Efficient timed diagnosis
using automata with timed domains

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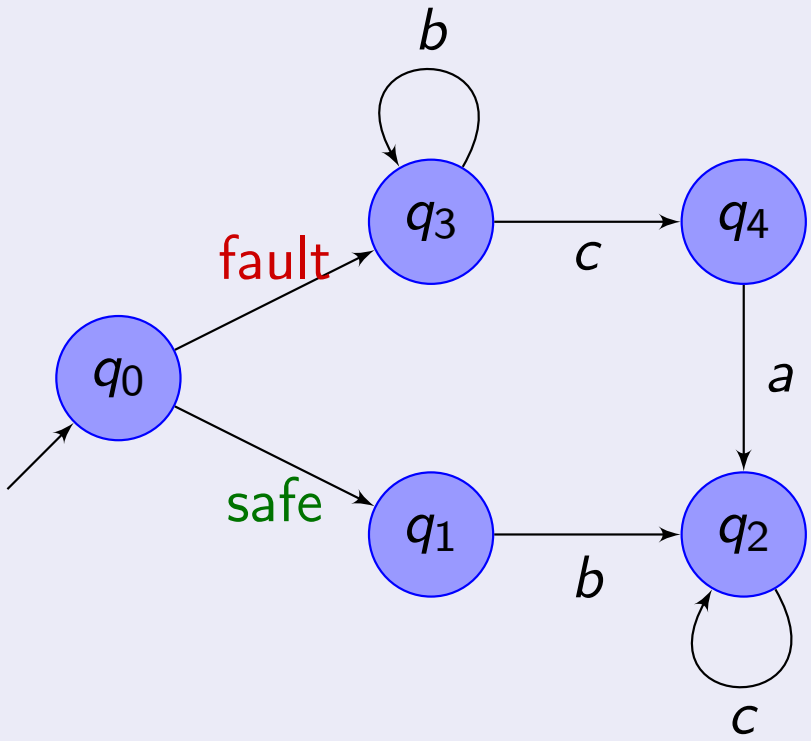
EQualIS

Fault diagnosis [SSL+95]

Partially-observable systems with faults



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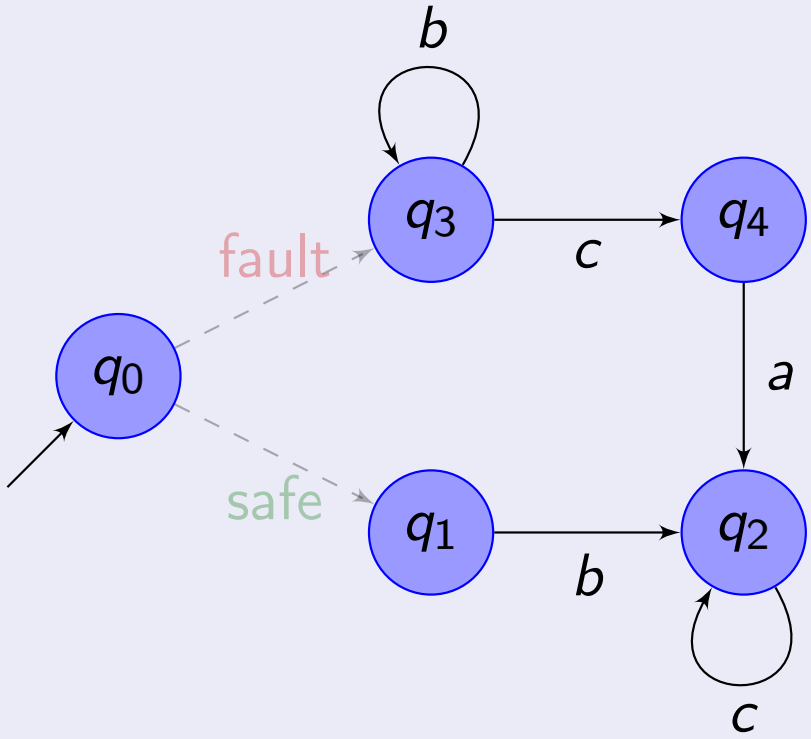

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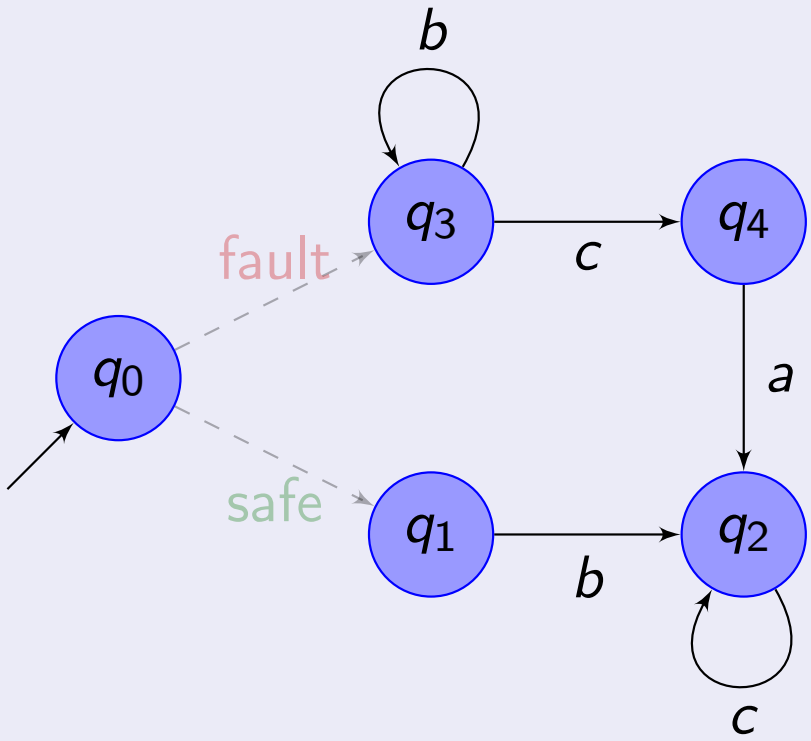
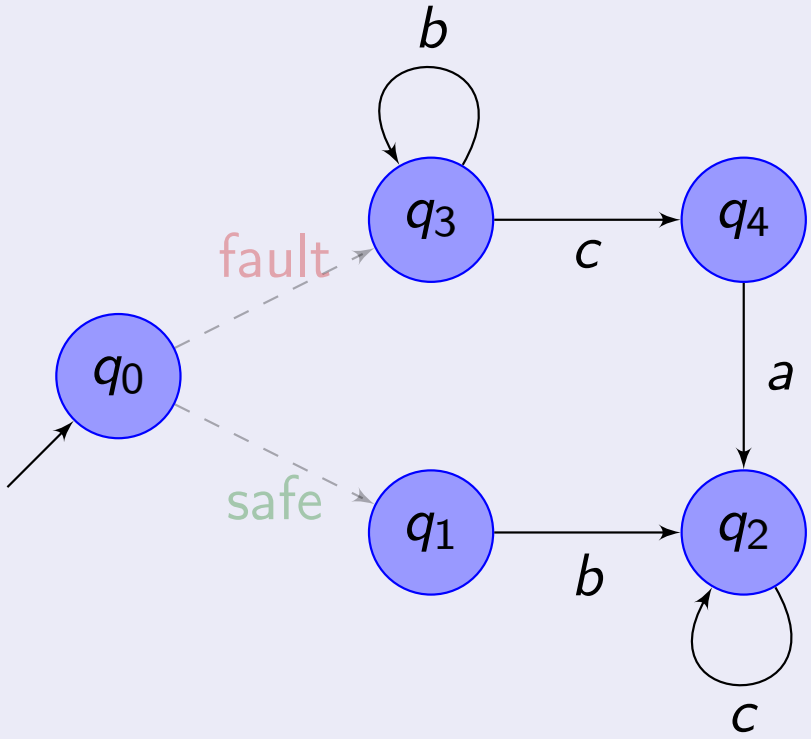


- observation *bca* corresponds to a faulty run;
- observation *bcc* corresponds to a safe run;
- observation *bc* is ambiguous.

[SSL+95] Sampath *et al.* Diagnosability of discrete-event systems. TAC, 1995.

Fault diagnosis [SSL+95]

Partially-observable systems with faults



Diagnozer

A Δ-diagnozer is a mapping $\Sigma^* \to \{\texttt{safe}, \texttt{fault}\}$ detecting a fault at most Δ steps after one occurred.

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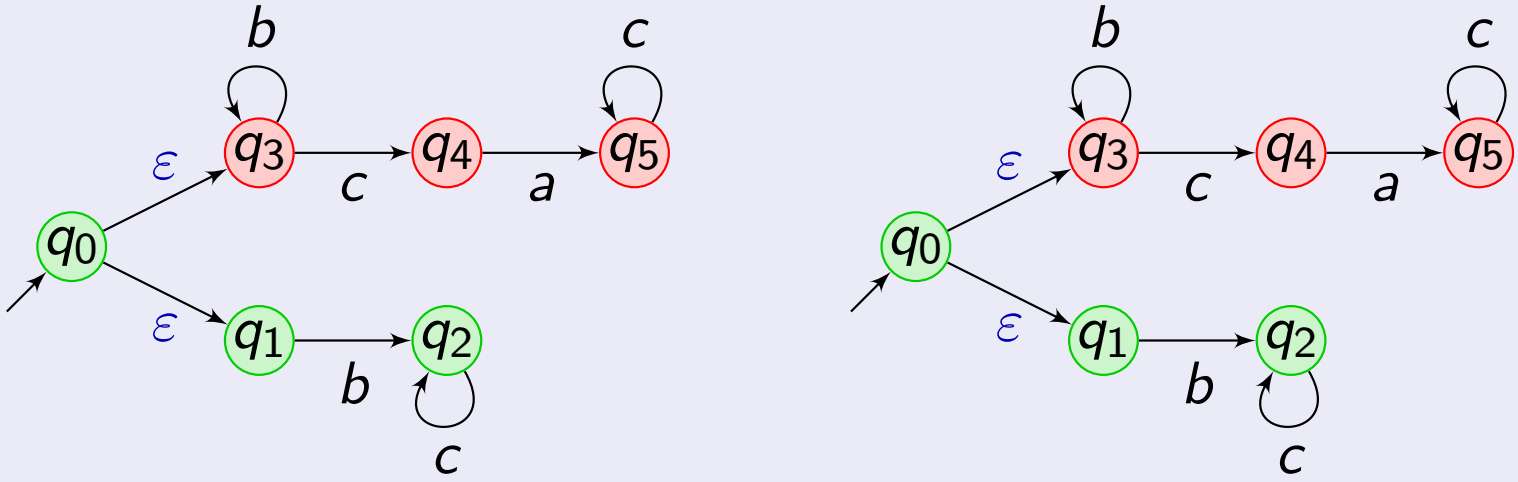
Deciding diagnozability

Diagnozability is decidable

Synchronized product of two (modified) copies of the automaton:

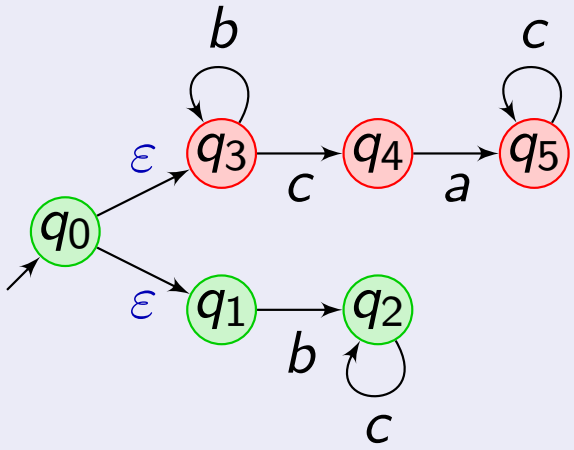


$\mathcal{A}$ is diagnozable iff $\mathcal{A}' \times \mathcal{A}'$ has no infinite ◯ × ◯ run.

