Optimal strategies in weighted timed games: undecidability and approximation

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(joint work with Patricia Bouyer and Samy Jaziri)

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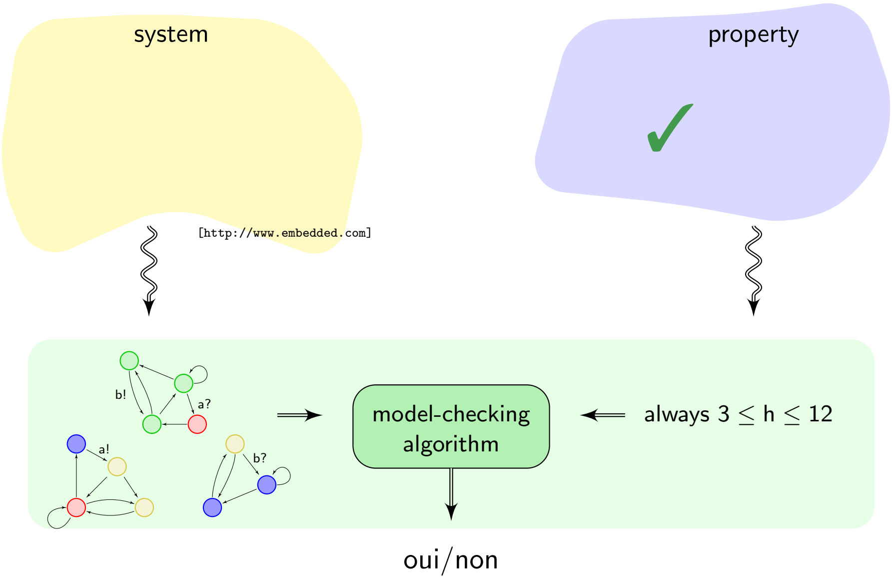
December 19, 2015

Model checking and *synthesis*



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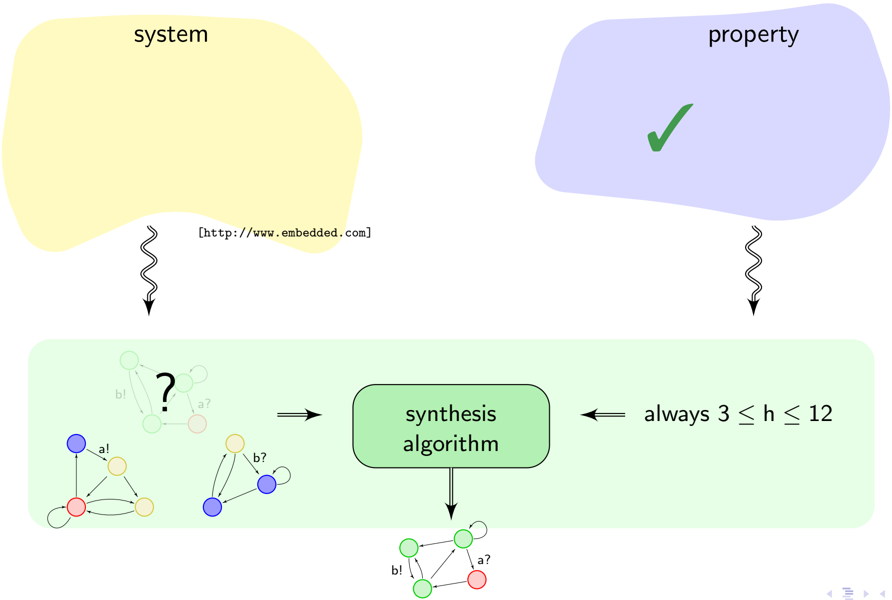
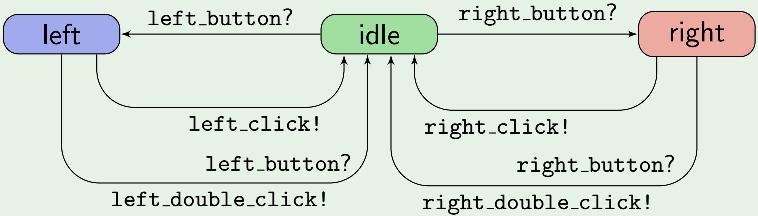


Reasoning about real-time systems

Example (A computer mouse)



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Definition ([AD90])

A timed automaton is made of

- a transition system,

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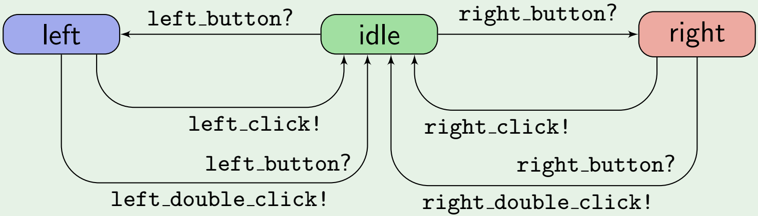

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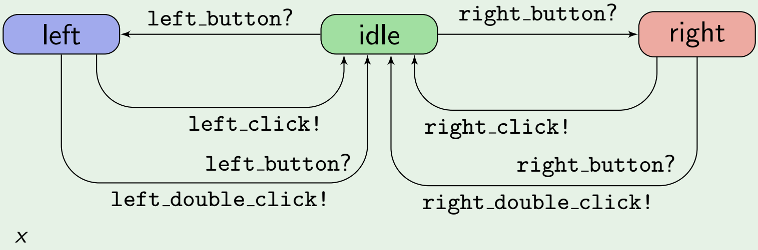

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Reasoning about real-time systems

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- timing constraints on states and transitions.

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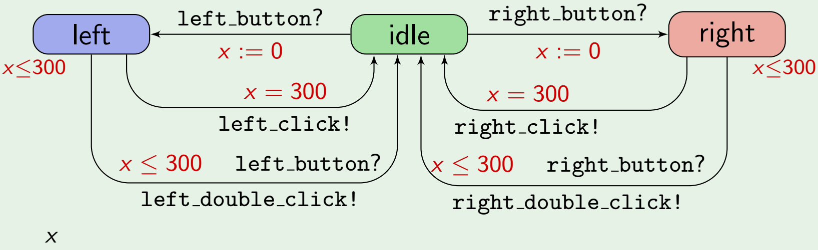

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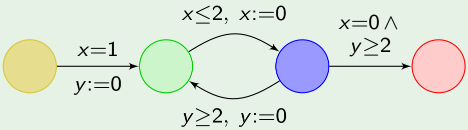
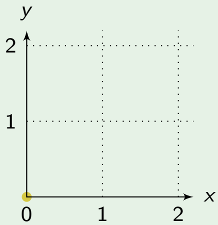
Continuous-time semantics

Example





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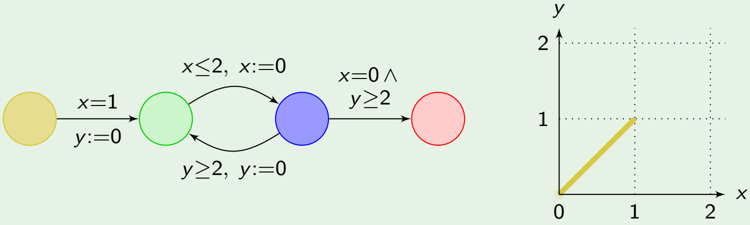

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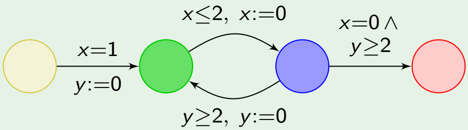

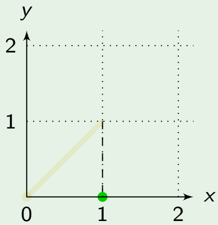

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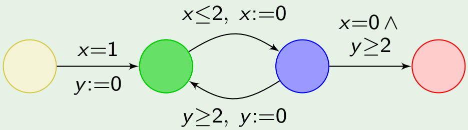

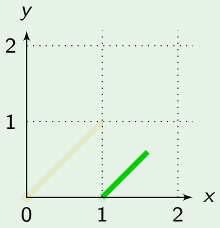

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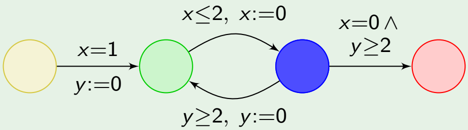

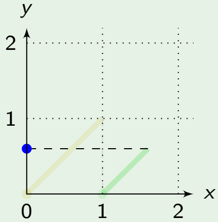

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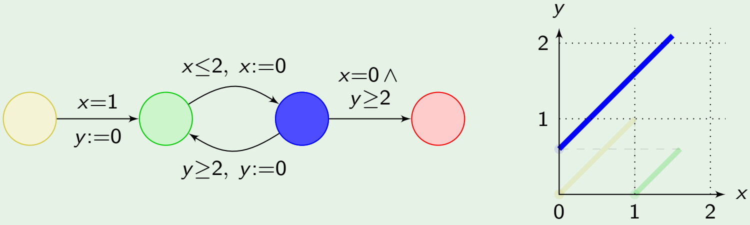

