ATL with strategy contexts

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LSV – ENS Cachan

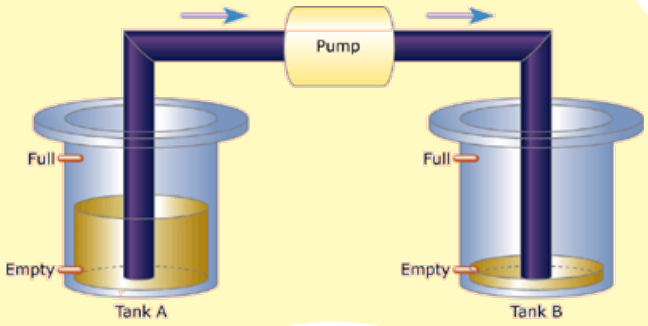
(based on joint works with Thomas Brihaye,
Arnaud Da Costa-Lopes, François Laroussinie)

Laboratoire d'Algorithmique, Complexité et Logique

Créteil, November 9, 2015

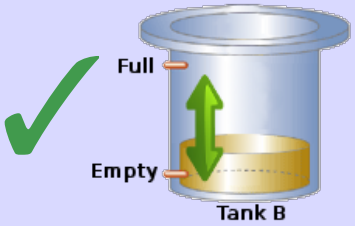
Model checking and synthesis

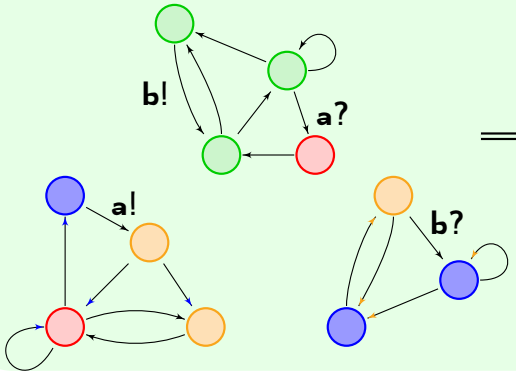
system:



[http://www.embedded.com]

property

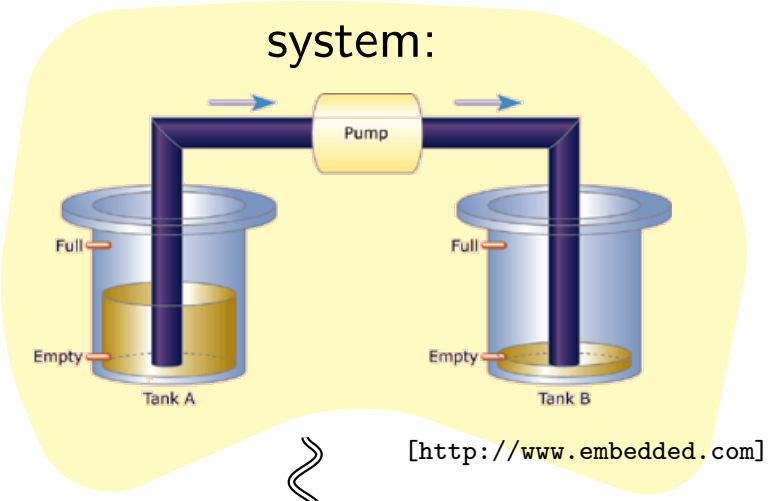


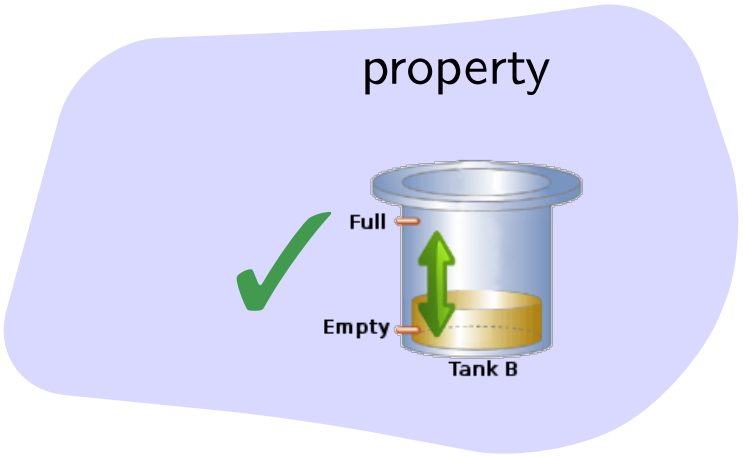


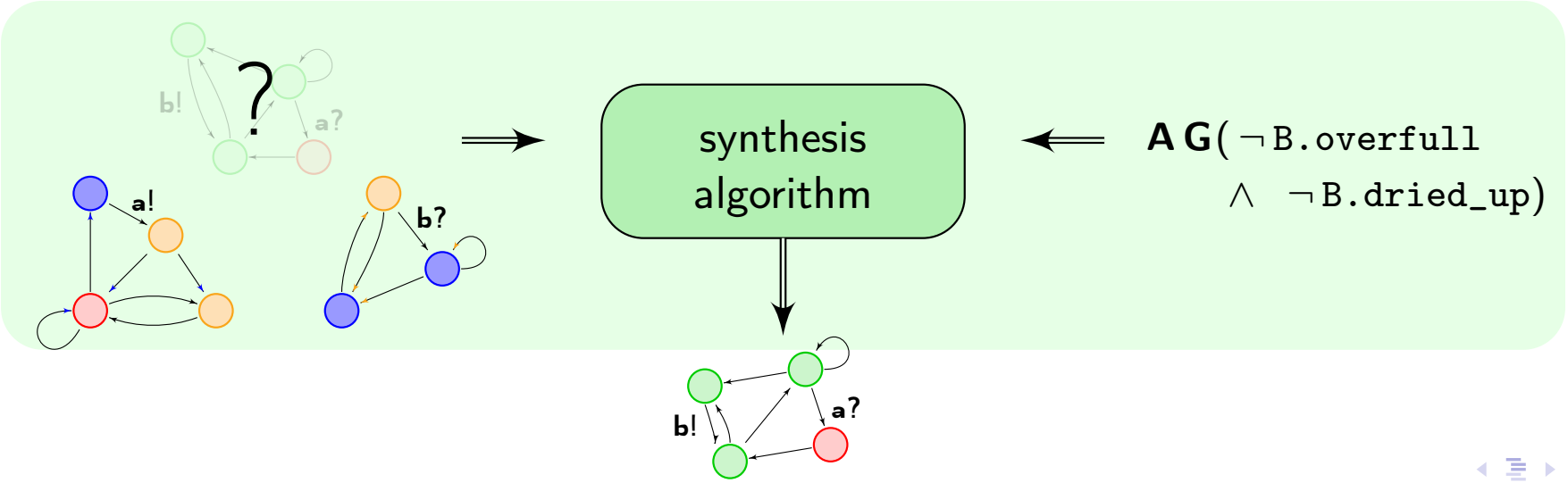
$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ $\mathbf{A}\,\mathbf{G}(\neg\,\texttt{B.overfull} \wedge \neg\,\texttt{B.dried\_up})$

yes/no

Model checking and synthesis







Outline of the presentation

1. Introduction
2. Basics of CTL and ATL
   - expressing properties of reactive systems
   - efficient verification algorithms
3. Temporal logics for multi-agent systems
   - specifying properties of complex interacting systems
   - expressive power of ATL_{sc}
   - translation into Quantified CTL (QCTL)
   - algorithms for ATL_{sc}
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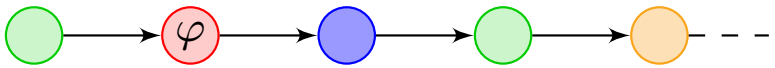
$\mathbf{X}\,\varphi$ 

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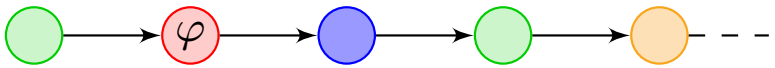

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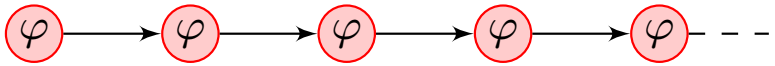

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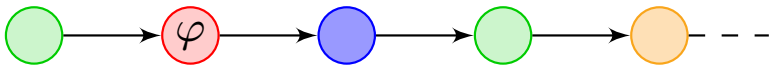

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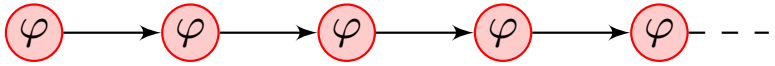
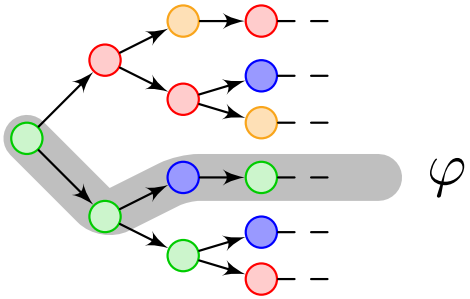


"always φ"

- **path quantifiers:**

$\mathbf{E}\varphi$



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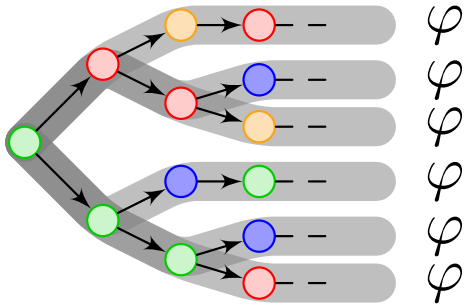

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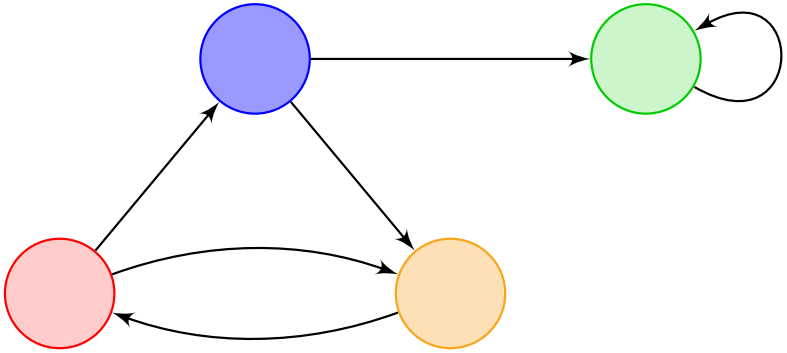

