Energy games

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IRISA, team SUMO

Based on joint works with

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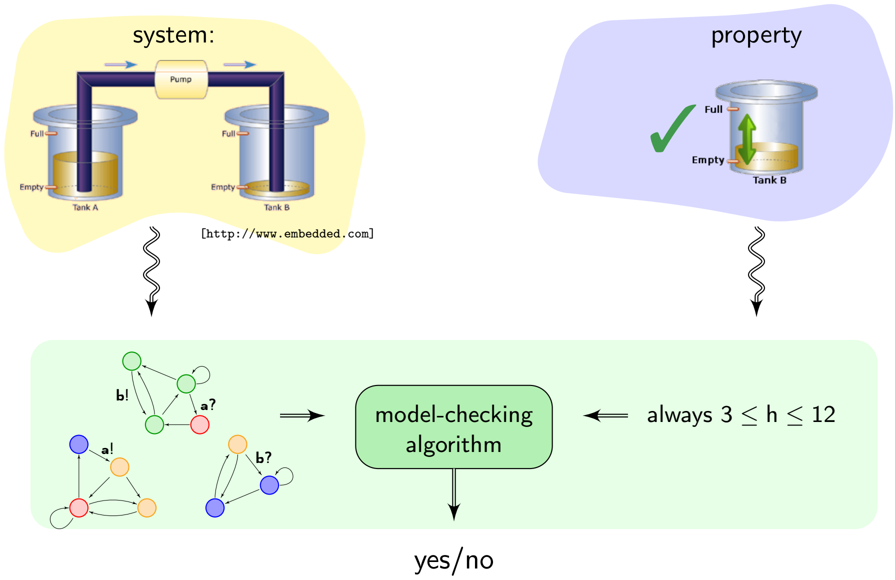
Kim G. Larsen   Simon Laursen   Mickael Randour
Jiri Srba   Martin Zimmermann

June 29, 2017

Model checking and synthesis



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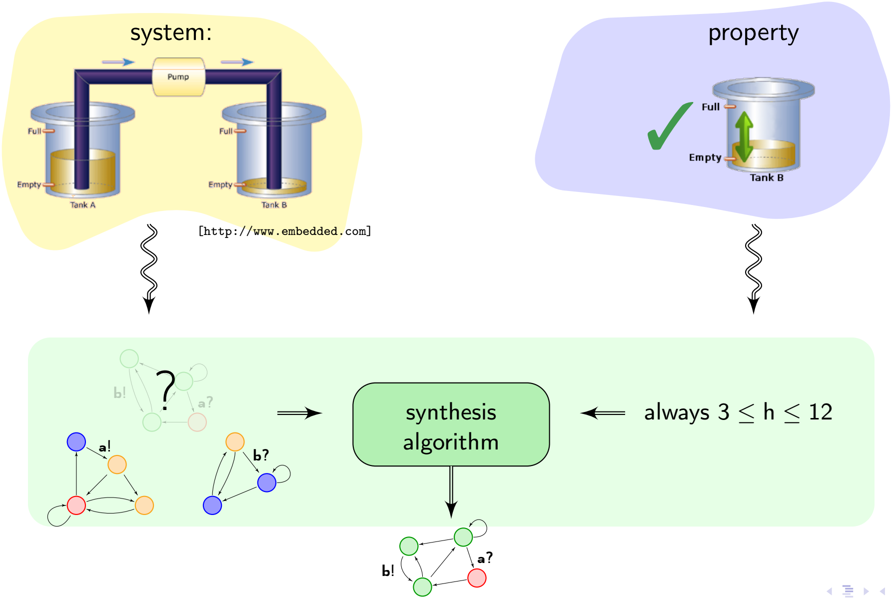
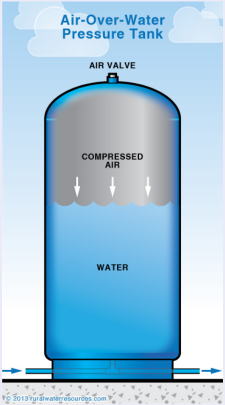


(Average-)energy objectives: an example

Pressure-tank case study [CJL+09]