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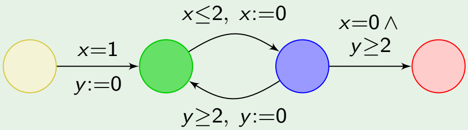

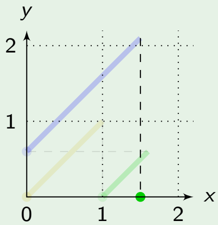

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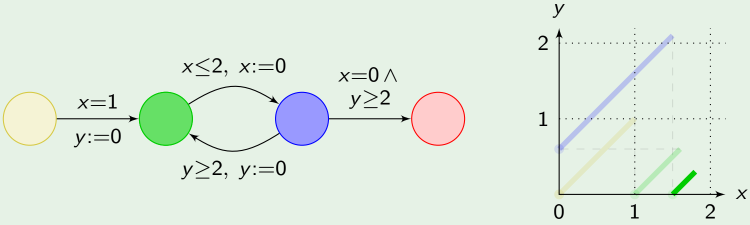

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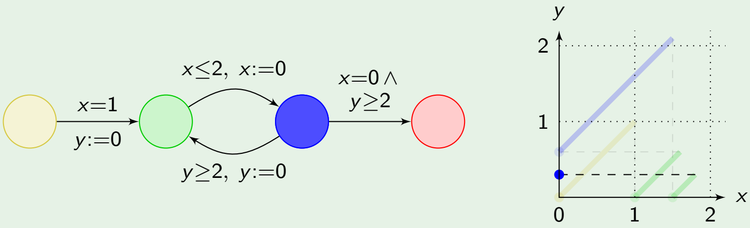

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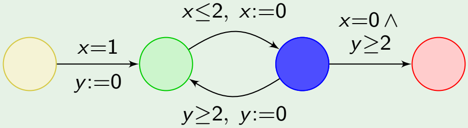

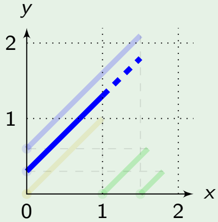

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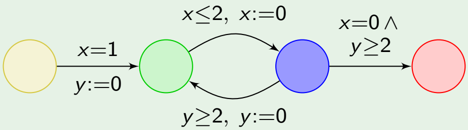

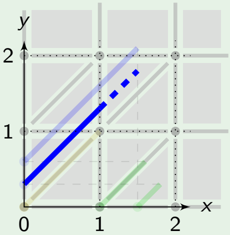

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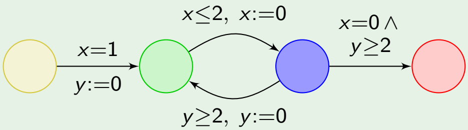
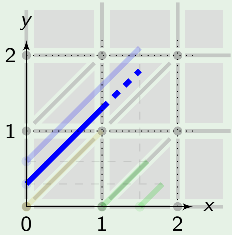




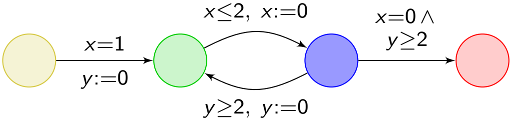
Theorem ([AD90,ACD93, ...])

Reachability in timed automata is decidable (as well as many other important properties).

[AD90] Alur, Dill. Automata For Modeling Real-Time Systems. ICALP, 1990.

[ACD93] Alur, Courcoubetis, Dill. Model-Checking in Dense Real-Time. Inf. & Comp., 1993.

Region automaton



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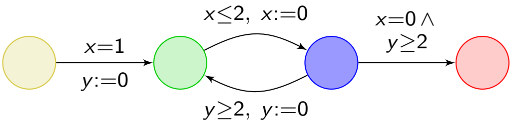

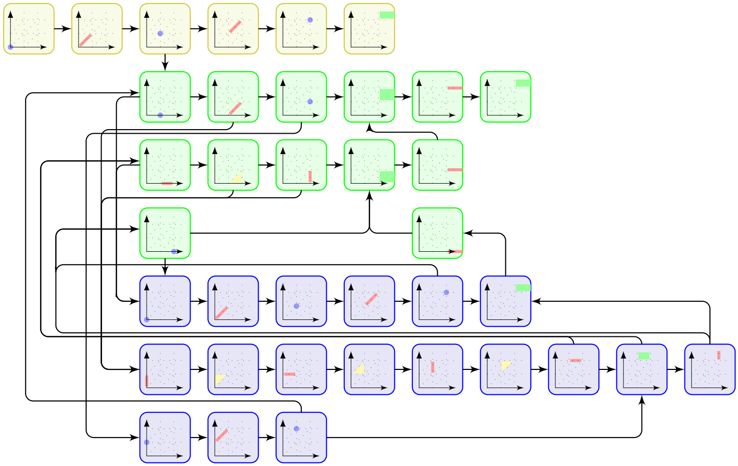

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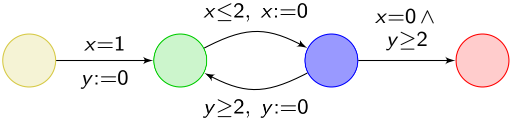
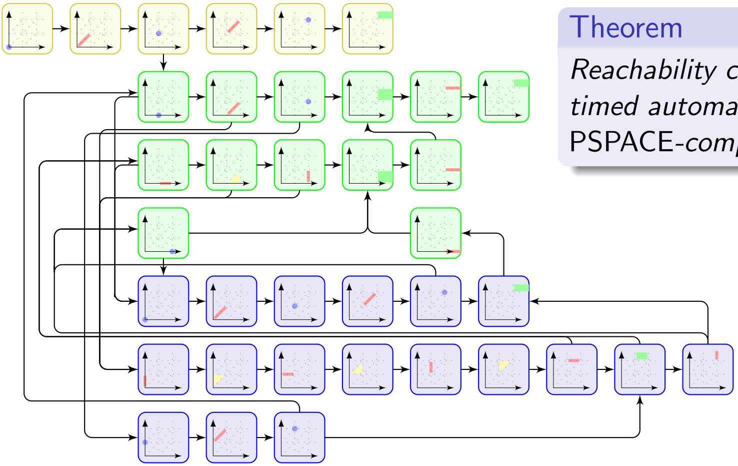




Theorem

Reachability checking in timed automata is PSPACE-*complete.*

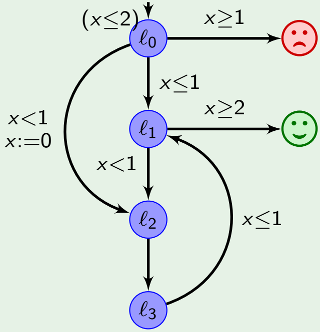
Timed games

Definition

A timed game is made of

- a timed automaton;

Example



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A timed game is made of

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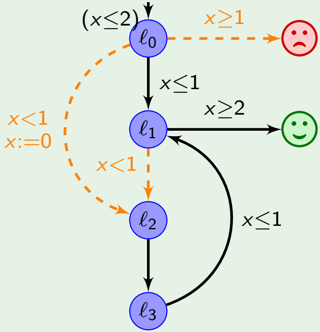

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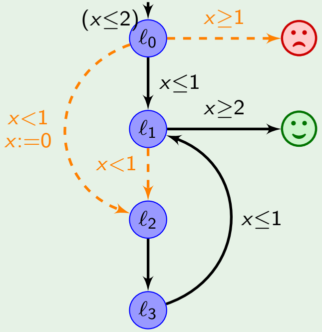


a memoryless strategy

in $(\ell_0, x=0)$: wait 0.5
goto ℓ_1

in (ℓ_1, x): wait until $x=2$
goto ☺

in $(\ell_2, x\leq 1)$: wait until $x=1$
goto ℓ_3

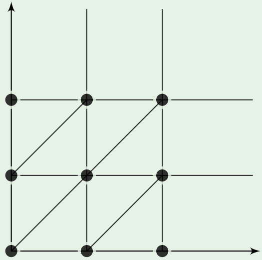
in $(\ell_3, x\leq 1)$: wait until $x=1$
goto ℓ_1

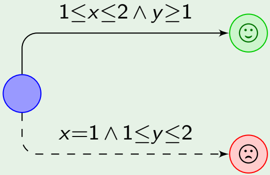
Timed games

Theorem ([AMPS98])

Deciding the winner in a timed game (e.g. for reachability objectives) is EXPTIME*-complete.*

Proof





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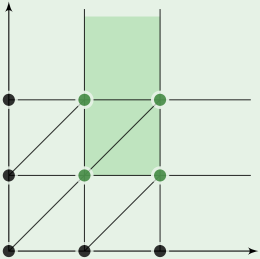