Building a diagnozer

Building a diagnozer as a deterministic finite-state automaton

Perform state estimation using a powerset construction:



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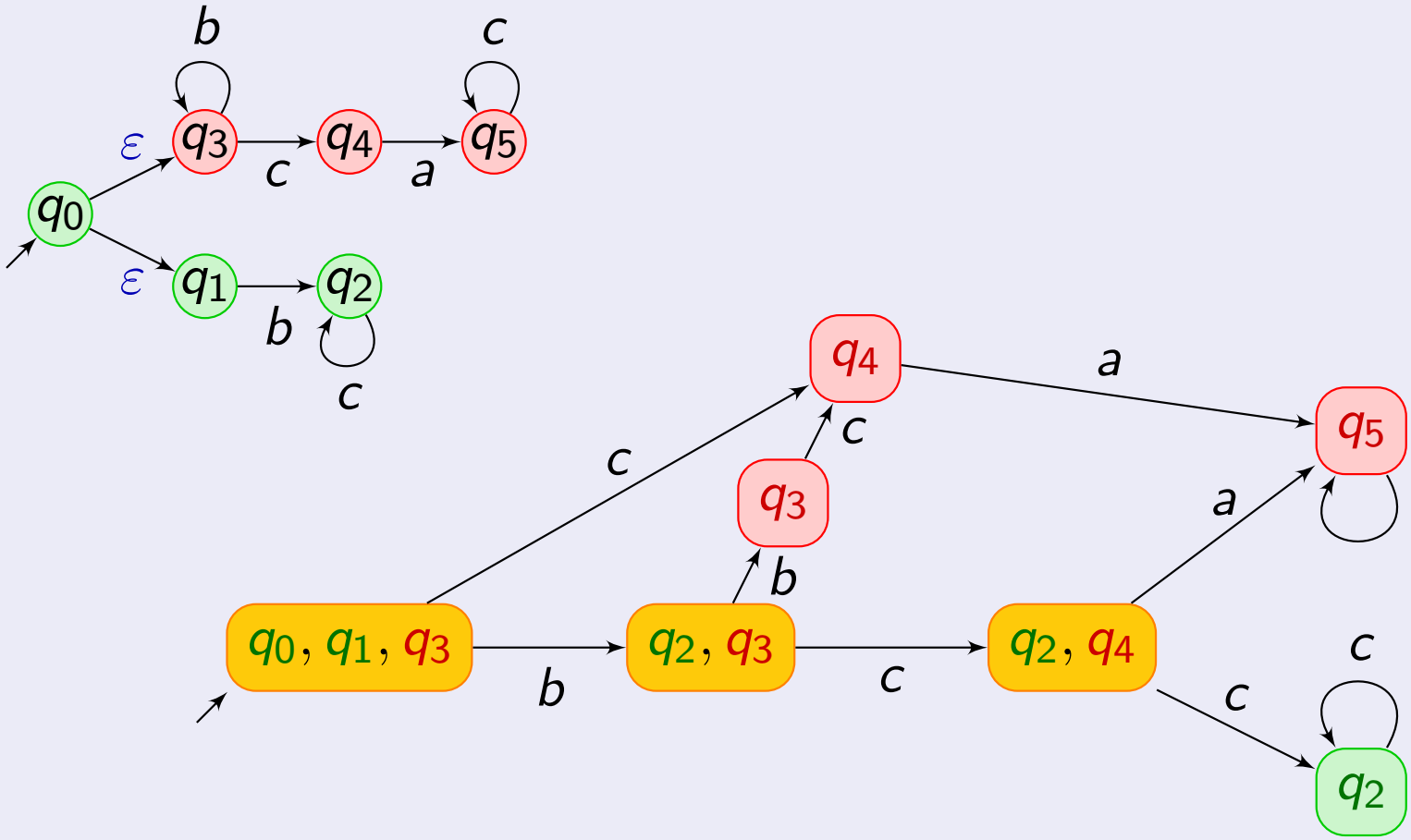
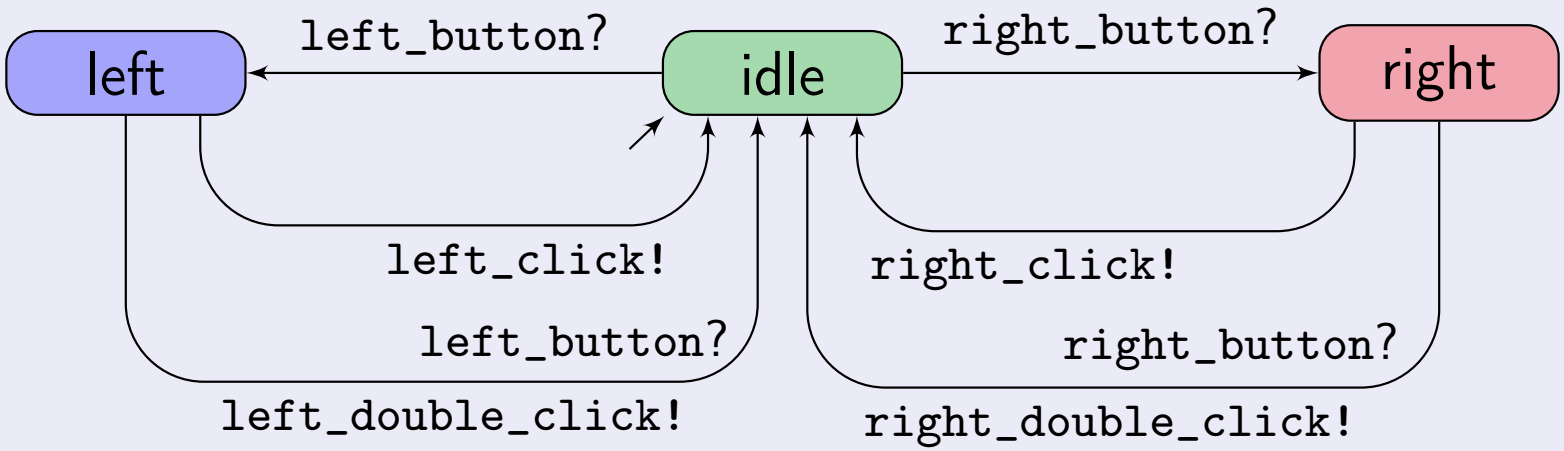


Timed automata [AD90]

Example of a computer mouse



[AD90] Alur and Dill. Automata For Modeling Real-Time Systems. ICALP 1990.

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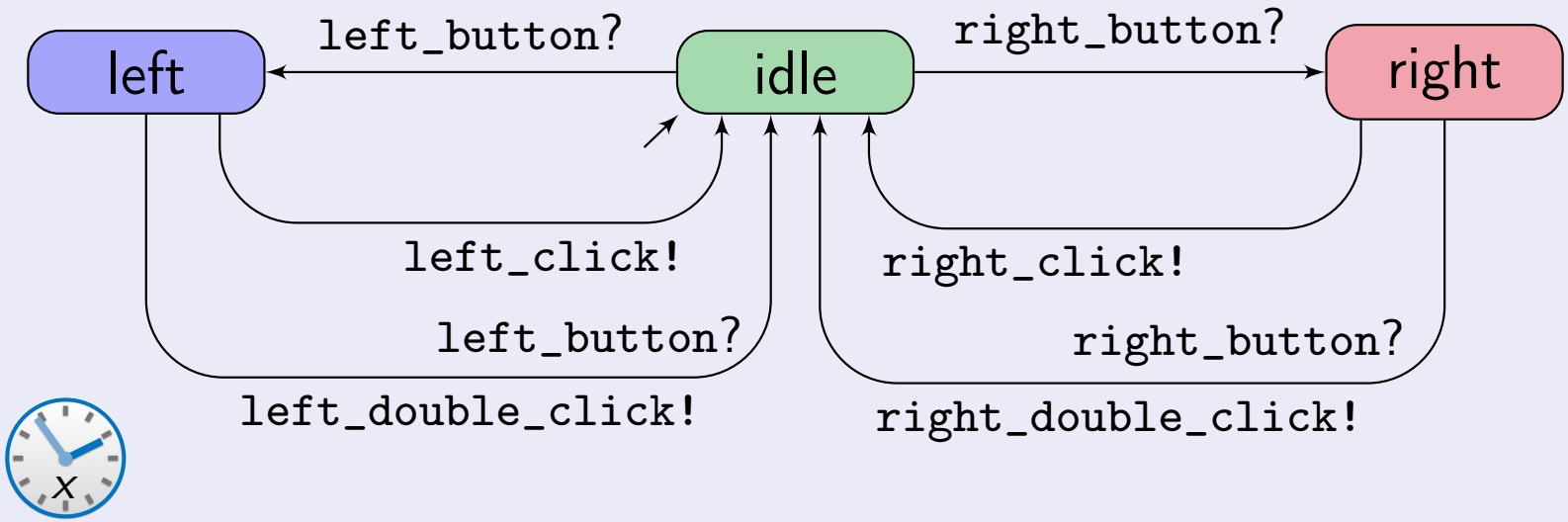

