A quantitative semantics for Strategy Logic

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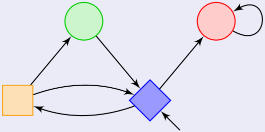
joint work (published at IJCAI'19) with Patricia Bouyer, Orna Kupferman, Bastien Maubert, Aniello Murano, Giuseppe Perelli.

Reasoning about multi-agent systems

Games on graphs



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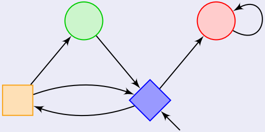

Strategy for player 

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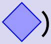
(starting with ).

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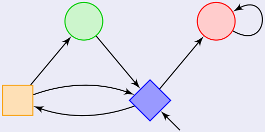

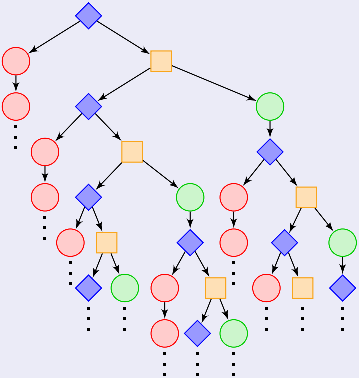

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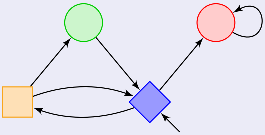

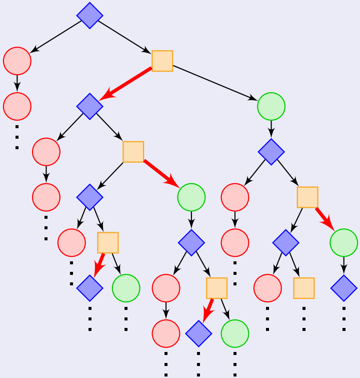

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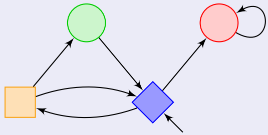

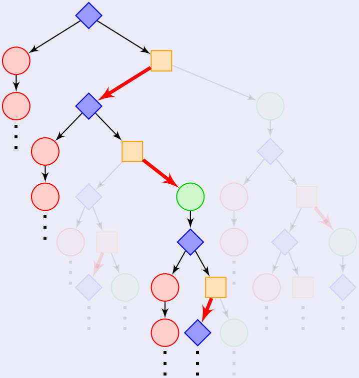

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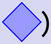
(starting with ).

Reasoning about strategic behaviours

Strategy logic [CHP07,MMV10]

LTL + explicit quantification and assignment of strategies:

$$\exists\sigma_A.\,\forall\sigma_B.\,\mathsf{assign}(A \to \sigma_A, B \to \sigma_B).\,\varphi_A$$

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Quantification over strategies

Assignment of strategies to players

LTL property

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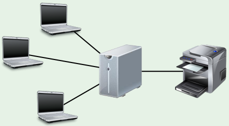
Quantification over strategies — Assignment of strategies to players — LTL property

Example (Client-server interaction)



$$\exists\sigma_S.\ \exists\sigma_c.\ \mathrm{assign}(S \to \sigma_S).$$
$$(\mathbf{G}\ \text{mutual exclusion} \wedge$$
$$\mathbf{G} \bigwedge_C \mathrm{assign}(C \to \sigma_c).\ \mathbf{F}\ \mathrm{access}_C)$$

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$$\underbrace{\exists \sigma_A.\ \forall \sigma_B}_{\text{Quantification over strategies}}.\ \underbrace{\mathsf{assign}(A \to \sigma_A, B \to \sigma_B)}_{\text{Assignment of strategies to players}}.\ \underbrace{\varphi_A}_{\text{LTL property}}$$

Example (Characterisation of Nash equilibria)

$(\sigma_1, \ldots, \sigma_n)$ is a Nash equilibrium iff

$$\mathsf{assign}(\mathsf{Agt} \to (\sigma_1, \ldots \sigma_n)).\ \bigwedge_{A_i \in \mathsf{Agt}} ((\exists \sigma_i'.\mathsf{assign}(A_i \to \sigma_i').\ \varphi_i) \Rightarrow \varphi_i.)$$

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LTL property

Theorem ([DLM10])

SL model-checking is decidable (TOWER-complete).

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Quantified CTL [ES84,Kup95,Fre01,DLM12]

QCTL extends CTL with propositional quantifiers

$\exists p.\, \varphi$ means that there exists a labelling of the model with p under which φ holds.