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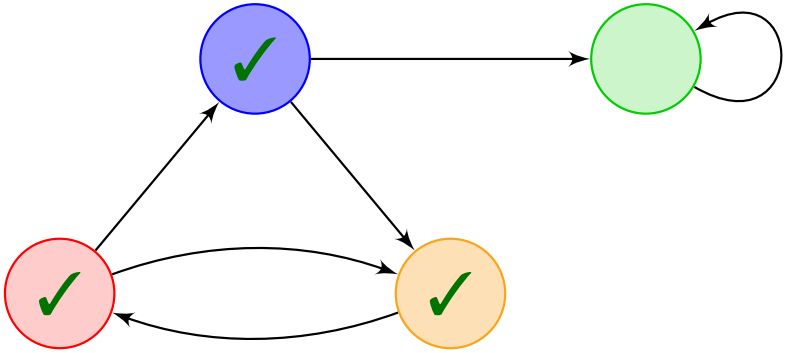

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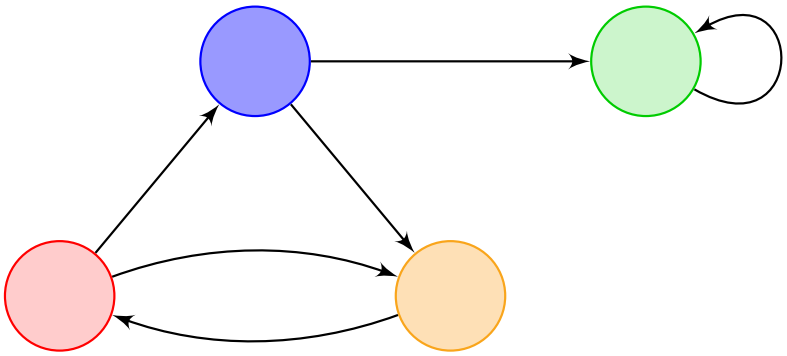

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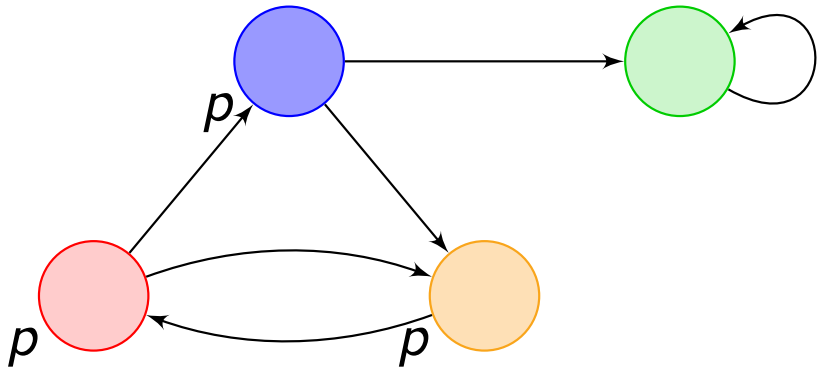

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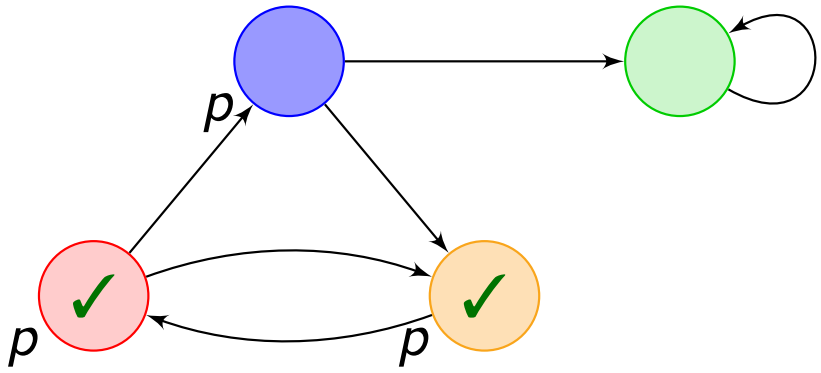

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Theorem ([KVW94])

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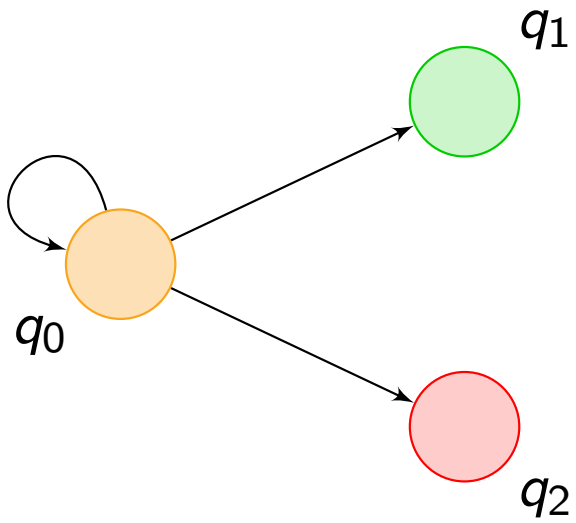
Reasoning about open systems

Reasoning about open systems

Concurrent games

A concurrent game is made of

- a transition system;



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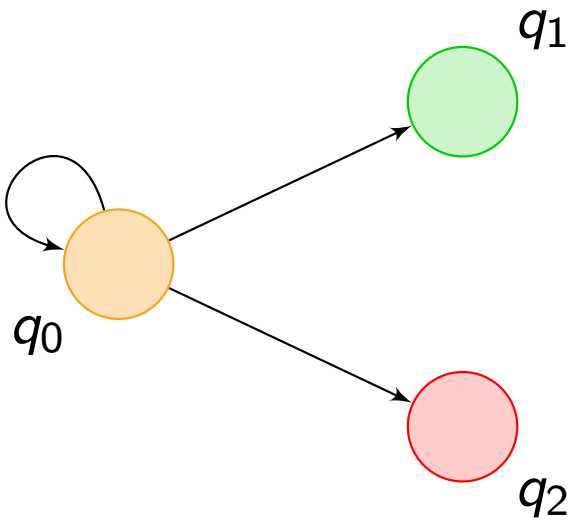

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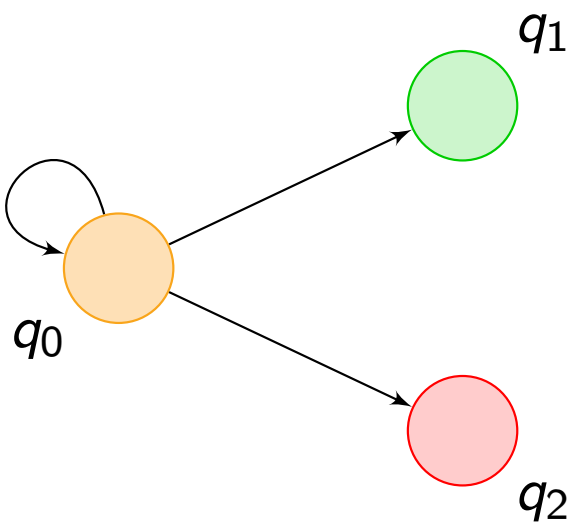

		player 1		
				
player 2		q_0	q_2	q_1
		q_1	q_0	q_2
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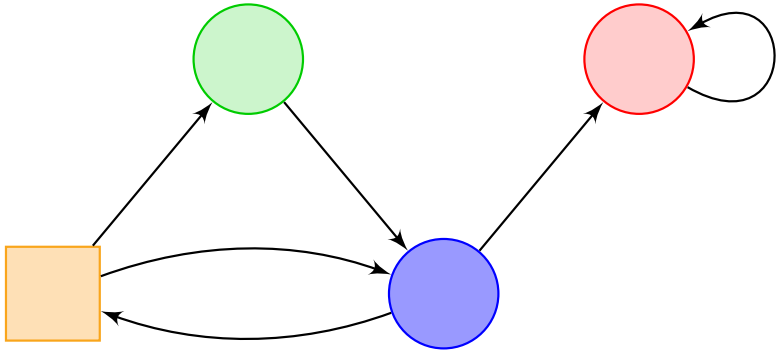
Turn-based games

A turn-based game is a game where only one agent plays at a time.



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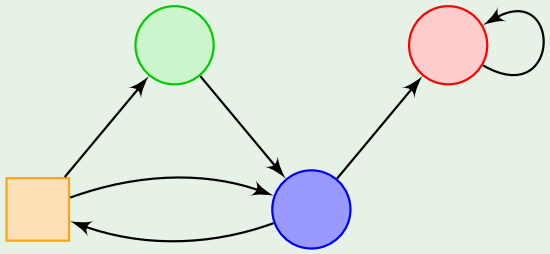

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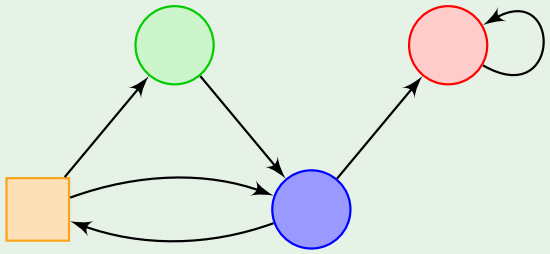
Strategy for player □



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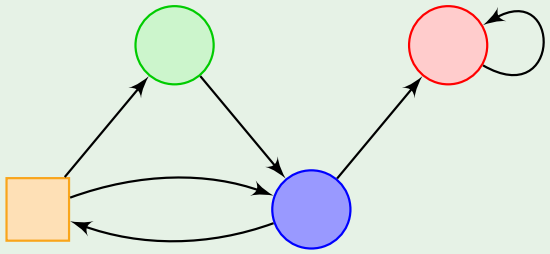

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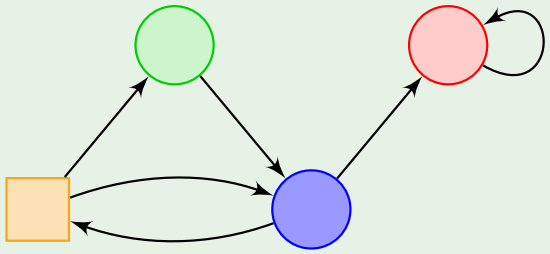

