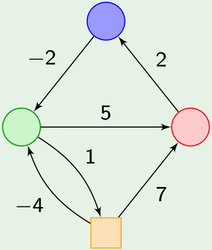
Average-energy games

Patricia Bouyer Nicolas Markey Mickael Randour
Kim G. Larsen Simon Laursen
Piotr Hofman Martin Zimmermann

January 27, 2017

Games on weighted graphs

Example



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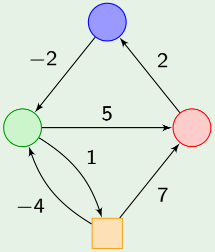

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- **weighted transitions**: $T \subseteq S \times \mathbb{Z} \times S$

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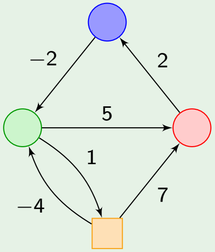

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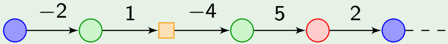

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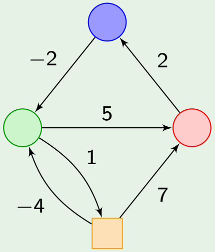

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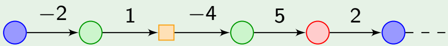

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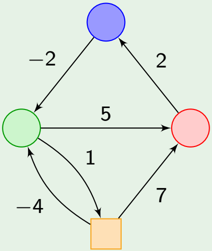

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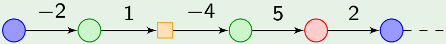

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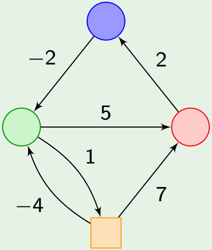

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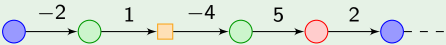

- **strategy**: transition to take depending on state/history:

  $\sigma_{\bigcirc}$: always go to  (from )

  $\sigma'_{\bigcirc}$: alternate between  and  (from )

Quantitative objectives

Relevant quantities to control

- energy level: $EL(\pi_{\leq n}) = \sum_{i \leq n} w(s_i \to s_{i+1})$ [aka. total payoff]
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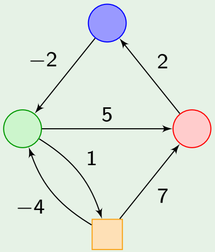

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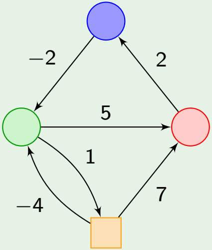

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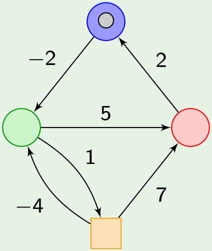

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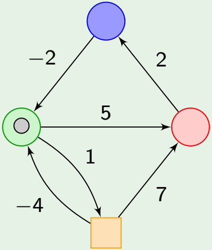

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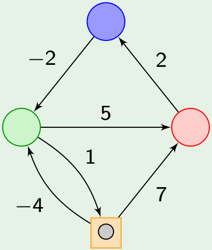

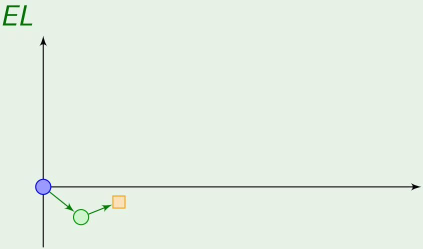

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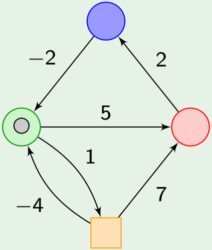

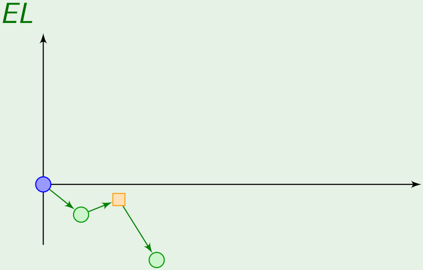

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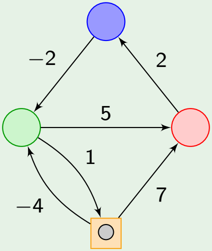

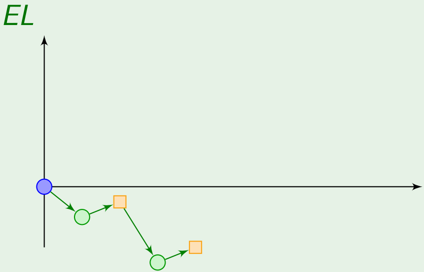

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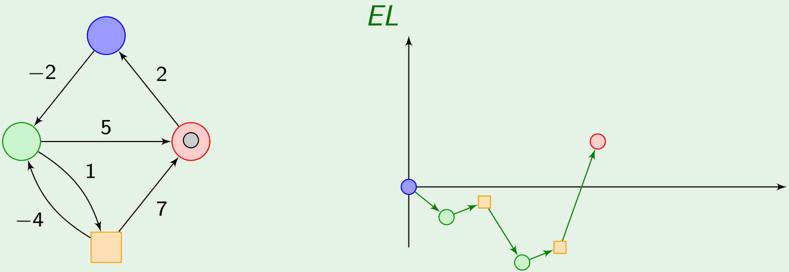

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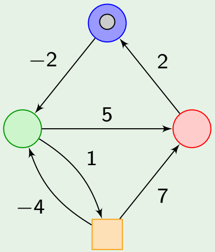

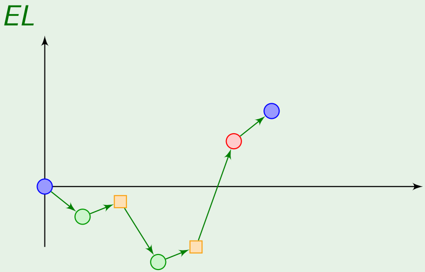

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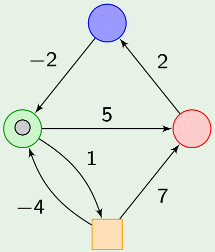

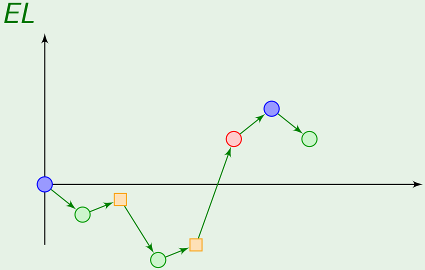

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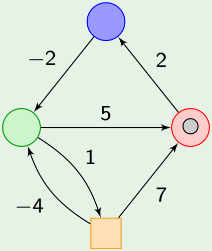

