Optimal strategies in weighted timed games: undecidability and approximation

Nicolas Markey

LSV, CNRS & ENS Cachan, France

(joint work with Patricia Bouyer and Samy Jaziri)

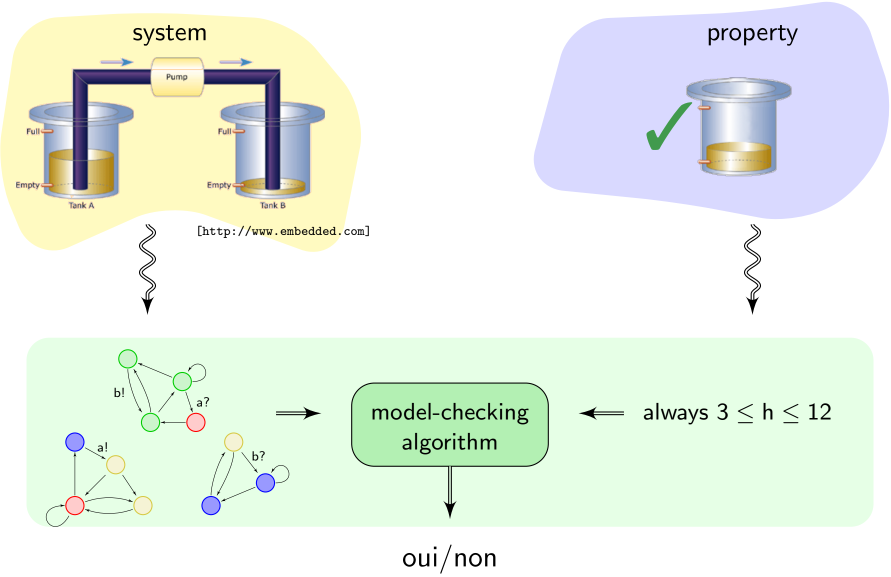
68 NQRT seminar – Rennes, France

October 1, 2015

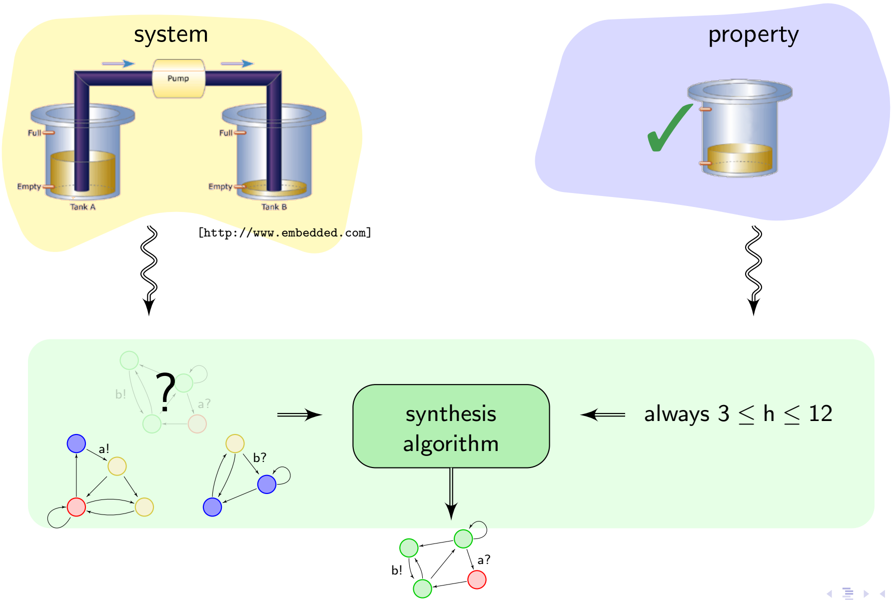
EQualIS

Cassting

Model checking and *synthesis*

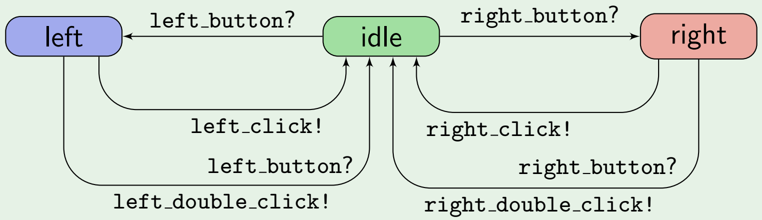


Model checking and *synthesis*



Reasoning about real-time systems

Example (A computer mouse)



