history-deterministic automata

Nicolas Markey
CNRS – IRISA (Univ. Rennes, France)

based on joint discussions with Uli, Sophie, Dylan, Hervé

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Nondeterminism in the Presence of a Diverse or Unknown Future*

Udi Boker[1], Denis Kuperberg[2], Orna Kupferman[2], and Michał Skrzypczak[3]

[1] IST Austria, Klosterneuburg, Austria
University, Jerusalem, Israel

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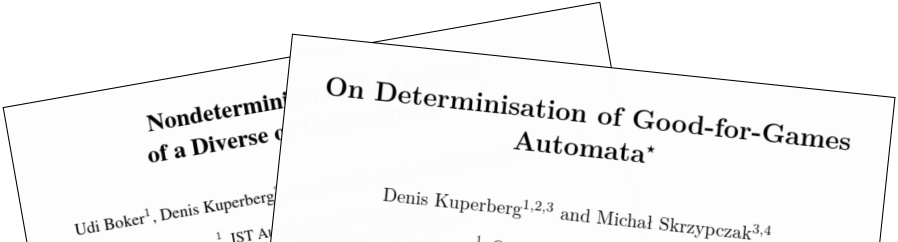

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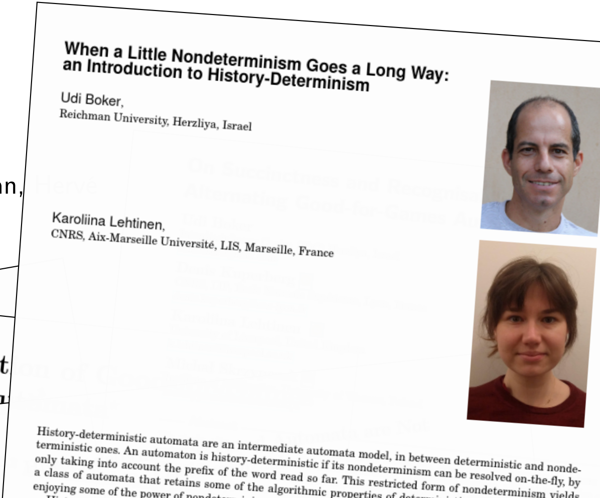
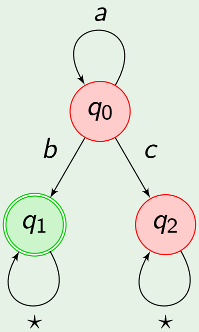


Automata on infinite words and trees

Example (Büchi word automata)



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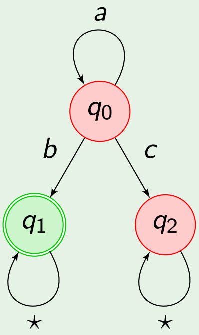

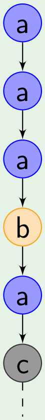

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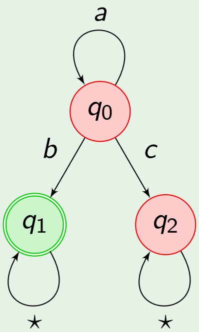

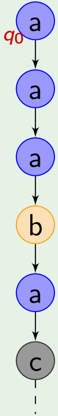

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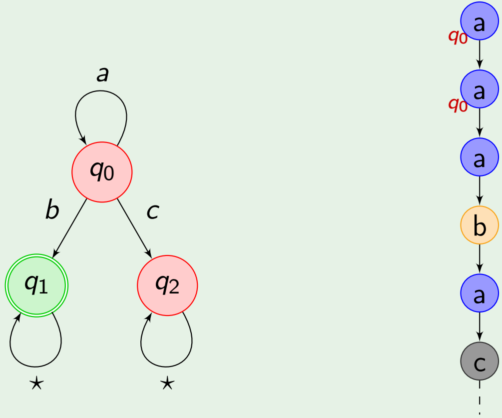

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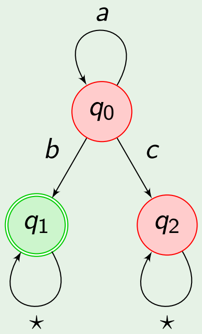

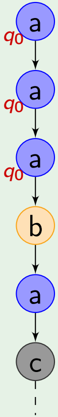

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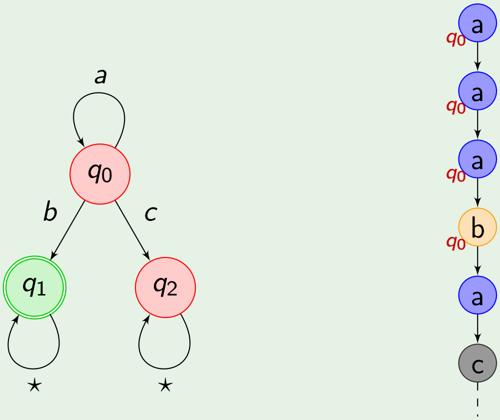

