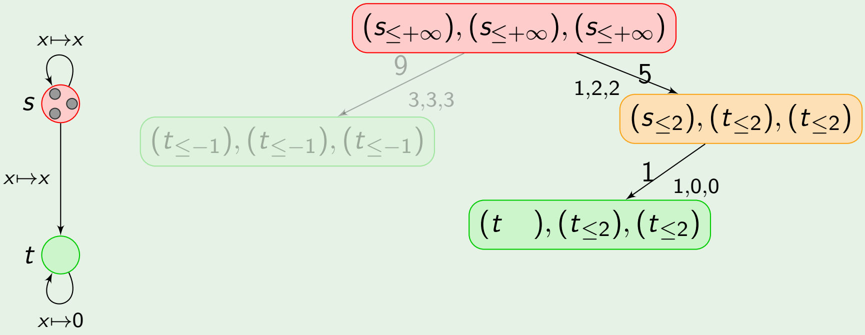

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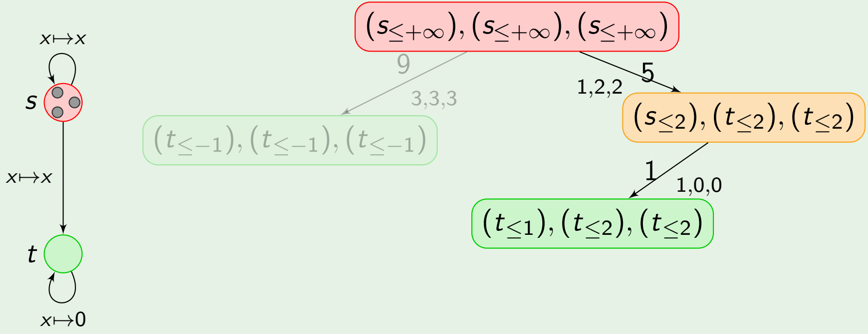

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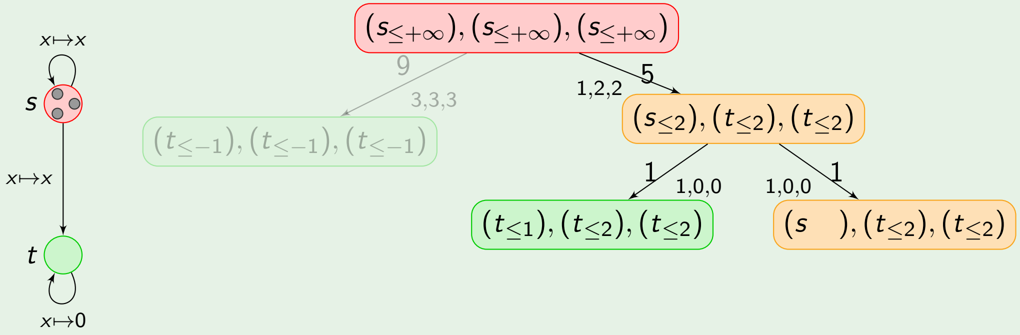

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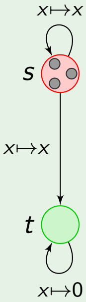

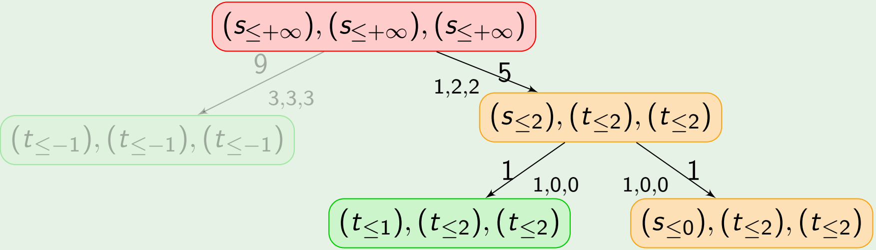

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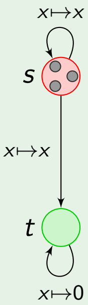