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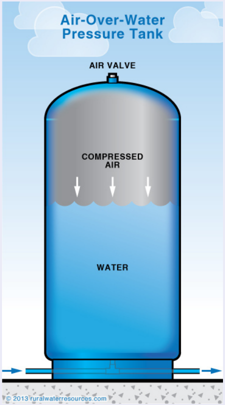

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- keep water level within given bounds
- minimize average level

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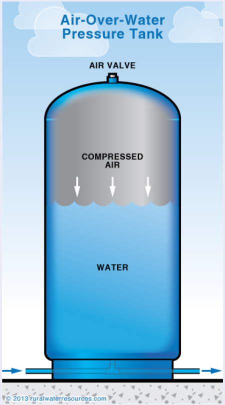

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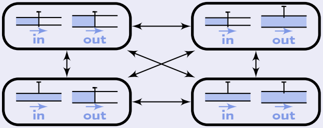

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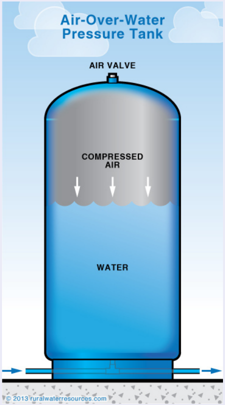

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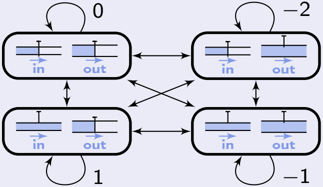

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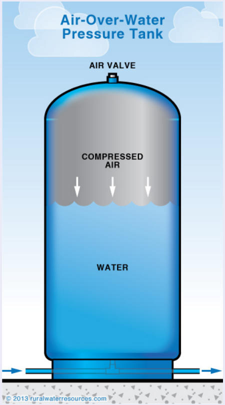

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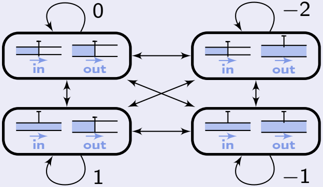

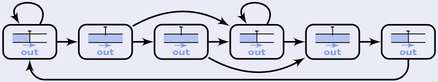

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Games on weighted graphs

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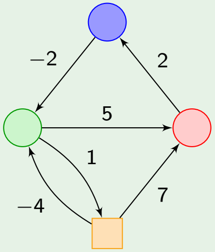

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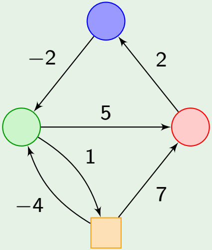

- **states**: $\mathcal{S} = \mathcal{S}_{\bigcirc} \uplus \mathcal{S}_{\square}$
- **weighted transitions**: $T \subseteq S \times \mathbb{Z} \times S$

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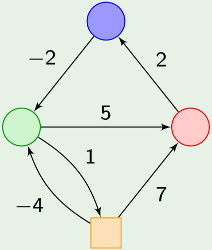

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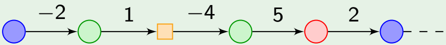

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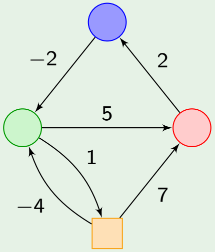

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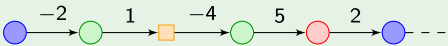

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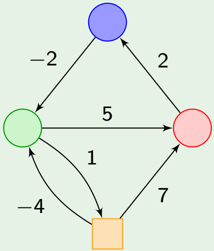

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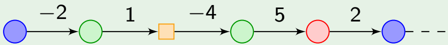

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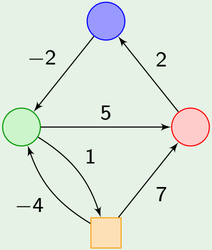

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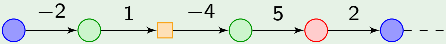

- **strategy**: transition to take depending on state/history:

  $\sigma_{\bullet}$: always go to ■ (from ●)

  $\sigma'_{\bullet}$: alternate between ● and ■ (from ●)

Quantitative objectives

Decision problems

- **shortest path**:
  is it possible to go from source to target with accumulated weight less than a given threshold?