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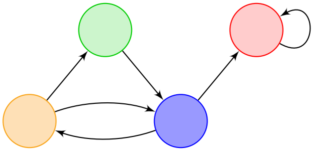
$$\mathbf{E\,F}\,\bigcirc \wedge \neg\left(\exists p.\ \left[\mathbf{E\,F}(\bigcirc \wedge p) \wedge \mathbf{E\,F}(\bigcirc \wedge \neg p)\right]\right)$$



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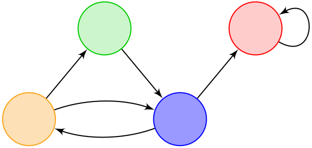
$$\mathbf{E\,F}\bigcirc \wedge \neg \left(\exists p.\ \left[\mathbf{E\,F}(\bigcirc \wedge p) \wedge \mathbf{E\,F}(\bigcirc \wedge \neg p)\right]\right) \equiv \mathrm{uniq}(\bigcirc)$$



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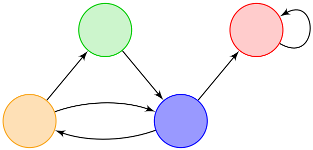

- $\rightsquigarrow$ true if we label the Kripke structure;
- $\rightsquigarrow$ false if we label the computation tree;

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Strategy logic can be translated into QCTL*.

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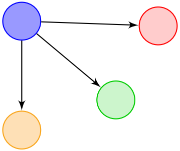

- players has moves m_1, ..., m_n;
- from the transition table, we can compute the set Next(, A, m_i) of states that can be reached from  when player A plays m_i.

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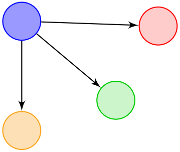

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SL can be translated as follows:

- encoding of $\exists\sigma.\ \psi$:

$$\exists m_1^\sigma\ \exists m_2^\sigma \ldots \exists m_k^\sigma.\ \mathbf{A}\,\mathbf{G}(m_i^\sigma \Leftrightarrow \bigwedge \neg\, m_j^\sigma) \wedge \hat{\psi}$$

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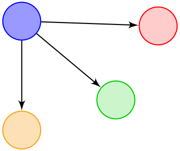

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- encoding of assign(α). φ (for full binding α: Agt $\to$ Strat):

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^{\alpha(A)} \Rightarrow \mathbf{X}\,\mathrm{Next}(q, A, m_i^{\alpha(A)})) \Rightarrow \hat{\varphi}\big]$$

QCTL with tree semantics

Theorem

- *Model checking QCTL with k quantifiers in the tree semantics is k-EXPTIME-complete.*
- *Satisfiability of QCTL with k quantifiers in the tree semantics is $(k+1)$-EXPTIME-complete.*

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Proof

Using (alternating) parity tree automata:

$$\delta(q_0, {\color{blue}\bigcirc}) = (q_0, q_1) \vee (q_1, q_0)$$
$$\delta(q_0, {\color{orange}\bigcirc}) = (q_1, q_1)$$
$$\delta(q_0, {\color{gray}\bigcirc}) = (q_2, q_2)$$
$$\delta(q_1, \textcircled{\star}) = (q_1, q_1)$$
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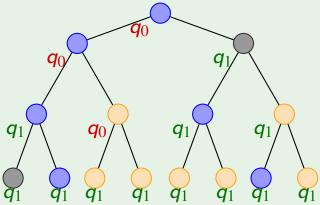

This automaton corresponds to **E**${\color{blue}\bigcirc}$ **U** ${\color{orange}\bigcirc}$

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Proof

- polynomial-size automata for CTL;
- quantification is handled by projection, which first requires removing alternation (exponential blowup);
- an automaton equivalent to a QCTL formula can be built inductively;
- emptiness of an alternating parity tree automaton can be decided in exponential time.

A fuzzy extension of Strategy Logic

Quantitative models

In our models, atomic propositions take values in $[0; 1]$:

- quality of satisfaction of a proposition
- (discretised, normalised) quantities, e.g. energy, distance, ...

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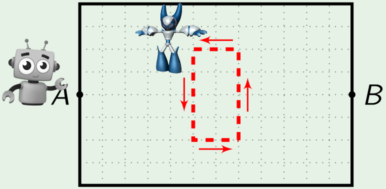
How to reach *B* while staying as far as possible from opponent?



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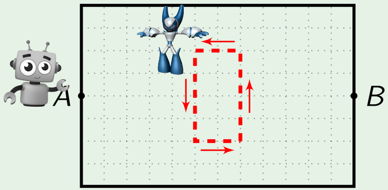
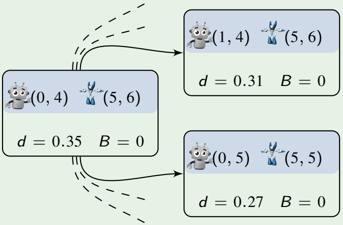


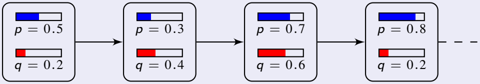


A fuzzy extension of Strategy Logic

LTL[$\mathcal{F}$]: formally reasoning about quality [ABK16]

- quantitative semantics for LTL [FPS14]:

$$[\![\pi, \varphi \ \mathbf{U} \ \psi]\!] = \sup_i \Big(\min([\![\pi_i, \psi]\!], \min_{j<i} (\min([\![\pi_j, \varphi]\!]) \Big).$$