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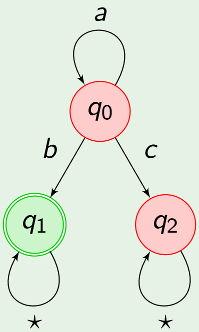

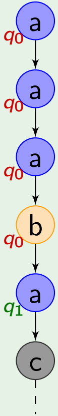

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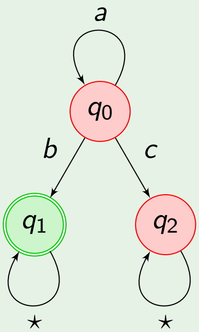

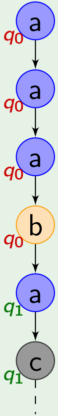

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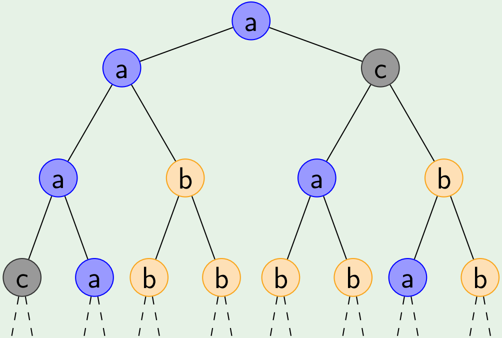

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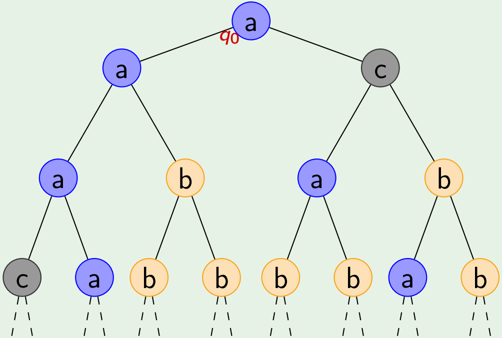

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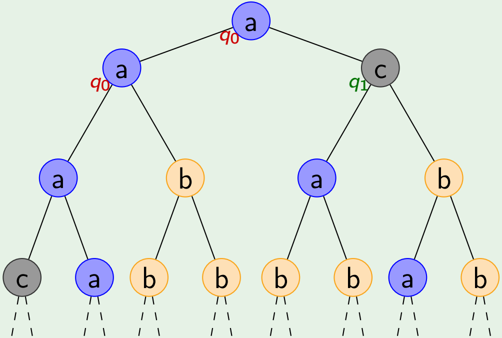

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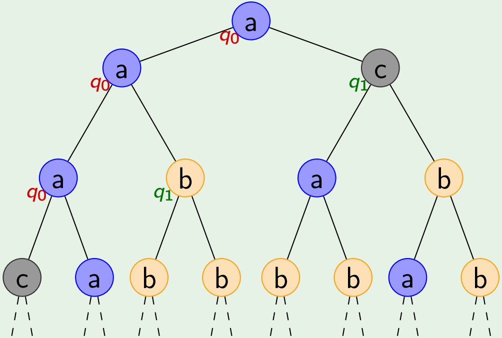

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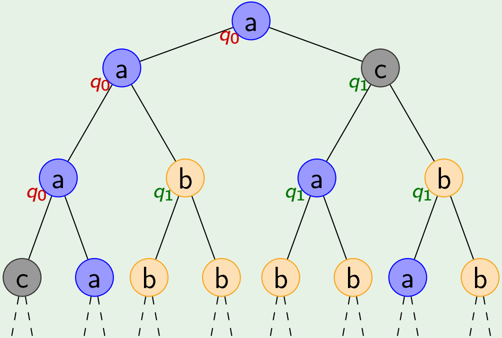

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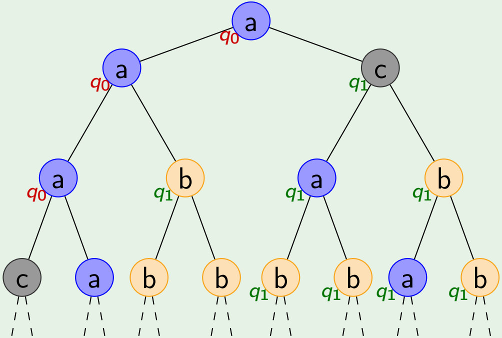