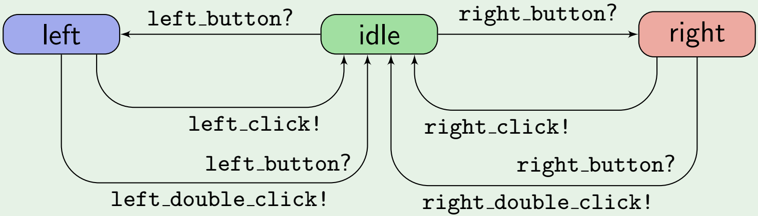
Reasoning about real-time systems

Definition ([AD90])

A timed automaton is made of

- a transition system,

Example (A computer mouse)



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Reasoning about real-time systems

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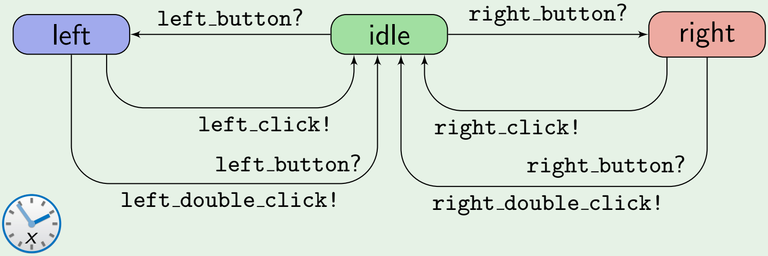

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Reasoning about real-time systems

Definition ([AD90])

A timed automaton is made of

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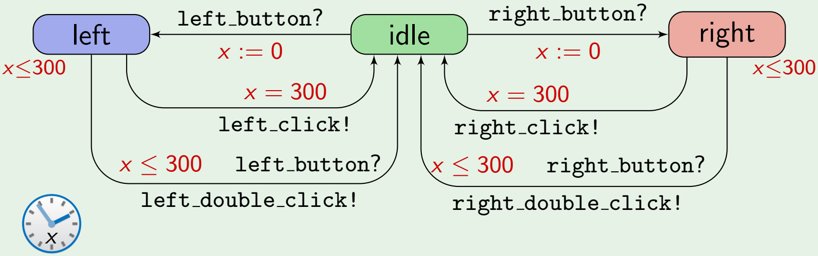
- a set of clocks,
- timing constraints on states and transitions.

Example (A computer mouse)



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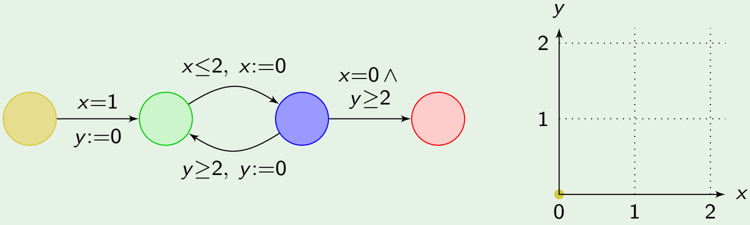
Continuous-time semantics

Example



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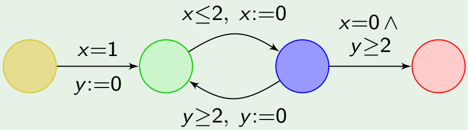

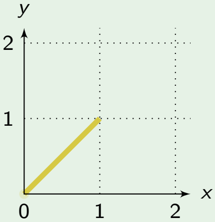

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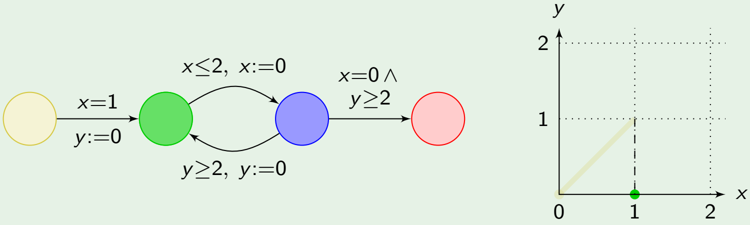