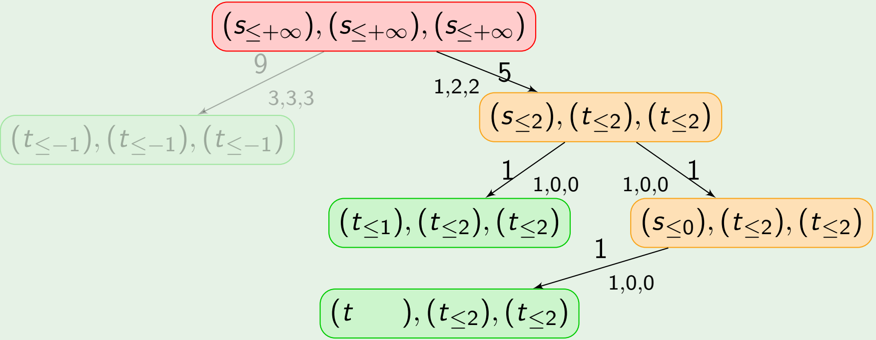

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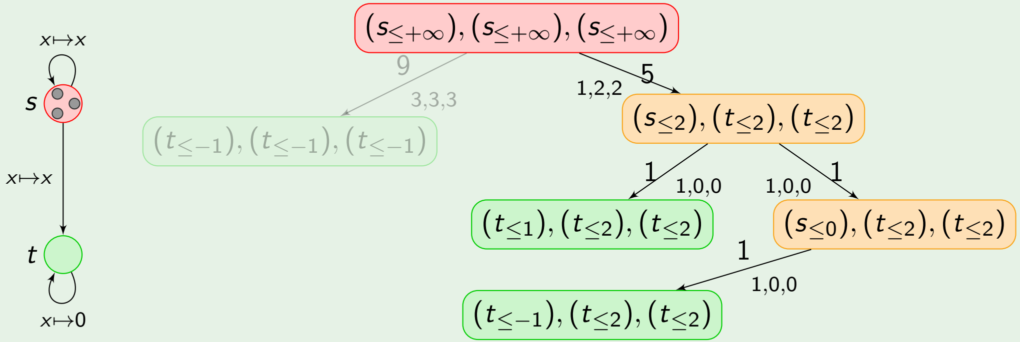

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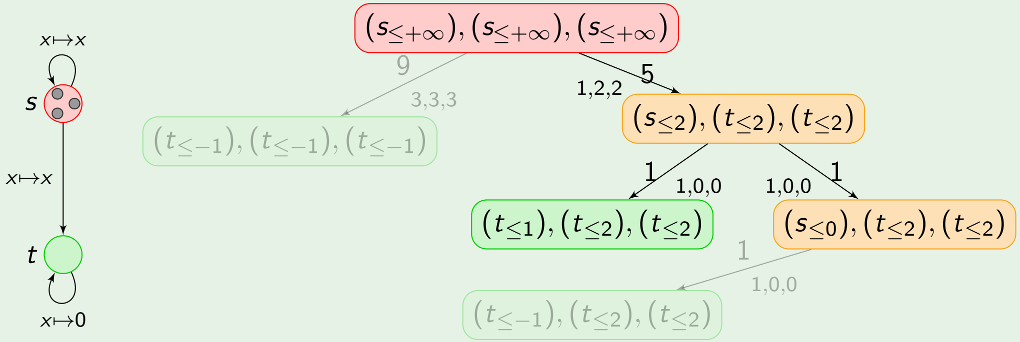

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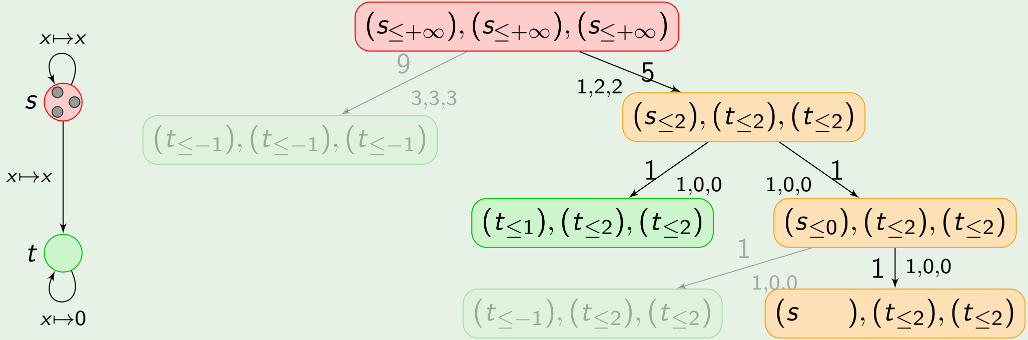