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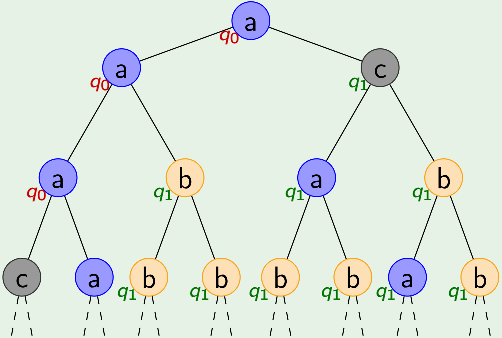

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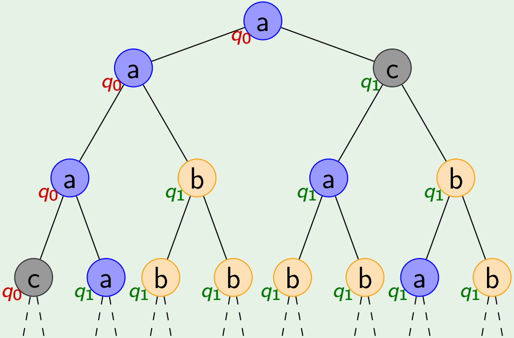
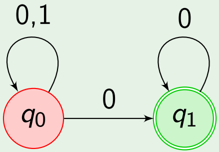


Deterministic vs. non-deterministic Büchi automata

Example



accepts all infinite words
having finitely many 1s

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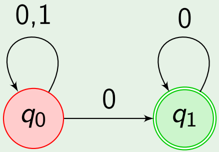

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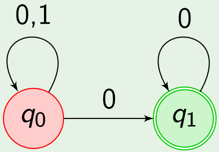

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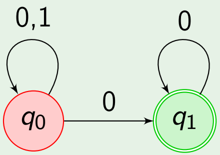

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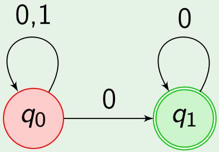

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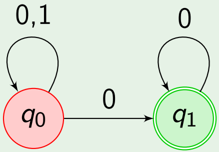

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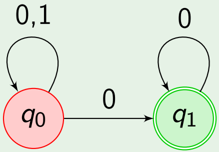

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From word to tree automata

Derived language [KSV06]

Given $L \subseteq \Sigma^\omega$, we let ΔL be the set of trees all of whose infinite branches are labelled with words in L.

[KSV06] Kupferman, Safra, Vardi. Relating word and tree automata. Annals Pure & Applied Logic, 138(1-3):126-146, 2006.
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Natural construction for deterministic automata

If $\delta_{\mathcal{B}}(q, \sigma) = q'$, we let $\delta_{\mathcal{C}}(q, \sigma) = (q', q')$. ✓

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We let $\delta_{\mathcal{C}}(q, \sigma) = \{(q', q'') \mid q', q'' \in \delta_{\mathcal{B}}(q, \sigma)\}$. ✗

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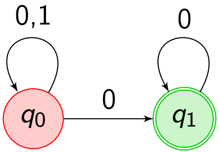

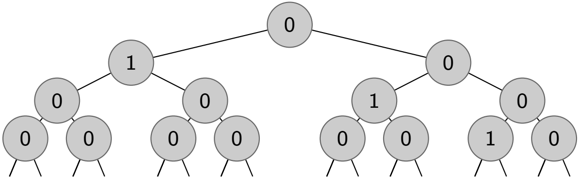

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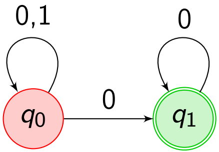

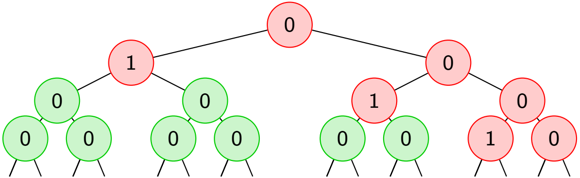

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Good-for-trees automata [BKKS13]

A word automaton $\mathcal{B}$ accepting some language L is good-for-trees if the associated tree automaton $\mathcal{C}$ accepts ΔL.