[BGHM17] Brihaye *et al.*. Pseudopolynomial iterative algorithm to solve [...]. Acta Inf, 2017.
[KBB+08] Khachiyan *et al.*. On short paths interdiction problems:[...]. TCSys, 2008.

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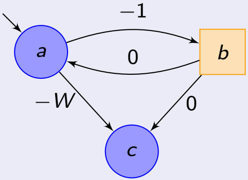
  $\rightsquigarrow$ in polynomial time for 1-player games (e.g. Bellman-Ford algorithm)

  $\rightsquigarrow$ in pseudo-polynomial time for 2-player games [BGHM17]
  in NP $\cap$ coNP, PTIME-hard
  PTIME-complete with nonnegative weights [KBB+08]



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[Kar78] Karp. A characterization of the minimum cycle mean in a digraph. Discr.Math., 1978.
[ZP96] Zwick, Paterson. The complexity of mean payoff games on graphs. TCS, 1996.

Energy objectives

Energy level

- energy level: $EL(\pi_{\leq n}) = \sum_{i \leq n} w(s_i \to s_{i+1})$ [aka. total payoff]

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Quantitative objectives

- **shortest path**: minimize energy level when reaching a target
- **mean-payoff**: minimize ratio (energy level/path length) in the long run
- **energy objectives**: maintain energy level within some bounds
  - above lower bound L
  - between bounds L and U

Solving lower-bounded energy games

1-player case [BFLMS08]

- aim: maintain energy level above $L \rightsquigarrow$ maximize energy level
- Bellman-Ford-like algorithm to compute maximal remaining energy after k steps

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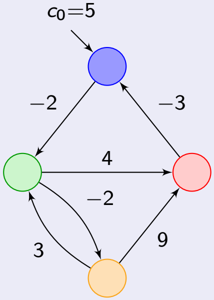

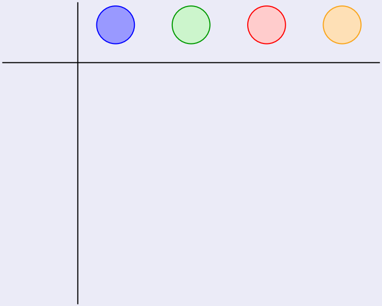

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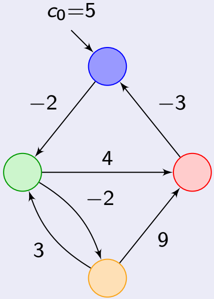

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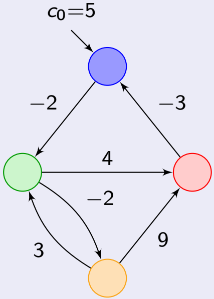

0	5	$-\infty$	$-\infty$	$-\infty$
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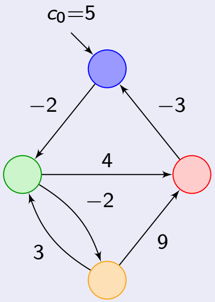

0	5	$-\infty$	$-\infty$	$-\infty$
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2	5	3	7	1

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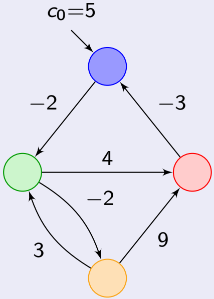

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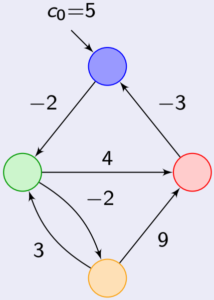

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Solving lower-bounded energy games

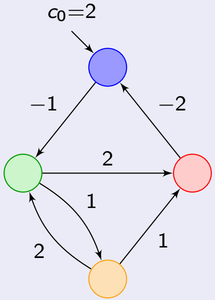
2-player case

- aim: maximize (resp. minimize) energy level
- both players have memoryless optimal strategies
- deciding the winner is in NP ∩ coNP
- mean-payoff games are logspace-reducible to L-energy games

