Synchronizing automata with LTL constraints

Nicolas Markey
IRISA, CNRS & Inria & Univ. Rennes, France

joint work with Nathalie Bertrand and Hugo Francon.

Coffee machine

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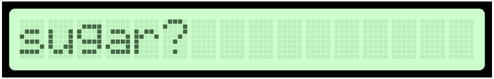

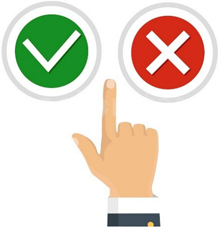

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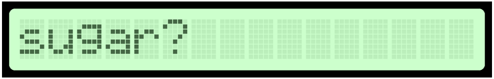

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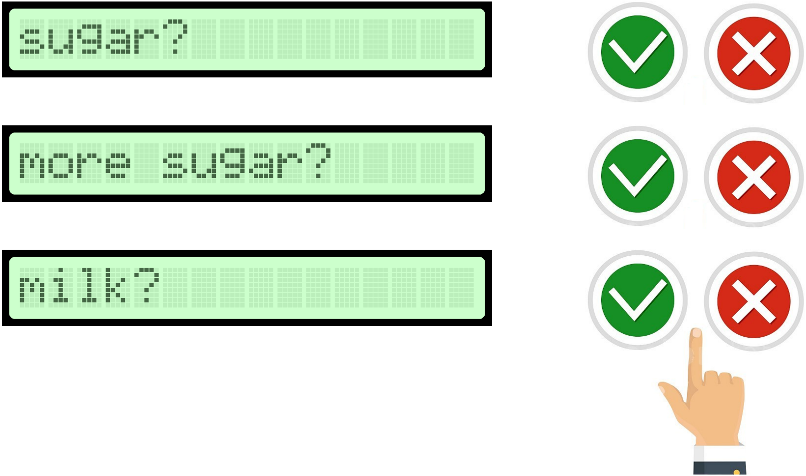

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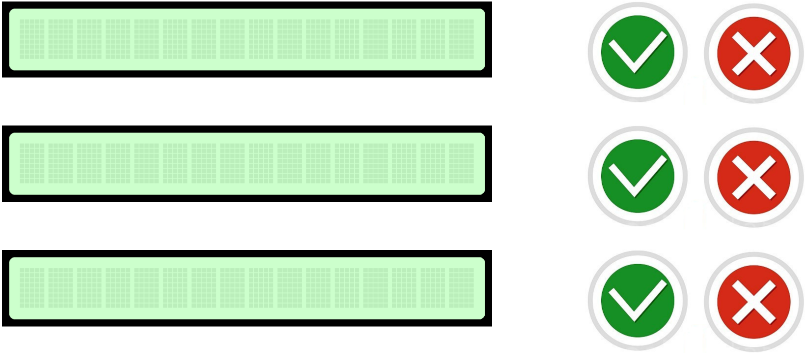

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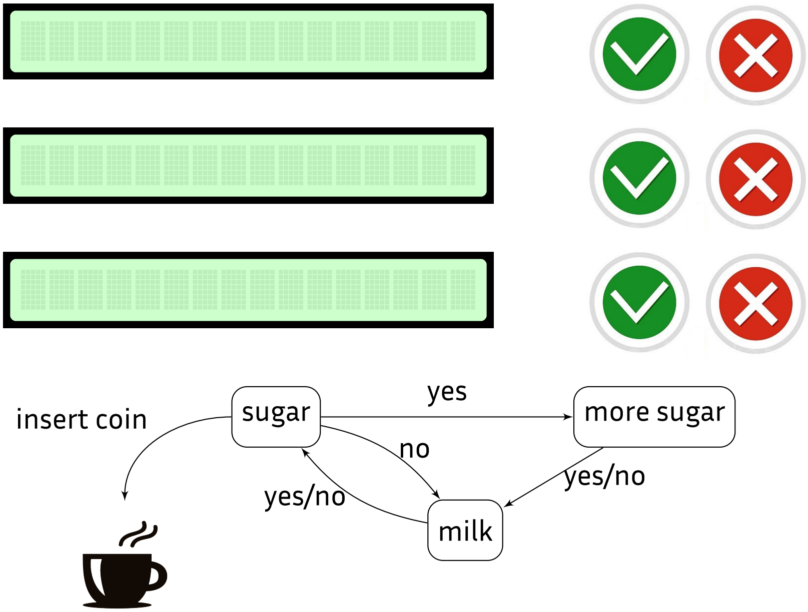