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Reactive synthesis

Environment
Controller

Reactive synthesis

Environment $i_1 = a$

Controller

Reactive synthesis

Environment	$i_1 = a$
Controller	$o_1 = \alpha$

Reactive synthesis

Environment	$i_1 = a$	$i_2 = b$
Controller	$o_1 = \alpha$	

Reactive synthesis

Environment	$i_1 = a$	$i_2 = b$
Controller	$o_1 = \alpha$	$o_2 = \alpha$

Reactive synthesis

Environment	$i_1 = a$	$i_2 = b$	$i_3 = b$
Controller	$o_1 = \alpha$	$o_2 = \alpha$	

Reactive synthesis

Environment	$i_1 = a$	$i_2 = b$	$i_3 = b$
Controller	$o_1 = \alpha$	$o_2 = \alpha$	$o_3 = \beta$

Reactive synthesis

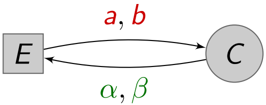
Environment	$i_1 = a$	$i_2 = b$	$i_3 = b$	$\cdots$
Controller	$o_1 = \alpha$	$o_2 = \alpha$	$o_3 = \beta$	$\cdots$

Reactive synthesis

Environment $i_1 = a \quad i_2 = b \quad i_3 = b \quad \cdots$

Controller $o_1 = \alpha \quad o_2 = \alpha \quad o_3 = \beta \quad \cdots$

$\mathcal{G}$



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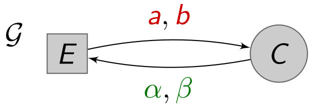

Environment	$i_1 = a$	$i_2 = b$	$i_3 = b$	$\cdots$
Controller	$o_1 = \alpha$	$o_2 = \alpha$	$o_3 = \beta$	$\cdots$

Controller wins if, and only if, $i_1 \; o_1 \; i_2 \; o_2 \; i_3 \; o_3 \; \ldots \in L$

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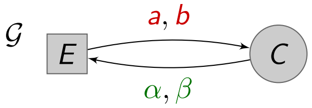

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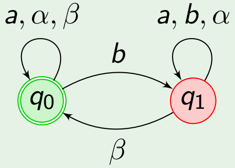
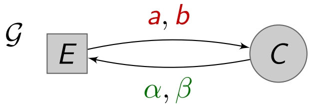
Example

if L is defined by a determinstic automaton $\mathcal{D}$: $(\mathbf{G}(b \Rightarrow \mathbf{F}\,\beta))$



Reactive synthesis



Environment	$i_1 = a$	$i_2 = b$	$i_3 = b$	$\cdots$
Controller	$o_1 = \alpha$	$o_2 = \alpha$	$o_3 = \beta$	$\cdots$

Controller wins if, and only if, $i_1\ o_1\ i_2\ o_2\ i_3\ o_3\ \ldots \in L$

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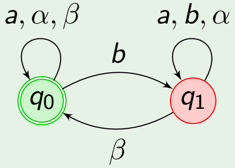

- take product of $\mathcal{D}$ and $\mathcal{G}$
- solve the resulting game (in polynomial time).

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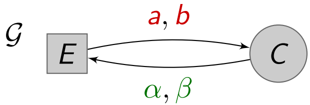

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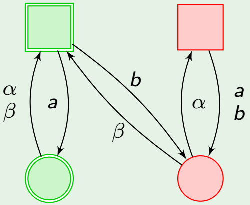

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Reactive synthesis

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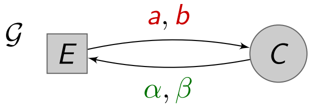

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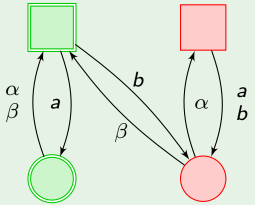

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✓

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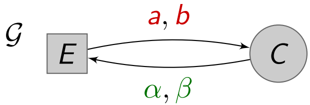

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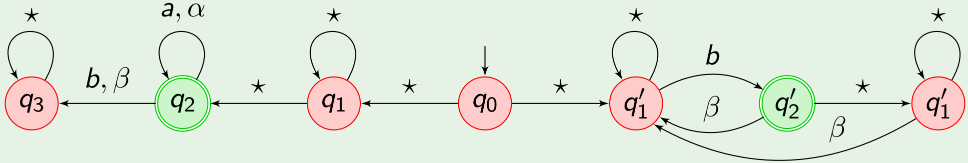
if L is defined by non-determinstic automaton $\mathcal{N}$: $(\mathbf{G}\,\mathbf{F}\, b \iff \mathbf{G}\,\mathbf{F}\, \beta)$



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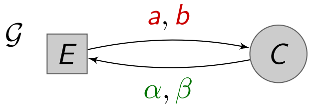