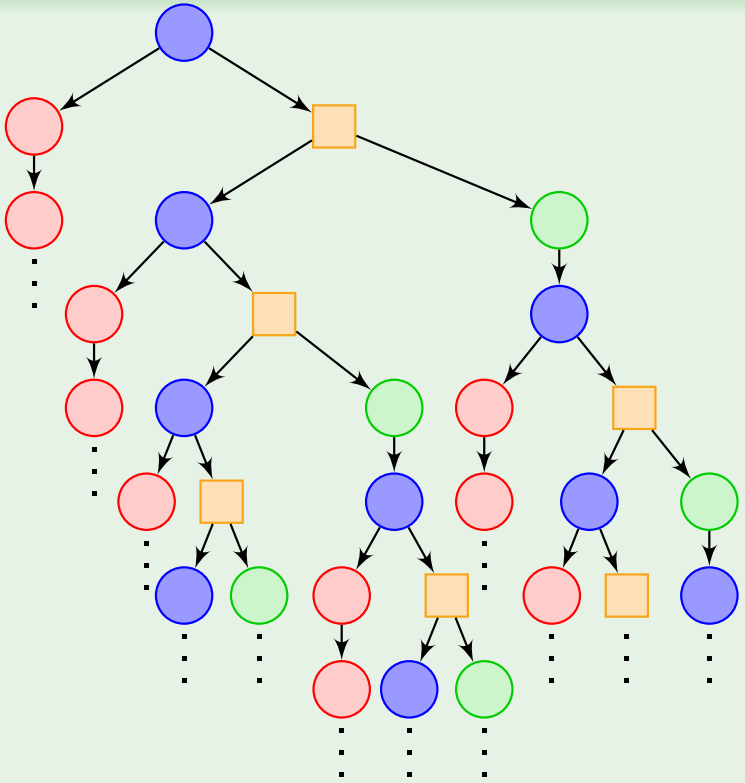

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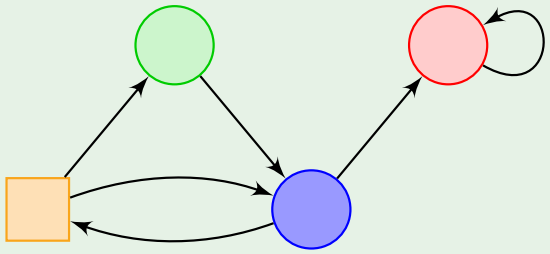

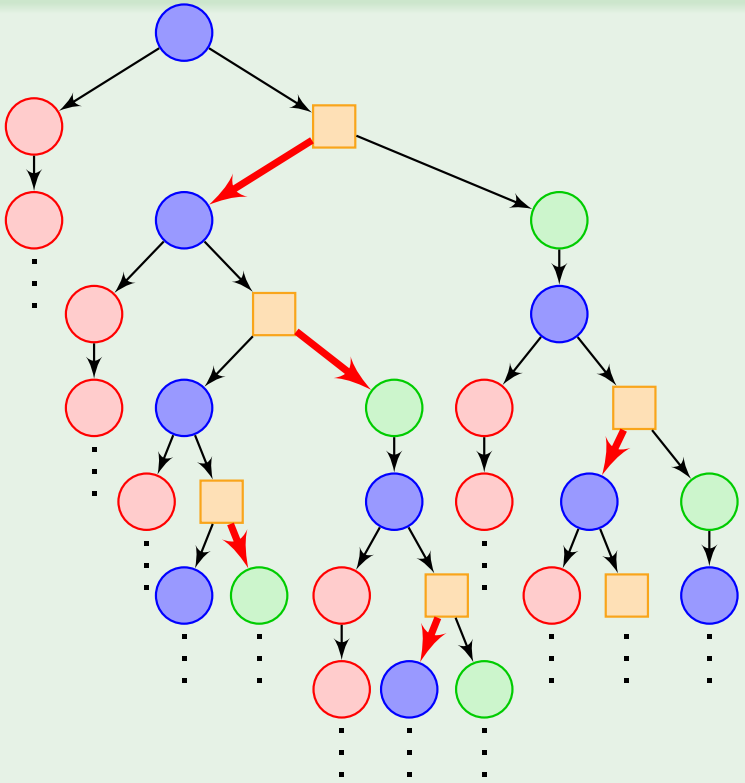

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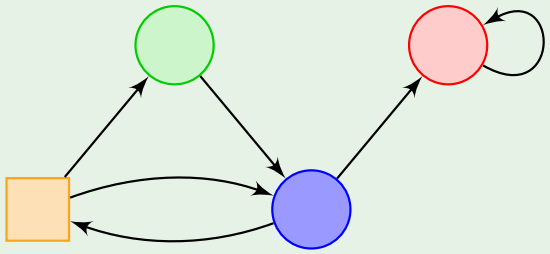

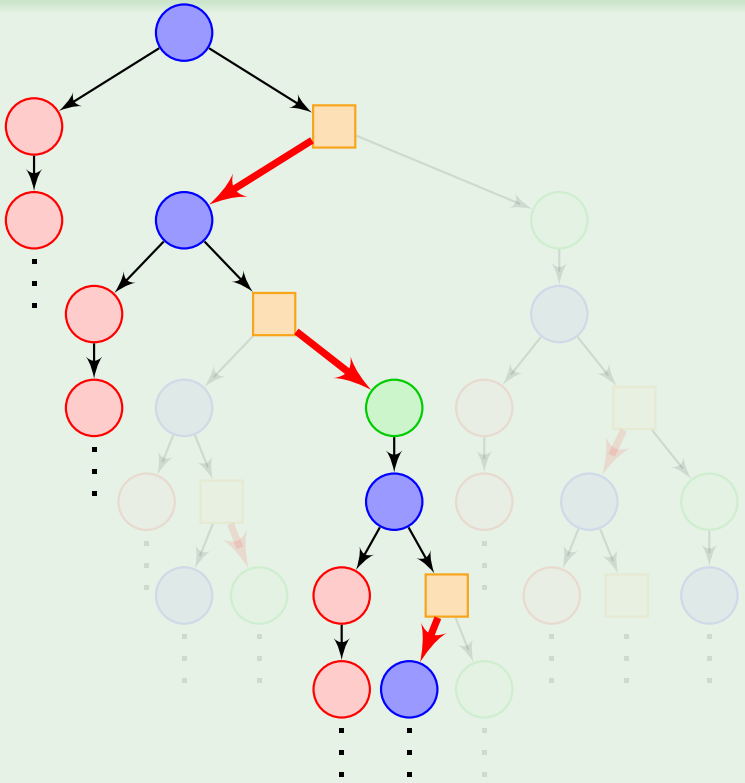

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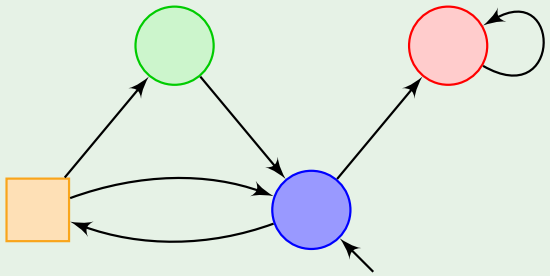

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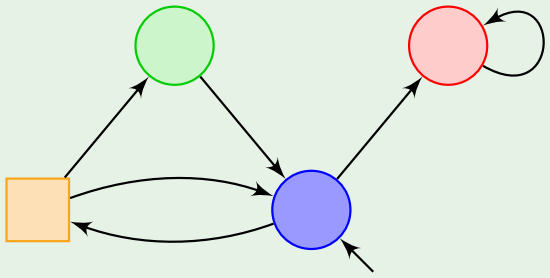

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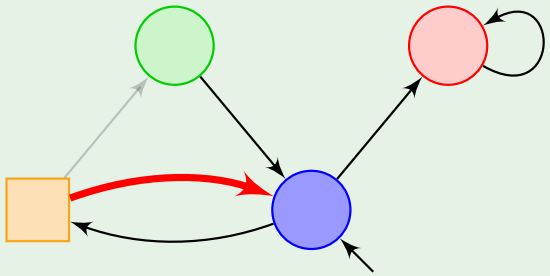

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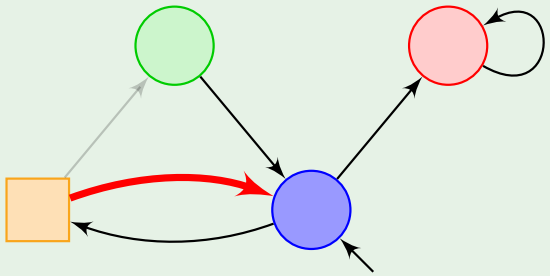

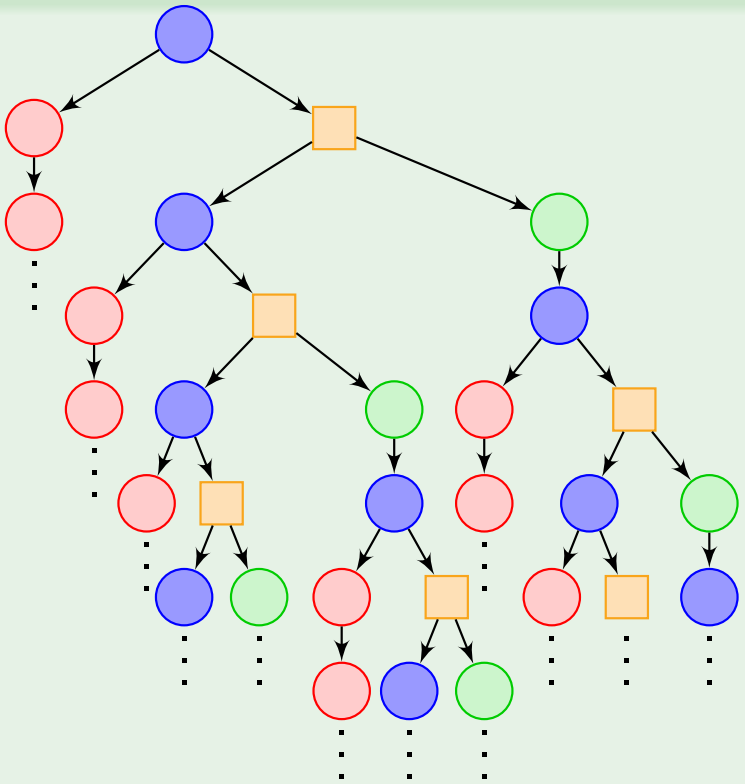

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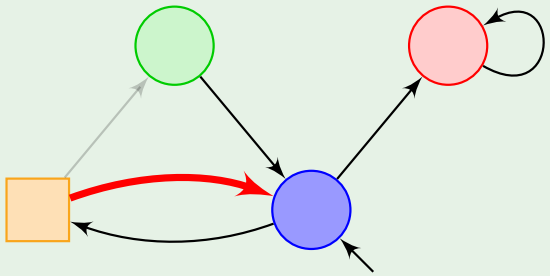