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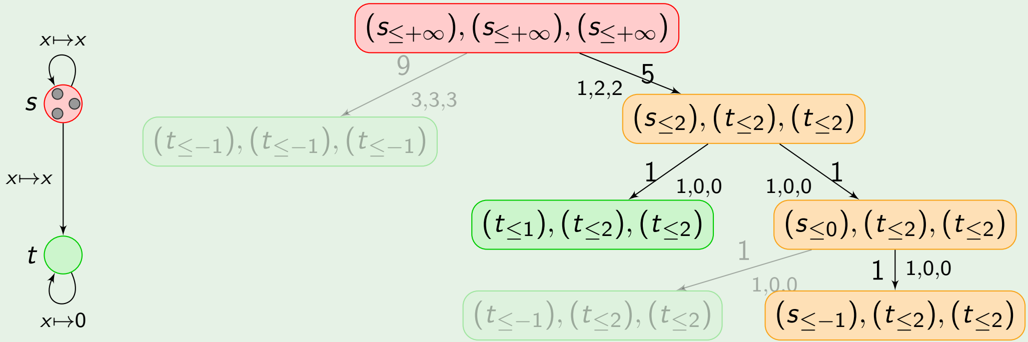

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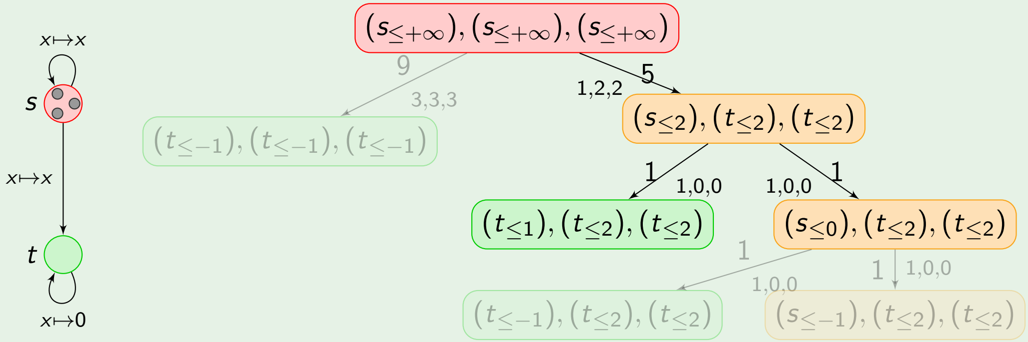

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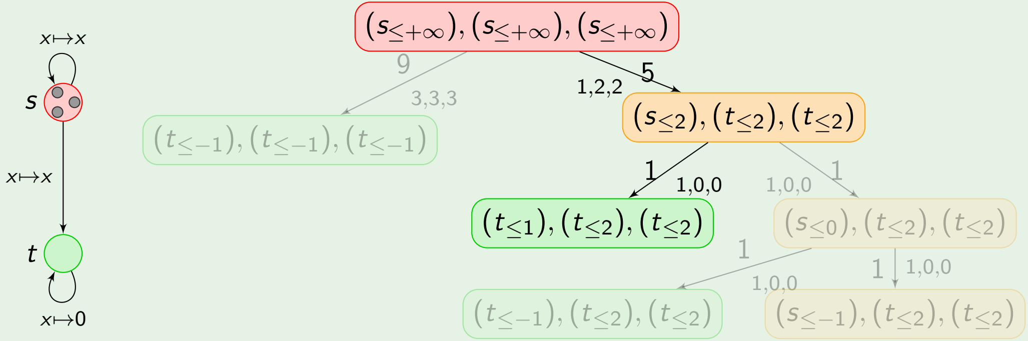