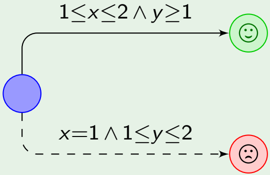

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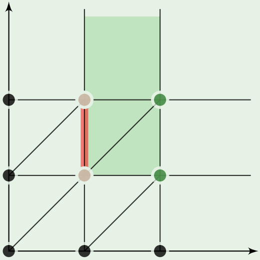

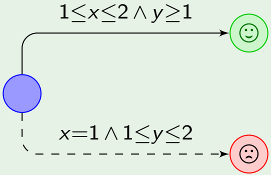

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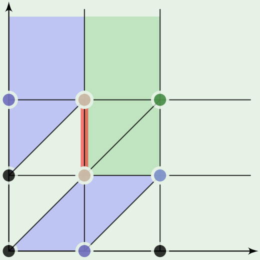

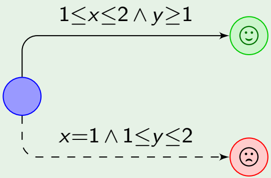

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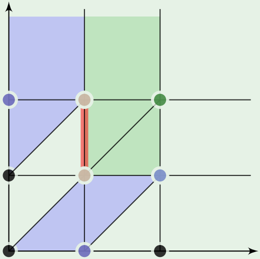

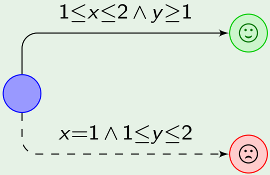

- regions are sufficient;
- the computation terminates.

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Outline of the talk

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Example: task graph scheduling

Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):

time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
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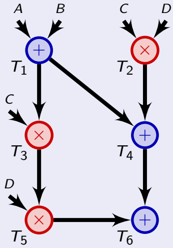

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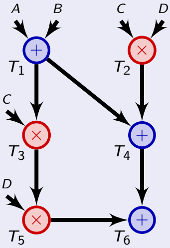

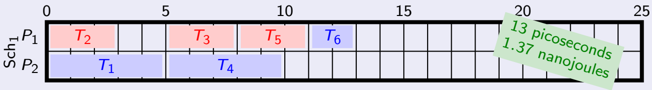

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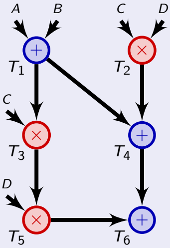

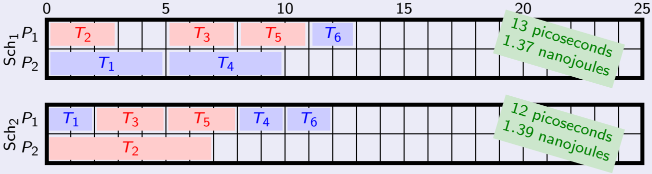

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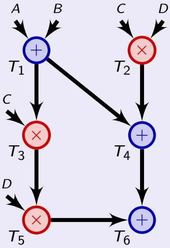

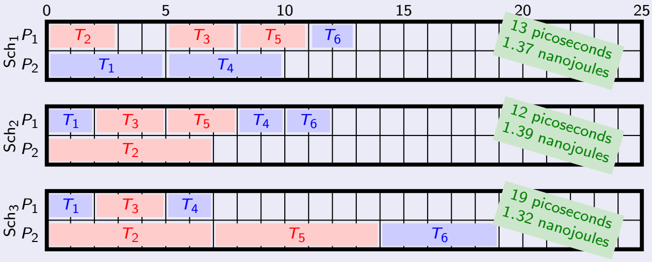

Linear hybrid automata

Definition

A linear hybrid automaton is made of

- a timed automaton;

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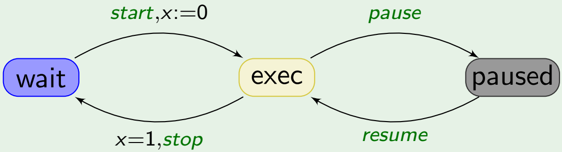

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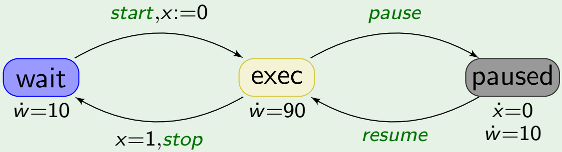

Reachability in linear hybrid automata

Theorem ([Čer92])

Reachability in linear hybrid automata is undecidable.

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Encode a two-counter machine using four stopwatches:

$$c_1 = a_1 - b_1 \qquad\qquad c_2 = a_2 - b_2$$

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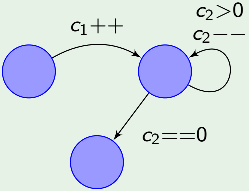

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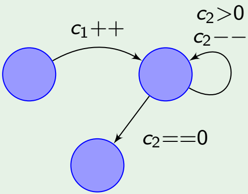

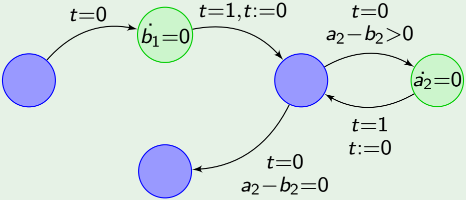

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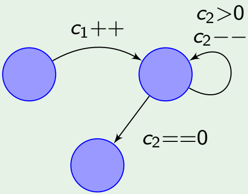
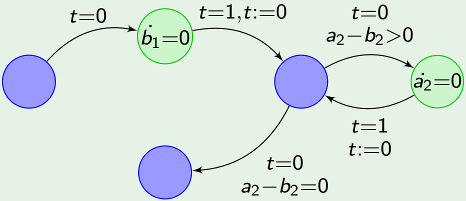




Already undecidable for one stopwatch and no diagonal constraints.

[Čer92] Čerāns. Algorithmic problems in Analysis of Real-Time System Specification. PhD Thesis, U. Latvia. 1992.

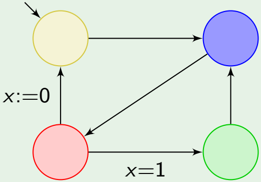
Priced timed automata

Definition ([KPSY99,ALP01,BFH$^+$01])

A priced timed automaton is made of

- a timed automaton;

Example



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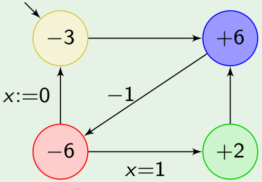

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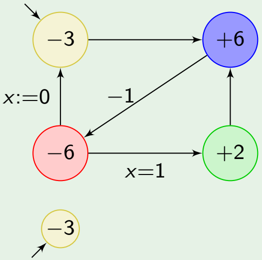

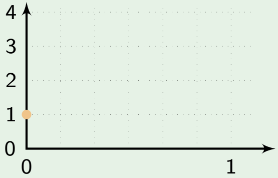

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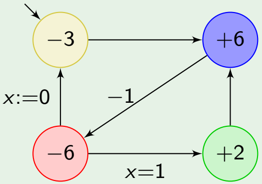

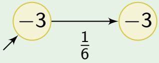

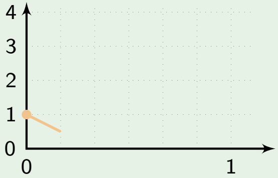

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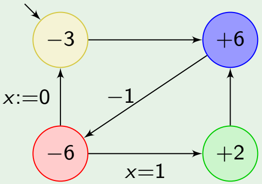

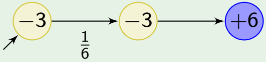

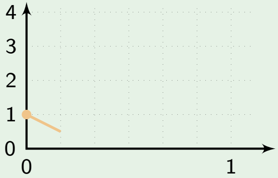

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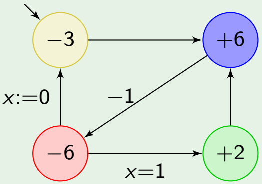

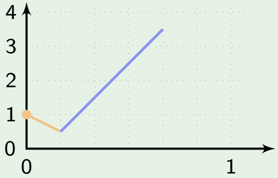

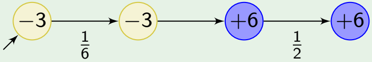

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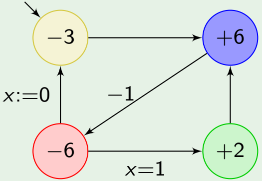

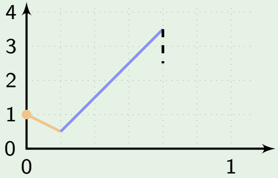

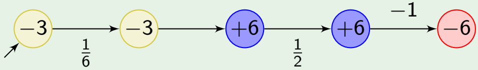

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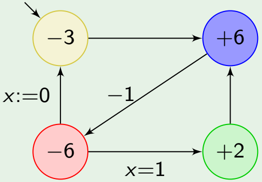