Solving interval-bounded energy games

Reduction to safety condition in pseudo-polynomial graph

Objective: keep energy level
between $L = 0$ and $U = 3$



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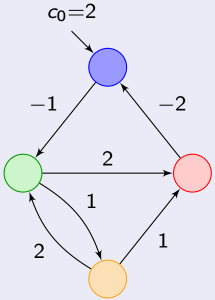

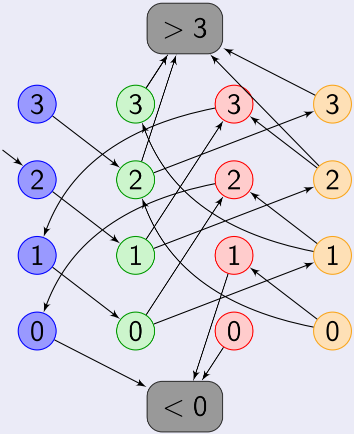

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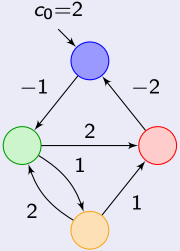
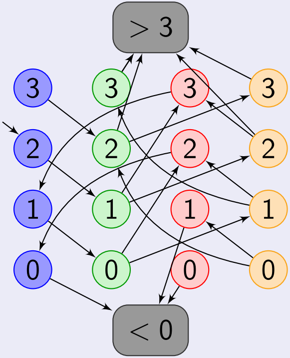




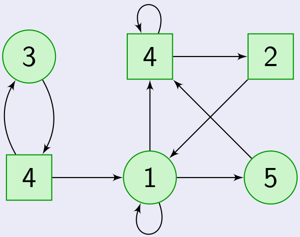
Theorem ([BFLMS08,FJ13])

1-player interval-bounded energy games are PSPACE-*complete.*
2-player interval-bounded energy games are EXPTIME-*complete.*

[BFLMS08] Bouyer *et al.* Infinite Runs in Weighted Timed Automata [...] FORMATS, 2008.
[FJ13] Fearnley, Jurdziński. Reachability in two-clock timed automata is PSPACE-complete. ICALP, 2013.

Energy parity games

Parity games



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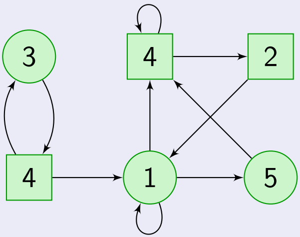

$M(\gamma)$ = maximal value seen along γ.

Objective of Player 1:

make $M(\gamma)$ even for any outcome.

Energy parity games

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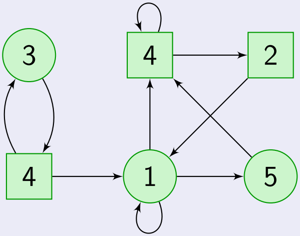

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Theorem

Both players have memoryless optimal strategies.
Deciding the winner is in NP $\cap$ coNP.

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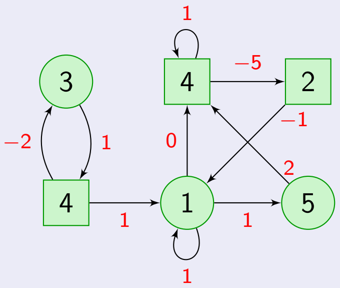

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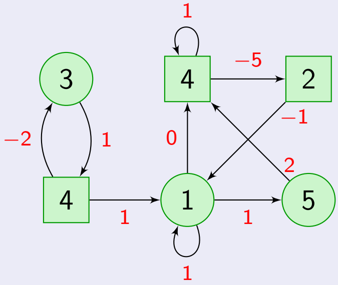

Objective of Player 1: for any γ

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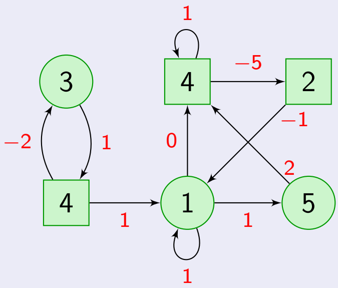

Objective of Player 1: for any γ

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Theorem ([CD12])

Player 2 has memoryless optimal strategies.
Player 1 has optimal strategies combining 2 memoryless strategies.
Deciding the winner is in NP ∩ coNP.