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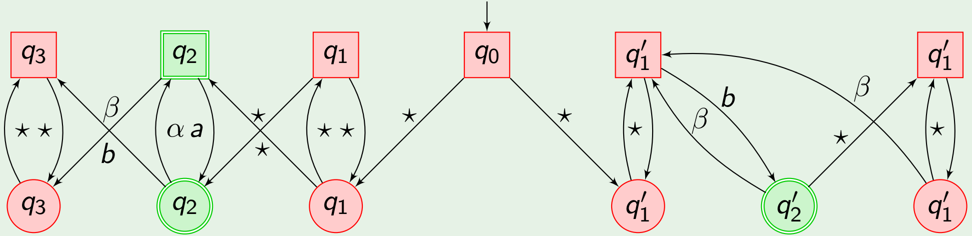

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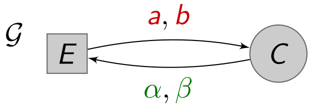

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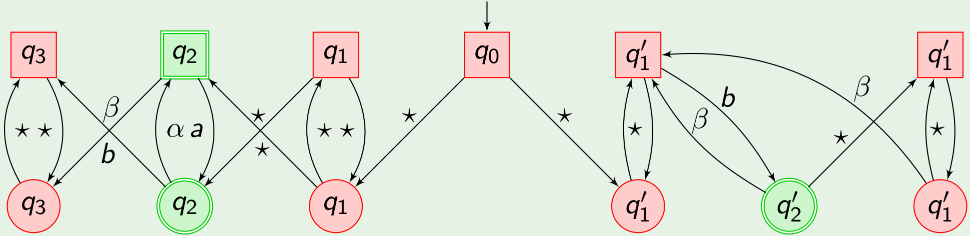

✗

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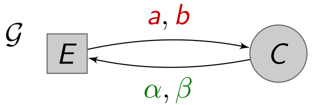

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Good-for-games automata [HP06]

An automaton $\mathcal{A}$ on alphabet Σ is good-for-games if for any two-player zero-sum game $\mathcal{G}$ with Σ-labelled transitions and winning condition $\mathcal{L}(\mathcal{A})$, the games $\mathcal{G}$ and $\mathcal{G} \times \mathcal{A}$ have the same winner.

[HP06] Henzinger, Piterman. Solving Games Without Determinization. In CSL 2006, p. 395-410. Springer, 2006.

History-deterministic automata

Letter game (aka. monitor game)

The letter game ona word automaton $\mathcal{A}$ is a 2-player game played as follows:

- initially, a token is placed on the (unique, wlog) initial state of $\mathcal{A}$;
- iteratively, and *ad infinitum*:
  - Adam proposes a letter σ in Σ;
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  In this process, Adam defines a word w, and Eve builds an run π of $\mathcal{A}$ on w.
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History-deterministic automata [HP06,Col09]

Automaton $\mathcal{A}$ is history-deterministic if Eve has a winning strategy (called resolver) in the letter game.

[HP06] Henzinger, Piterman. Solving Games Without Determinization. In CSL 2006, p. 395-410. Springer, 2006.

[Col09] Colcombet. The Theory of Stabilisation Monoids and Regular Cost Functions. In ICALP'09, p. 139-150. Springer, 2009.

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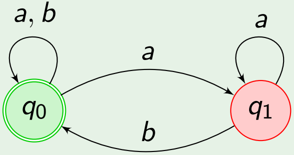
Example

This co-Büchi automaton accepting $\{w \mid |w|_b < \infty\}$ is history-deterministic:



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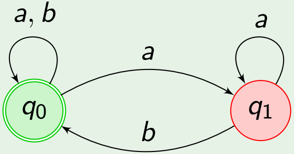

Resolver: always go to q_1 when reading a in q_0.

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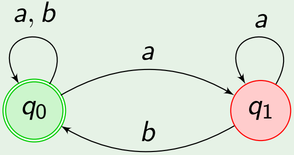

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Determinizable-by-pruning automata [AKL10]

An automaton $\mathcal{A}$ if determinizable-by-pruning if a language-equivalent deterministic automaton can be obtained from $\mathcal{A}$ by removing some transitions.

[AKL10] Aminof, Kupferman, Lampert. Reasoning about online algorithms with weighted automata. ACM Trans. Alg. 6(2):28. ACM, 2010

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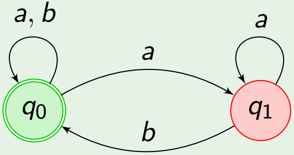

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Obviously, determinizability-by-pruning implies history-determinism.