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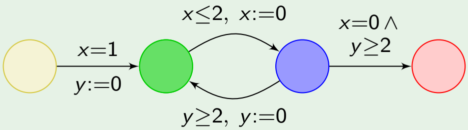

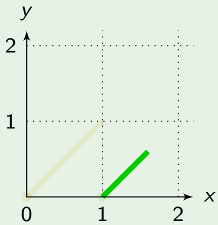

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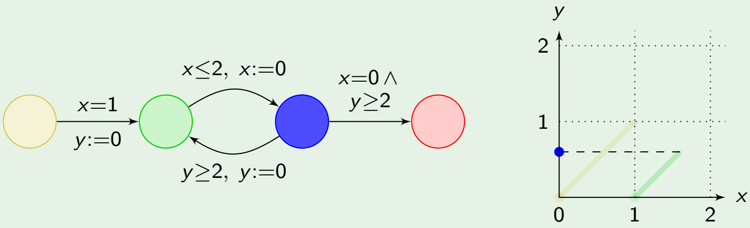

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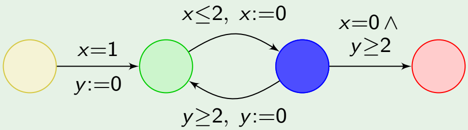

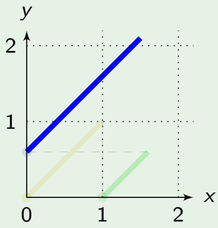

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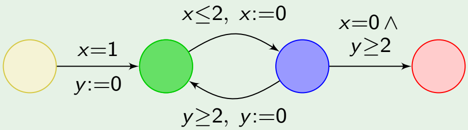

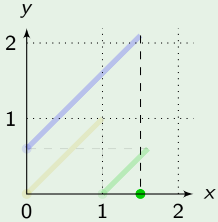

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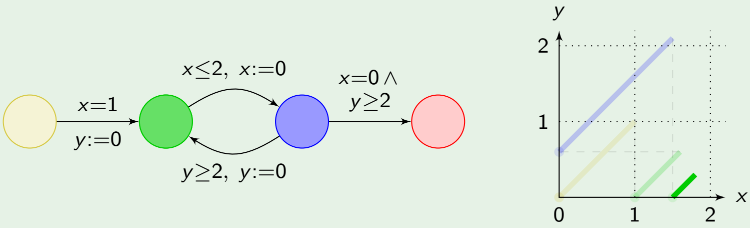

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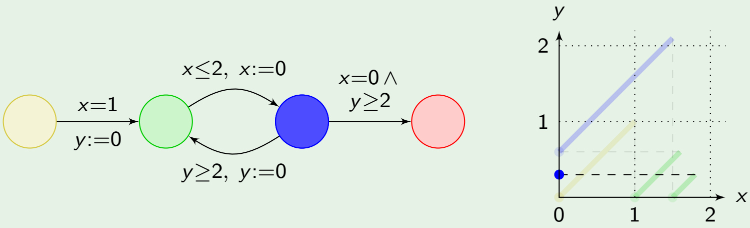

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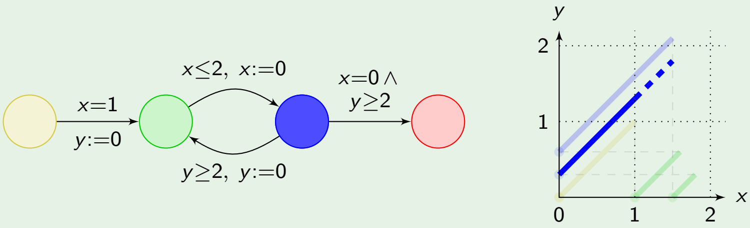

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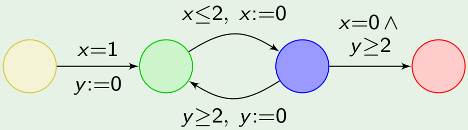

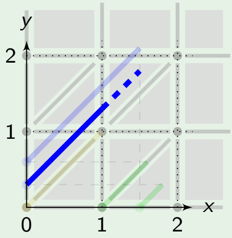

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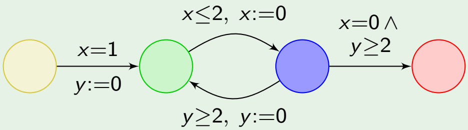
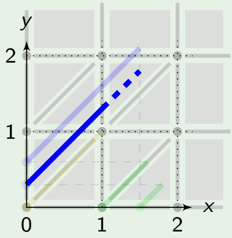


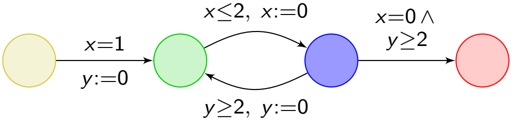


Theorem ([AD90,ACD93, ...])