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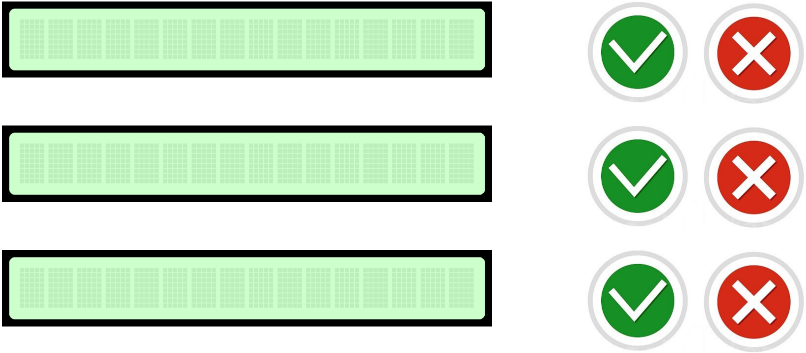

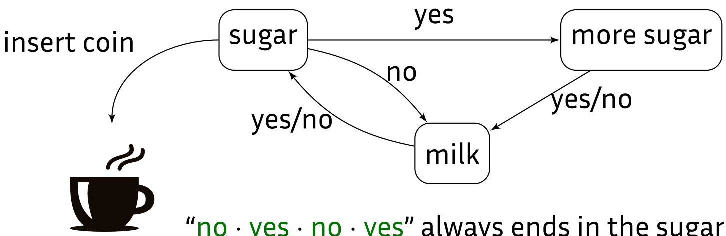

"no · yes · no · yes" always ends in the sugar state

Synchronizing words

Definition

A finite-state automaton $\mathcal{A} = (S, \Sigma, \delta)$ is synchronizable if there exists $w \in \Sigma^*$ such that $\delta(s, w) = \delta(s', w)$ for all $s, s' \in S$.

In (most of) the sequel, automata are deterministic and complete.

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Synchronizability can be decided in polynomial space.

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Proposition

Synchronizability can be decided in polynomial space.

Proof

Consider the power automaton $\mathcal{P}(\mathcal{A}) = (S', \Sigma, \delta')$:

- $S' = 2^S$;
- $\delta'(P, \sigma) = \{\delta(s, \sigma) \mid s \in P\}$.

Non-deterministically look for a path from S to a singleton state.

Deciding the existence of a synchronizing word

Theorem (Čer64)

Synchronizability can be decided in non-deterministic log. space.

[Čer64] J. Černý. Poznámka k homogénnym eksperimentom s konečnými automatami. 1964.

Deciding the existence of a synchronizing word

Theorem ([Čer64])

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Proof

Lemma

$\mathcal{A}$ is synchronizable iff

for all s_i, s_j, there exists w_{ij} such that $\delta(s_i, w_{ij}) = \delta(s_j, w_{ij})$.

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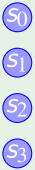

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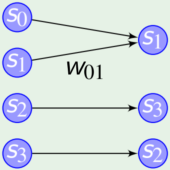

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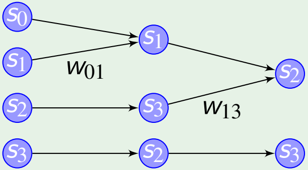

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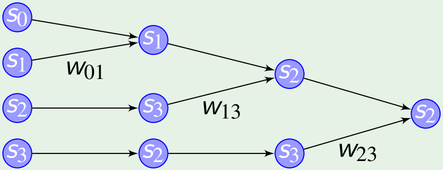

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for all s_i, s_j, there exists w_{ij} such that $\delta(s_i, w_{ij}) = \delta(s_j, w_{ij})$.

This gives a synchronizing word (if any) of size $O(n^3)$.

Černý's Conjecture

The shortest synchronizing word (if any) has size at most $(n-1)^2$.