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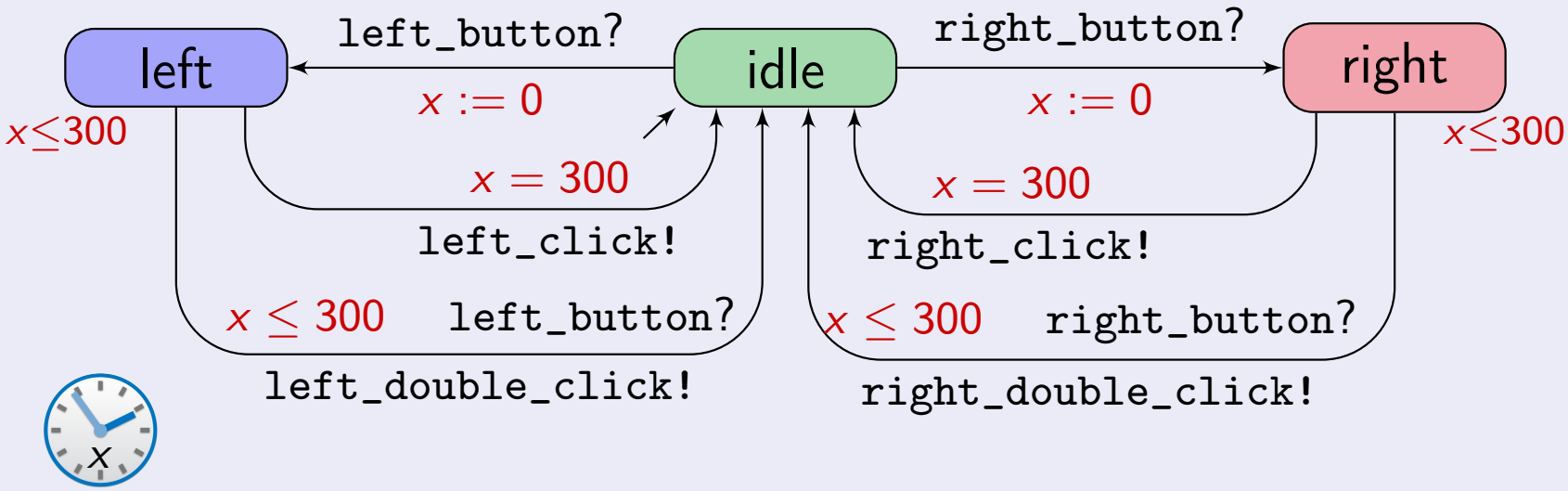

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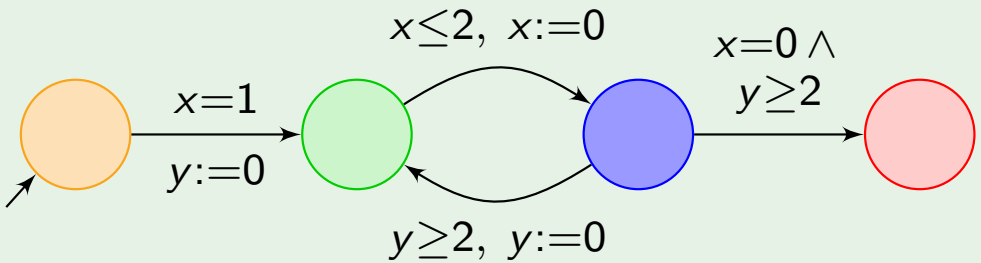
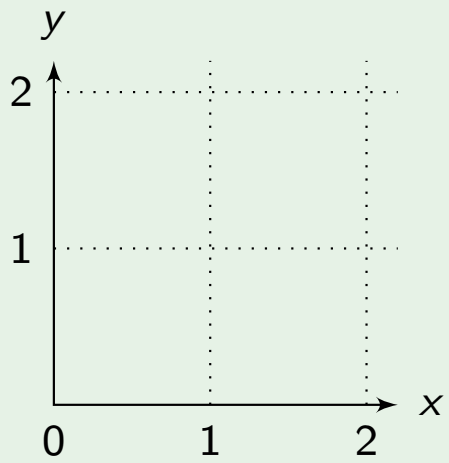
Semantics of timed automata

Example





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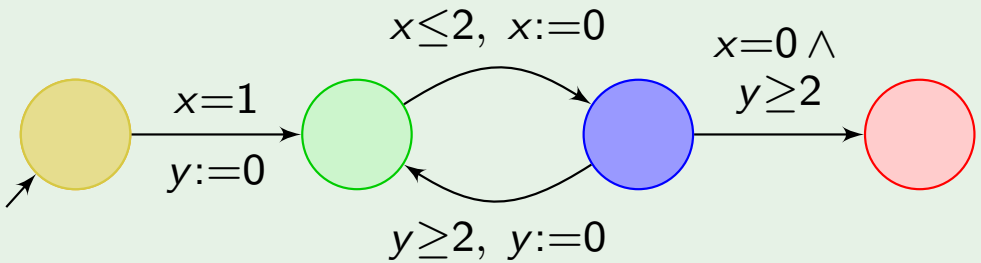

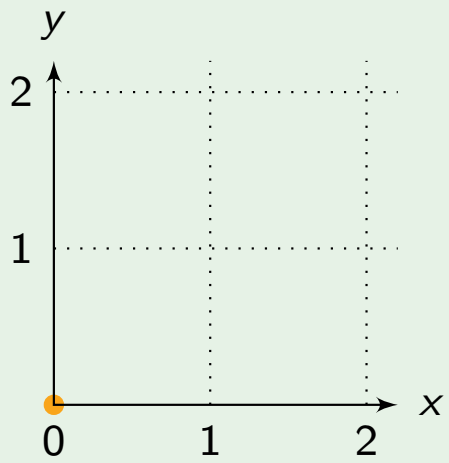

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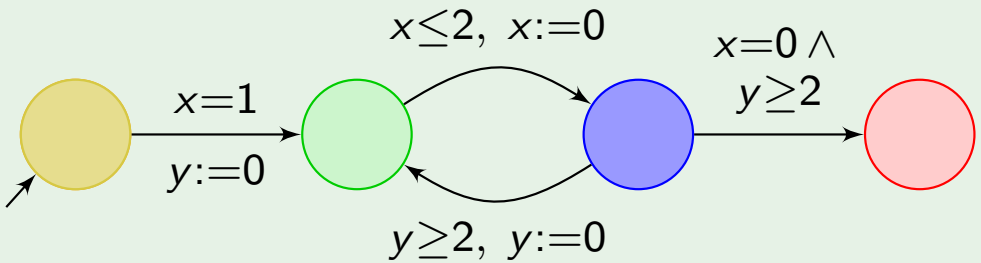

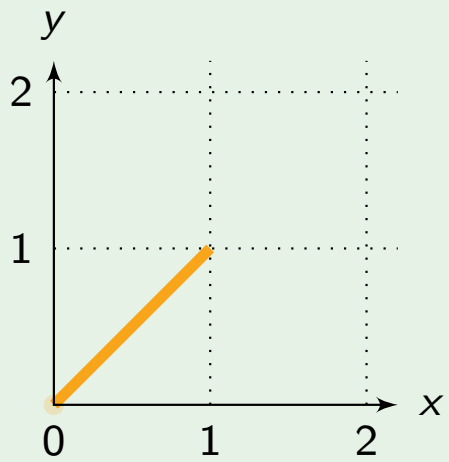

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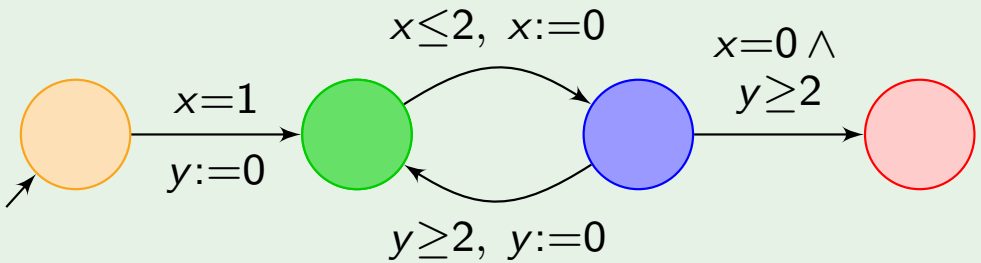

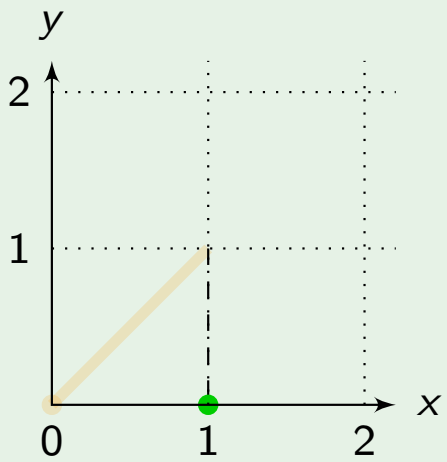

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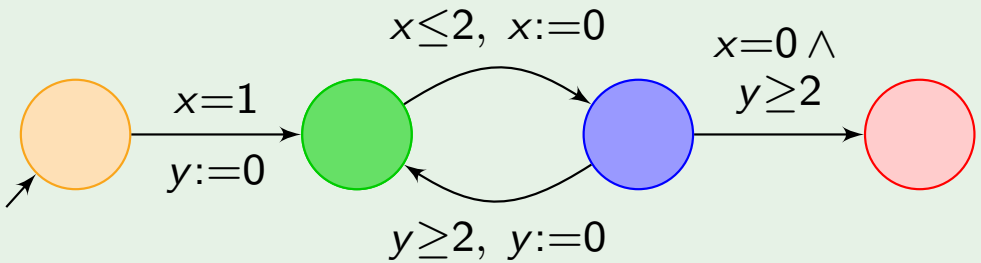

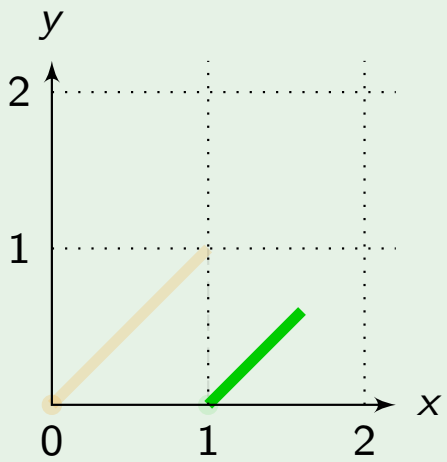

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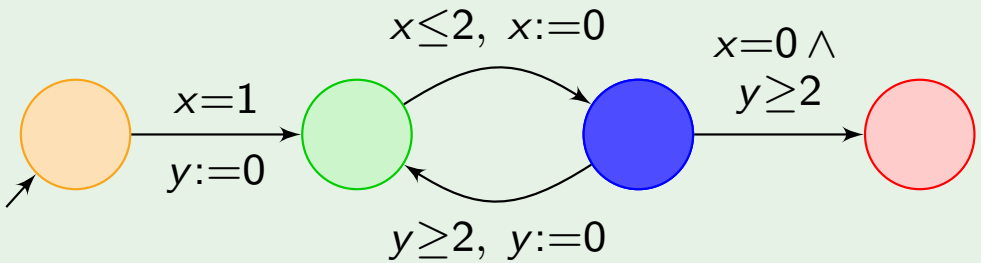