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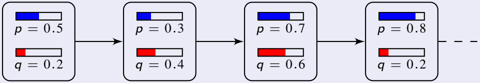

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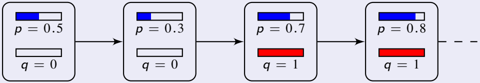

$$[\![p \ \mathbf{U} \ q]\!] = 0.3$$

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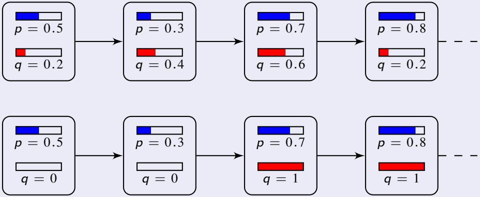

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- extension with functions:
  - max (disjunction), min (conjunction);
  - any other function $f : [0; 1]^m \to [0; 1]$.

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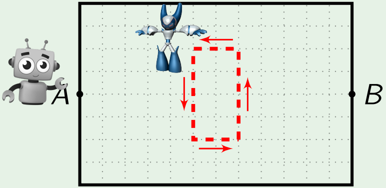
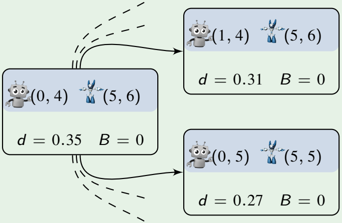
Example (Robot example)

How to reach B while staying as far as possible from opponent?





$\exists\sigma.$ assign( $\to \sigma$). d **U** B

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Example (Characterisation of Nash equilibria)

Formula Φ_{NE} expresses the fact that $(\sigma_1, ..., \sigma_n)$ is a NE:

$$\Phi_{NE} = \text{assign}(\text{Agt}{\to}(\sigma_1, ...\sigma_n)). \bigwedge_{A_i\in\text{Agt}} \left[(\exists d.\text{assign}(A_i{\to}d).\varphi_i)\right] \preceq \varphi_i$$

where the function $\preceq\colon [0;1]^2 \to \{0,1\}$ is such that

$$\preceq(\alpha, \beta) = 1 \text{ whenever } \alpha \leq \beta.$$

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Example (Characterisation of ε-Nash equilibria)

Formula $\Phi_{\overline{NE}}$ measures how far $(\sigma_1, ..., \sigma_n)$ is from being a NE:

$$\Phi_{\overline{NE}} = \text{assign}(\text{Agt}\to(\sigma_1, ...\sigma_n)). \bigvee_{A_i\in\text{Agt}} \left[(\exists d.\text{assign}(A_i\to d).\varphi_i) - \varphi_i\right]$$

Proposition

$(\sigma_1, ..., \sigma_n)$ is an ε-Nash equilibrium iff $[\![\Phi_{\overline{NE}}]\!] \leq \varepsilon$.

Model checking SL[$\mathcal{F}$]

Theorem

Model checking SL[$\mathcal{F}$] is decidable (and TOWER-complete).

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Lift the classical approach for SL to SL[$\mathcal{F}$], using QCTL*.

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Definition (Booleanly-Quantified CTL (BQCTL$^*[\mathcal{F}]$))

$$\varphi ::= p \mid \exists p.\, \varphi \mid \mathbf{E}\psi \mid f(\varphi, \ldots, \varphi)$$
$$\psi ::= \varphi \mid \mathbf{X}\, \psi \mid \psi\ \mathbf{U}\ \psi \mid f(\psi, \ldots, \psi)$$

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Remark

BQCTL* [$\mathcal{F}$] is interpreted over quantitative trees, but

quantification is boolean!

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Key lemma:

Lemma

For any $\varphi \in$ BQCTL[$\mathcal{F}$] and any finite $V \subseteq [0;1]$, the set*

$$V_\varphi = \{[\![t, \varphi]\!] \mid t \text{ quantitative tree with values in } V\}$$

is finite.

Model checking SL[$\mathcal{F}$]

Theorem

Model checking SL[$\mathcal{F}$] is decidable (and TOWER-complete).

Sketch of proof

Key lemma:

Lemma

For any $\varphi \in$ BQCTL$^*[\mathcal{F}]$ and any finite $V \subseteq [0; 1]$, the set

$$V_\varphi = \{[\![t, \varphi]\!] \mid t \text{ quantitative tree with values in } V\}$$

is finite.

$\rightsquigarrow$ For any φ and $P \subseteq [0; 1]$, we can build tree automata characterizing V-trees t for which $[\![t, \varphi]\!] \in P$.

Conclusions and future works

Contributions

- quantitative extension of Strategy Logic;
- (semi-)quantitative extension of QCTL*;
- model checking remains decidable.

Future works

- more applications, analysis of expressive power;
- specialized efficient algorithms for fragments of SL[$\mathcal{F}$];
- fully-quantitative extension of QCTL*.