Temporal logic with forgettable past

Nicolas Markey
CNRS – IRISA (Univ. Rennes, France)

joint work with my former PhD advisors
François Laroussinie and Philippe Schnoebelen
Temporal logic with forgettable past

Nicolas Markey
CNRS – IRISA (Univ. Rennes, France)

joint work with my former PhD advisors
François Laroussinie and Philippe Schnoebelen
(Linear-time) temporal logic with forgettable past
(Linear-time) temporal logic with forgettable past
(Linear-time) temporal logic with forgettable past

- atomic propositions: $\circ, \circ, \ldots$

- Boolean combinators: $\neg \varphi, \varphi \lor \psi, \varphi \land \psi, \ldots$
(Linear-time) temporal logic with forgettable past

- Atomic propositions: \( \bigcirc, \bigcirc, ... \)
- Boolean combinators: \( \neg \varphi, \varphi \lor \psi, \varphi \land \psi, ... \)
- Temporal modalities:
  - \( X \varphi \) \( \text{“next } \varphi \text{”} \)
  - \( \varphi \mathbf{U} \psi \) \( \text{“}\varphi \text{ until } \psi\text{”} \)
(Linear-time) temporal logic with forgettable past

- atomic propositions: $\circ$, $\circ$, ...
- Boolean combinators: $\neg \varphi$, $\varphi \lor \psi$, $\varphi \land \psi$, ...
- temporal modalities:

  - $\text{X} \varphi$ \quad \text{"next } \varphi\text{"}
  - $\varphi \text{ U } \psi$ \quad \text{"} \varphi \text{ until } \psi\text{"}
  - $\text{F } \varphi \equiv \text{true U } \varphi$ \quad \text{"eventually } \varphi\text{"}
  - $\text{G } \varphi \equiv \neg \text{F } \neg \varphi$ \quad \text{"always } \varphi\text{"}
(Linear-time) temporal logic with forgettable past

- atomic propositions: $\bigcirc, \bigcirc, ...$
- Boolean combinators: $\neg \varphi, \varphi \lor \psi, \varphi \land \psi, ...$
- temporal modalities:
  
  - $\text{X } \varphi$  
  - $\varphi \text{ U } \psi$ 

  "next $\varphi$"

  "$\varphi$ until $\psi$"
(Linear-time) temporal logic with forgettable past

- atomic propositions: \( \bigcirc, \bigcirc, \ldots \)

- Boolean combinators: \( \neg \varphi, \varphi \lor \psi, \varphi \land \psi, \ldots \)

- temporal modalities:

```
\begin{array}{c}
\varphi \\
\varphi U \psi \\
\varphi S \psi \\
\end{array}
```

```
\begin{array}{c}
X \varphi \\
\varphi \text{ until } \psi \\
\text{"next } \varphi\text{"} \\
\varphi \text{ since } \psi \\
\text{"previously } \varphi\text{"} \\
\end{array}
```
Linear-time temporal logic with forgettable past

- atomic propositions: $\circ, \bigcirc, \ldots$
- Boolean combinators: $\neg \phi, \phi \lor \psi, \phi \land \psi, \ldots$
- temporal modalities:
  
  - $X \phi$ \quad \text{“next $\phi$”}
  
  - $\phi \mathbf{U} \psi$ \quad \text{“$\phi$ until $\psi$”}

Example

You can’t win if you don’t play.

Mark Manson
(Linear-time) temporal logic with forgettable past

- atomic propositions: \( \bigcirc, \bigcirc, \ldots \)
- Boolean combinators: \( \neg \varphi, \varphi \lor \psi, \varphi \land \psi, \ldots \)
- temporal modalities:

\[
\begin{align*}
&X \varphi \\
&\varphi U \psi
\end{align*}
\]

“next \( \varphi \)”

“\( \varphi \) until \( \psi \)”

Example

\[G(\neg \text{submit}) \Rightarrow G(\neg \text{accepted})\]
(Linear-time) temporal logic with forgettable past

- **atomic propositions:** $\bigcirc, \bigcirc, ...$
- **Boolean combinators:** $\neg \varphi, \varphi \lor \psi, \varphi \land \psi, ...$
- **temporal modalities:** $\varphi$, $\varphi \lor \psi$, $\varphi \land \psi$, ...