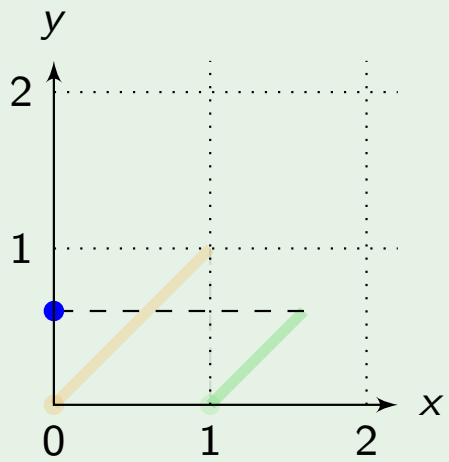

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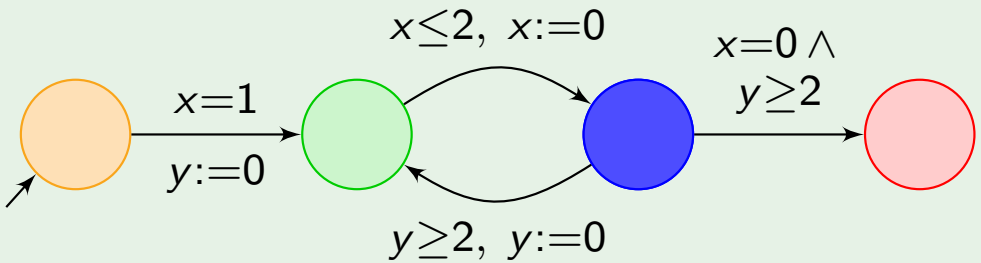

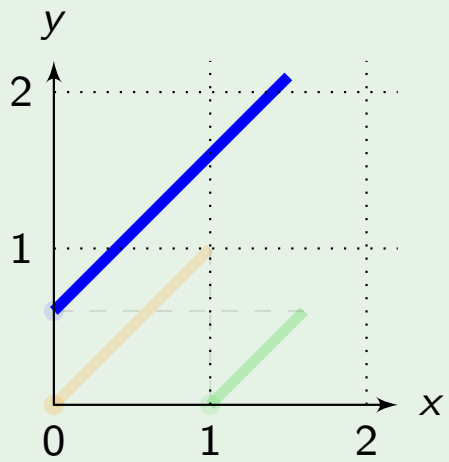

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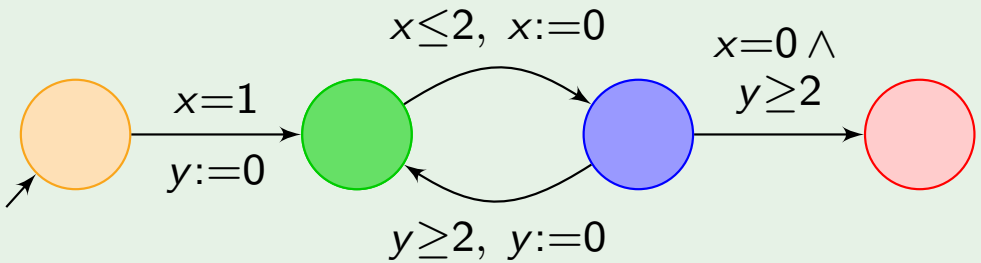

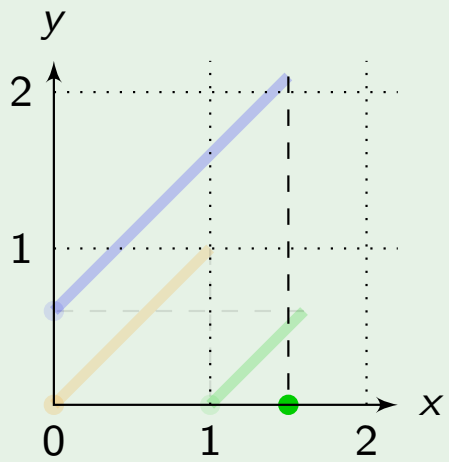

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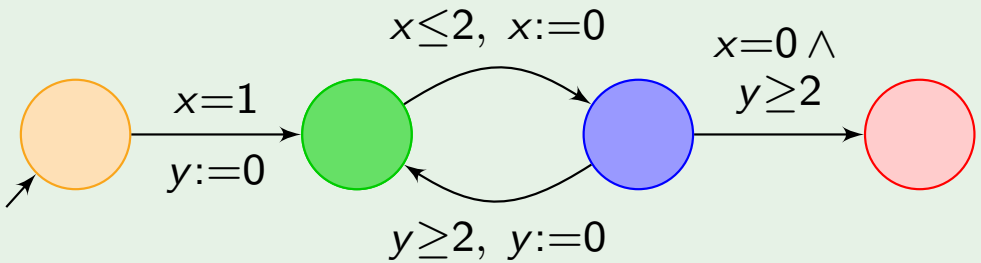

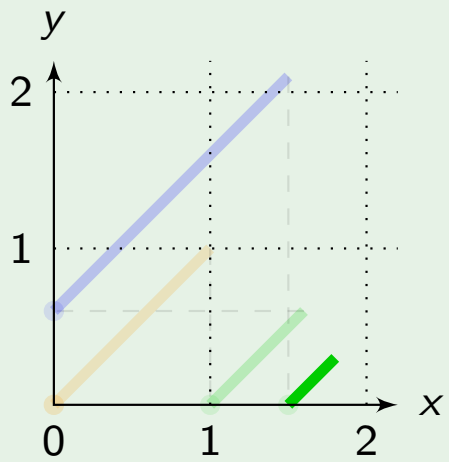

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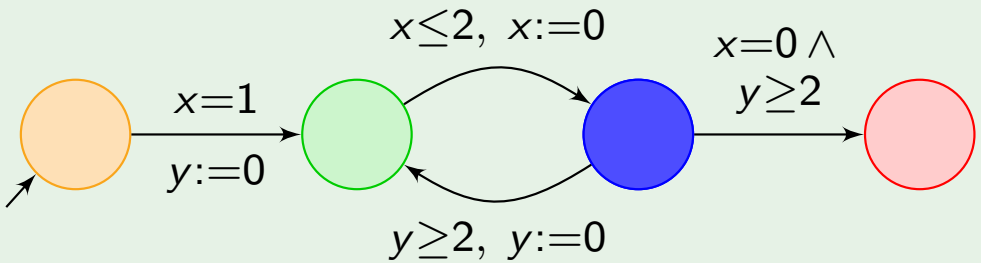

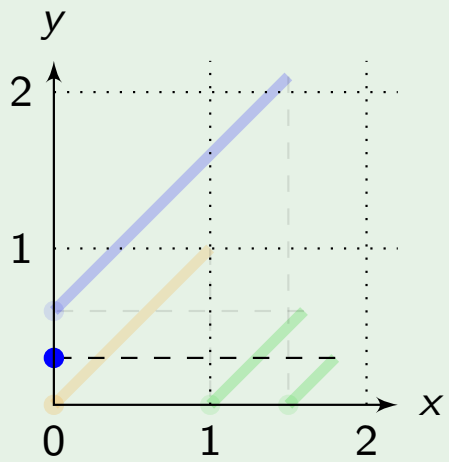

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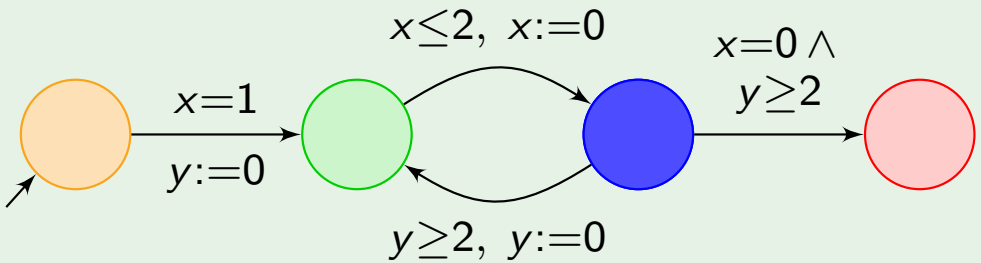

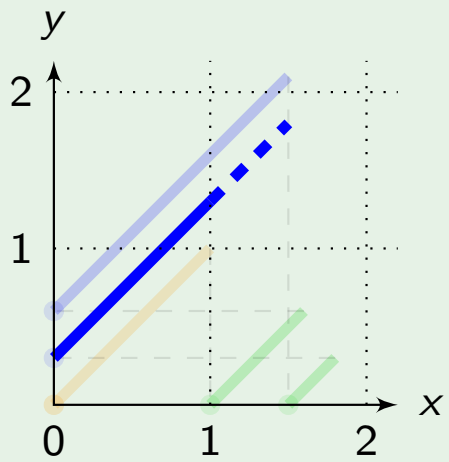

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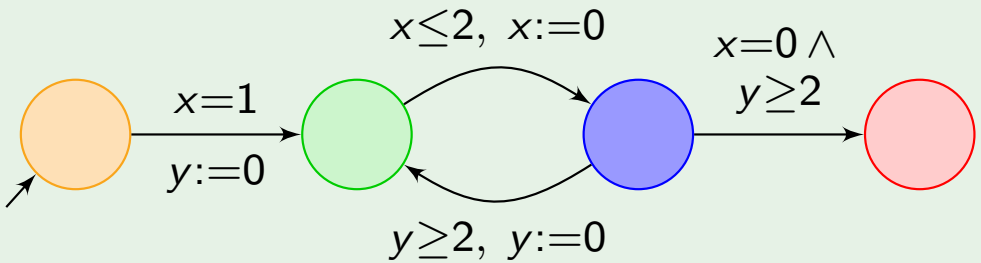

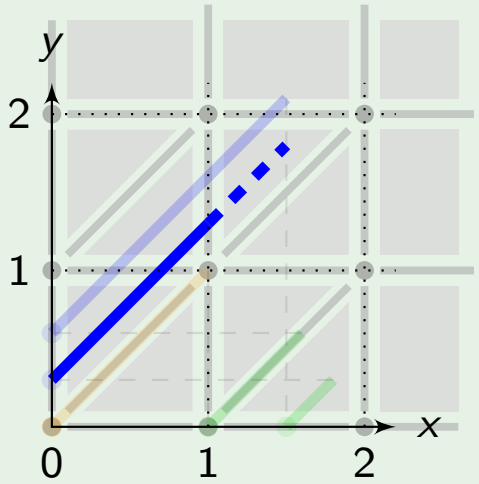

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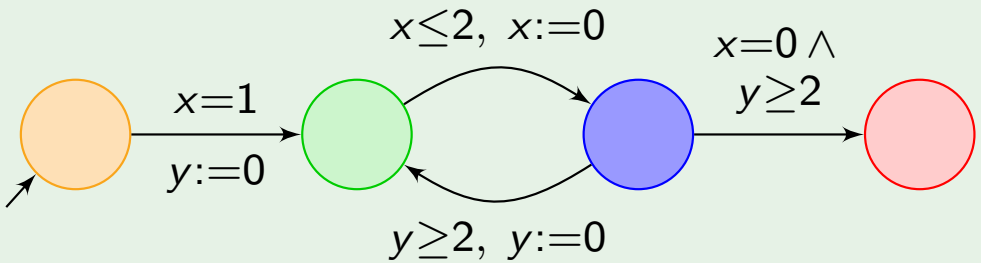

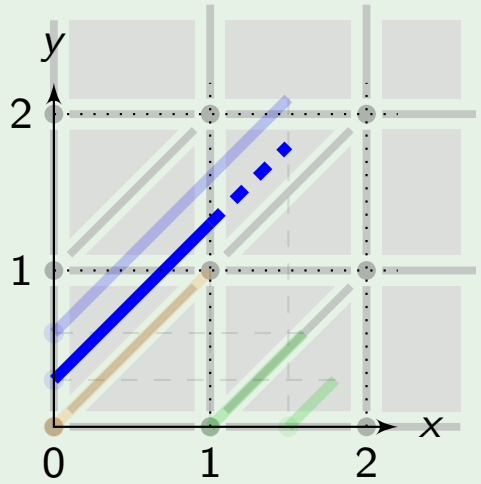

Theorem ([AD90])

Reachability in timed automata is decidable.