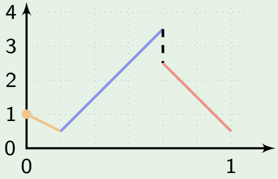

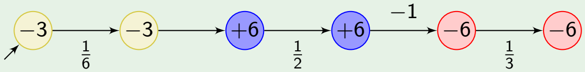

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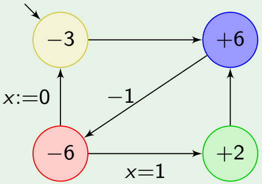

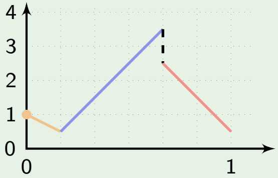

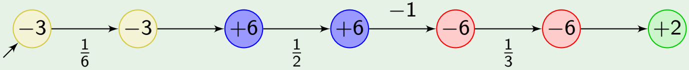

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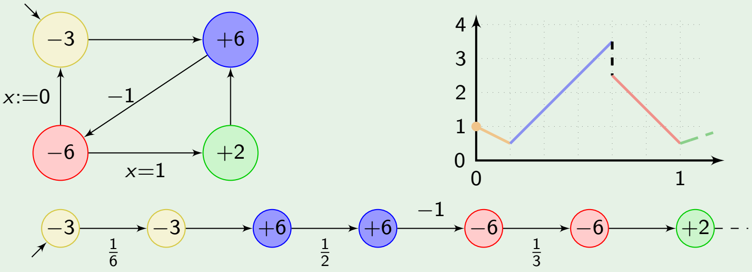

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Example: task graph scheduling

Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

P_1 (fast):

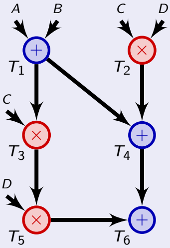
time	
$+$	2 picoseconds
$\times$	3 picoseconds

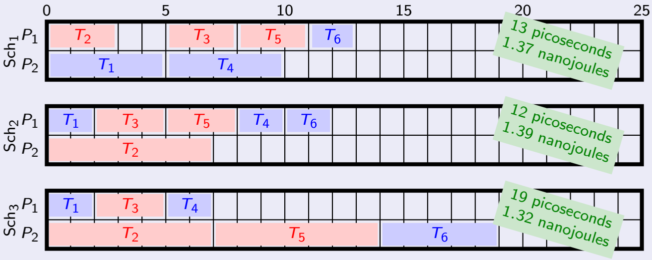
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):

time	
$+$	5 picoseconds
$\times$	7 picoseconds

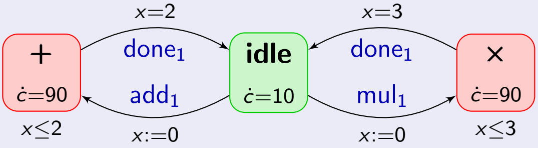
energy	
idle	20 Watts
in use	30 Watts





Modelling the task graph scheduling problem

Processors:



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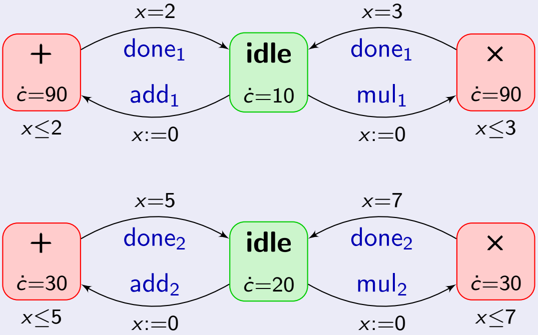

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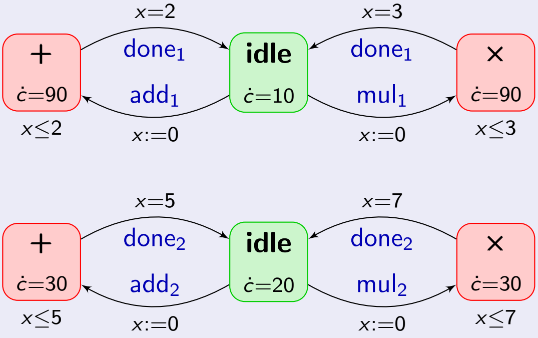
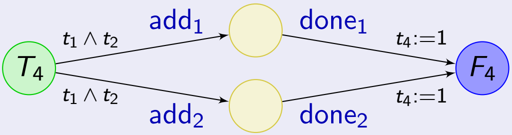


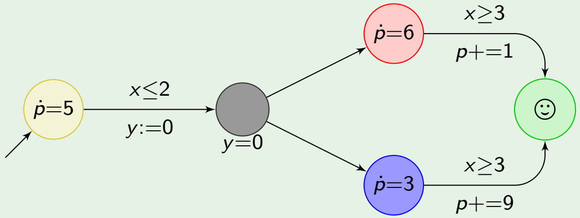
Tasks:



Outline of the talk

Cost-optimal reachability in priced timed automata

Example



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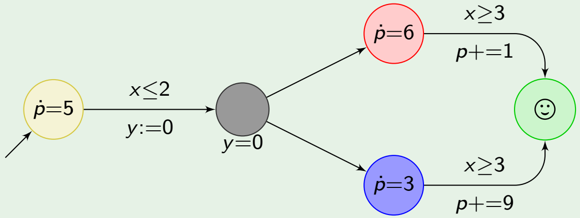

Minimal cost for reaching ☺:

Cost-optimal reachability in priced timed automata

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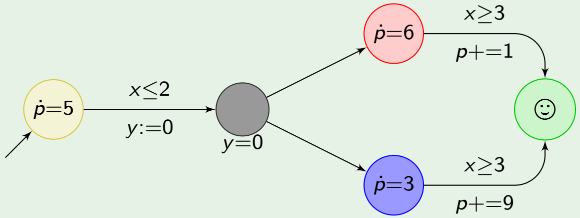
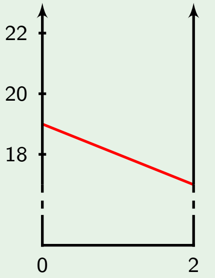


Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$



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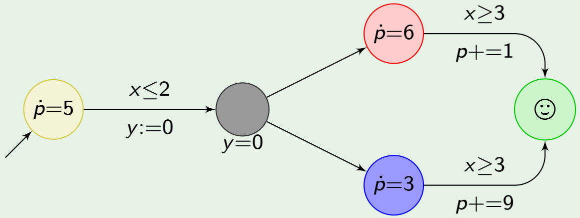
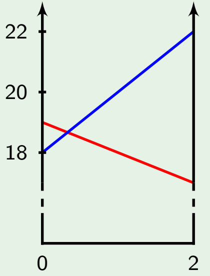


Minimal cost for reaching ☺:

$$5t + 6(3-t) + 1$$
$$5t + 3(3-t) + 9$$



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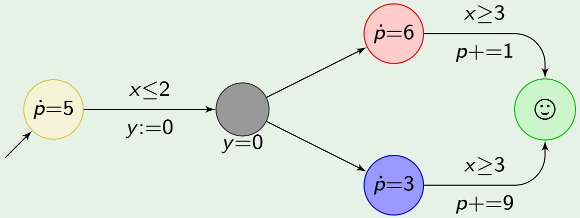
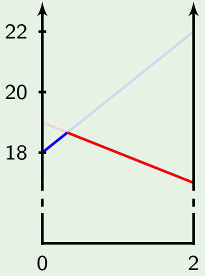


Minimal cost for reaching ☺:

$$\min \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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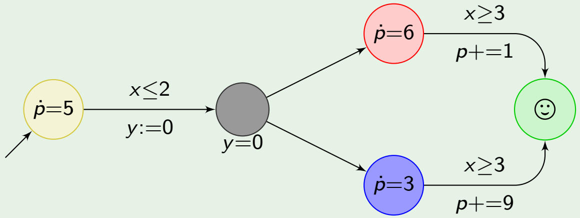
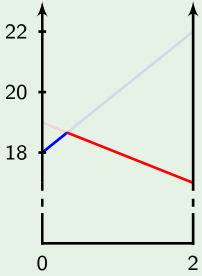


Minimal cost for reaching ☺:

$$\inf_{0 \leq t \leq 2} \min \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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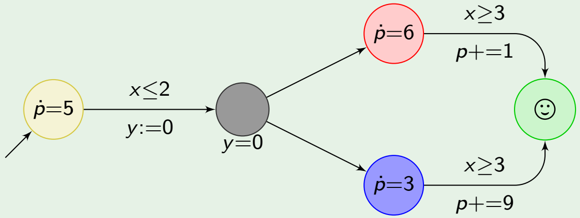
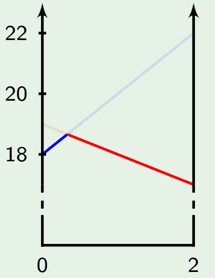


Minimal cost for reaching ☺:

$$\inf_{0\leq t\leq 2} \min \begin{pmatrix} 5t+6(3-t)+1 \\ 5t+3(3-t)+9 \end{pmatrix} = 17$$



Cost-optimal reachability in priced timed automata

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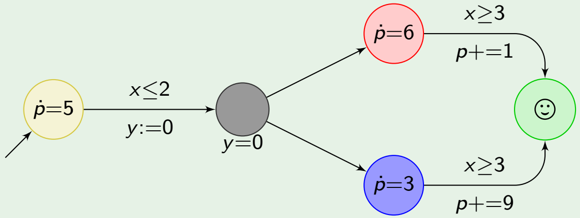

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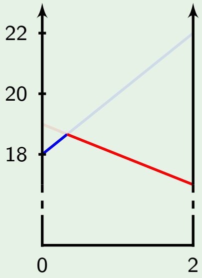

The optimal *schedule* consists in

- waiting 2 time units in ◯;
- going through ◯.