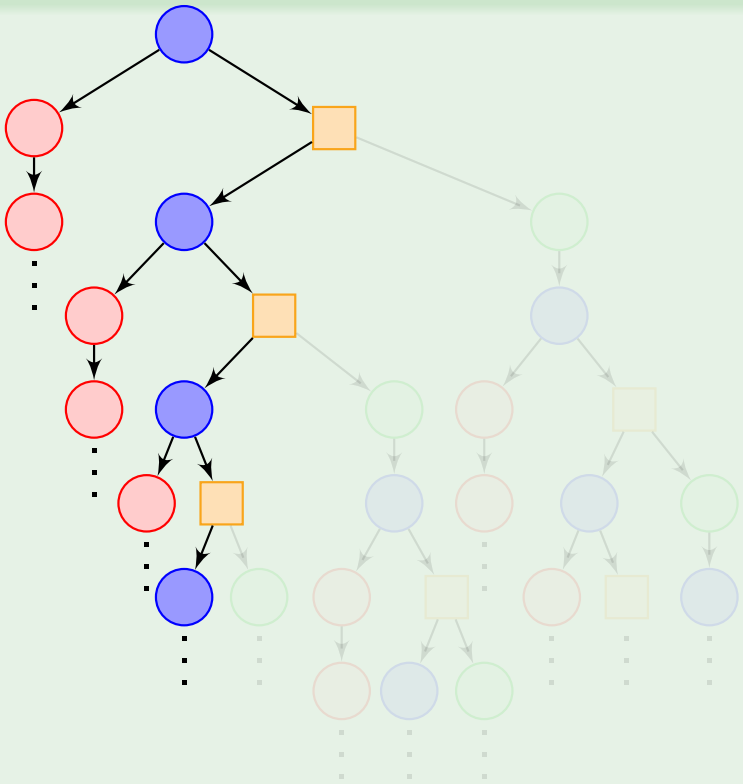

Temporal logics for games: ATL

ATL extends CTL with strategy quantifiers

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ.

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Semantics of $\langle\langle A \rangle\rangle\, \varphi$

Existential quantification (over strategies) implicitly includes a universal quantification (over outcomes):

$$\mathcal{G}, \bigcirc \models \langle\langle A \rangle\rangle\, \varphi \quad \Longleftrightarrow \quad \exists \sigma_A.\ \forall \pi \in \mathrm{Out}(\bigcirc, \sigma_A).\ \pi \models \varphi.$$

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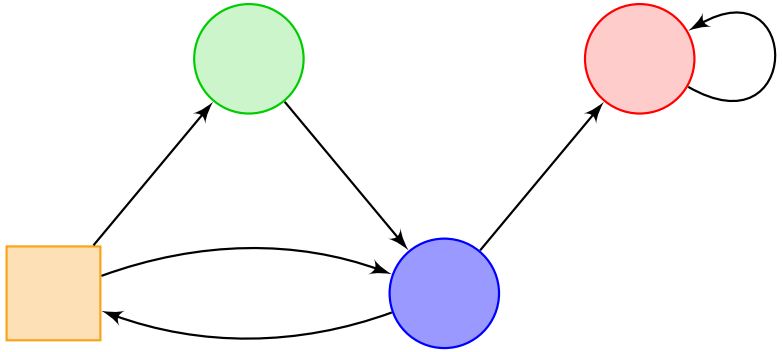

- $\langle\langle \bigcirc \rangle\rangle\, \mathbf{F}$

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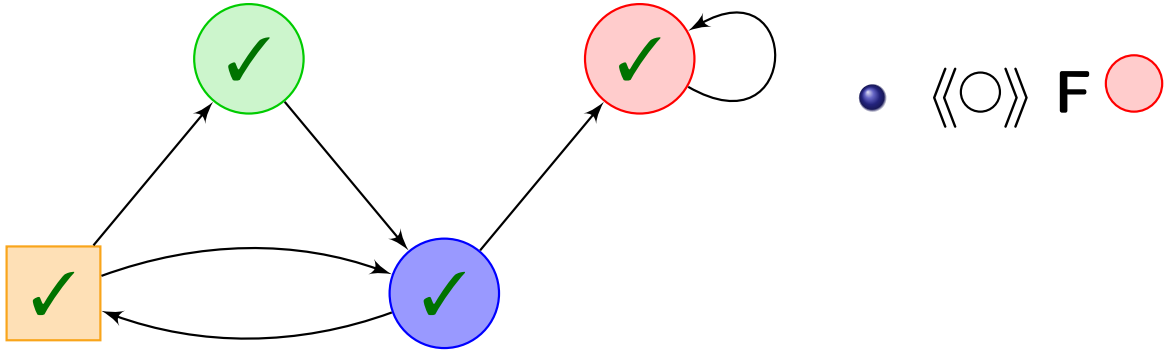

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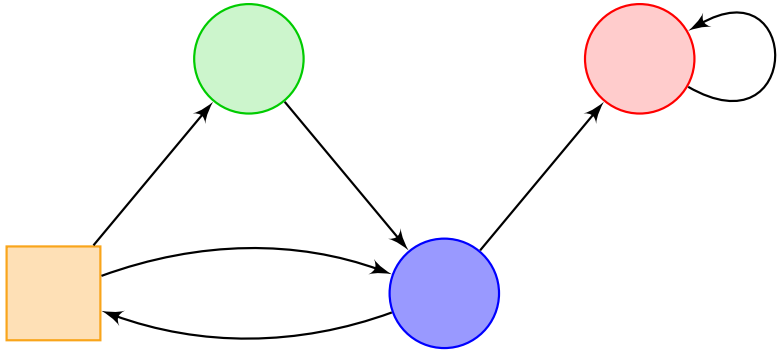

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ATL extends CTL with strategy quantifiers

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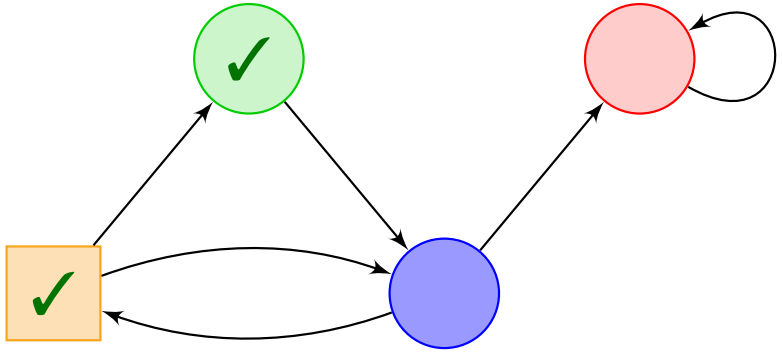

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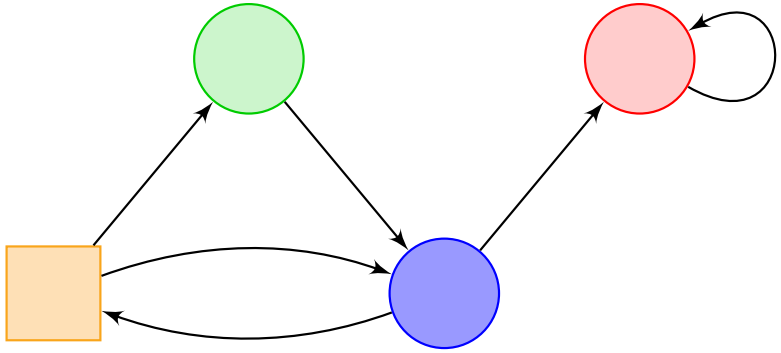

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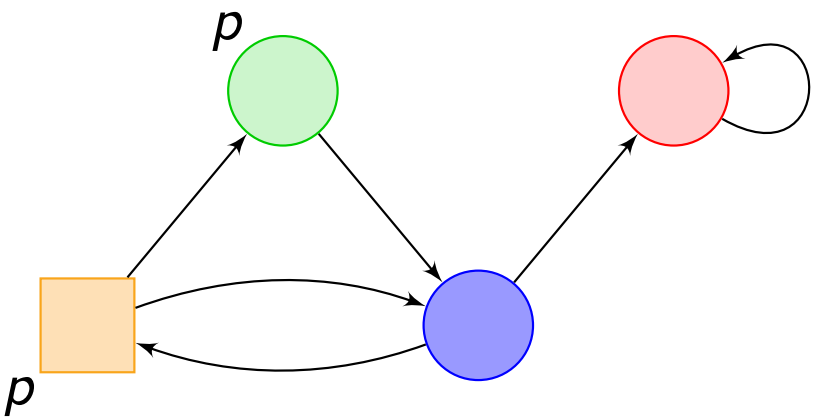

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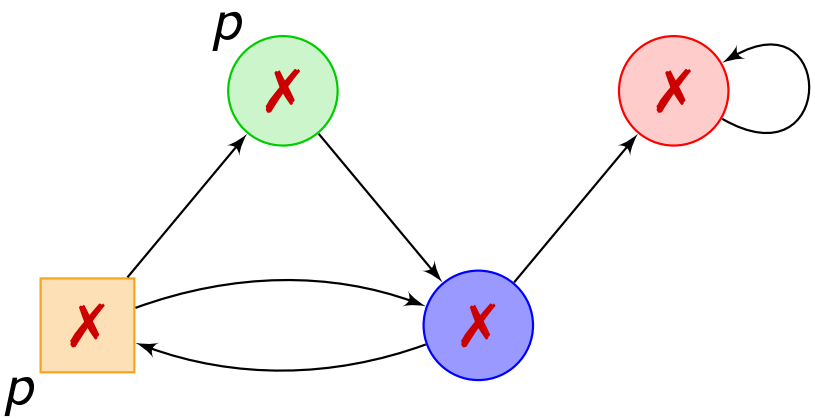

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ATL with strategy contexts [BDLM09,DLM10]