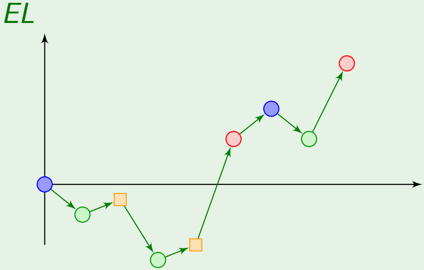

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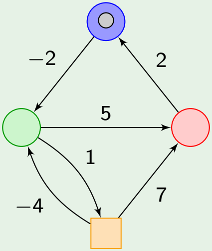

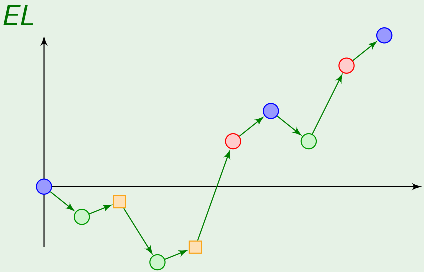

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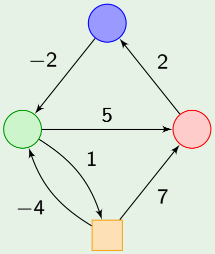

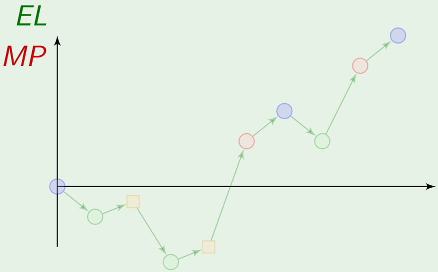

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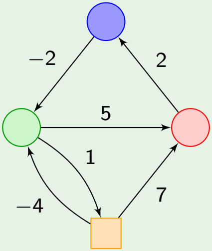

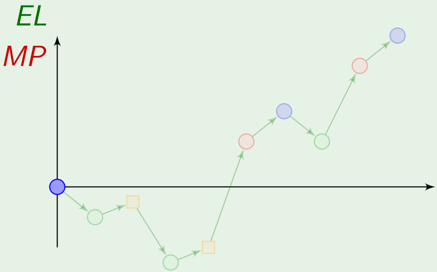

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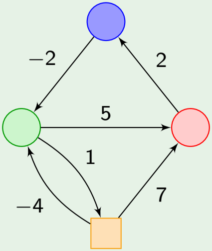

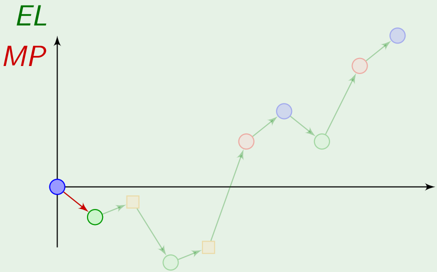

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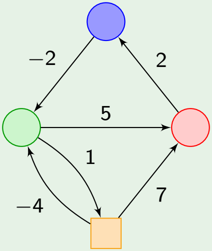

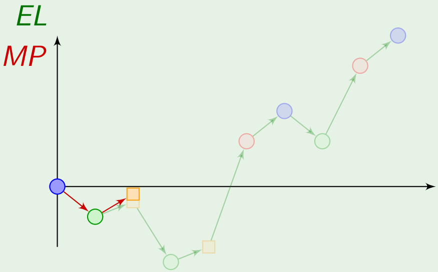

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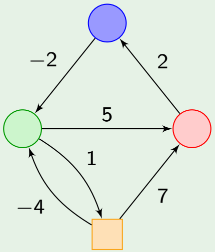

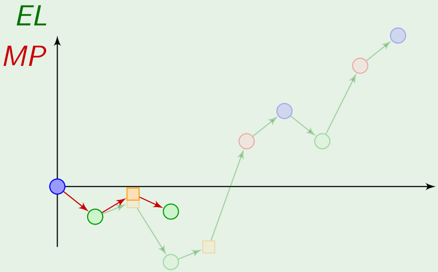

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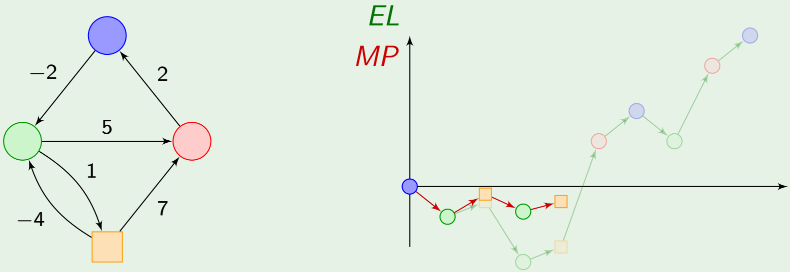

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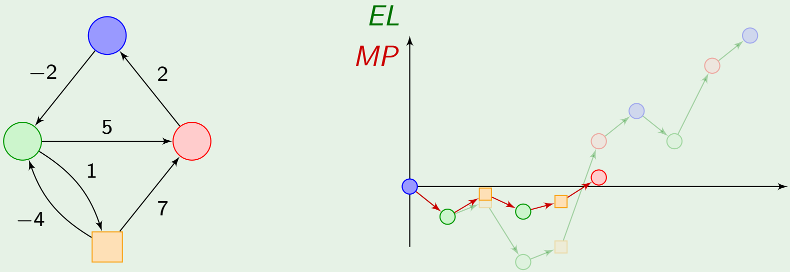

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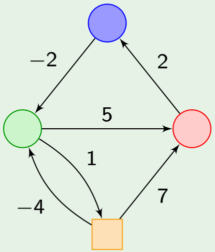

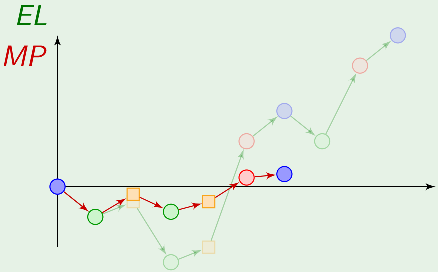

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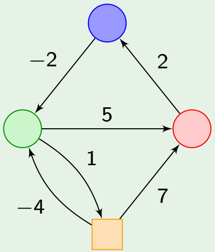