[CD12] Chatterjee and Doyen. Energy Parity Games. TCS, 2012.

Average-energy games

Average energy is not mean-payoff

- **mean payoff** = average of weight on transitions

$$MP(\pi_{\leq n}) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi_{\leq n})$$

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- **average energy** = average of accumulated weight

$$AE(\pi_{\leq n}) = \limsup_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} EL(\pi_{\leq i})$$

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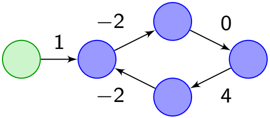

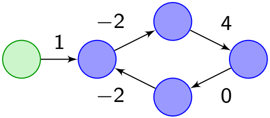

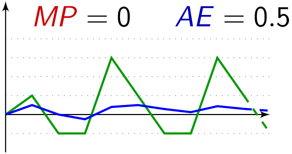

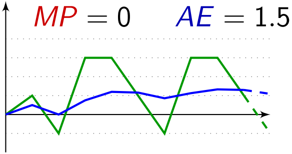

Average-energy games

Theorem ([BMRLL15])

- *1-player AE games can be solved in* PTIME *(memoryless stategies are sufficient)*

[BMRLL15] Bouyer *et al.* Average-energy games. GandALF, 2015.

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    - *compute* $\overline{AE}(\mathcal{C}_{k,s})$ *for all simple cycles of length* k *on* s*;*
    - *minimize* $EL(\rho_{s_0 \to s}) + \overline{AE}(\mathcal{C}_s)$*.*

[BMRLL15] Bouyer *et al.* Average-energy games. GandALF, 2015.

Average-energy games

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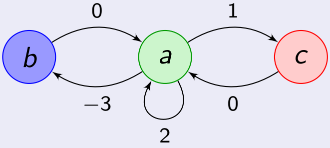
  - *minimize* $EL(\rho_{s_0 \to s}) + \overline{AE}(\mathcal{C}_s)$*.*
- *in 2-player AE games, both players have memoryless optimal strategies*
- *deciding the winner is in* NP $\cap$ coNP
- *mean-payoff games are logspace-reducible to AE games*

[BMRLL15] Bouyer *et al.* Average-energy games. GandALF, 2015.

Energy- and average-energy constraints

Mixing average-energy and interval constraints



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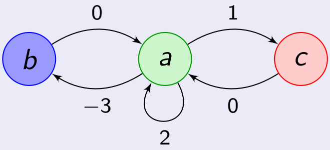

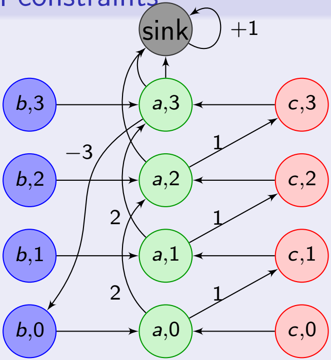

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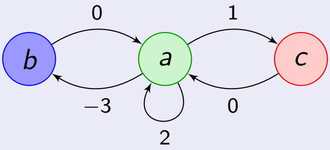

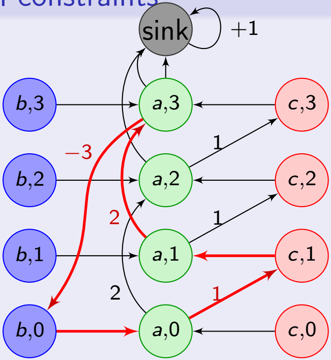

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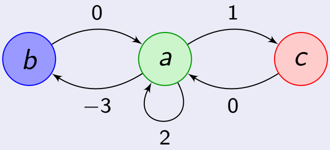
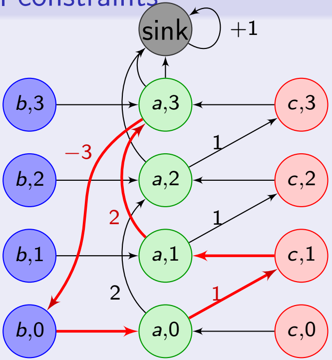




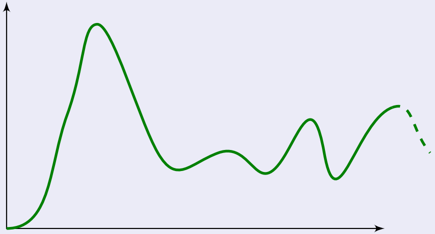
Theorem ([BMRLL15])

1-player AELU-games are in EXPTIME, and PSPACE-hard.
2-player AELU-games are EXPTIME-complete.