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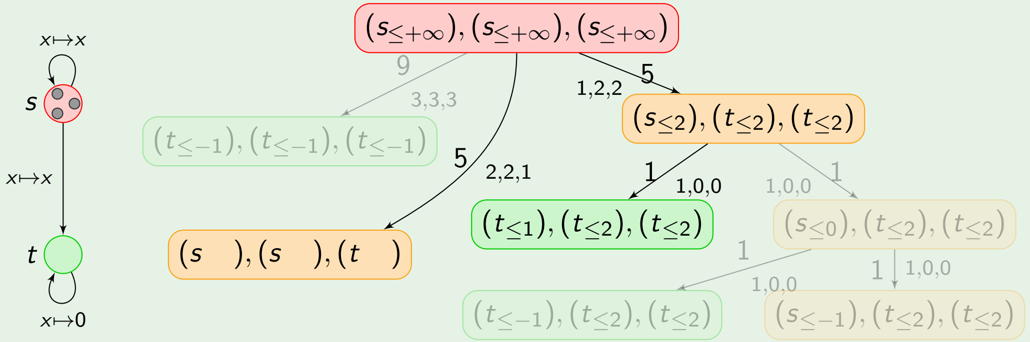

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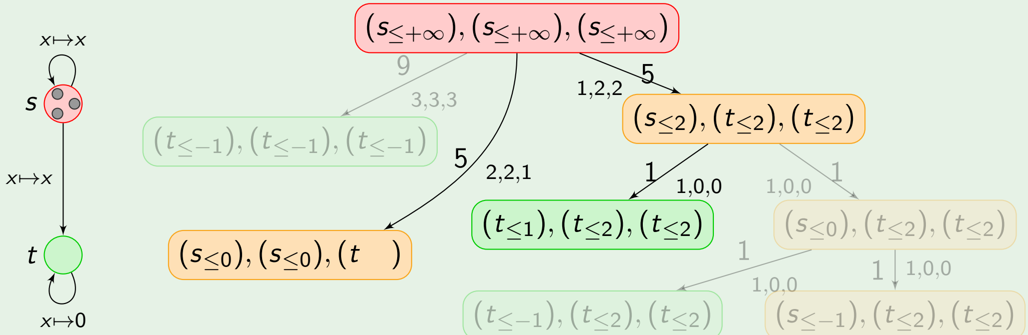

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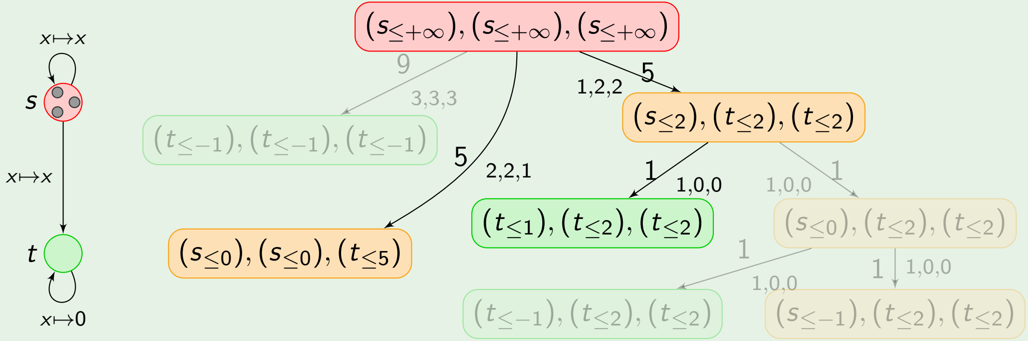

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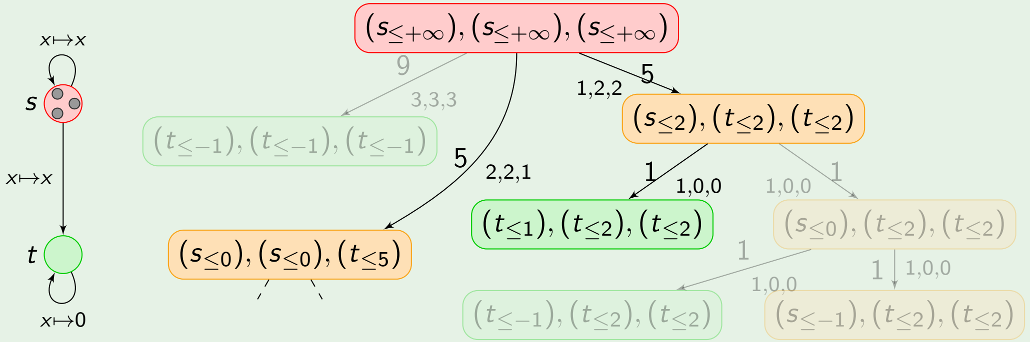