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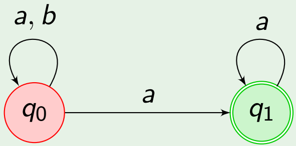
This Büchi automaton accepting $\{w \mid |w|_b < \infty\}$ is not history-deterministic:



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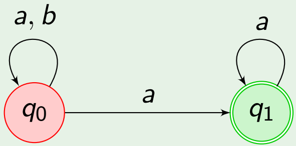

Eve does not win the letter game:

- Adam plays letter a as long as the token is in q_0;
- If Eve never move to q_1, she looses; otherwise, from q_1, Adam propose suffix ba^ω.

Results for ω-automata

Theorem ([HP06,BKKS13,BL19])

Good-for-treeness, good-for-gameness and history-determinism are equivalent.

Determinizability-by-pruning is stronger.

[HP06] Henzinger, Piterman. Solving Games Without Determinization. In CSL 2006, p. 395-410. Springer, 2006.
[BKKS13] Boker *et al.* Nondeterminism in the Presence of a Diverse or Unknown Future. In ICALP'13, p. 89-100. Springer, 2013.
[BL19] Boker, Lehtinen. Good for Games Automata: From Nondeterminism to Alternation. In CONCUR'19, p 19:1-19:16. LZI, 2019.

Results for ω-automata

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Proof

- history-determinism implies good-for-gameness and game-for-treeness: the resolver can be used to resolve non-determinism.
- The other direction is a bit more involved.
- That determinizability-by-pruning is stronger is shown with the next example.

[HP06] Henzinger, Piterman. Solving Games Without Determinization. In CSL 2006, p. 395-410. Springer, 2006.
[BKKS13] Boker *et al.* Nondeterminism in the Presence of a Diverse or Unknown Future. In ICALP'13, p. 89-100. Springer, 2013.
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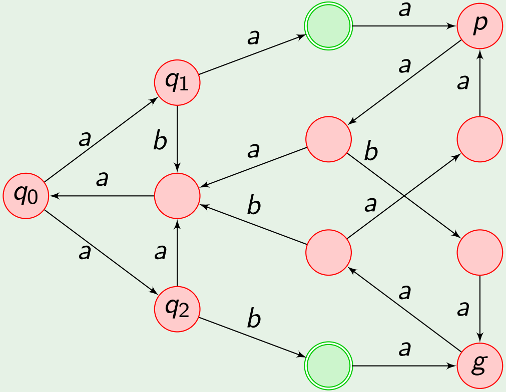
History-deterministic automata

Example

This Büchi automaton accepting $[(aaa + aba)^* \cdot (aaa\ aaa + aba\ aba)]^\omega$ is history-deterministic, but not determinizable-by-pruning:



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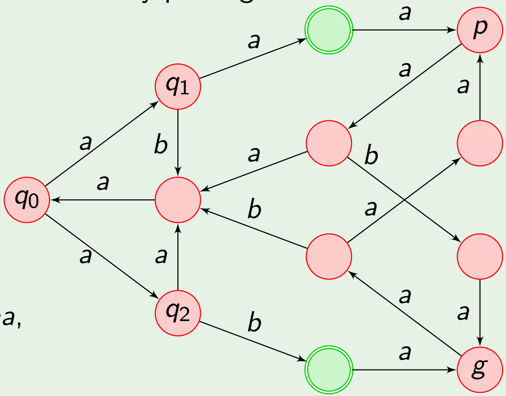


Resolver:

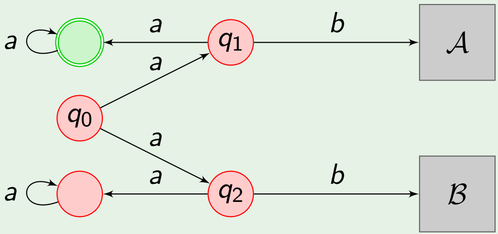
In q_0, go to q_1 if we just read aaa, otherwise, go to q_2.

[BKKS13] Boker *et al.* Nondeterminism in the Presence of a Diverse or Unknown Future. In ICALP'13, p 89-100. Springer, 2013.

Deciding history-determinism

Example (Hardness)

If $\mathcal{A}$ is deterministic, the automaton below is history-deterministic (and determinizable-by-pruning) if, and only if, $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$.



[LZ22] Lehtinen, Zimmermann. Good-for-games ω-pushdown automata. LMCS 18(1):3. 2022.

Deciding history-determinism: k-token games

k-token games

Given an automaton $\mathcal{A}$, the k-token game $\mathcal{G}_k(\mathcal{A})$ on $\mathcal{A}$ runs as follows:

- initially, k tokens of Adam and one token of Eve are on the initial state of $\mathcal{A}$;
- repeatedly, *ad infinitum*:
  - Adam proposes a letter $\sigma \in \Sigma$;
  - Eve moves her token along a σ-transition;
  - Adam moves his k tokens along σ-transitions.
- Eve wins if either Adam fails to produce an accepting run, or if she produces an accepting run.