[AD90] Alur and Dill. Automata For Modeling Real-Time Systems. ICALP 1990.

Diagnosability for timed automata

Diagnozer for timed automata

A Δ-diagnozer is a mapping $T\Sigma^* \to \{\texttt{safe}, \texttt{fault}\}$ detecting a fault at most Δ time units after one occurred.

Diagnosability for timed automata

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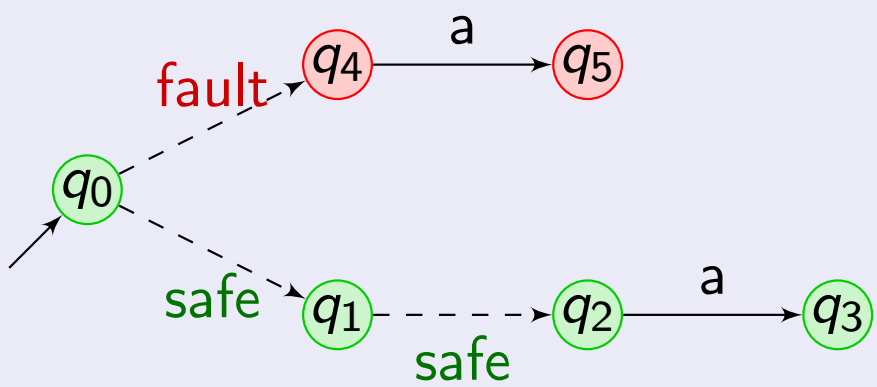
Time can be observed

- even discrete-time makes a difference:



- we want to perform state estimation in real time, even when no events occur.

Diagnosability for timed automata

Diagnozer for timed automata

A Δ-diagnozer is a mapping $T\Sigma^* \to \{\texttt{safe}, \texttt{fault}\}$ detecting a fault at most Δ time units after one occurred.

We can still compute synchronized products and check reachability:

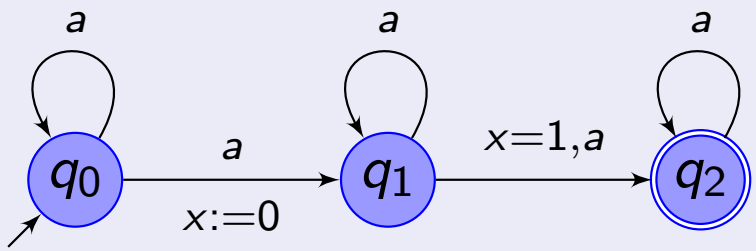
Theorem ([Tri02])

Diagnosability for timed automata is decidable.

[Tri02] Tripakis. Fault diagnosis for timed automata. FTRTFT 2002.

How about building a diagnozer?

Some timed automata cannot be determinized



No deterministic timed automata accept the same timed language.

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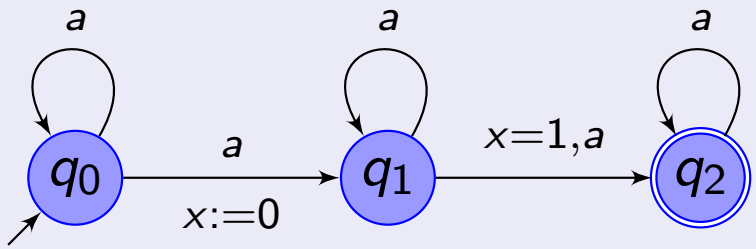
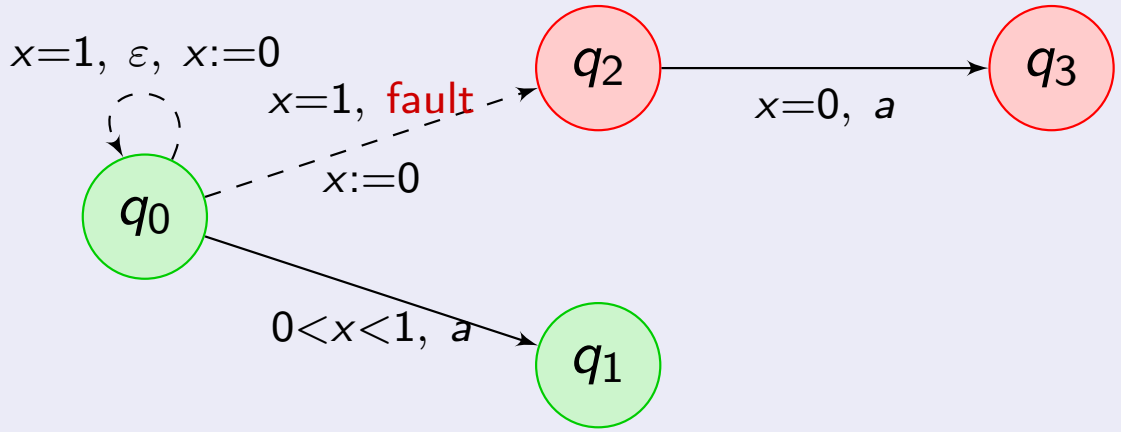


No deterministic timed automata accept the same timed language.

Some diagnozable timed automata admit no DTA as diagnozers



How about building a diagnozer?

Building a diagnozer as a deterministic timed automaton [BCD05]

Look for DTA-diagnozers with

- limited number of clocks
- limited granularity and range of clocks
- known delay Δ.

Theorem ([BCD05])

*DTA-diagnozability under such restrictions is (effectively) decidable in 2*EXPTIME.

[BCD05] Bouyer, Chevalier, D'Souza. Fault diagnosis using timed automata. FoSSaCS 2005.

How about building a diagnozer?

Building a diagnozer as a deterministic timed automaton [BCD05]

Building an algorithm performing state estimation [Tri02]

$$R_{\text{obs}}(S, a) = \{(q', v') \mid \exists (q, v) \in S.\ (q, v) \xrightarrow{a} (q', v')\}$$

$$R_{\text{unobs}}(S, \delta) = \{(q', v') \mid \exists (q, v) \in S.\ (q, v) \xrightarrow{\delta}{}^{*}\ (q', v')\}$$

- compute state estimation when observing a after delay δ:

$$R_{\text{obs}}(R_{\text{unobs}}(S, \delta), a)$$

- return fault if only faulty states are obtained.

Computing R_{unobs} is expensive!

[BCD05] Bouyer, Chevalier, D'Souza. Fault diagnosis using timed automata. FoSSaCS 2005.
[Tri02] Tripakis. Fault diagnosis for timed automata. FTRTFT 2002.

Our contribution

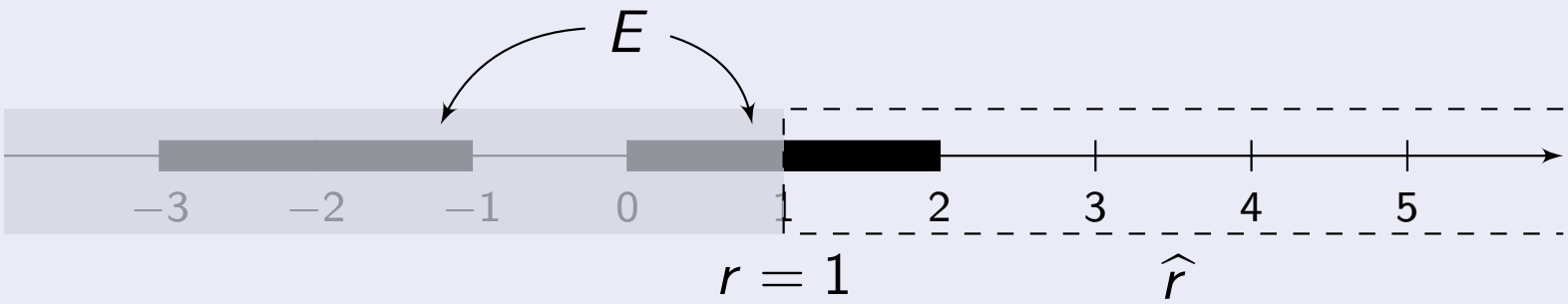
For timed automata with one clock, we

- use a powerset construction for timed automata (borrowed from [BJM17]) in order to represent the diagnozer;
- compute a convenient representation for this diagnozer; in particular, (part of) R_{unobs} is pre-computed, so that state estimation is much more efficient for delay transitions.
- implement our approach and compare to [Tri02].

[BJM17] Bouyer, Jaziri, Markey. On the determinization of timed systems. FORMATS 2017.
[Tri02] Tripakis. Fault diagnosis for timed automata. FTRTFT 2002.

Timed sets

Atomic timed sets

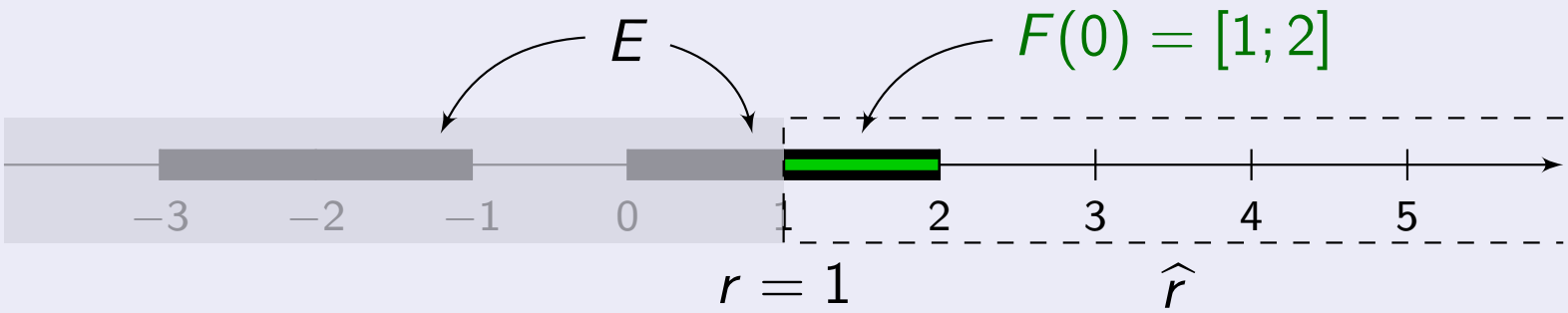


Given a finite union of intervals E and a right-infinite interval $\widehat{r}$, the timed set $F = (E; \widehat{r})$ is defined as