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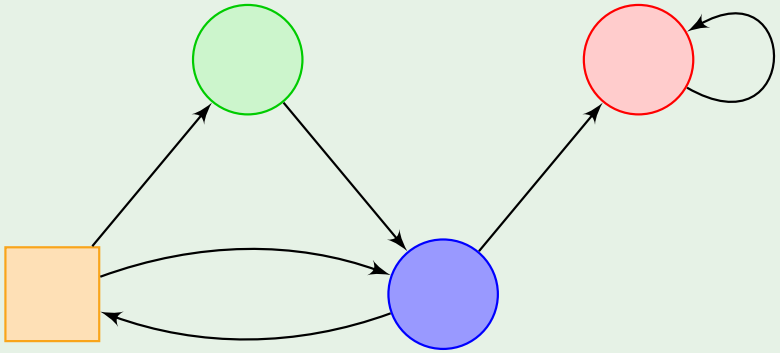

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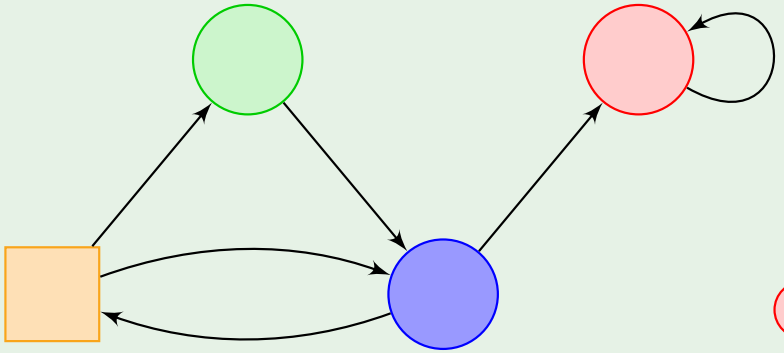

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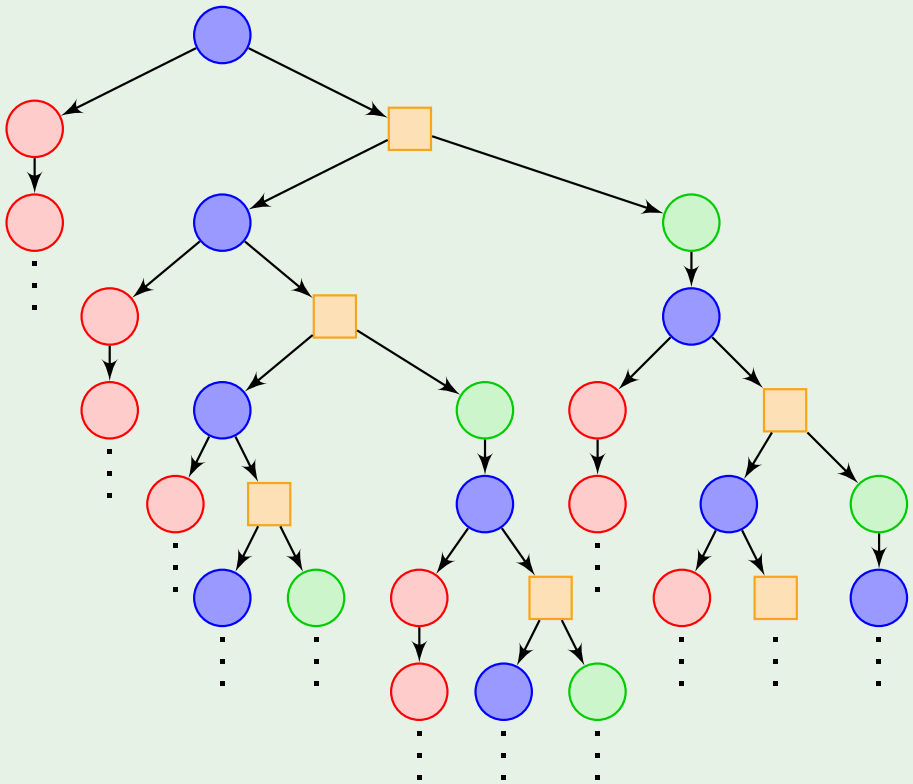

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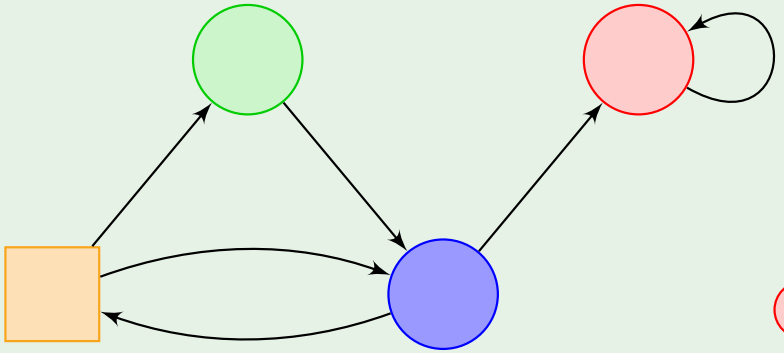

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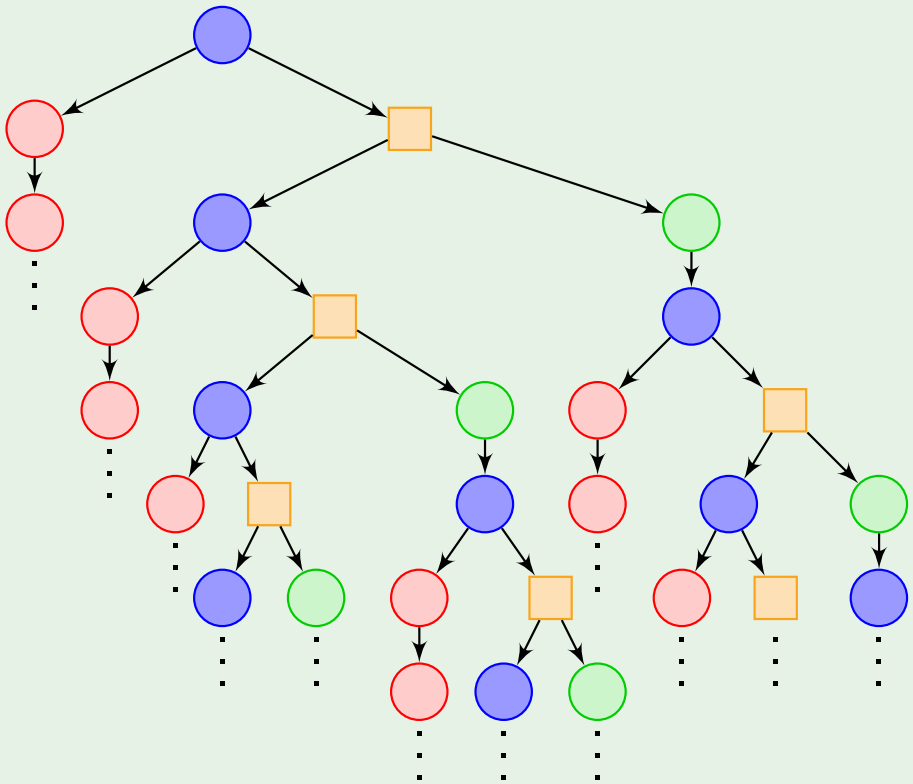

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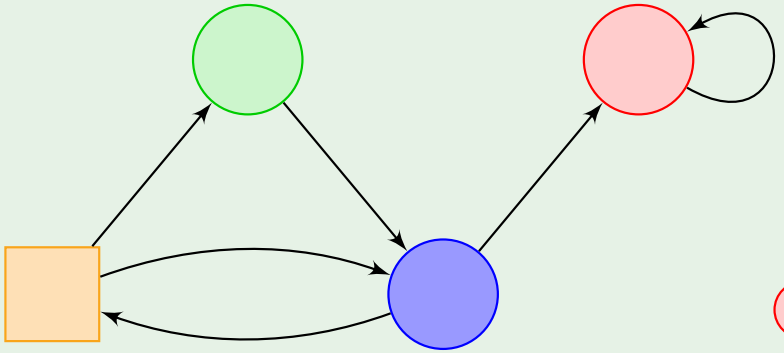

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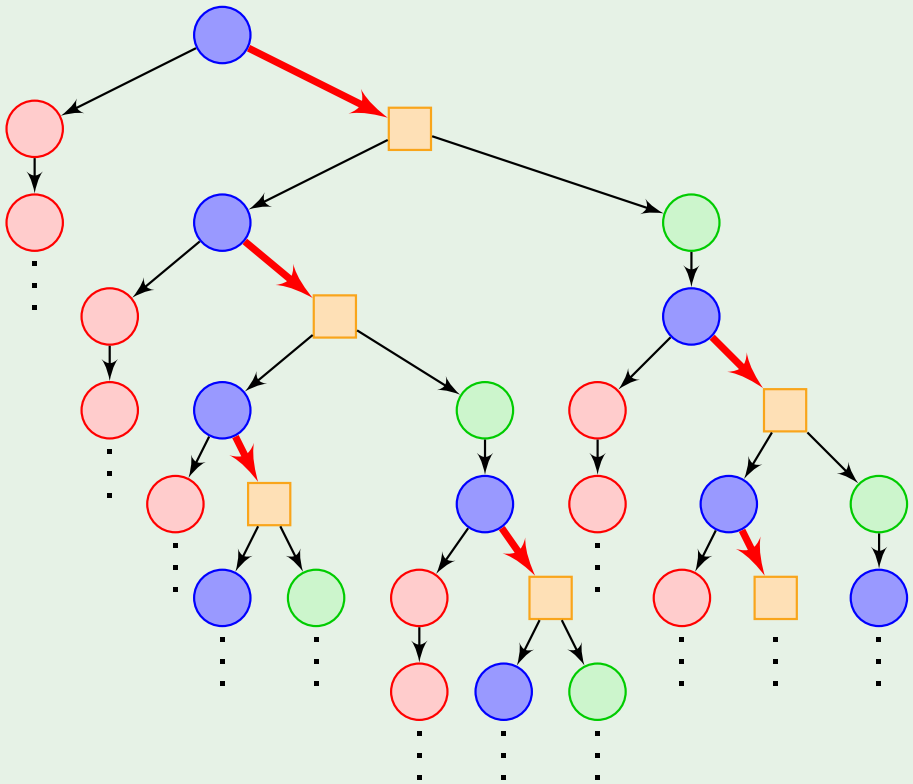