Cost-optimal reachability in priced timed automata

Theorem ([BBBR07])

Optimal reachability in priced timed automata is PSPACE*-complete.*

[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem. FMSD, 2007.

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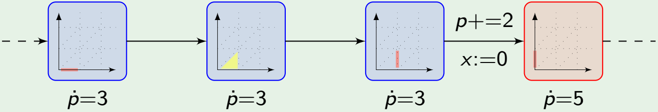
Proof

- Regions are not precise enough;



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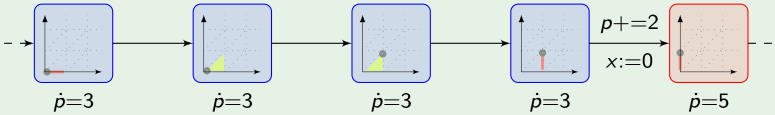

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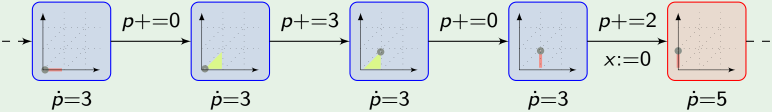

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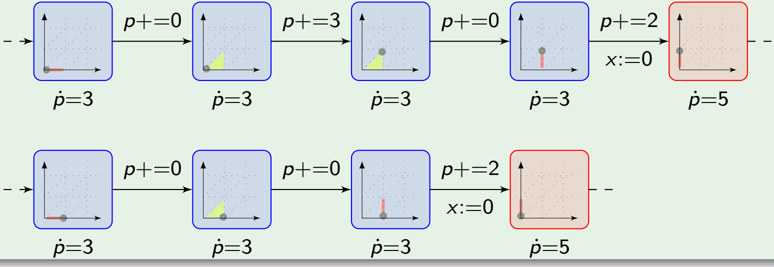

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Outline of the talk

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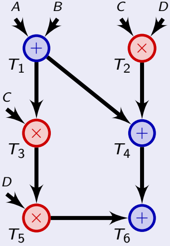
$\times$	3 picoseconds

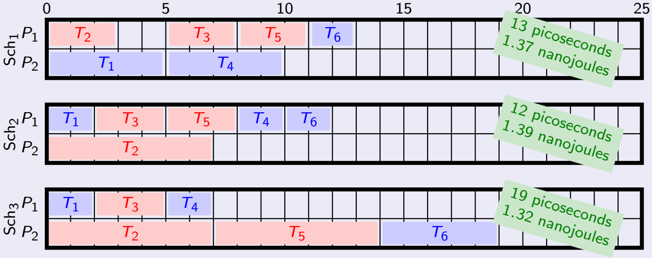
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):

time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts

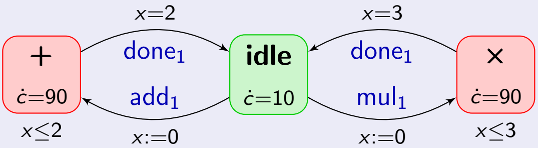




Cost-optimal reachability in priced timed games

Using games to model uncertainty over delays

Processors with exact delays:



Cost-optimal reachability in priced timed games

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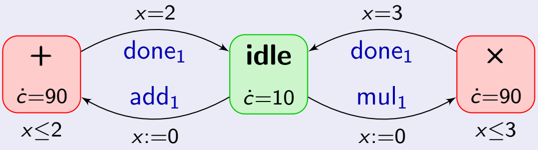
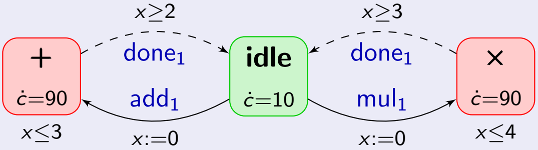
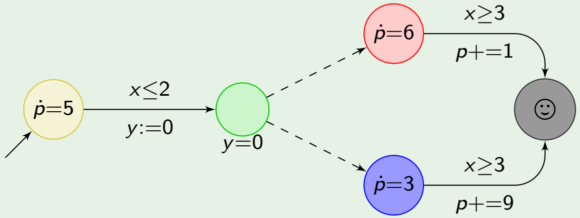


Processors with approximate delays:



Cost-optimal reachability in priced timed games

Example



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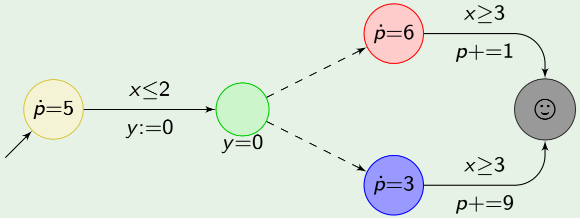
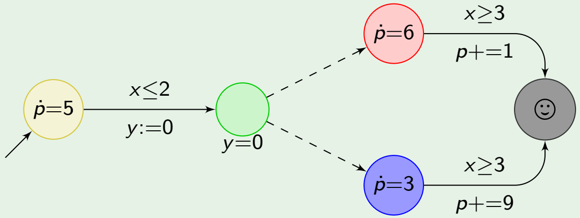


Minimal cost for reaching ☺:

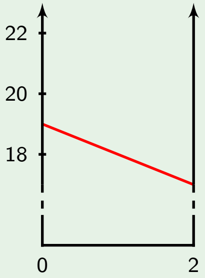
Cost-optimal reachability in priced timed games

Example



Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$



Cost-optimal reachability in priced timed games

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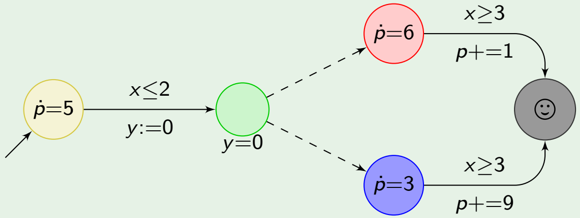
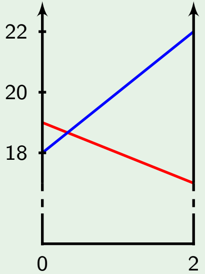


Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$
$$5t + 3(3 - t) + 9$$



Cost-optimal reachability in priced timed games

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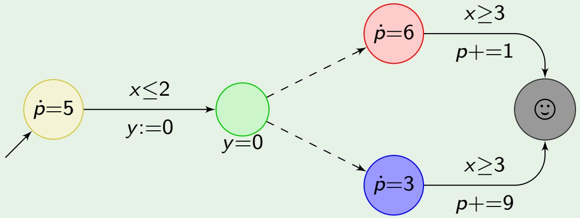
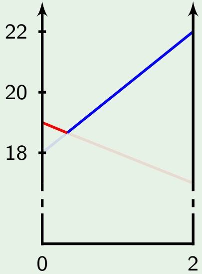


Minimal cost for reaching ☺:

$$\max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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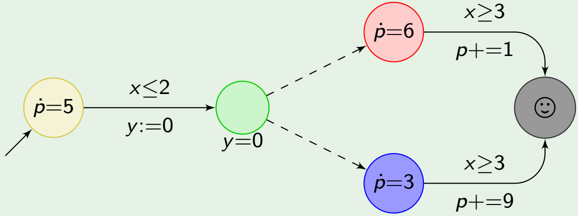
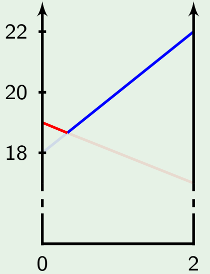


Minimal cost for reaching ☺:

$$\inf_{0 \leq t \leq 2} \max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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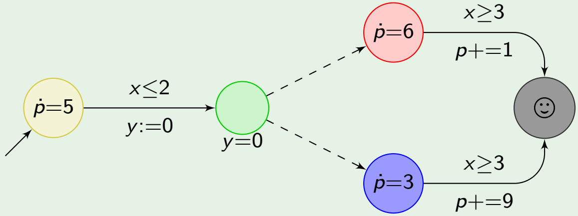

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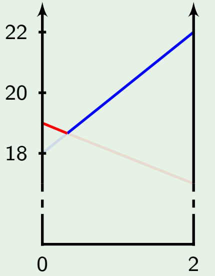
$$\inf_{0\leq t\leq 2} \max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix} = 18.66$$



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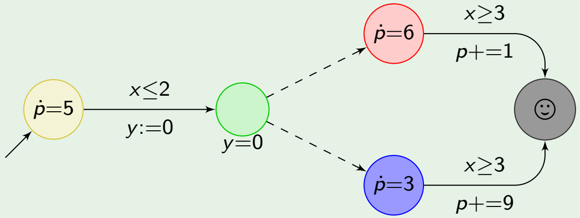

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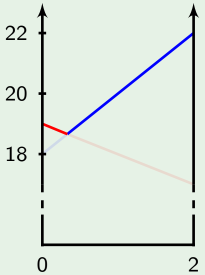
$$\text{(with } t_{\text{opt}} = \frac{1}{3} \text{)}$$



Cost-optimal reachability in priced timed games

Theorem ([BBR05,BBM06])

Optimal reachability in priced timed games is undecidable.