[BK18] Bagnol and Kuperberg. Büchi Good-for-game Automata are Efficiently Recognizable. In FSTTCS'18, p. 16:1-16:14. LZI, 2018.

Deciding history-determinism: k-token games

Lemma

If $\mathcal{A}$ is history-deterministic, then Eve wins $\mathcal{G}_k(\mathcal{A})$.

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Lemma

If $\mathcal{A}$ is history-deterministic, then Eve wins $\mathcal{G}_k(\mathcal{A})$.

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There exists a non-history-deterministic automaton $\mathcal{B}$ for which Eve wins $\mathcal{G}_1(\mathcal{B})$.

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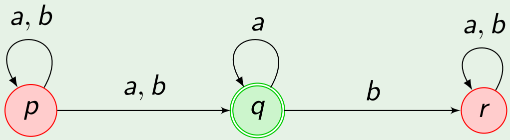

- Adam wins the letter game: play a until Eve moves to q; then play ba^ω;
- Eve wins the 1-token game: follow Adam's token.

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Deciding history-determinism: k-token games

Lemma

Eve wins $\mathcal{G}_2(\mathcal{A})$ if, and only if, she wins $\mathcal{G}_k(\mathcal{A})$ for all $k \geq 2$.

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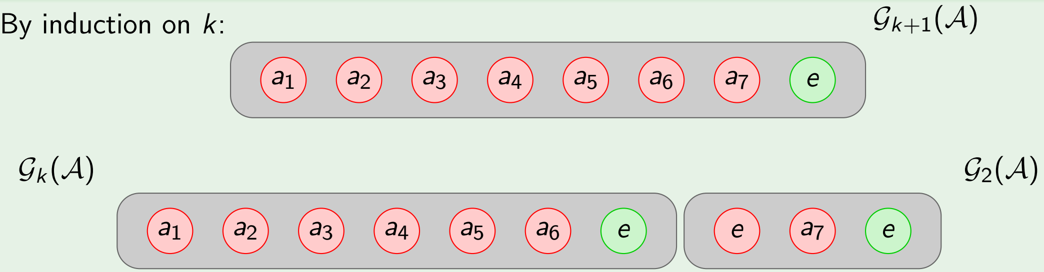

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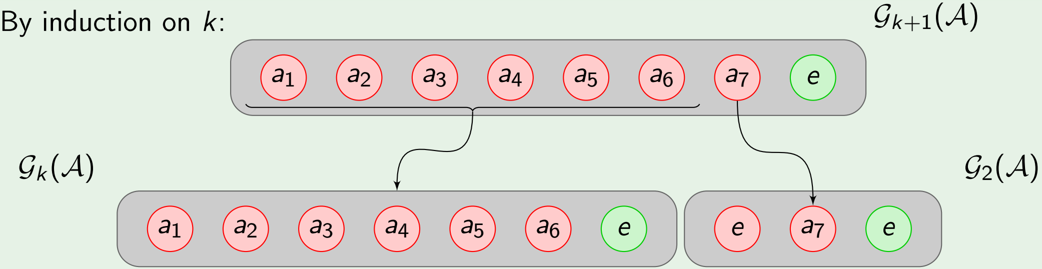

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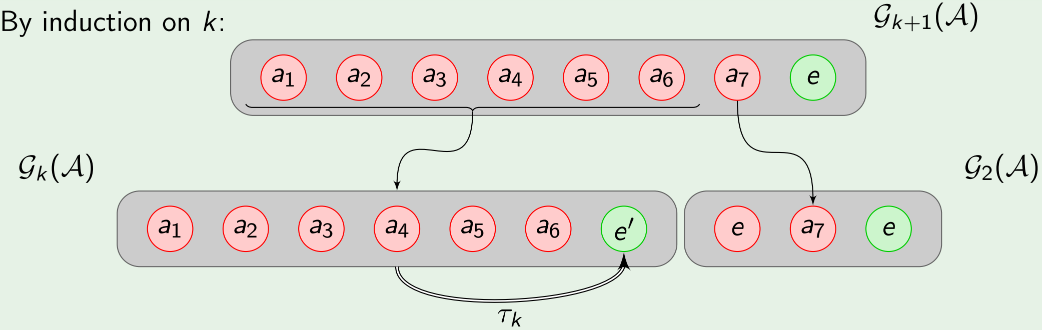