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Synchronizing words in different contexts

- Subset-synchronization;

Theorem (Rys83,Mar10)

Subset-synchronizability in complete DFAs is PSPACE-complete.
Synchronizability in NFAs is PSPACE-complete.
Careful-synchronizability in incomplete DFAs is PSPACE-complete.

[Rys83] I.K. Rystsov. Polynomial complete problems in automata theory. IPL, 1983.
[Mar10] P.V. Martyugin. Complexity of Problems Concerning Carefully Synchronizing Words [...] CSR, 2010.

Synchronizing words in different contexts

- Subset-synchronization;
- Synchronization in MDPs and probabilistic automata;

Theorem (Shi14)

Almost-sure synchronizability in MDPs is PSPACE-complete.
Almost-sure synchronizability in PAs is undecidable.

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Synchronizing words in different contexts

- Subset-synchronization;
- Synchronization in MDPs and probabilistic automata;
- Synchronization in timed automata;

Theorem (Shi14)

Synchronizability in timed automata is PSPACE-complete.

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Synchronizing words in different contexts

- Subset-synchronization;
- Synchronization in MDPs and probabilistic automata;
- Synchronization in timed automata;
- Synchronization with energy constraints;

Theorem (Shi14)

Synchronizability under energy constraints is PSPACE-complete.

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Synchronizing words in different contexts

- Subset-synchronization;
- Synchronization in MDPs and probabilistic automata;
- Synchronization in timed automata;
- Synchronization with energy constraints;
- Synchronization with dynamic constraints;
  Synchronization with conditions on the order of states
  (e.g. last occurrence of s must preceed first occurrence of s').

Theorem (Wol20)

Synchronizability with constraint $last(s) < last(s')$ is in NP.
Synchronizability with constraint $last(s) < first(s')$ is PSPACE-complete.

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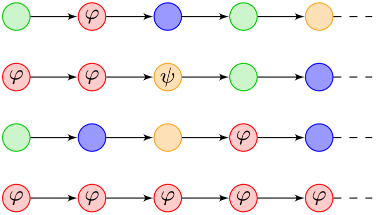
Linear-time Temporal Logic

- **atomic propositions:** , , ...
- **Boolean combinators:** $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...
- **temporal modalities:**

$\mathbf{X}\,\varphi$	"next φ"
$\varphi\ \mathbf{U}\ \psi$	"φ until ψ"
$\text{true}\ \mathbf{U}\ \varphi \equiv \mathbf{F}\,\varphi$	"eventually φ"
$\neg\,\mathbf{F}\,\neg\varphi \equiv \mathbf{G}\,\varphi$	"always φ"



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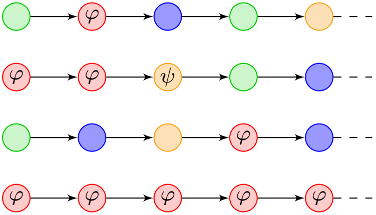

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Example

last(s) $<$ first(s') can be expressed (roughly) as

$$(\neg s')\ \mathbf{U}\ (s \wedge \mathbf{X}\,\mathbf{G}\,\neg s)$$

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Synchronizing words with LTL constraints

Theorem (VW86,DV13)

For any LTL formula φ, there exists an exponential-size NFA $\mathcal{F}_\varphi$ (with a single accepting state q_f) accepting exactly the finite traces satisfying φ.

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Proof

- (Non-deterministically) look for a path in the powerset automaton $\mathcal{P}(\mathcal{A} \otimes \mathcal{F}_\varphi)$ to a singleton-state $(\{s, q_f\})$.
- Hardness follows from [Wol20].

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Conclusion and future work

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- synchronizablity with LTL constraints
- PSPACE-complete in the general case;
- remains PSPACE-complete for
  - subset-synchronizability;
  - fixed LTL formulas;
  - small fragments of LTL (such as $L^+(\mathbf{G})$).

Possible directions for future work

- suitable restrictions of LTL with lower complexity;
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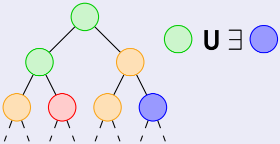

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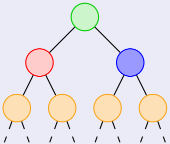

$\mathbf{F}\,\forall$ 

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