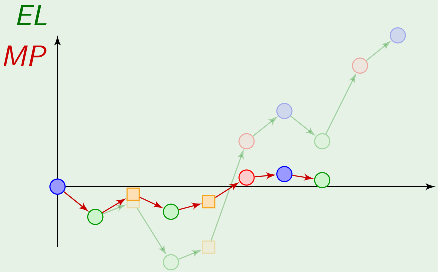

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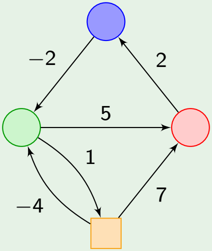

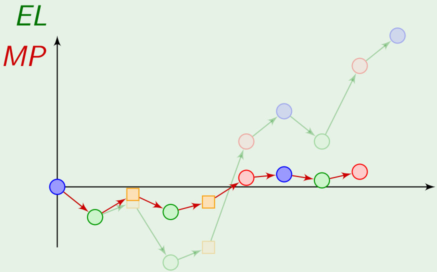

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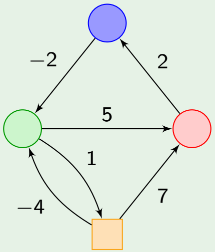

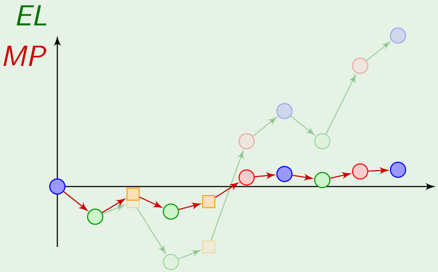

Decision problems and known results

Decision problems

- **total payoff**: is there a strategy to have $\limsup \mathsf{EL} \leq t$?
- **mean payoff**: is there a strategy to have $\limsup \mathsf{MP} \leq t$?
- **energy constraint**: is there a strategy to keep $\mathsf{EL} \in [l, u)$?

Decision problems and known results

Known results

objective	1 player	2 players
Mean Payoff	PTIME [Kar78]	NP $\cap$ coNP [ZP96]
Total Payoff	PTIME [FV97]	NP $\cap$ coNP [GS09]
Energy$_{[L,+\infty)}$	PTIME [BFL+08]	NP $\cap$ coNP [CdAHS03,BFL+08]
Energy$_{[L,U)}$	PSPACE-c. [FJ13]	EXPTIME-c [BFL+08]

For all except Energy$_{[L,U)}$, memoryless strategies are sufficient.

[Kar78] Karp. A characterization of the minimum cycle mean in a digraph. Discr.Math., 1978.
[ZP06] Zwick, Paterson. The complexity of mean payoff games on graphs. TCS, 1996.
[FV97] Filar, Vrieze. Competitive Markov decision processes. Springer, 1997.
[GS09] Gawlitza, Seidl. Games through nested fixpoints. CAV, 2009
[BFL+08] Bouyer *et al.* Infinite runs in weighted timed automata with energy constraints. FORMATS, 2008.
[CdAHS03] Chakrabarti *et al.* Resource interfaces. EMSOFT, 2003.
[FJ13] Fearnley, Jurdziński. Reachability in two-clock timed automata is PSPACE-complete. ICALP, 2013.

Outline of the presentation

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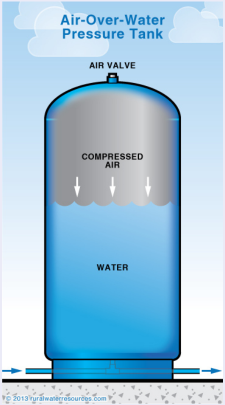
Average-energy objectives: example

Pressure-tank case study [CJL+09]



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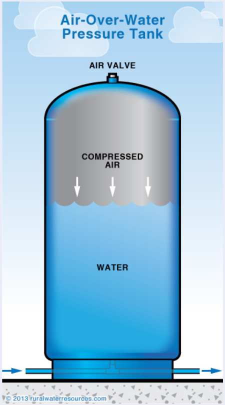

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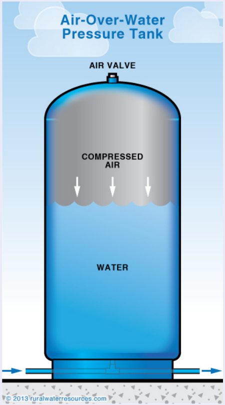

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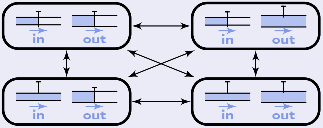

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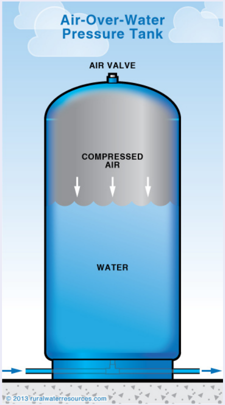

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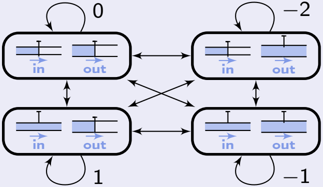

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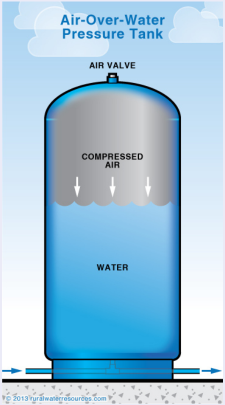

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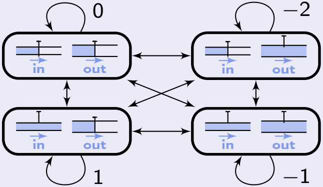

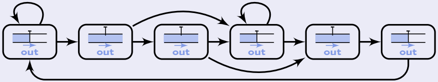

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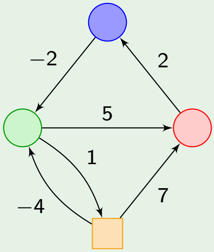

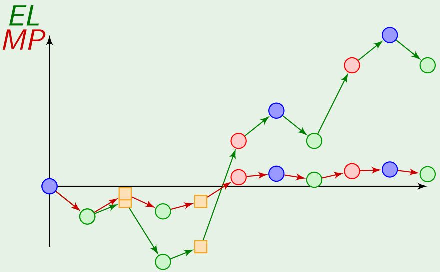

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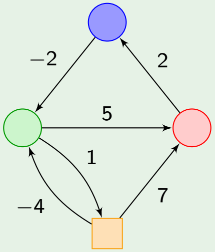