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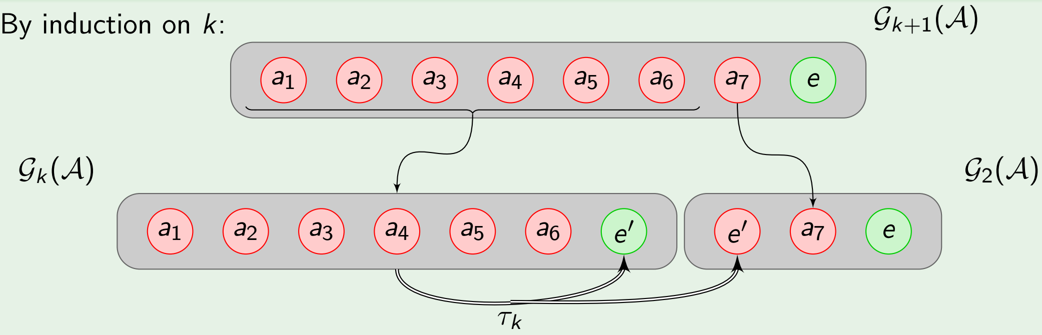

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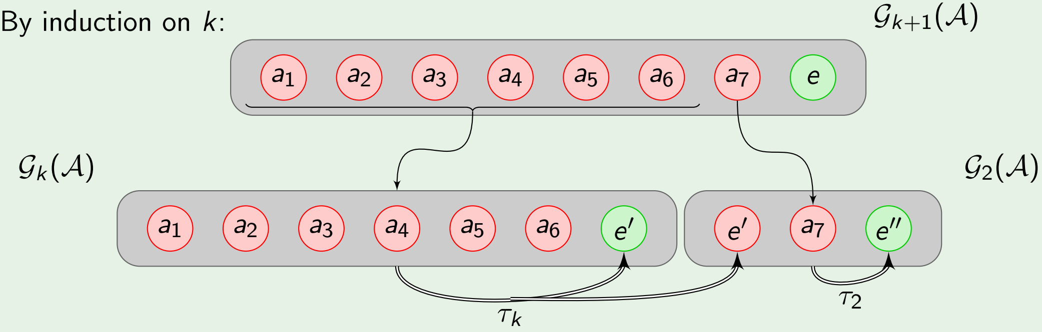

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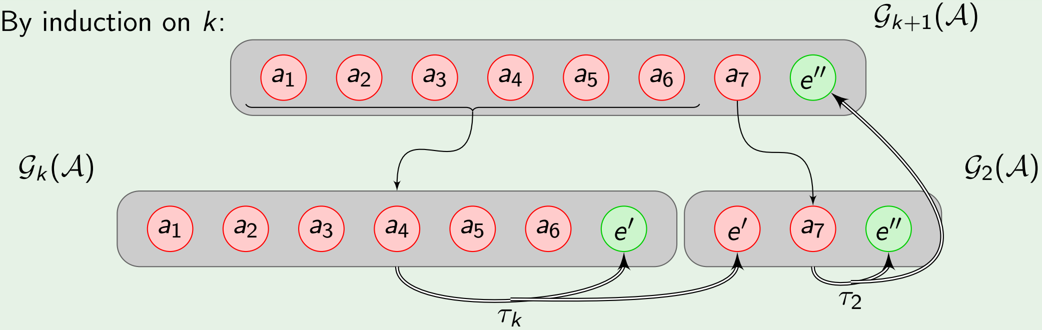

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Theorem (BK19,BKLS20)

For Büchi and co-Büchi acceptance conditions, $\mathcal{A}$ is history-deterministic if, and only if, Eve wins the 2-token game.

[BK18] Bagnol and Kuperberg. Büchi Good-for-game Automata are Efficiently Recognizable. In FSTTCS'18, p. 16:1-16:14. LZI, 2018.
[BKLS20] Boker *et al.* On Succinctness and Recognisability of Alternating Good-for-Games Automata. arXiv preprint 2002.07278.

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Corollary

History-determinism is decidable in polynomial time for Büchi and co-Büchi automata.

[BK18] Bagnol and Kuperberg. Büchi Good-for-game Automata are Efficiently Recognizable. In FSTTCS'18, p. 16:1-16:14. LZI, 2018.
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More results and open problems on history-determinism

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[KS15] Kuperberg and Skrzypczak. On determinisation of good-for-games automata. In ICALP'15, p. 299-310. Springer, 2015.

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Extensions of automata [LZ20,BHL$^+$22]

- history-determinism is undecidable for pushdown automata;
- history-deterministic timed automata are not determinizable.

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