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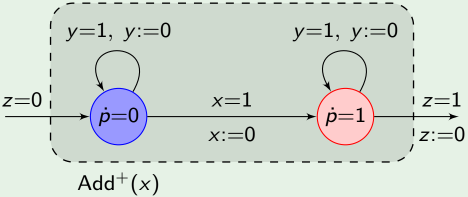
Encode a two-counter machine as a priced timed game.

- add the value of clock x to the accumulated cost



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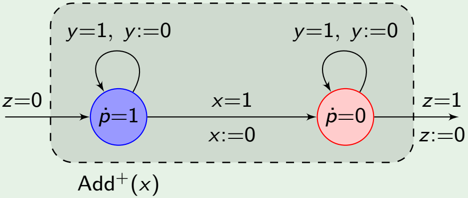
- add the value of clock x to the accumulated cost
- add $1 - x$ to the accumulated cost



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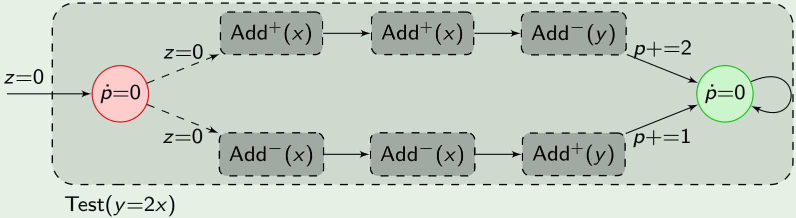
- check that $y = 2x$



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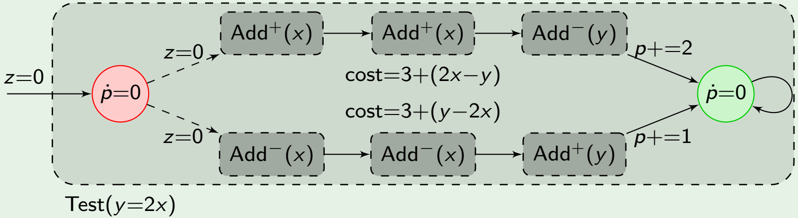

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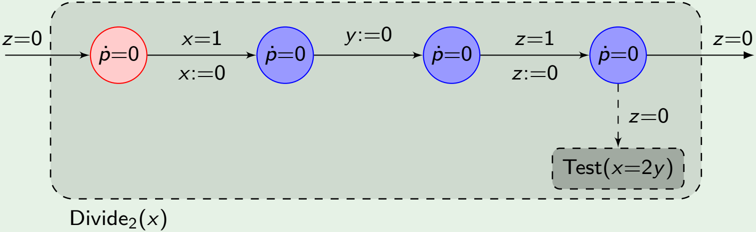
- divide clock x by 2



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$\rightsquigarrow$ We can use the following encoding:

$$x_1 = \frac{1}{2^{c_1}} \qquad x_2 = \frac{1}{2^{c_2}}$$

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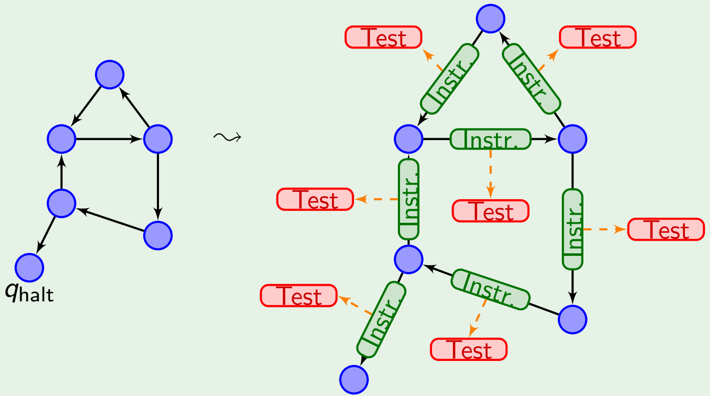

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Encode a two-counter machine as a priced timed game.

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if, and only if,
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Wouldn't almost-optimal strategies be sufficient?

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Optimal cost in a priced timed game:

$\mathsf{optcost}_{\mathcal{G}} = \inf\{\mathsf{cost}(\sigma) \mid \sigma \text{ winning strategy in } \mathcal{G}\}$

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Optimal cost in a priced timed game:

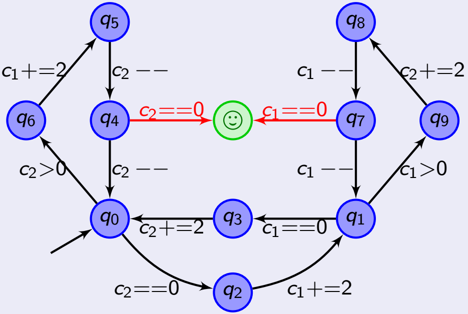
$\mathrm{optcost}_{\mathcal{G}} = \inf\{\mathrm{cost}(\sigma) \mid \sigma \text{ winning strategy in } \mathcal{G}\}$

The existence of a strategy with cost less than k is undecidable.

What about deciding if $\mathrm{optcost}_{\mathcal{G}} \leq k$?

Undecidability of the value problem

Trying to reuse the previous reduction...



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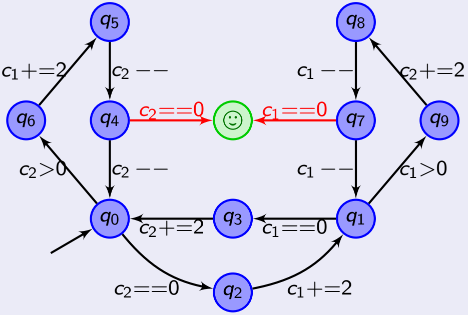

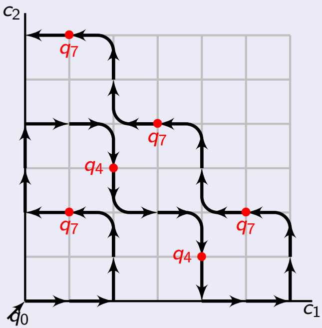

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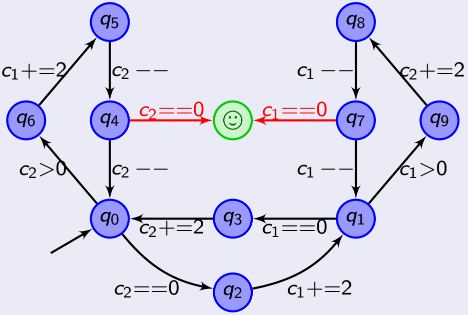

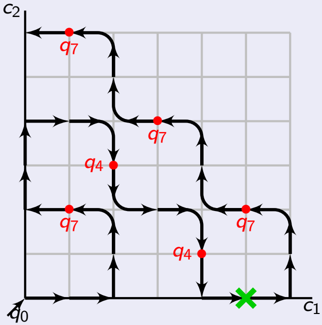

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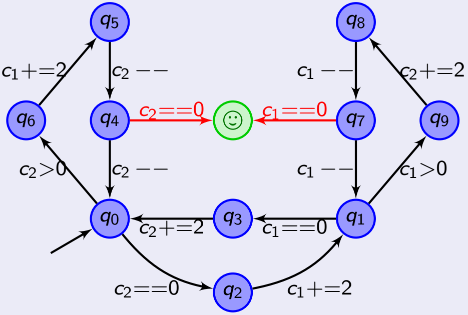