Reachability in timed automata is decidable (as well as many other important properties).

[AD90] Alur, Dill. Automata For Modeling Real-Time Systems. ICALP, 1990.
[ACD93] Alur, Courcoubetis, Dill. Model-Checking in Dense Real-Time. Inf. & Comp., 1993.

Region automaton



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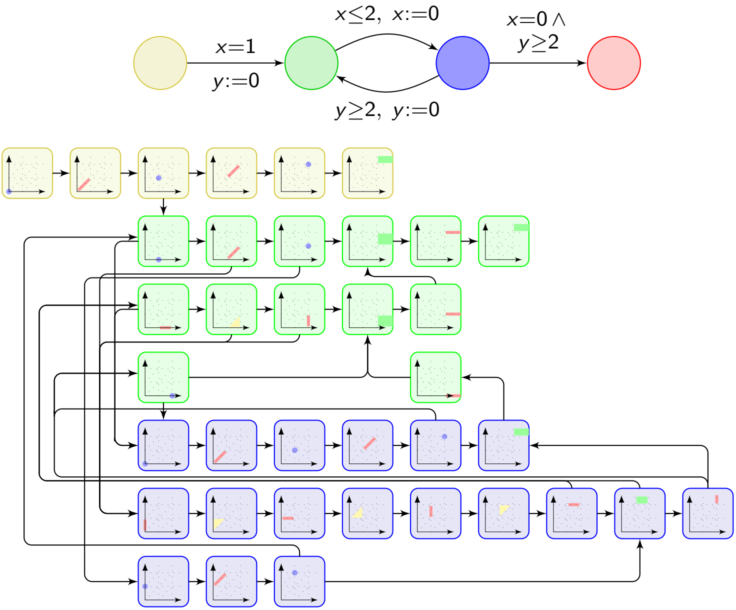

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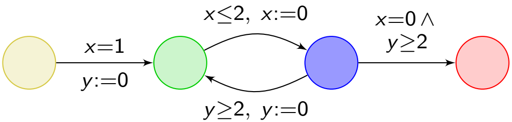
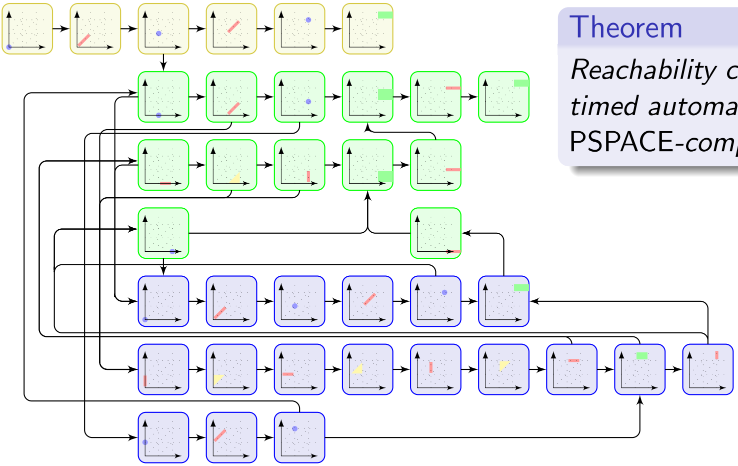




Theorem

Reachability checking in timed automata is PSPACE-*complete.*

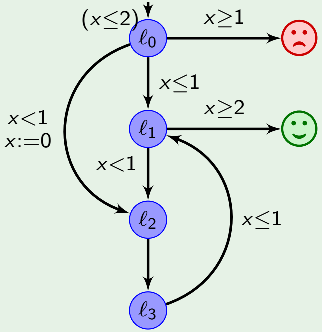
Timed games

Definition

A timed game is made of

- a timed automaton;