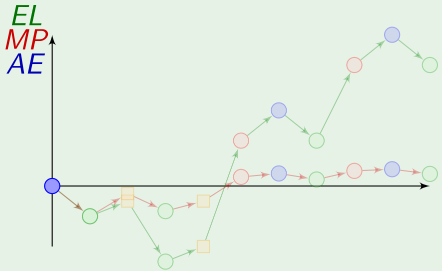

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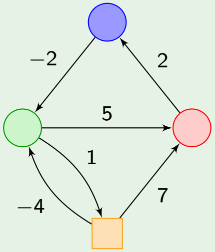

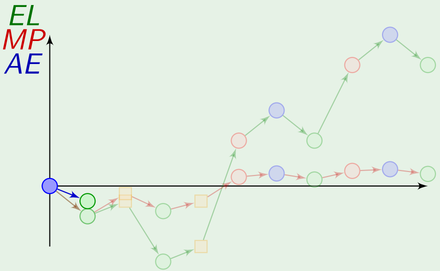

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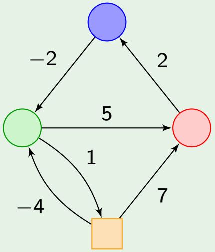

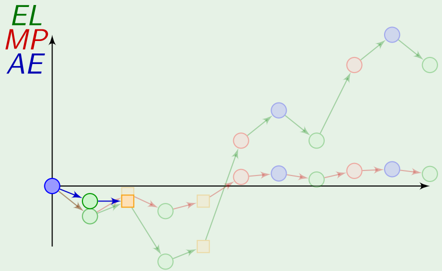

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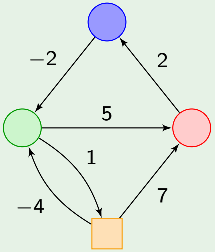

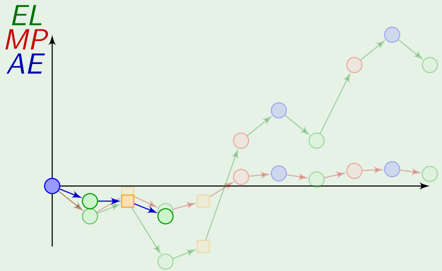

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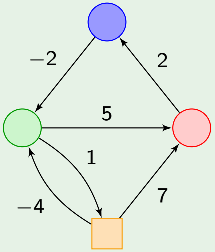

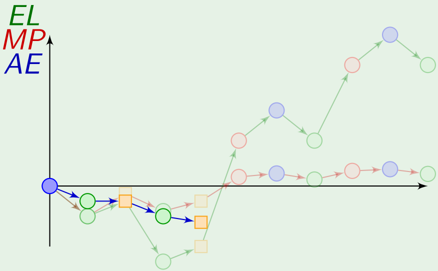

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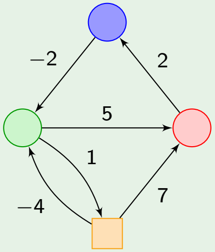

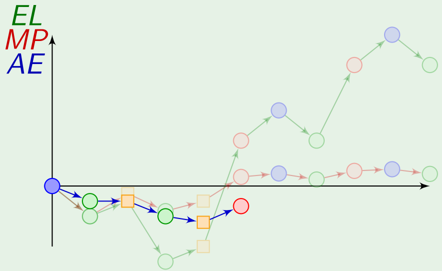

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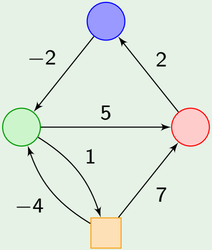

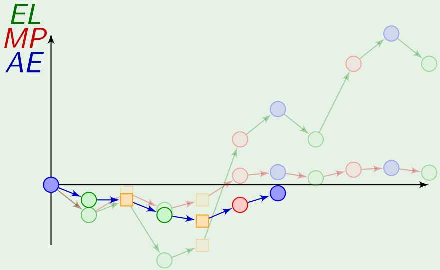

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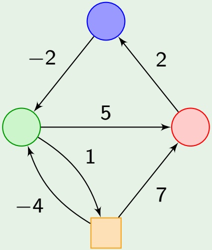

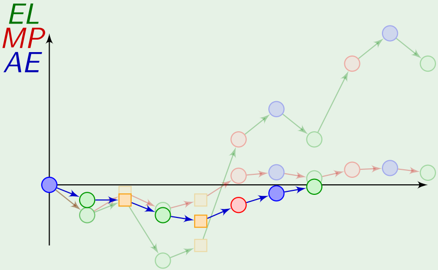

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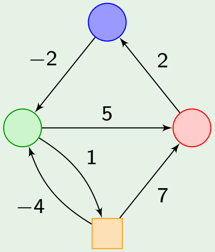

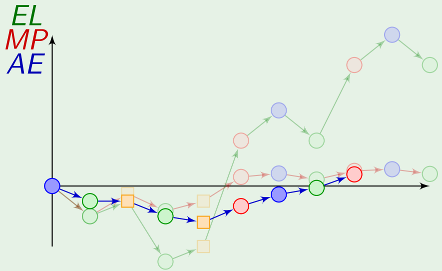

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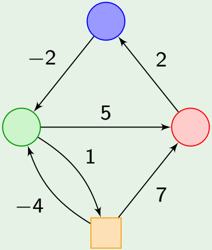

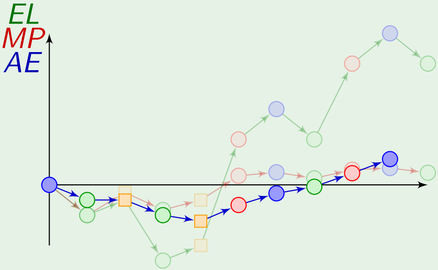

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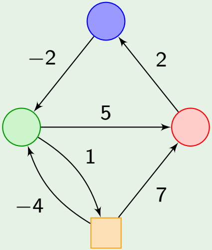

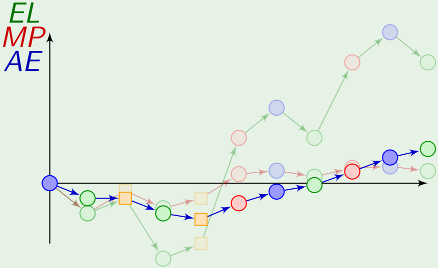

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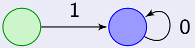

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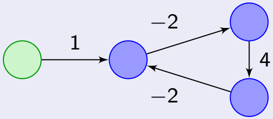