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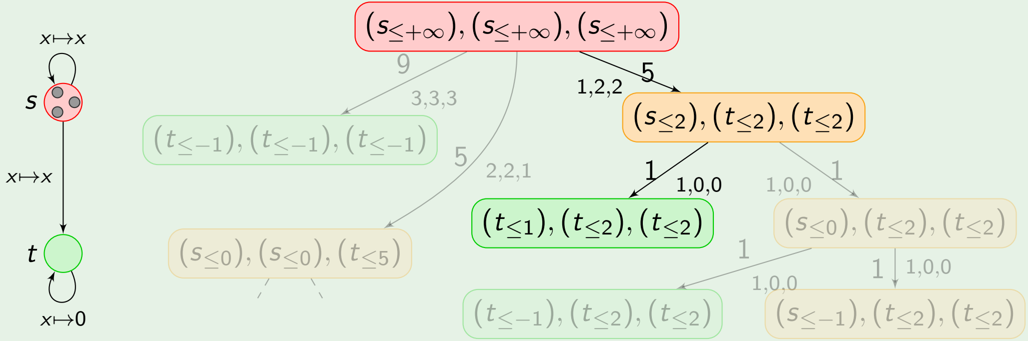

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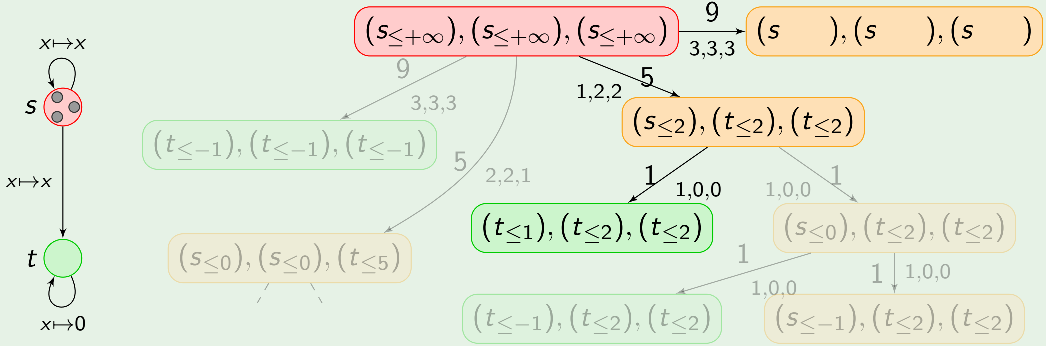

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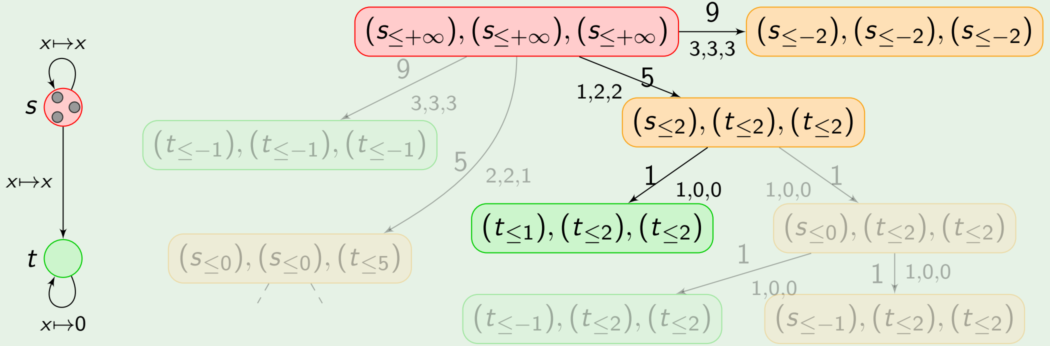