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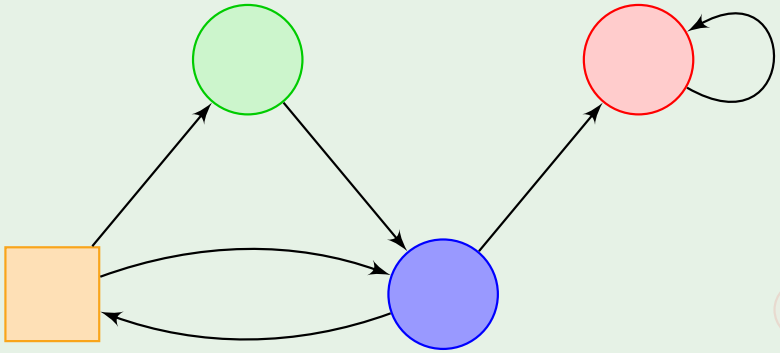

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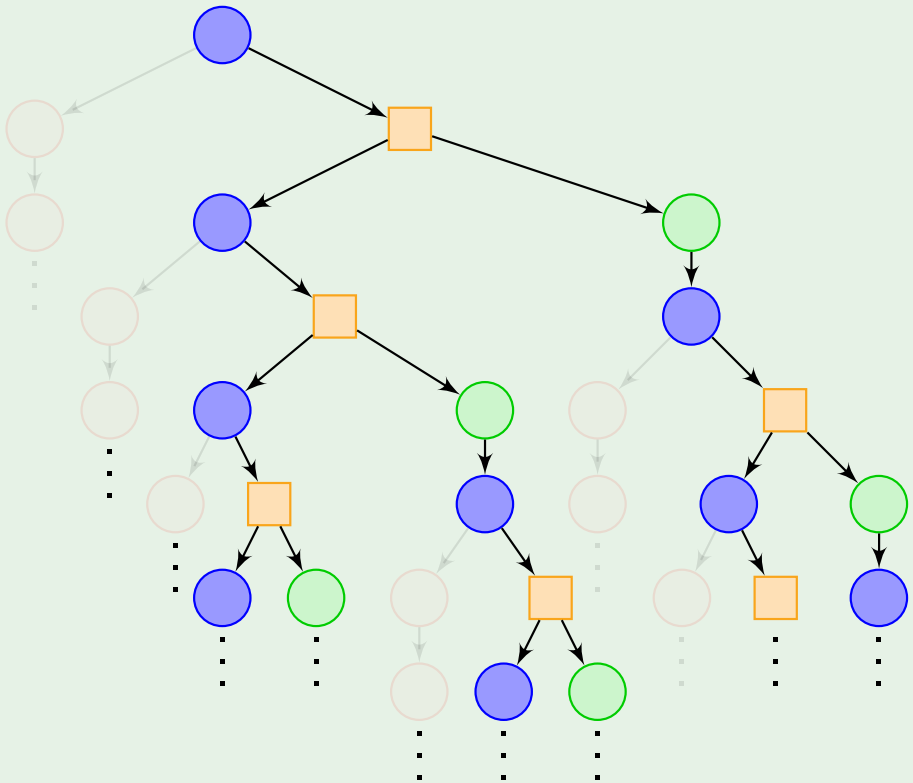

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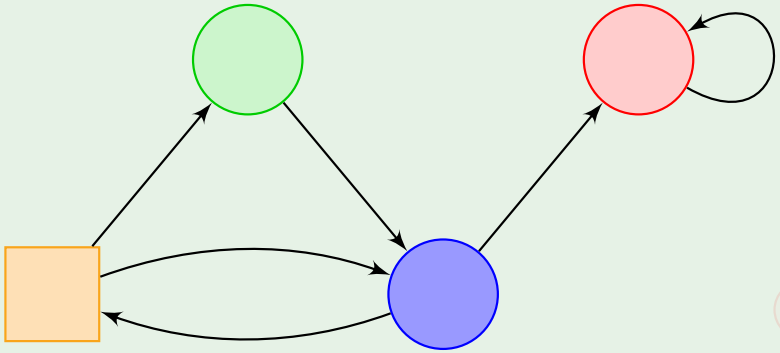

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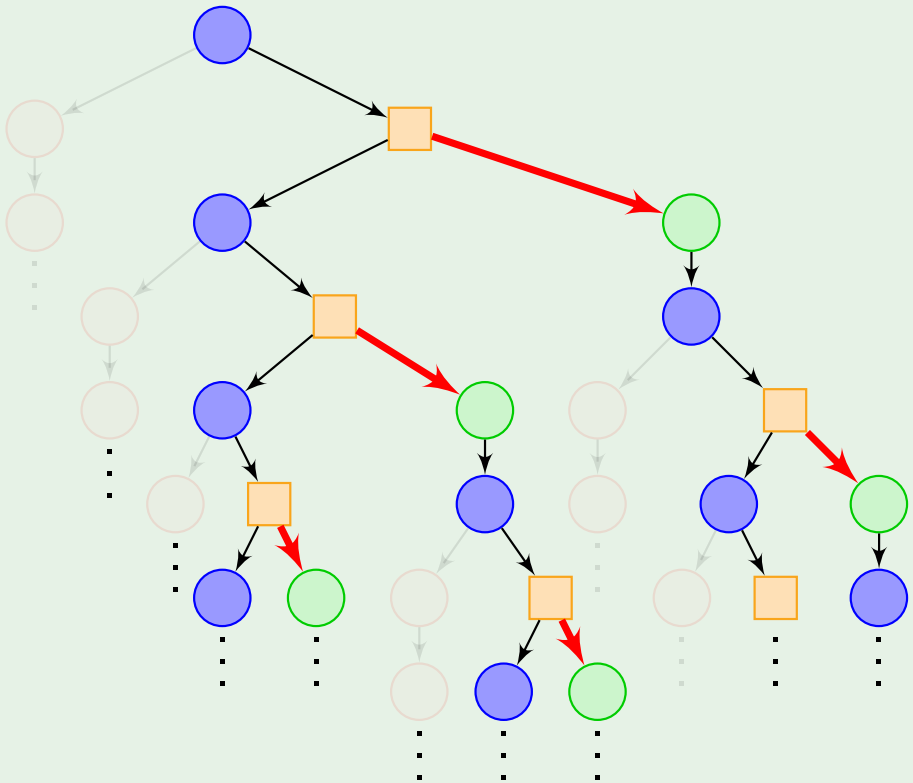

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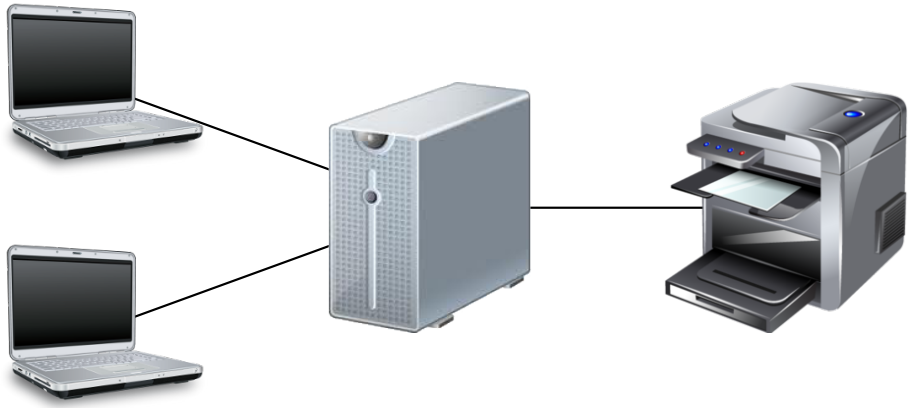
$\rightsquigarrow$ newly selected strategies added to the context:

$$\sigma_A \circ \sigma_B : \quad \begin{array}{ll} a \mapsto \sigma_A(a) & \text{if } a \in A \setminus B \\ b \mapsto \sigma_B(b) & \text{if } b \in B \setminus A \\ c \mapsto \sigma_A(c) & \text{if } c \in B \cap A \end{array}$$

What ATL_{sc} can express

- Client-server interactions for accessing a shared resource:

$$\langle\cdot \mathrm{Server} \cdot\rangle\ \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \mathrm{Clients}} \langle\cdot c \cdot\rangle\ \mathbf{F}\ \mathrm{access}_c \\ \wedge \\ \neg \bigwedge_{c \neq c'} \mathrm{access}_c \wedge \mathrm{access}_{c'} \end{array} \right]$$



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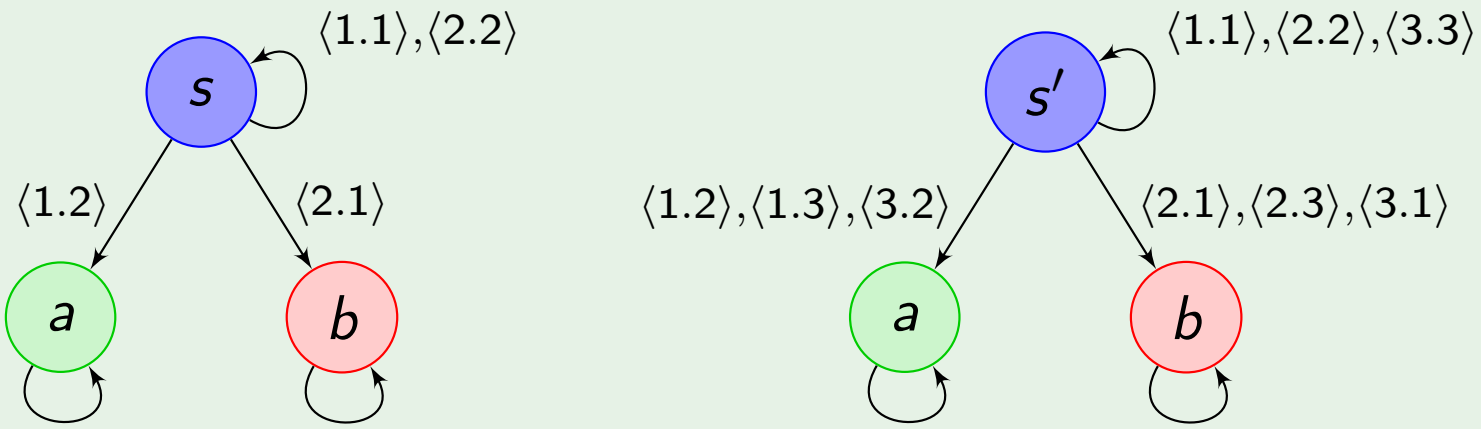
Proof

$\langle\cdot 1\cdot\rangle\,(\,\langle\cdot 2\cdot\rangle\ \mathbf{X}\ a \wedge\ \langle\cdot 2\cdot\rangle\ \mathbf{X}\ b)$ is only true in the second game.
But ATL cannot distinguish between these two games.