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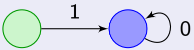

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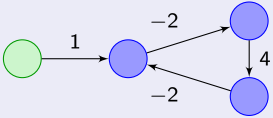

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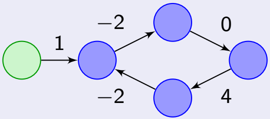

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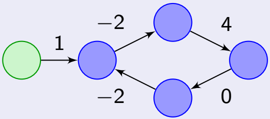

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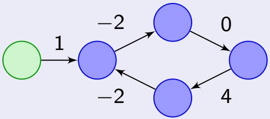

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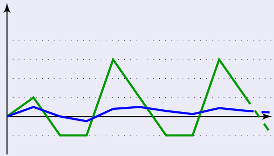

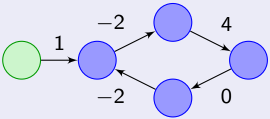

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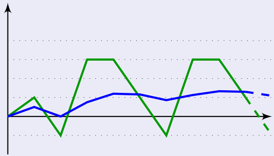

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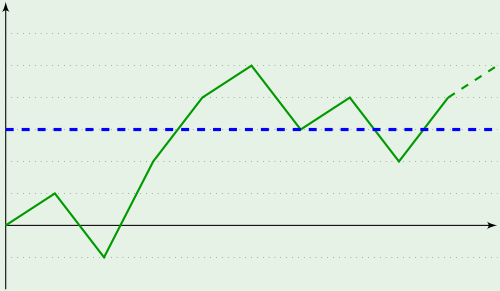

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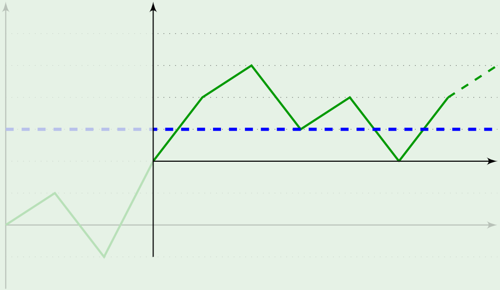

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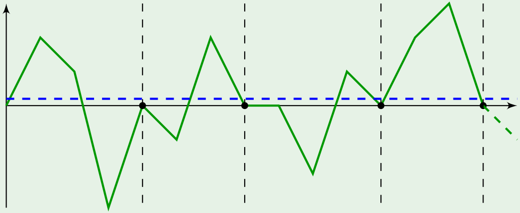


Some properties of average-energy objectives

Lemma

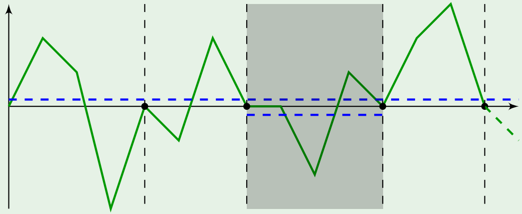
$$\overline{AE}(\pi_1 \cdot \pi_2) = EL(\pi_1) + \overline{AE}(\pi_2)$$

Lemma

Let $\pi = \pi_1 \cdot \pi_2 \cdots$ be a sequence of bounded-length bounded-weight zero-cycles. Then

$$\exists i.\ AE(\pi_i) \leq \overline{AE}(\pi).$$

Example



1-player case: memoryless optimal strategy

Lemma

Memoryless strategies are sufficient for 1-player AE games.

Theorem

1-player AE games can be solved in PTIME.

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Algorithm

- compute $\overline{AE}(\mathcal{C}_{k,s})$ for all simple cycles of length k on s;
- minimize $EL(\rho_{s_0 \to s}) + \overline{AE}(\mathcal{C}_s)$.

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- for each pair (s, k), can be solved in PTIME via a linear programming (see next slide).

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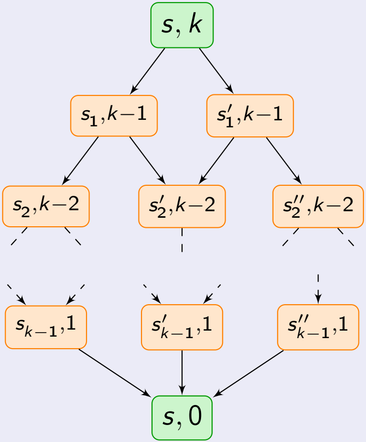
Proof

- for each pair (s, k), can be solved in PTIME via a linear programming (see next slide).
- optimal reachability; solvable in PTIME.

1-player case: computing $\overline{AE}(\mathcal{C}_{k,s})$



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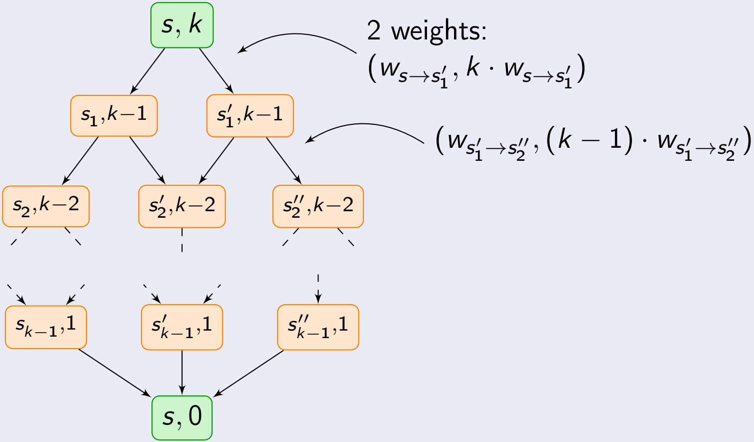

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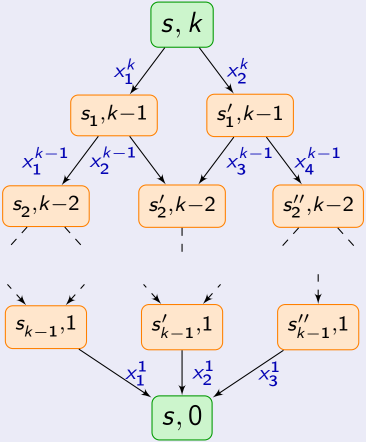
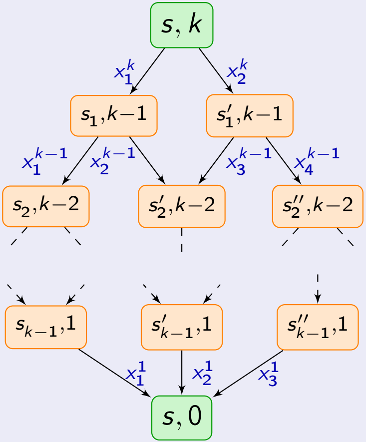