$$F : t \in \mathbb{R}_{\geq 0} \mapsto (E + t) \cap \widehat{r}.$$

Timed sets

Atomic timed sets



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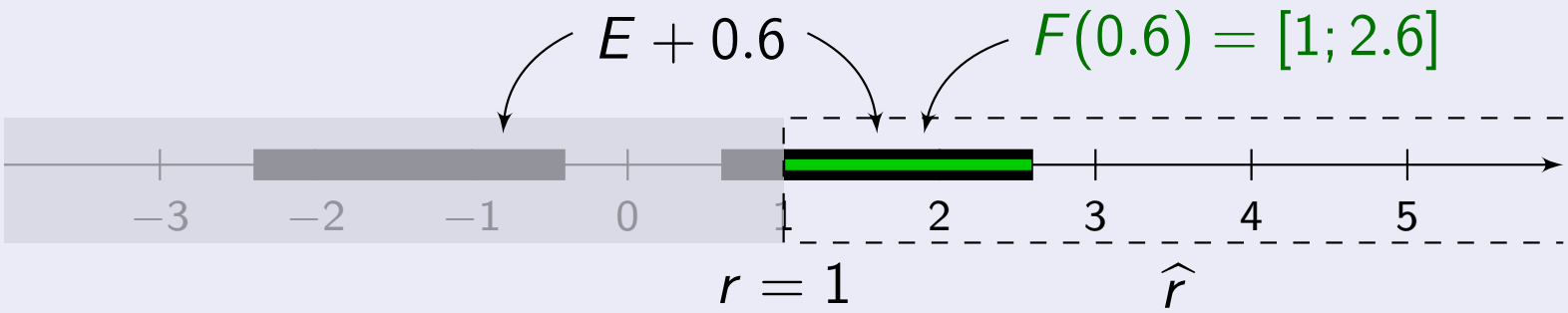

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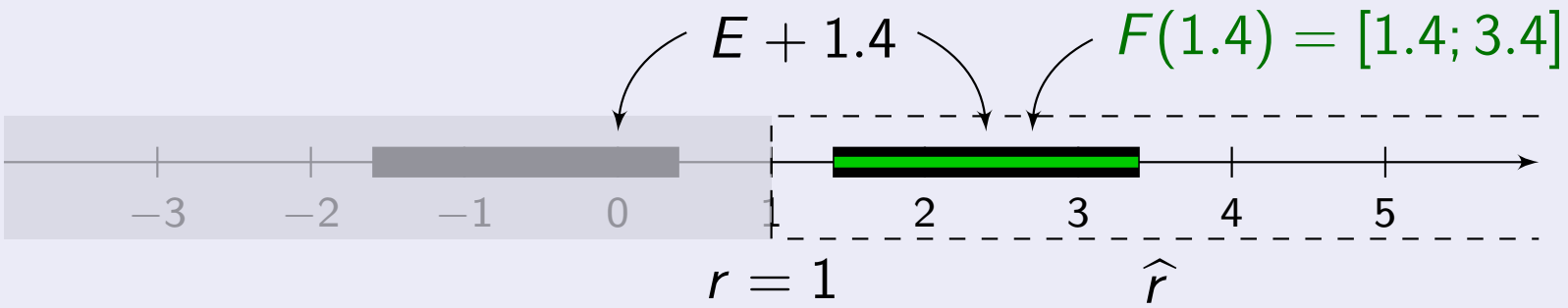

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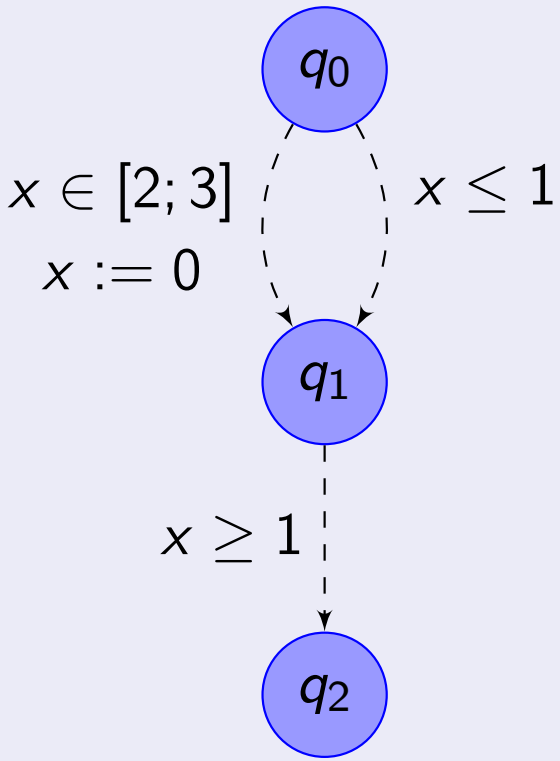
Timed sets

Timed sets are used to compute the sets of reachable configurations.

Example



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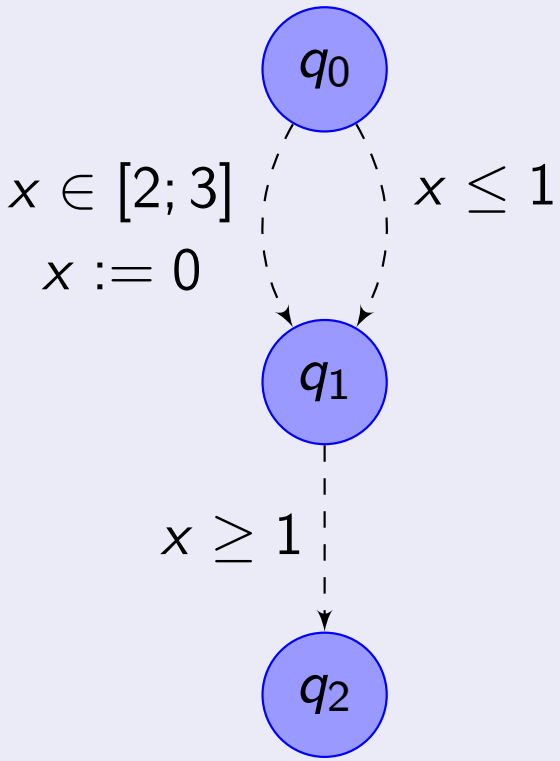

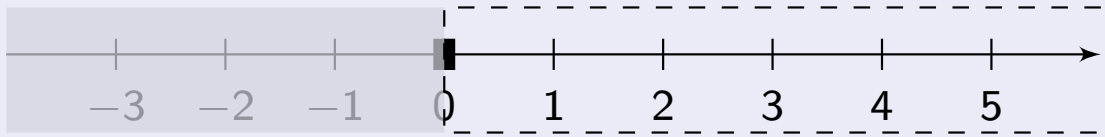

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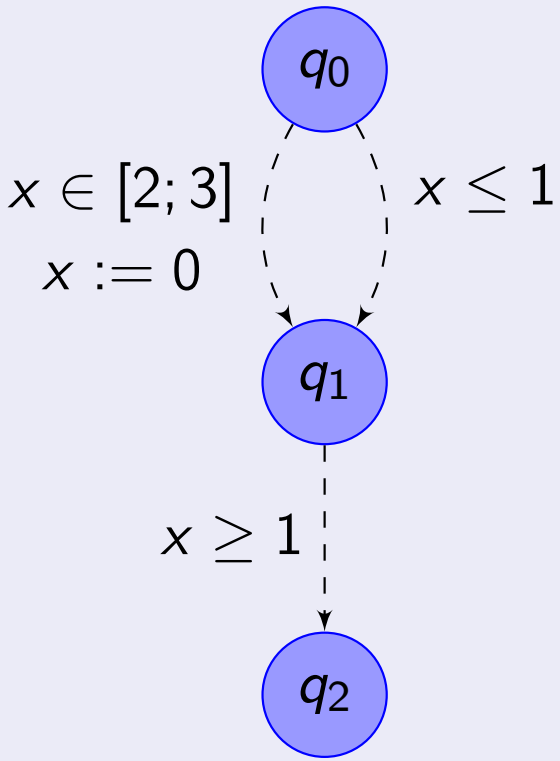
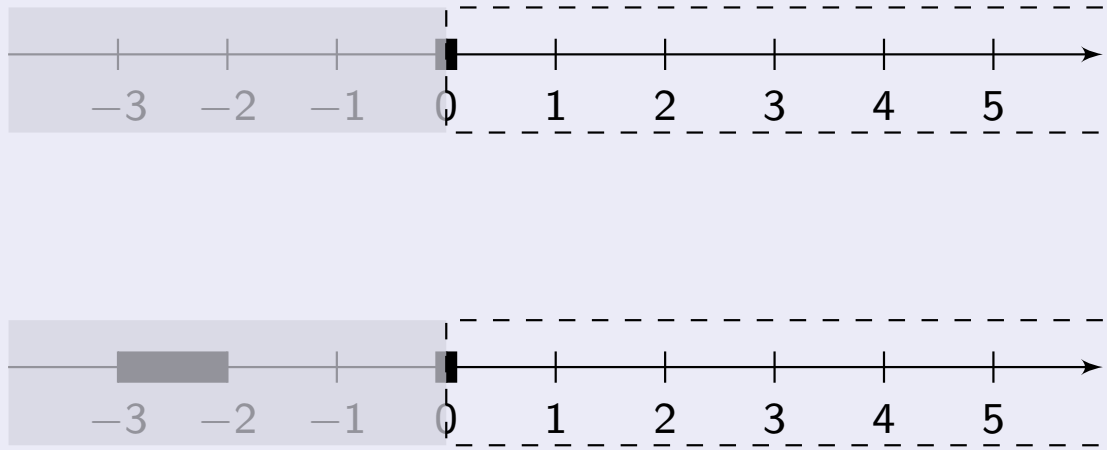
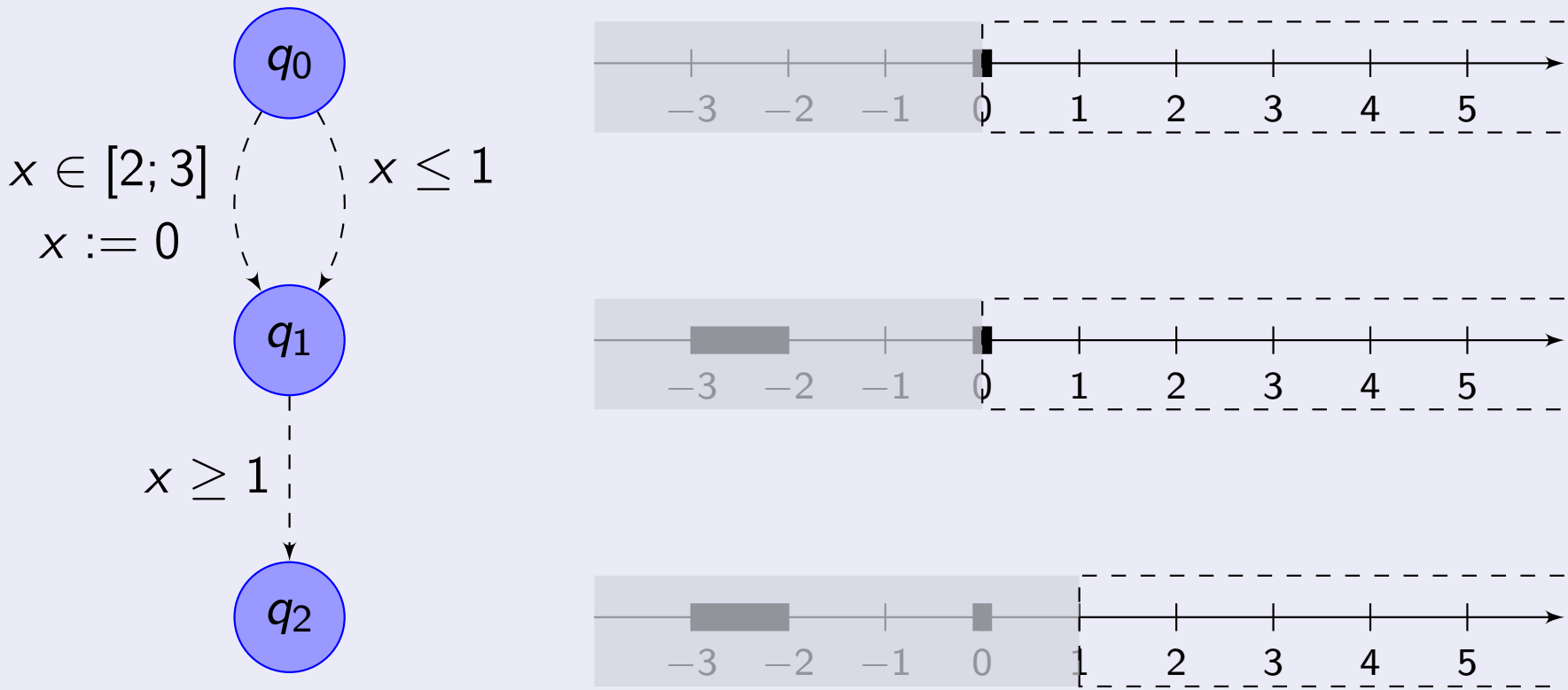