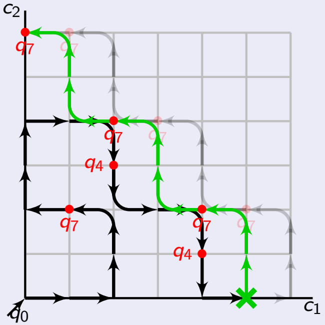

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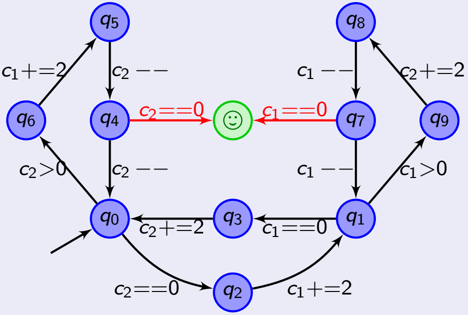

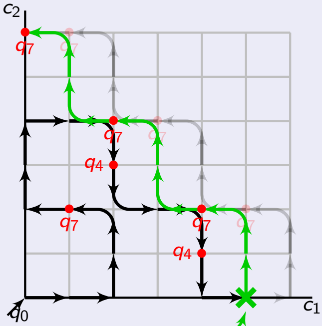

final cost: $3+\left|2\cdot\frac{1}{2^5}-\frac{1}{2^5}\right|$

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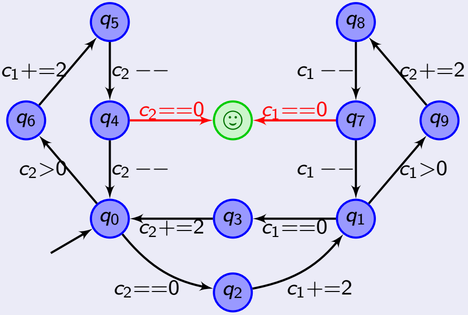
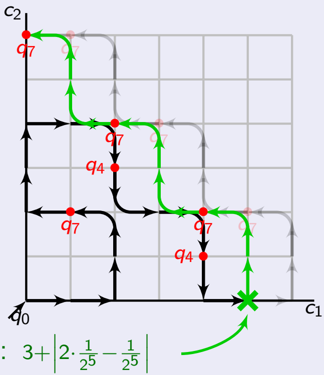
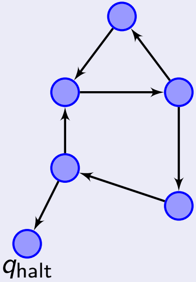




The value of the game is 3, but there is no optimal strategy...

Undecidability of the value problem

Adapting the previous reduction...



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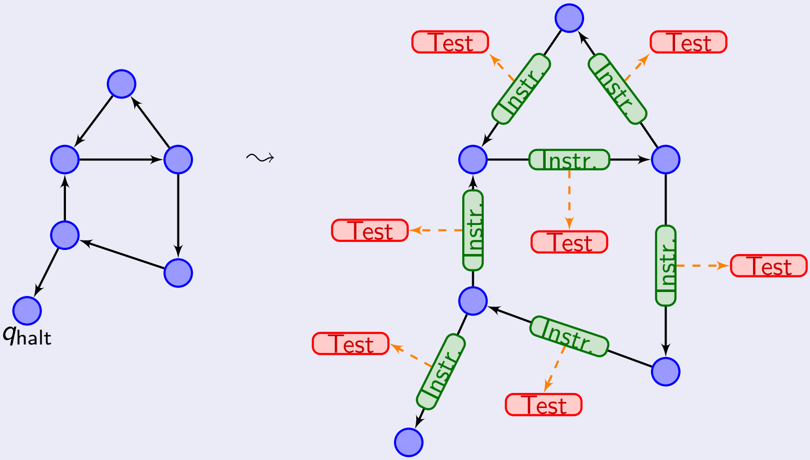

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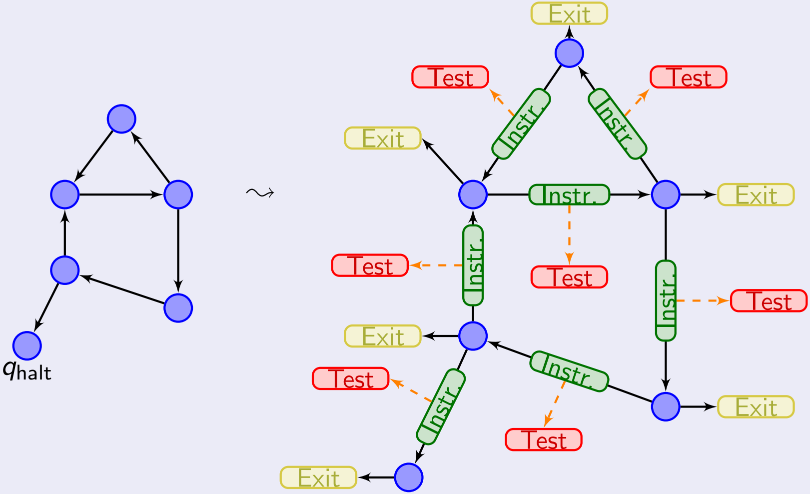

exit nodes: cost $3 + \frac{1}{2^n}$
(n = length of path)

Undecidability of the value problem

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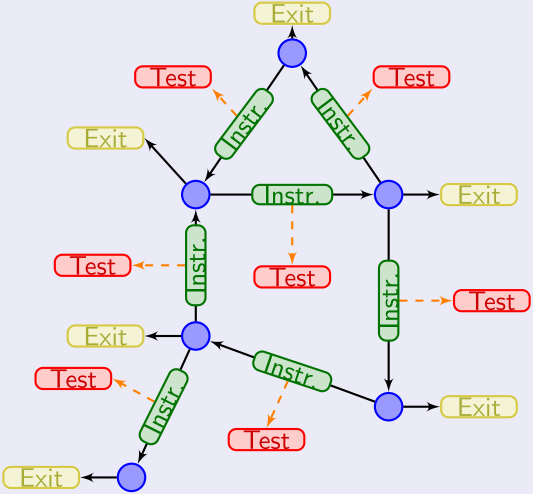

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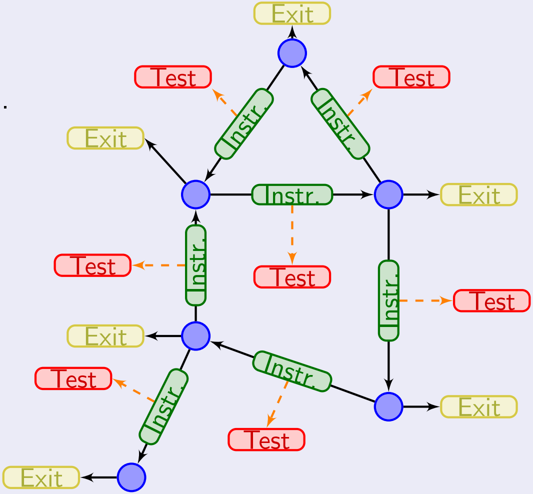
  $\rightsquigarrow$ $\text{cost}(\sigma) \leq 3 + \epsilon$



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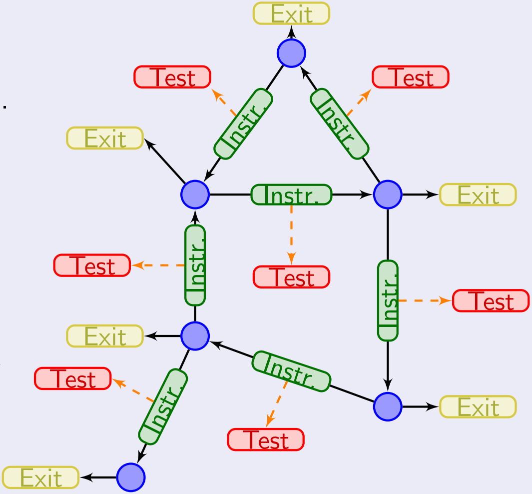
- **if $\mathcal{M}$ halts:** correct simulation for finite duration.

  $\rightsquigarrow \mathrm{cost}(\sigma) \geq 3 + \alpha_{\mathcal{M}}$ for all σ



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Undecidability of the value problem

Theorem ([BJM15])

The value problem is undecidable in priced timed games.

[BJM15] Bouyer, Jaziri, Markey. On the Value Problem in Weighted Timed Games. CONCUR, 2015.

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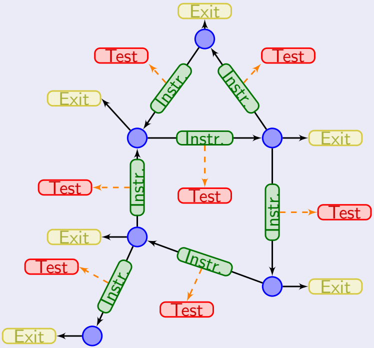
Remark

- blue nodes and intermediary instruction modules have cost zero everywhere;
- positive weights only occur in acyclic parts.