1-player case: computing $\overline{AE}(\mathcal{C}_{k,s})$

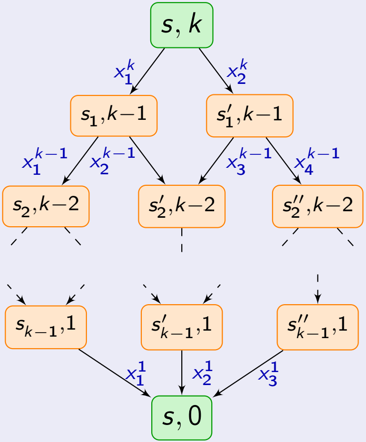
Linear program: minimize



$$\sum_{i=1}^{k} \sum_{j} x_j^i \cdot (i \cdot w_j^i)$$

1-player case: computing $\overline{AE}(\mathcal{C}_{k,s})$

Linear program: minimize



$$\sum_{i=1}^{k} \sum_{j} x_j^i \cdot (i \cdot w_j^i)$$

subject to:

- $0 \leq x_j^i \leq 1$
- $\sum x_j^i > 0$
- incoming flow = outgoing flow
- flow from (s, k)=1
- flow to $(s, 0)$=1
- total weight is zero

2-player case: memoryless determinacy

Lemma

AE-games are memoryless determined.

2-player case: memoryless determinacy

Lemma

AE-games are memoryless determined.

Proof

2-player case: memoryless determinacy

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Proof

- optimal positional strategy in the one-player games

2-player case: memoryless determinacy

Lemma

AE-games are memoryless determined.

Proof

- optimal positional strategy in the one-player games
- result follows from [GZ05].

[GZ05] Gimbert, Zielonka. Games Where You Can Play Optimally Without Any Memory. CONCUR, 2004.

2-player case: memoryless determinacy

Lemma

AE-games are memoryless determined.

Corollary

AE-games can be solved in $\mathsf{NP} \cap \mathsf{coNP}$.

2-player case: memoryless determinacy

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AE-games are at least as hard as MP-games.

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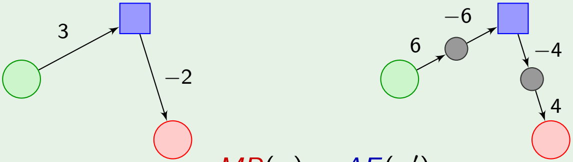


$$MP(\pi) = AE(\pi')$$

Average-energy games: summary

objective	1 player	2 players
Mean Payoff	PTIME [Kar78]	NP ∩ coNP [ZP06]
Total Payoff	PTIME [FV97]	NP ∩ coNP [GZ09]
Energy$_{[L,+\infty)}$	PTIME [BFL+08]	NP ∩ coNP [CdAHS03,BFL+08]
Energy$_{[L,U)}$	PSPACE-c. [FJ13]	EXPTIME-c [BFL+08]
AvgEnergy	PTIME	NP ∩ coNP

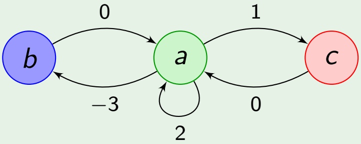
For all except Energy$_{[L,U)}$, memoryless strategies are sufficient.

Outline of the presentation

Mixing energy- and average-energy objectives

Example

Minimize AE (in the long run) while keeping EL between 0 and 3:



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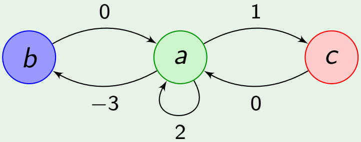

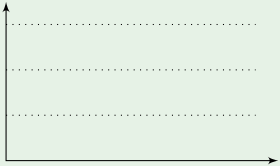

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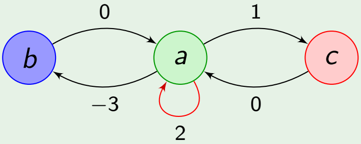

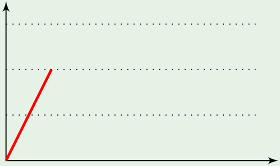

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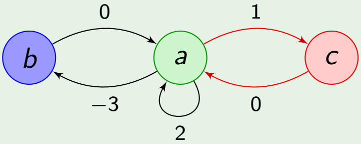

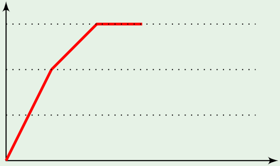

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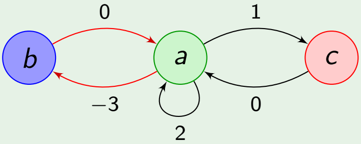

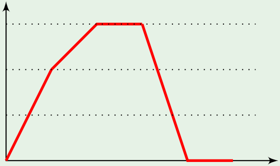

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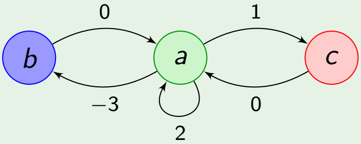

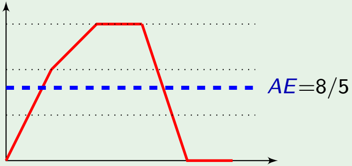

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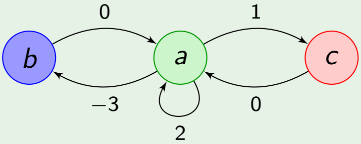

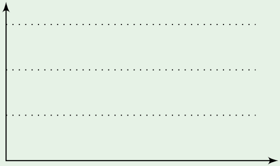

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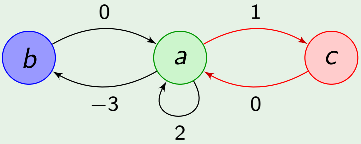

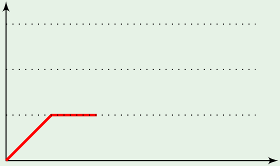

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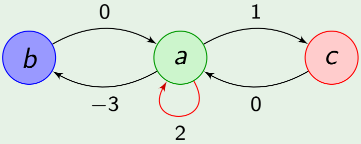

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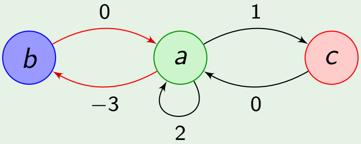