**Example**

$\neg F(accepted \land X^{-1} G^{-1} \neg submit)$
(Linear-time) temporal logic with forgettable past

- atomic propositions: $\bigcirc, \bigcirc, ...$
- Boolean combinators: $\neg \varphi, \varphi \lor \psi, \varphi \land \psi, ...$
- temporal modalities:

$$\neg F(accepted \land X^{-1} G^{-1} \neg submit) \equiv_i \neg((\neg submit) U accepted)$$

Example
Theorem (Sistla, Clarke (1982) + Vardi, Wolper (1986))

Model checking PastLTL and LTL is PSPACE-complete.

PastLTL and LTL formulas can be compiled into equivalent exponential-size Büchi automata.
(Linear-time) temporal logic with forgettable past

- **operator Now:**

  ![Diagram](image)

  \(\phi^N\) \(\equiv \) "from now on \(\phi\)"

Example

\[\neg N F (\text{accepted} \land X^{-1} G^{-1} \neg \text{submit})\]

**Theorem**

Any formula in PastLTL+Now can be compiled into an equivalent exponential-size alternating Büchi automaton.

Model checking PastLTL+Now is EXPSPACE-complete.
(Linear-time) temporal logic with forgettable past

- operator \textbf{Now}:

\[
\varphi \xrightarrow{N} \varphi
\]

"from now on \varphi"

\textbf{Example}

\[\neg N F (\text{accepted} \land X^{-1} G^{-1} \neg \text{submit})) \equiv \neg((\neg \text{submit}) U \text{accepted})\]
(Linear-time) temporal logic with forgettable past

- operator **Now**:

\[ \phi \]

"from now on \( \phi \)"

**Example**

\[ \neg N F(\text{accepted} \land X^{-1} G^{-1} \neg \text{submit}) \equiv \neg (\neg \text{submit} U \text{accepted}) \]

**Theorem**

Any formula in PastLTL+Now can be compiled into an equivalent exponential-size alternating Büchi automaton.

Model checking PastLTL+Now is EXPSPACE-complete.
(Linear-time) temporal logic with forgettable past

Theorem (Kamp (1968) + Gabbay, Pnueli, Shelah, Stavi (1980))

PastLTL and LTL are equally expressive.
(Linear-time) temporal logic with forgettable past

**Theorem**

\textit{PastLTL} can be exponentially more succinct than \textit{LTL}.
Theorem

*PastLTL* can be exponentially more succinct than *LTL*.

Proof

\( R_n: \) “any state that agrees with the initial state on propositions \( p_1 \) to \( p_n \) also agrees on \( p_0 \)”
(Linear-time) temporal logic with forgettable past

Theorem

*PastLTL* can be exponentially more succinct than *LTL*.

Proof

\( R_n \): "any state that agrees with the initial state on propositions \( p_1 \) to \( p_n \) also agrees on \( p_0 \)"

- property \( R_n \) can be expressed as a “small” *PastLTL* formula;
(Linear-time) temporal logic with forgettable past

Theorem

PastLTL can be exponentially more succinct than LTL.

Proof

\( R_n \): “any state that agrees with the initial state on propositions \( p_1 \) to \( p_n \) also agrees on \( p_0 \)”

- property \( R_n \) can be expressed as a “small” PastLTL formula;

\( R'_n \): “any two states that agree on propositions \( p_1 \) to \( p_n \) also agree on \( p_0 \)”

- property \( R'_n \) cannot be expressed as a “small” (Past)LTL formula.