Timed sets

Timed sets are used to compute the sets of reachable configurations.

Example



Timed sets

Timed sets are used to compute the sets of reachable configurations.

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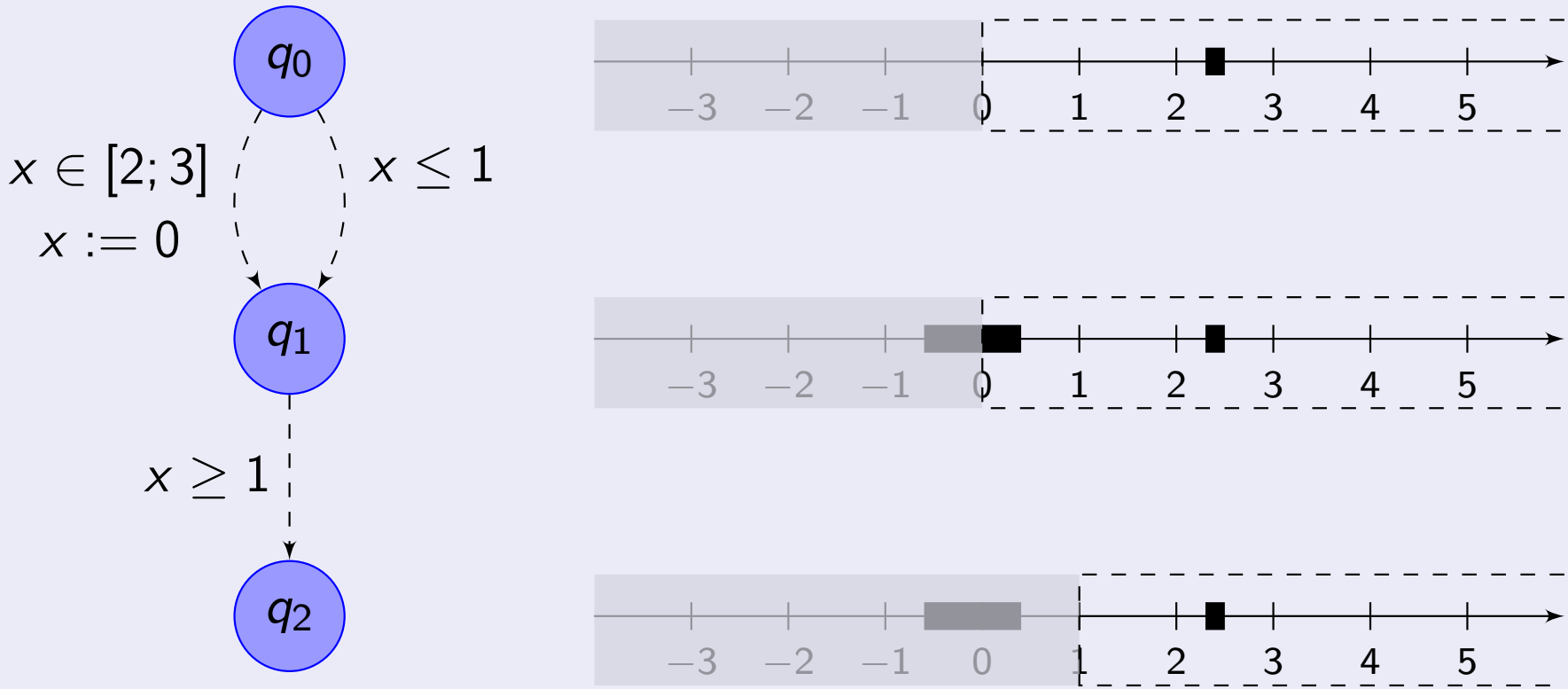
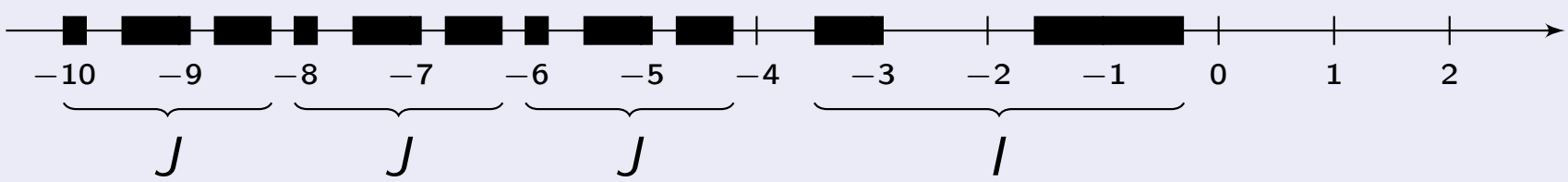


Timed sets

Regular timed sets

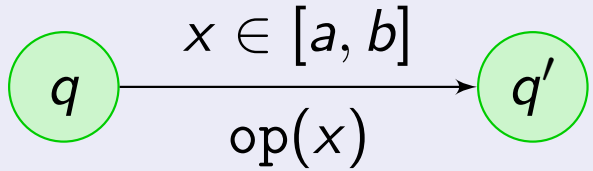
- **regular union of intervals**: sets of the form $I \cup \bigcup_{k=q}^{+\infty} J - k \cdot p$.

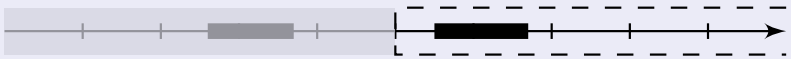


- **regular timed sets**: finite union of timed sets $(E_i, \widehat{r_i})$ for regular unions of intervals E_i.

Image of a timed set by a transition

Computing the effect of a transition

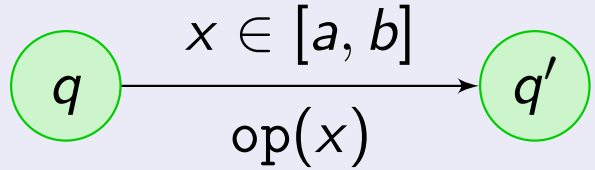




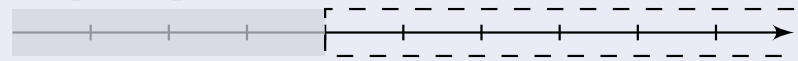
$(E, \widehat{r})$ in q

Image of a timed set by a transition

Computing the effect of a transition



if $[a; b] \cap \widehat{r} = \varnothing$:



$(\varnothing, [0; +\infty))$

$(E, \widehat{r})$ in q

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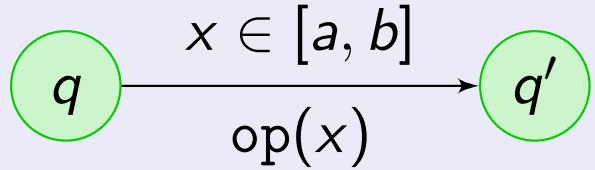
Computing the effect of a transition



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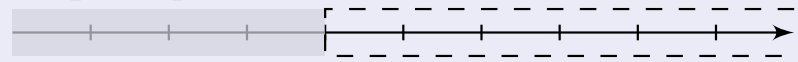
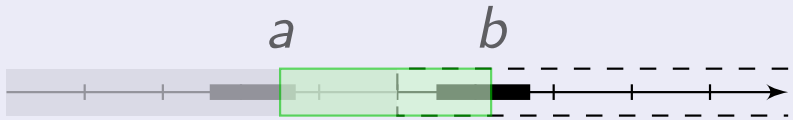
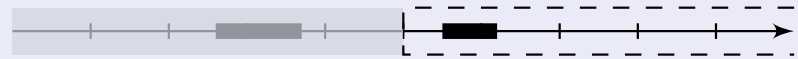


$(\varnothing, [0; +\infty))$



$(E, \widehat{r})$ in q

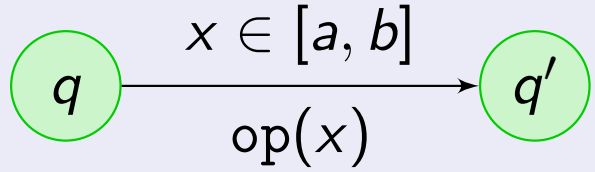
if $[a; b] \cap \widehat{r} \neq \varnothing$ and $\mathrm{op} = \mathtt{id}$:

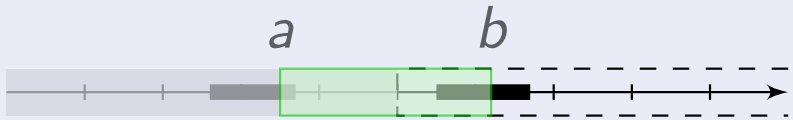


$(E \cap (-\infty; b], [\max\{a, r\}; +\infty))$

Image of a timed set by a transition

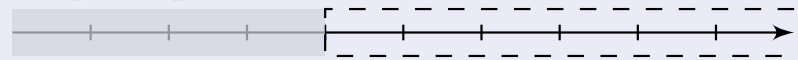
Computing the effect of a transition





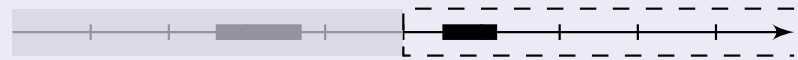
$(E, \widehat{r})$ in q

if $[a; b] \cap \widehat{r} = \varnothing$:



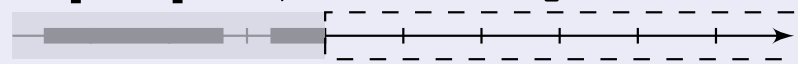
$(\varnothing, [0; +\infty))$

if $[a; b] \cap \widehat{r} \neq \varnothing$ and $\mathtt{op} = \mathtt{id}$:



$(E \cap (-\infty; b], [\max\{a, r\}; +\infty))$

if $[a; b] \cap \widehat{r} \neq \varnothing$ and $\mathtt{op} = \mathtt{reset}$:



$((E - [\max\{a, r\}; b]) \cap \mathbb{R}_{\leq 0}; [0; +\infty))$

ε-closure

Timed markings and their ε-closures

- A timed marking $M\colon s \mapsto \{(E_i, \widehat{r_i})\}$ assigns a timed set with each state.
- For a sequence w of silent transitions:

$$M^w\colon (s', d) \mapsto \left\{ v' \mid \exists s.\, \exists d_0 \leq d.\, \exists v \in M(s)(d_0).\, (s, v) \xrightarrow[w]{d}{}^{*} \ (s', v') \right\}$$

- The ε-closure of M is the union of M^w over all silent paths w.