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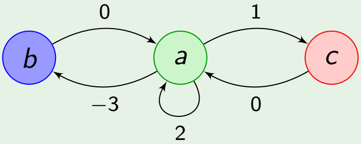

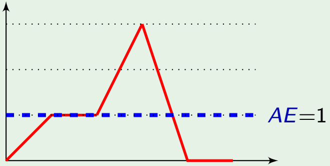

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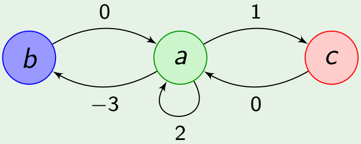
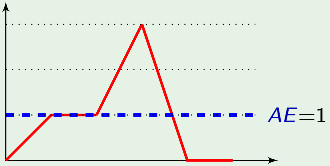


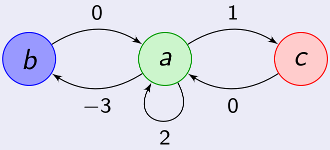


Lemma

Memory is needed to win AE+LU games.

Solving AELU games

Game expanded with energy level



Solving AELU games

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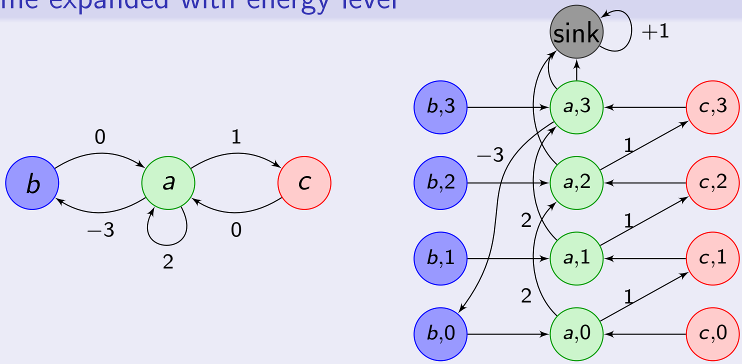

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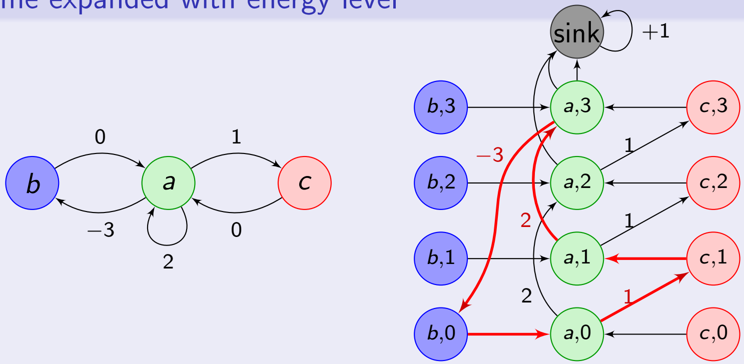

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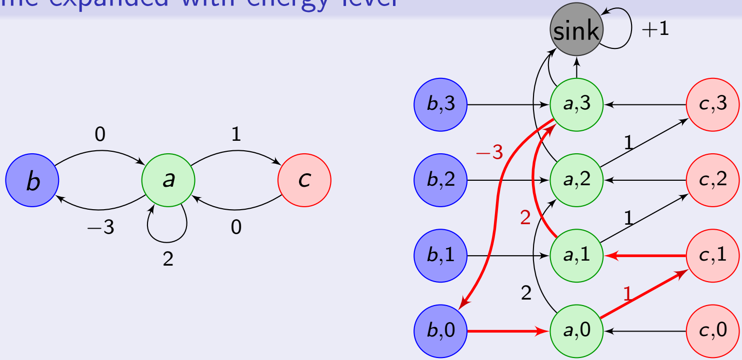

Lemma

AELU reduces to AE in expanded game.

Solving AELU games

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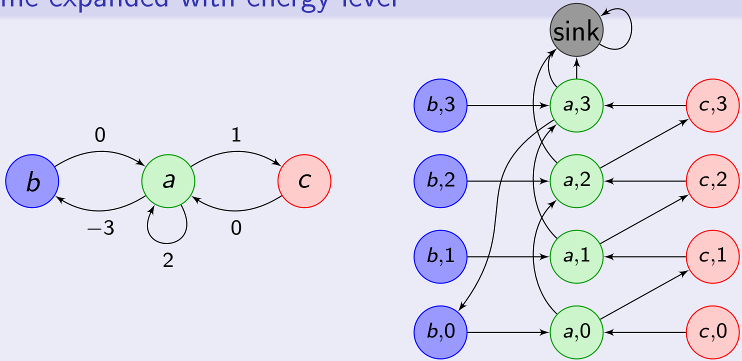

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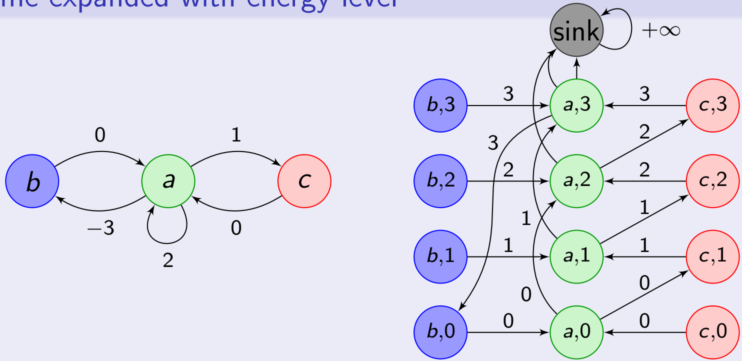

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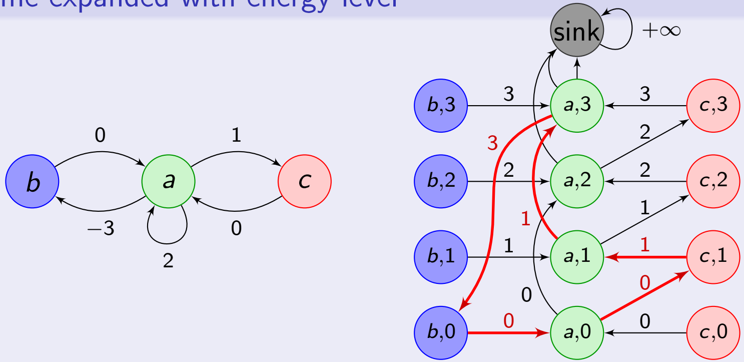