Approximation of the optimal cost

Definition

A priced timed game $\mathcal{G}$ is almost-strongly non-Zeno if there exists $\kappa > 0$ for any run ρ that starts and ends in the same region:

$$\mathsf{cost}(\rho) \geq \kappa \qquad \text{or} \qquad \mathsf{cost}(\rho) = 0$$

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The optimal cost of almost-strongly non-Zeno priced timed automata can be approximated: for every $\epsilon > 0$, we can compute

- *values v_ϵ^+ and v_ϵ^- such that*

$$|v_\epsilon^+ - v_\epsilon^-| < \epsilon \qquad v_\epsilon^- \leq optcost_{\mathcal{G}} \leq v_\epsilon^+$$

- *a strategy σ_ϵ such that*

$$optcost_{\mathcal{G}} \leq cost(\sigma_\epsilon) \leq optcost_{\mathcal{G}} + \epsilon.$$

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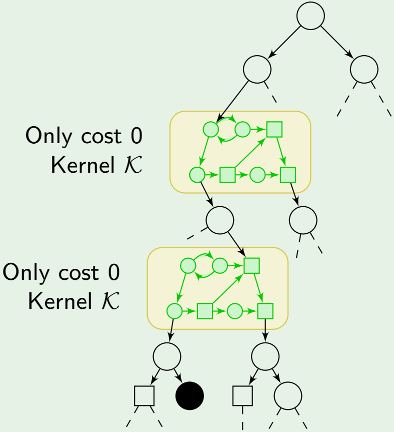
Approximation of the optimal cost

Proof

- **semi-unfolding** of region automaton (seen as a timed game)



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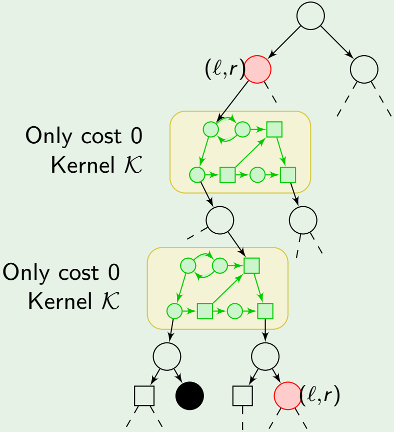

Approximation of the optimal cost

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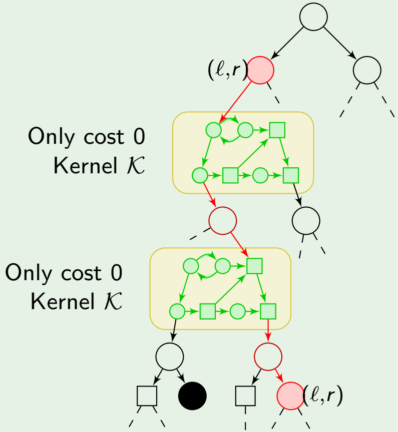

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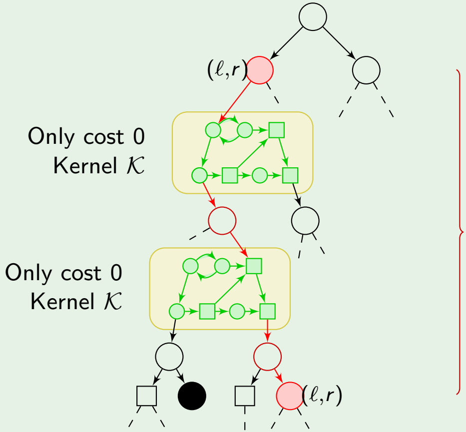

Hypothesis:

cost > 0

$\downarrow$

cost $\geq \kappa$

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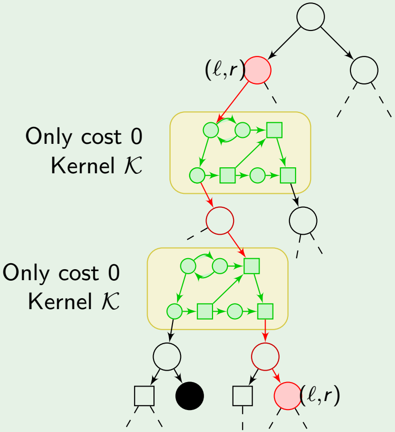

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$$\text{cost} > 0$$

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$\rightsquigarrow$ bounded depth

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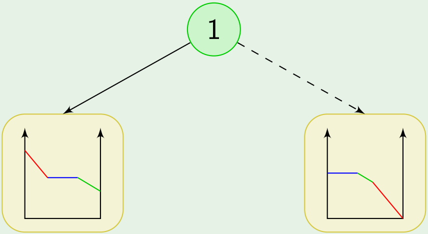
- **semi-unfolding** of region automaton (seen as a timed game)
- compute **exact** optimal cost in tree-like parts [LMM02,ABM04]



[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-Reachability and Control for Acyclic WTA. TCS, 2002
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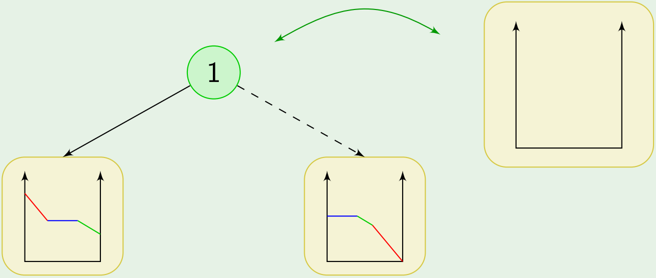

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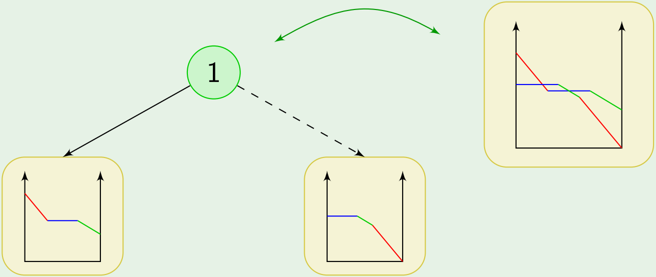

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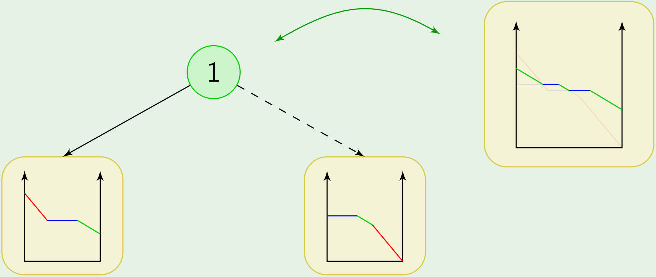

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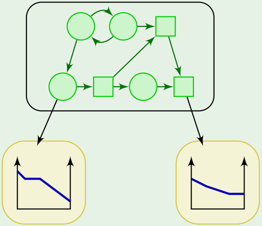
Approximation of the optimal cost

Proof

- **semi-unfolding** of region automaton (seen as a timed game)
- compute **exact** optimal cost in tree-like parts [LMM02,ABM04]
- compute **approximate** optimal cost in kernels



Output cost functions f

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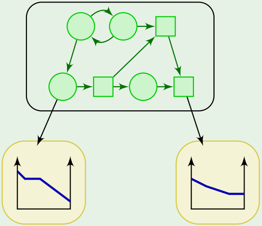
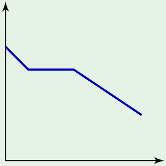


Output cost functions f

Under- and over-approximate by piecewise constant functions f_ϵ^- and f_ϵ^+



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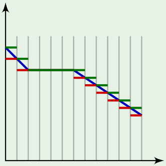

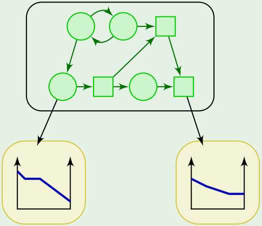

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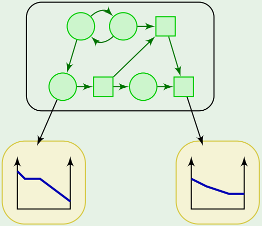

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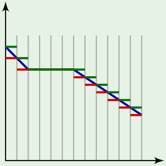

$\rightsquigarrow$ reachability timed game in small regions

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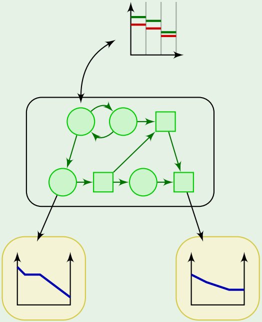

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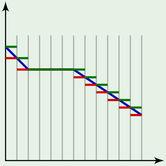

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Outline of the talk

Conclusions and future directions

Priced timed automata and games

- convenient for modelling resources;
- 1-player setting remains tractable (sort of);
- 2-player setting undecidable, but approximable.
- approximation algorithms are a convenient trade-off.

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- convenient for modelling resources;
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- 2-player setting undecidable, but approximable.
- approximation algorithms are a convenient trade-off.

Future work

- improve approximation technique (in terms of complexity);
- extend results to whole class of priced timed games;
- average energy and energy constraints;
- robust analysis of priced timed games;
- develop a tool.

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