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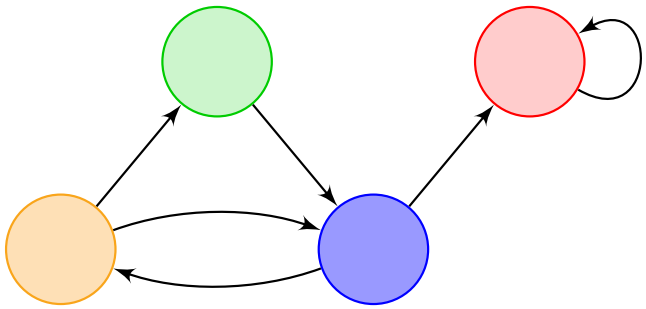

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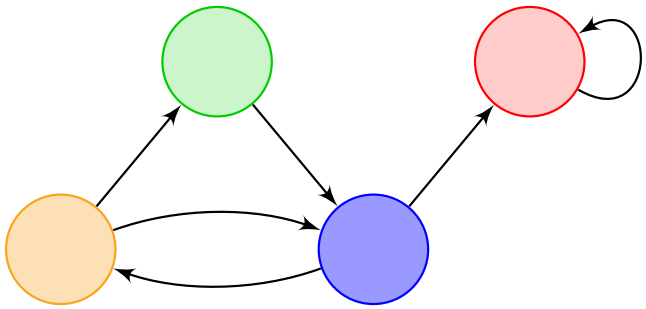

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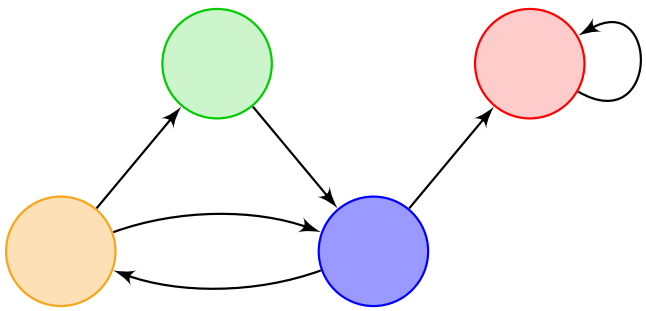

$\rightsquigarrow$ true if we label the Kripke structure;

$\rightsquigarrow$ false if we label the computation tree;

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Semantics of QCTL

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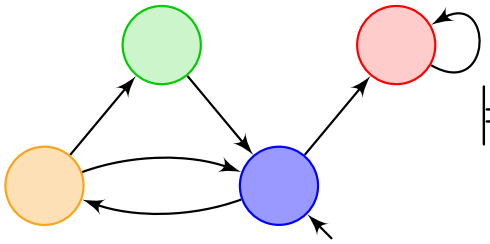

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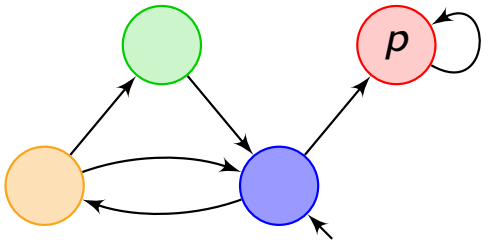

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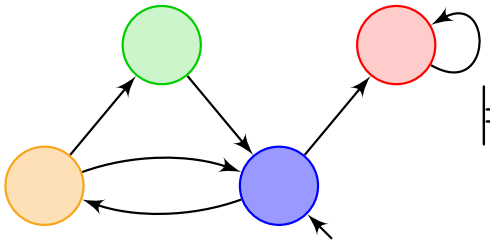

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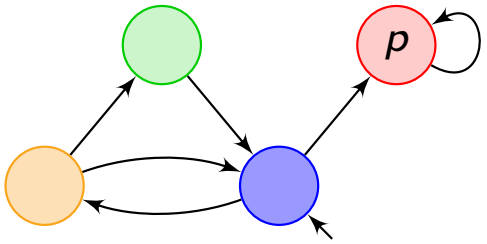

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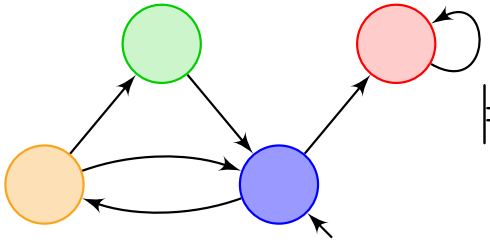

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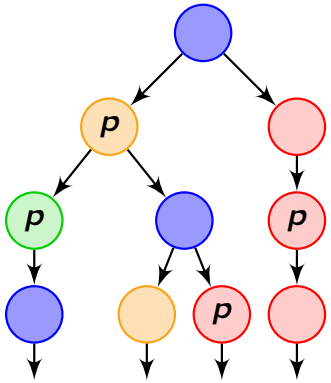

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Expressiveness of QCTL

- QCTL can "count":

$$\mathbf{E}\,\mathbf{X}_1\,\varphi \equiv \mathbf{E}\,\mathbf{X}\,\varphi \wedge \forall p.\ [\mathbf{E}\,\mathbf{X}(p \wedge \varphi) \Rightarrow \mathbf{A}\,\mathbf{X}(\varphi \Rightarrow p)]$$
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Theorem

QCTL, QCTL* and MSO are equally expressive (under both semantics).

[DLM12] Da Costa, Laroussinie, Markey. Quantified CTL: ... CONCUR, 2012.

QCTL with structure semantics

Theorem

Model checking QCTL for the structure semantics is PSPACE-*complete.*

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Membership: labelling algorithm.

Iteratively
- (nondeterministically) pick a labelling,
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Hardness:
QBF is a special case (without even using temporal modalities).

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QCTL satisfiability for the structure semantics is undecidable.

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- *Model checking QCTL with k quantifiers in the tree semantics is* k-EXPTIME-*complete.*
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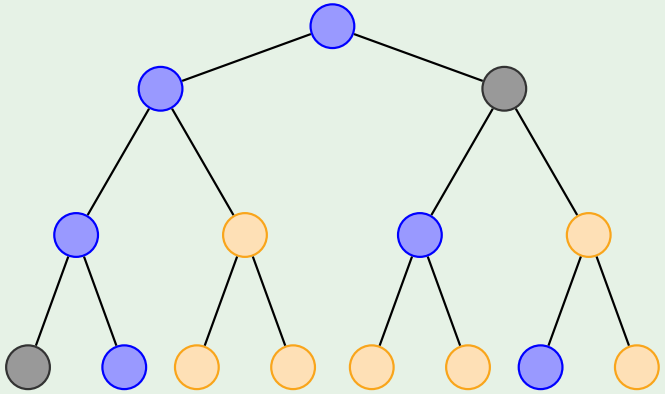

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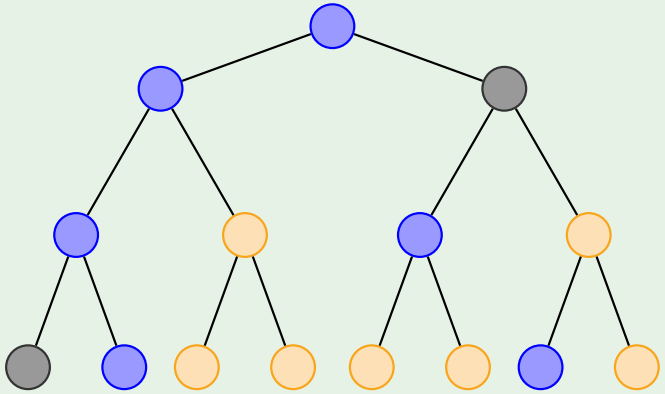

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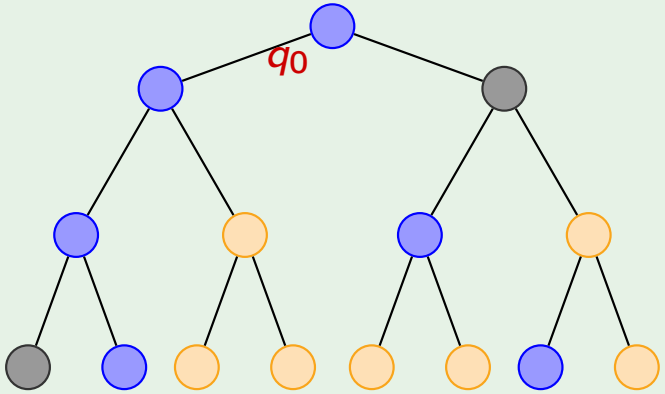

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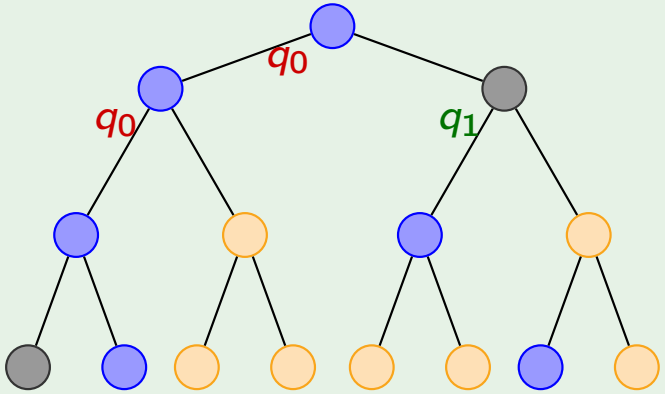