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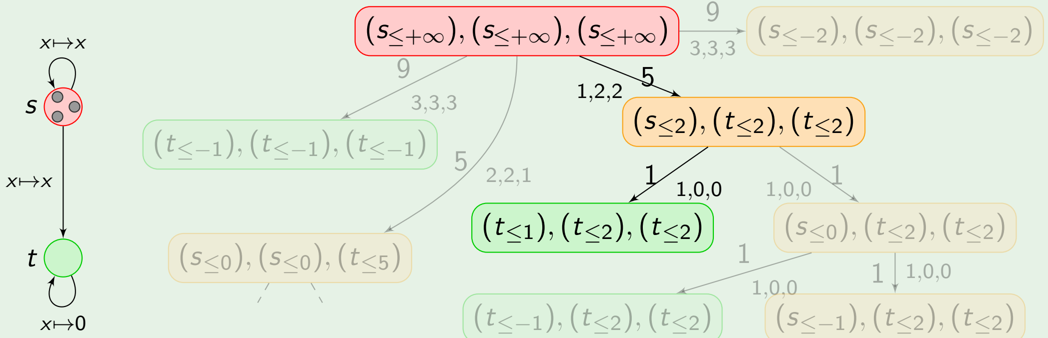

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Theorem

In ANCG with synchronous costs and non-blind strategies, the constrained Nash-equilibrium problem is in EXPSPACE.

Proof

Non-deterministically build a path in this graph:

- each $\mathrm{val}_i(c)$ can be computed in exponential time,
- storing a vertex of the graph requires exponential space;
- propagating constraints uses exponential time.

Subgame-perfect equilibria

In dynamic games, subgame perfect equilibria better reflect behaviours of rational players:

Definition

A strategy profile is a subgame perfect equilibrium if it is a Nash equilibrium in any subgame of $\mathcal{G}$.

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Proof

We adapt the PSPACE algorithm of [BBG+19] to (doubly-exp) concurrent games:

- use equivalent notion of very-weak SPEs (restricted deviations);
- define functions that bound the cost of outcomes of SPEs;
- compute those functions as fixpoints.

[BBG+19] Brihaye *et al.* The complexity of subgame perfect equilibria [...]. CONCUR'19, p. 13:1-13:16. LZI, 2019.

Timed network congestion games

Timed network congestion games

Assigning time-dependent costs to states

- costs are now assigned to states; transitions are guarded by timing constraints;
- the cost for a player depends on the amount of time spent in a state;
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[AGK17] Avni, Guha, Kupferman. Timed Network Games. MFCS'17, p. 37:1-37:16. LZI, 2017.

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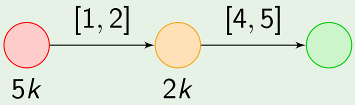

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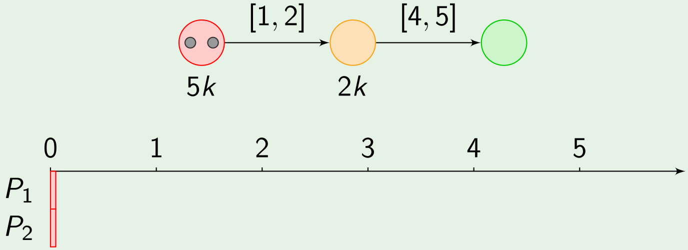

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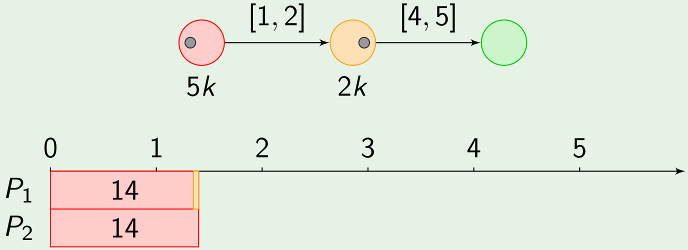

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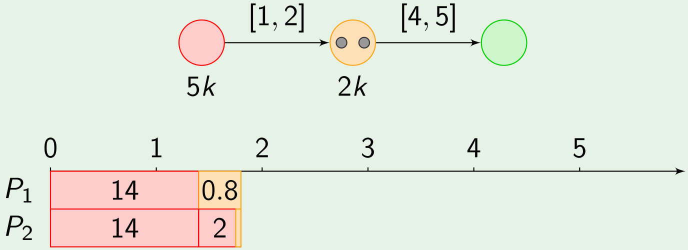

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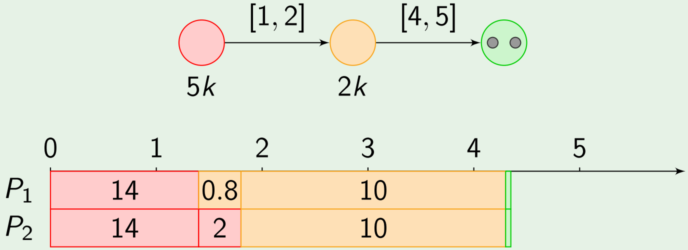