Solving AELU games

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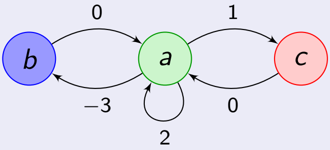
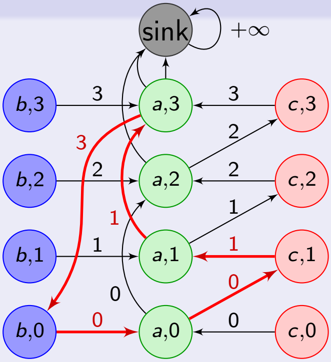


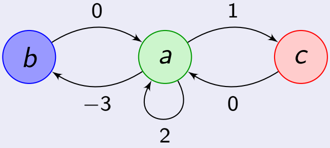


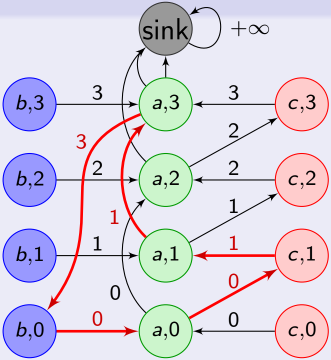
Lemma

AELU reduces to MP in expanded game.

Solving AELU games

Game expanded with energy level



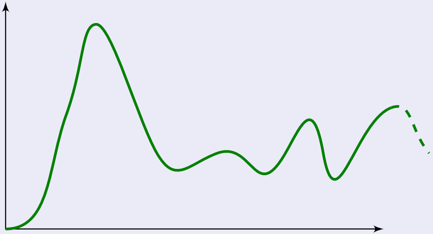


Theorem

1-player AELU-games are in EXPTIME, and PSPACE-hard.
2-player AELU-games are EXPTIME-complete.

Solving 1-player AEL games

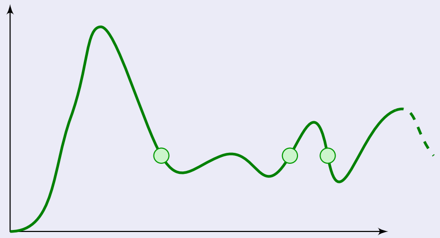
1-player AEL games



- Assume there is a winning path;

Solving 1-player AEL games

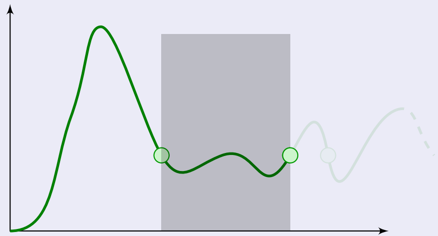
1-player AEL games



- Assume there is a winning path;
- There is a repeated configuration;

Solving 1-player AEL games

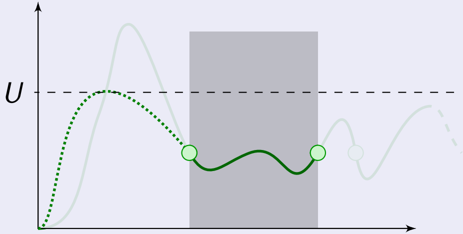
1-player AEL games



- Assume there is a winning path;
- There is a repeated configuration;
- One of the resulting cycle has low average energy;

Solving 1-player AEL games

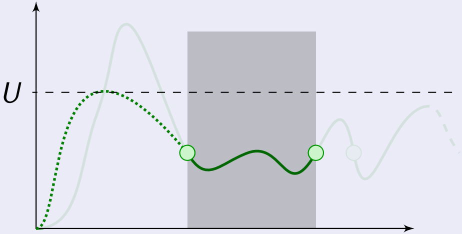
1-player AEL games



- Assume there is a winning path;
- There is a repeated configuration;
- One of the resulting cycle has low average energy;
- Reachability requires pseudo-polynomial peak height;

Solving 1-player AEL games

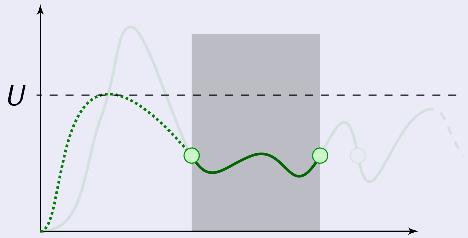
1-player AEL games



$\rightsquigarrow$ reduction to AELU, with $U = t + O(W^3 \cdot |S|^3)$.

Solving 1-player AEL games

1-player AEL games



$\rightsquigarrow$ reduction to AELU, with $U = t + O(W^3 \cdot |S|^3)$.

Theorem

The AEL-problem is in PSPACE, *and* NP-*hard, for 1-player games.*

Solving 2-player AEL games

- Expanded game (with MP objective) has infinite state space
  $\leadsto$ classical results/techniques for MP games fail.

Solving 2-player AEL games

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Solving 2-player AEL games

- Expanded game (with MP objective) has infinite state space
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Solving 2-player AEL games

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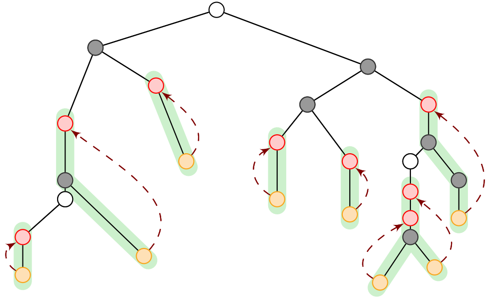
- Then modify strategy tree to bound global peak height.



Solving 2-player AEL games

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Theorem

*2-player AEL games are in 2-*EXPTIME*, and* EXPSPACE*-hard.*

Conclusions and future works

objective	1 player	2 players
Mean Payoff	PTIME [Kar78]	NP $\cap$ coNP [ZP06]
Total Payoff	PTIME [FV97]	NP $\cap$ coNP [GZ09]
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Energy$_{[L,U)}$	PSPACE-c. [FJ13]	EXPTIME-c [BFL+08]
AvgEnergy	PTIME	NP $\cap$ coNP
AE+E$_{[L,\infty)}$	EXPTIME, NP-h.	2-EXPTIME, EXPSPACE-h.
AE+E$_{[L,U)}$	EXPTIME, PSPACE-h.	EXPTIME-c.

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AE+E$_{[L,\infty)}$	EXPTIME, NP-h.	2-EXPTIME, EXPSPACE-h.
AE+E$_{[L,U)}$	EXPTIME, PSPACE-h.	EXPTIME-c.

Future work

- close complexity gaps;
- extend to stochastic setting;
- extend to timed setting.