[BMRLL15] Bouyer *et al.* Average-energy games. GandALF, 2015.

Energy- and average-energy constraints

Mixing average-energy and lower-bound constraints



Energy- and average-energy constraints

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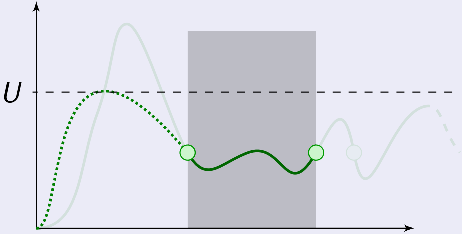

Bound peak height U:

- pseudo-polynomial for 1-player games
- 2-exponential for 2-player games

Energy- and average-energy constraints

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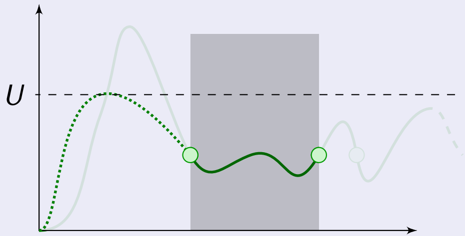


Bound peak height U:

- pseudo-polynomial for 1-player games
- 2-exponential for 2-player games

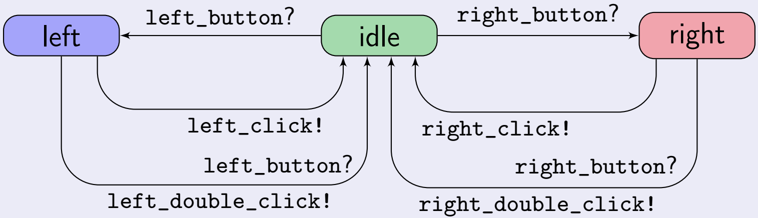
Theorem ([BHM+17])

1-player AEL-games are in EXPTIME, and NP-hard.
2-player AEL-games are in 2-EXPTIME, and EXPSPACE-hard.

[BHM+17] Bouyer *et al.* Bounding Average-energy Games. FoSSaCS, 2017.

Timed energy games

Timed automata: example of a computer mouse



Timed energy games

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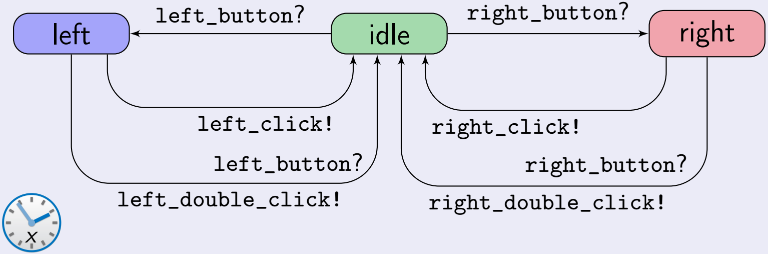

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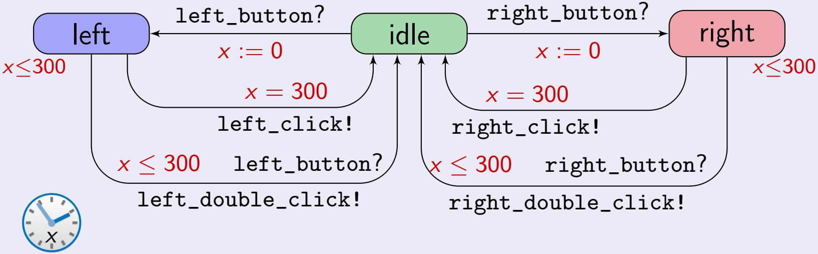