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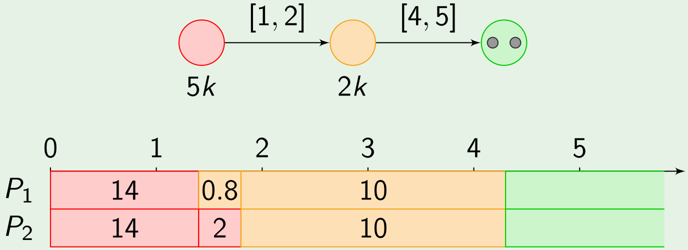

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Some results on timed network congestion games

For blind strategies:

- any timed ANCG can be transformed into an isomorphic ANCG (with asynchronous cost computation); a converse transformation exists for acyclic ANCG (for some relevant notion of isomorphic);
- the price of anarchy in timed ANCG with linear cost functions is at most $5/2$;
- computing a Nash equilibrium can be performed in polynomial time for symmetric timed ANCG; it is PLS-complete in the asymmetric case;
- all timed ANCG admit boundary social optima and Nash equilibria (boundary means that transitions are taken at bounds of timing intervals);
- there are timed ANCG in which worst Nash equilibria are not boundary;

[AGK17] Avni, Guha, Kupferman. Timed Network Games. MFCS'17, p. 37:1-37:16. LZI, 2017.

Non-blind strategies in discrete-time timed network congestion games

Lemma

For any timed ANCG $(\mathcal{G}, n)$*, in any Nash equilibrium, all players can reach their target state with cost at most* $\kappa_n \cdot (M + |V|)$*, where* κ_n *is the maximal cost that can appear in* $(\mathcal{G}, n)$ *and* M *is the maximal time bound appearing in clock constraints.*

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Remark

- we hope we can extend this result to continuous time:
  - compute $\text{val}_i(c')$ in 1-clock timed game (piecewise-affine functions);
  - propagate constraints using characterization of outcomes.
- little chance to extend this to timed network games with clocks [AGK18].

[AGK18] Avni, Guha, Kupferman. Timed Network Games with Clocks. MFCS'18, p. 23:1-23:18. LZI, 2018.

Future works

Congestion impacting travel times (with Stéphane Le Roux and Ocan Sankur)

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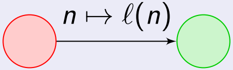
“water-dispenser” semantics (aka. “conveyor-belt” semantics [KP12])



[KP12] Koutsoupias, Papakonstantinopoulou. Contention Issues in Congestion Games. ICALP'12, p. 623-635. Springer, 2012.

Future works

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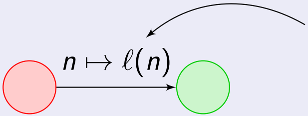

if n players are using this edge, they progress at speed $1/\ell(n)$.

[KP12] Koutsoupias, Papakonstantinopoulou. Contention Issues in Congestion Games. ICALP'12, p. 623-635. Springer, 2012.

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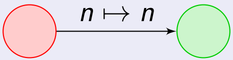
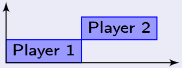


- social optimum: fill the bottle one after the other;



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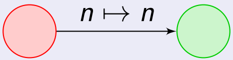
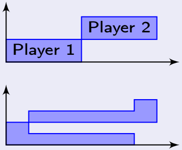


- social optimum: fill the bottle one after the other;
- Nash equilibria: one starts now, one during first t.u.



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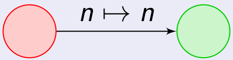

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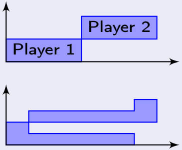
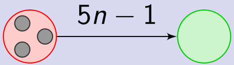


Nash equilibria need not exist in this setting.



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Future works

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Partial-observation strategies (with Arthur Dumas and Ocan Sankur)

Preliminary results:

- Nash equilibria always exist;
- the constrained Nash-equilibrium problem is decidable.

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Conjecture

There exists a unique symmetric mixed Nash equilibrium in series-parallel networks.

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