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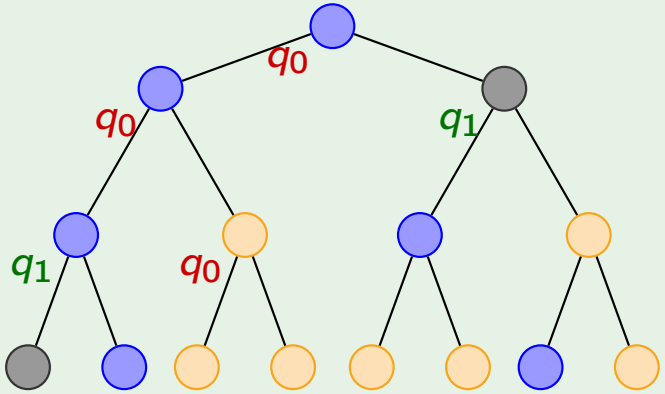

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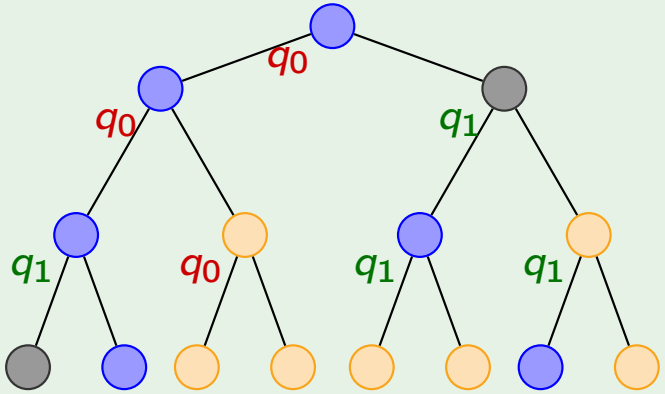

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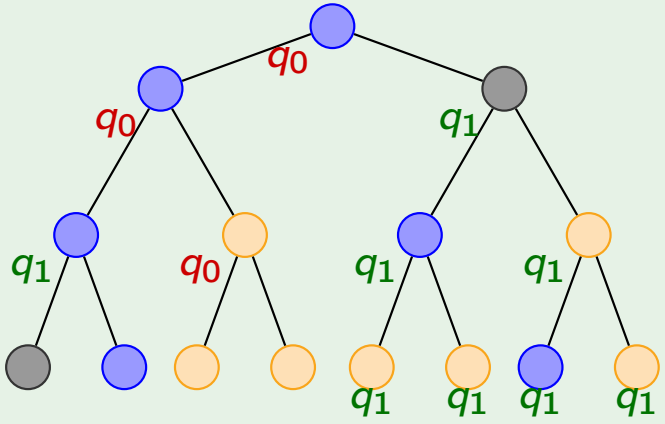

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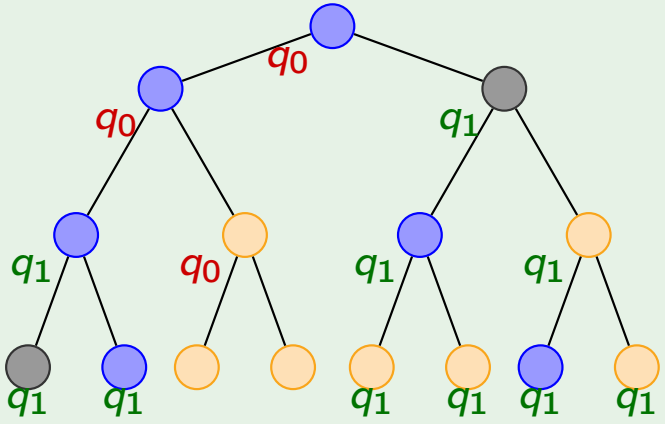

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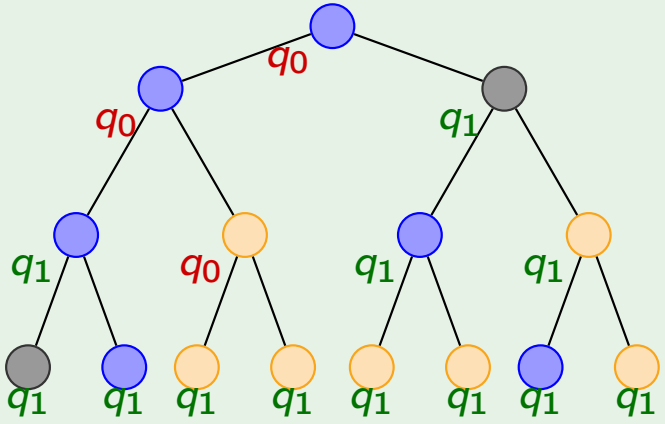

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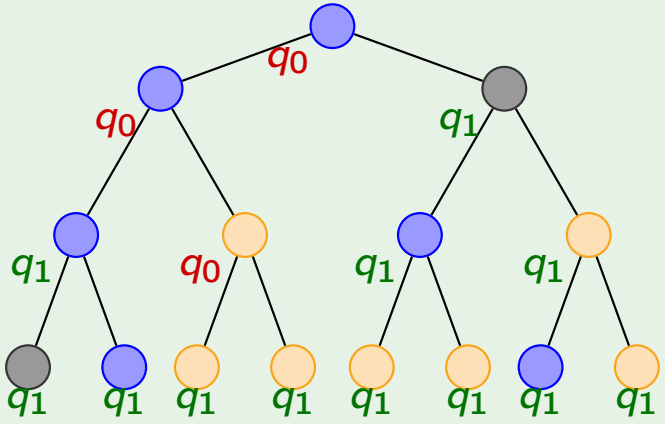

This automaton corresponds to **E** ● **U** ○

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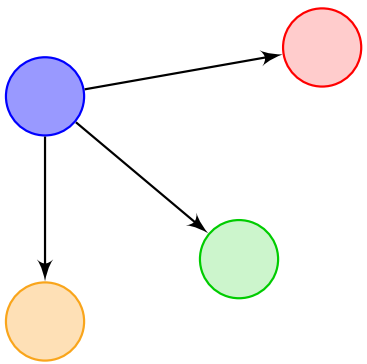
Proof

- polynomial-size tree automata for CTL;
- quantification is handled by projection, which first requires removing alternation (exponential blowup);
- an automaton equivalent to a QCTL formula can be built inductively;
- emptiness of an alternating parity tree automaton can be decided in exponential time.

Translating ATL_{sc} into QCTL



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(, A, m_i^A) of states that can be reached from when player A plays m_i^A.

[DLM12] Da Costa, Laroussinie, Markey. Quantified CTL: ... CONCUR, 2012.

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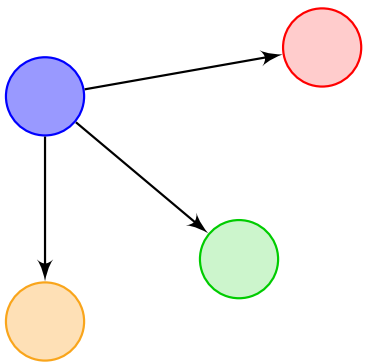

- player A has moves m_1^A, ..., m_n^A;
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$\langle \cdot A \cdot \rangle\, \varphi$ can be encoded as follows:

$\exists m_1^A.\ \exists m_2^A \ldots \exists m_n^A.$

- this corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(m_i^A \Leftrightarrow \bigwedge \neg\, m_j^A)$;
- the outcomes all satisfy φ:

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^A \Rightarrow \mathbf{X}\,\mathrm{Next}(q, A, m_i^A)) \Rightarrow \varphi\big].$$

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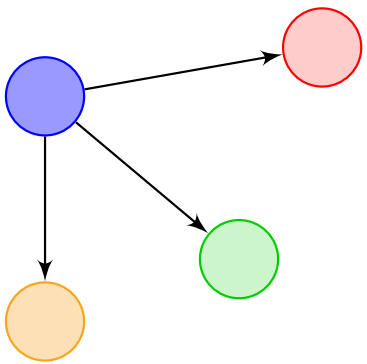

- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(●, A, m_i^A) of states that can be reached from ● when player A plays m_i^A.

Corollary

*ATL_{sc} model checking is decidable, with non-elementary complexity (*TOWER-*complete).*

Corollary

ATL_{sc}^0 (quantification restricted to memoryless strategies) model checking is PSPACE-*complete.*

[DLM12] Da Costa, Laroussinie, Markey. Quantified CTL: ... CONCUR, 2012.

What about satisfiability?

Theorem

QCTL satisfiability is decidable (for the tree semantics).

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Theorem ([TW12])

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Why?

The translation from ATL_{sc} to QCTL assumes that the game structure is given!

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Satisfiability for turn-based games

Theorem ([LM13])

When restricted to turn-based games, ATL_{sc} satisfiability is decidable.

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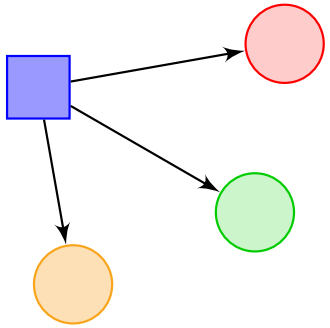

- player $\square$ has moves ●, ● and ●.
- a strategy can be encoded by marking some of the nodes of the tree with proposition mov_A.

$\langle\cdot A\cdot\rangle\,\varphi$ can be encoded as follows:

$\exists \mathrm{mov}_A.$

- it corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(\mathrm{turn}_A \Rightarrow \mathbf{E}\,\mathbf{X}_1\,\mathrm{mov}_A)$;
- the outcomes all satisfy φ: $\mathbf{A}\big[\mathbf{G}(\mathrm{turn}_A \wedge \mathbf{X}\,\mathrm{mov}_A) \Rightarrow \varphi\big]$.

[LM13] Laroussinie, Markey. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

What about Strategy Logic? [CHP07,MMV10]

Strategy logic

Explicit quantification over strategies + strategy assignement

Example

$$\langle \cdot A \cdot \rangle \, \varphi \equiv \exists \sigma_1 . \mathsf{assign}(\sigma_1, A) . \varphi$$

Strategy logic can also be translated into QCTL.

Theorem

- *Strategy-logic model-checking is decidable.*
- *Strategy-logic satisfiability is decidable when restricted to turn-based games.*

[CHP07] Chatterjee, Henzinger, Piterman. Strategy Logic. CONCUR, 2007.
[MMV10] Mogavero, Murano, Vardi. Reasoning about strategies. FSTTCS, 2010.

Conclusions and future works

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- ATL_{sc} is a very expressive, yet decidable extension of ATL;
- QCTL is a powerful extension of CTL;
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Future directions

- Defining interesting fragments of those logics;
- Obtaining practicable algorithms.
- Considering partial observation;
- Considering randomised strategies.