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- The ε-closure of M is the union of M^w over all silent paths w.

Theorem

- *The ε-closure of a regular timed marking is a regular timed marking.*
- *ε-closures are finite unions of timed sets of the form $(E - K; \widehat{r'})$ for regular unions of intervals K that can be pre-computed.*

Implementation and experimentations

Implementation of our diagnozer

- prototype in Python (ca. 2000 lines of code)
- precompute sets for ε-closures
- simulate and perform state-estimation for trace entered by user; also state-prediction!
- prototype of the approach of [Tri02] for comparison.

Comparison of the results

Example. over 400 random runs of 10-20 actions on a 7-state TA:

	action	delay	precomputation
[BJM18]	0.17 s	10^{-5} s	11 s / 12 ko
[Tri02]	0.26 s	0.30 s	-

[Tri02] Tripakis. Fault diagnosis for timed automata. FTRTFT 2002.

[BJM18] Bouyer, Jaziri, Markey. Efficient timed diagnosis using automata over timed domains. RV 2018.

Conclusions and future work

Conclusions

- novel approach for building diagnozers for (one-clock) timed automata;
- performs pre-computation to speed-up computation of silent runs;
- can also predict occurrence of states

Future work

- improve implementation
  understand why pre-computation is sometimes very long;
- take invariants into account;
- extension to n clocks: from timed sets to timed zones?
- cost estimation in priced timed automata with unobservable cost.

A powerset construction for timed automata [BJM17]

Timed domains

A timed domain is a pair $(\mathcal{V}, \hookrightarrow)$ where $\mathcal{V}$ is a set and $\hookrightarrow : \mathcal{V} \times \mathbb{R}_{\geq 0} \to \mathcal{V}$ s.t. for all v, d and d',

$$\hookrightarrow(v, d + d') = \hookrightarrow(\hookrightarrow(v, d), d').$$

Example. $\mathcal{V} = [0; M] \cup \{+\infty\}$ and $\hookrightarrow(v, d) = \begin{cases} v + d & \text{if } v + d \leq M \\ +\infty & \text{otherwise.} \end{cases}$

Example. For a timed automaton $\mathcal{A}$ with states Q, the set of *timed markings* of $\mathcal{A}$, equipped with $\hookrightarrow^Q(M, d) : s \mapsto M^\epsilon(s, d)$, is a timed domain.

Update

An update is a mapping $\lambda : \mathcal{V} \to \mathcal{V}$.

[BJM17] Bouyer, Jaziri, Markey. On the determinization of timed systems. FORMATS 2017.

A powerset construction for timed automata [BJM17]

Automata over timed domains

An automaton over timed domain $(\mathcal{V}, \hookrightarrow)$, alphabet Σ and updates Λ is a tuple $(Q, (q_0, v_0), T, F)$ with $T \subseteq Q \times \mathcal{V} \times \Sigma \times \Lambda \times Q$.

Theorem

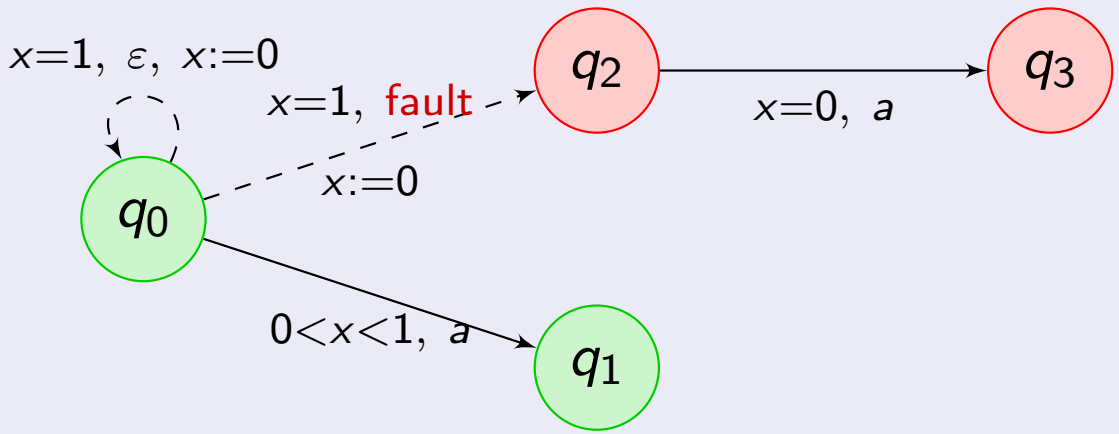
Any automaton over a timed domain $\mathcal{V}$ can be determinized as an automaton over timed domain $\mathcal{P}(\mathcal{V})^{|Q|}$.

Our diagnozer is the automaton we get by this determinization procedure.

[BJM17] Bouyer, Jaziri, Markey. On the determinization of timed systems. FORMATS 2017.

Running the tool

Small example




```
# States.  Size :  4
q0;q1;q2;q3
# Transitions
q2;[0,0];0;q3;a
q0;]0,1[;0;q1;a
q0;[1,1];0;q0;e
q0;[1,1];0;q2;e
```

State evaluation: initially

State : q0:

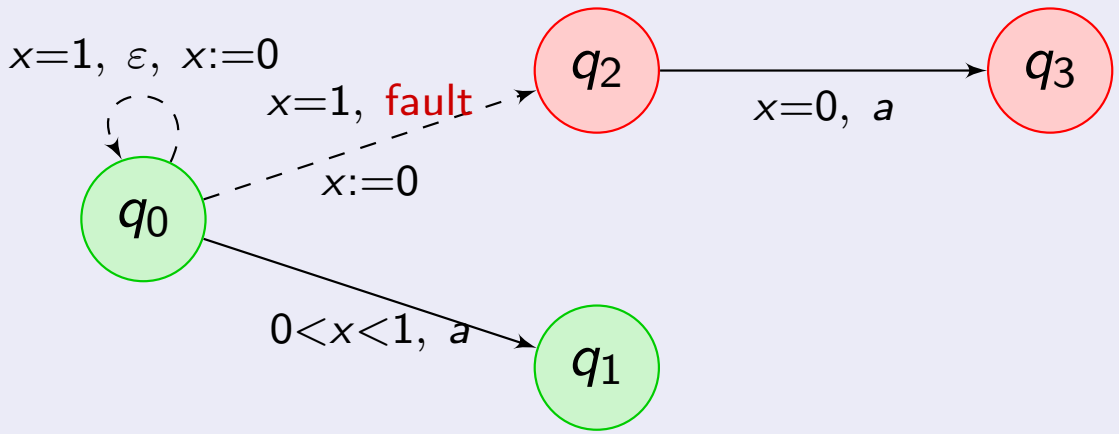
```
(( [0.00,0.00] ) u U_k in N ( [0.00,0.00] - 1.00 -k1.00 ) )
n [0.00,inf[
```

State : q2:

```
(( [-2.00,-2.00] U [-1.00,-1.00] ) u U_k in N ( [0.00,0.00]
- 3.00 -k1.00 ) ) n [0.00,inf[
```

Running the tool

Small example

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q0;q1;q2;q3
# Transitions
q2;[0,0];0;q3;a
q0;]0,1[;0;q1;a
q0;[1,1];0;q0;e
q0;[1,1];0;q2;e
```

State evaluation: after delay 0.3

State : q0:

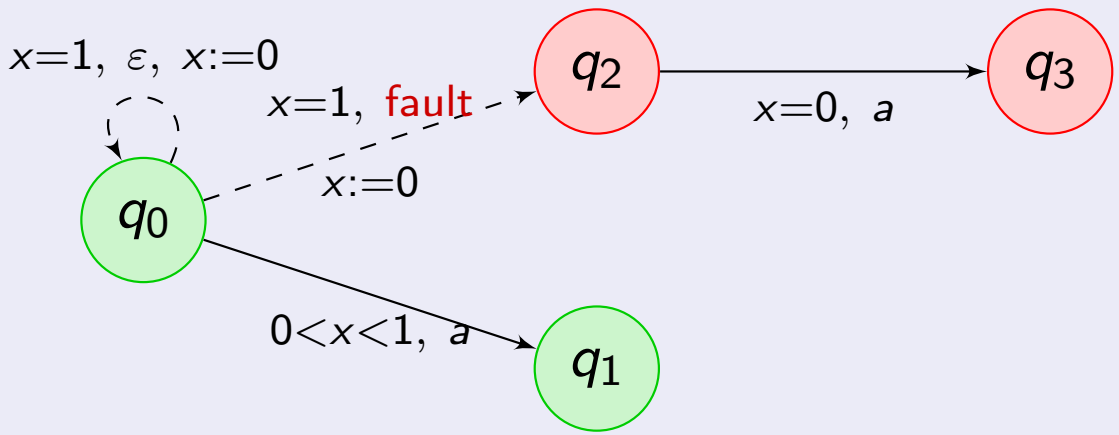
```
(( [0.30,0.30] ) u U_k in N ( [0.00,0.00] - 0.70 -k1.00 ) )
n [0.00,inf[
```

State : q2:

```
(( [-1.70,-1.70] U [-0.70,-0.70] ) u U_k in N ( [0.00,0.00]
- 2.70 -k1.00 ) ) n [0.00,inf[
```

Running the tool

Small example




```
# States.  Size :  4
q0;q1;q2;q3
# Transitions
q2;[0,0];0;q3;a
q0;]0,1[;0;q1;a
q0;[1,1];0;q0;e
q0;[1,1];0;q2;e
```

State evaluation: after delay 0.3 and action a

```
State :  q1:
([0.00,0.00]) n [0.00,inf[
```