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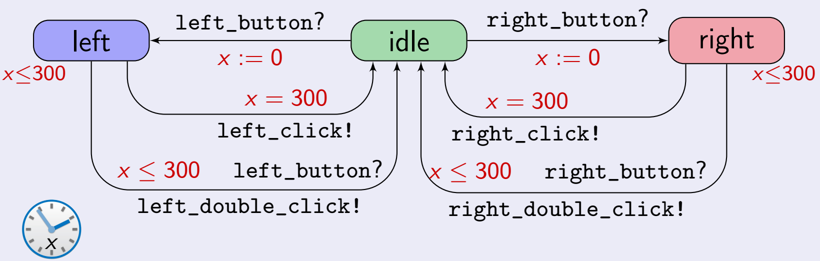


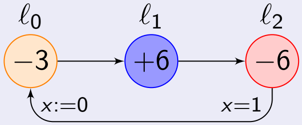
Theorem ([AD94,AMP$^+$98])

Reachability in timed automata is PSPACE*-complete.*
Reachability in timed games is EXPTIME*-complete.*

[AD94] Alur and Dill. A Theory of Timed Automata. TCS, 1994.
[AMP$^+$98] Asarin *et al.*. Controller Synthesis for Timed Automata. SSC, 1998.

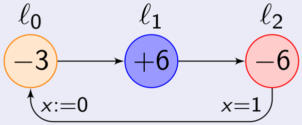
Timed energy games

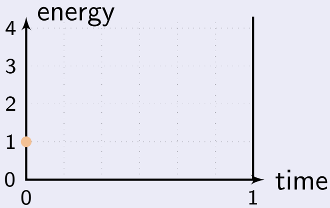
Weighted timed automata



Timed energy games

Weighted timed automata





Timed energy games

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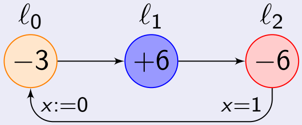

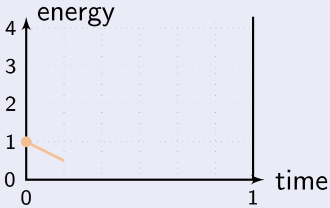

Timed energy games

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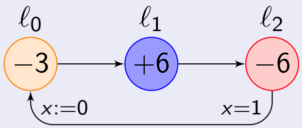

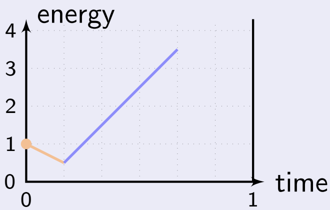

Timed energy games

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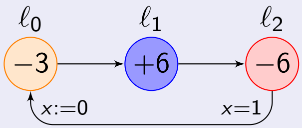

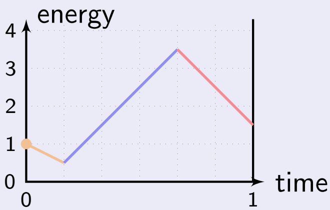

Timed energy games

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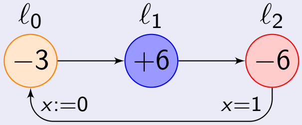

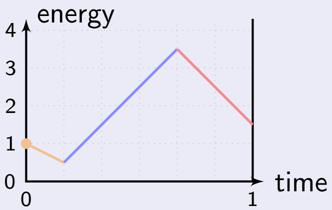

Theorem ([BFLMS08,BFLM10])

- *For 1 player:*
  - *lower-bound problem for 1-clock timed automata is in* EXPTIME
  - *interval problem for 2-clock timed automata is undecidable*
- *For 2 players:*
  - *interval problem for 1-clock timed automata is undecidable*

[BFLMS08] Bouyer *et al.* Infinite Runs in Weighted Timed Automata [...] FORMATS, 2008.
[BFLM10] Bouyer *et al.* Timed Automata with Observers under Energy Constraints. HSCC, 2010.

Conclusion and future works

Conclusion

- Weighted games, in particular energy games, conveniently model resource-management problems;
- they are rather well-understood, but with significant open problems.
- no real tools available, only prototypes.

Future works

- extend to stochastic strategies, stochastic games
- multiple-player quantitative games
- combine weighted timed games with imprecisions