Example



Timed games

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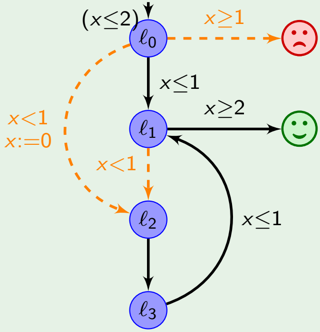

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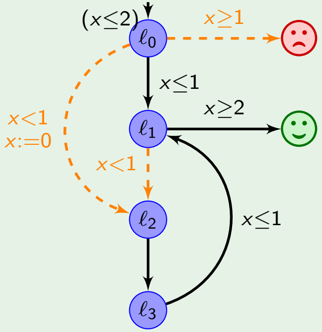


a memoryless strategy

in $(\ell_0, x = 0)$: wait 0.5
goto ℓ_1

in (ℓ_1, x): wait until $x = 2$
goto ☺

in $(\ell_2, x \leq 1)$: wait until $x = 1$
goto ℓ_3

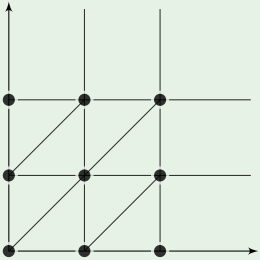
in $(\ell_3, x \leq 1)$: wait until $x = 1$
goto ℓ_1

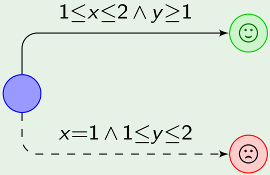
Timed games

Theorem ([AMPS98])

Deciding the winner in a timed game (e.g. for reachability objectives) is EXPTIME-complete.

Proof





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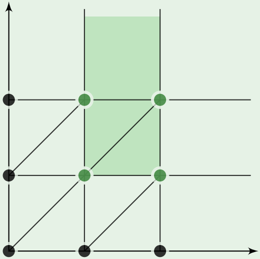

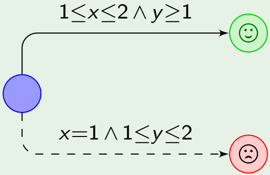

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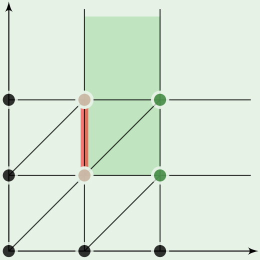

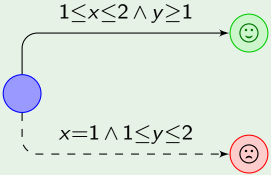

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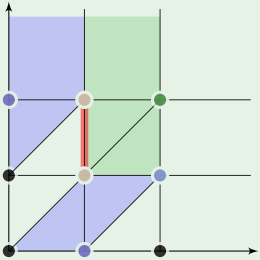

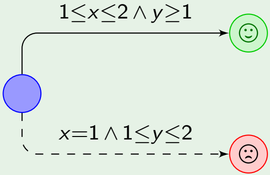

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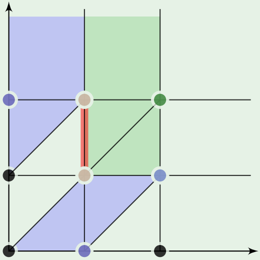

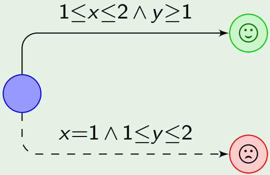

- regions are sufficient;
- the computation terminates.

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Outline of the talk

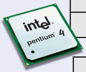
1. Introduction: timed automata and timed games
2. Measuring extra quantities in timed automata
   - Example: task graph scheduling
   - Timed automata with observer variables
3. Cost-optimal strategies
   - Optimal reachability in priced timed automata
   - Optimal reachability in priced timed games
4. Conclusions and future works

Outline of the talk

Example: task graph scheduling

Compute $D{\times}(C{\times}(A{+}B)){+}(A{+}B){+}(C{\times}D)$ using two processors:

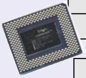
P_1 (fast):



time	
$+$	2 picoseconds
$\times$	3 picoseconds

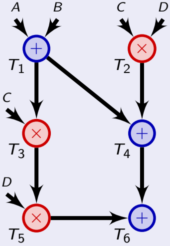
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
$+$	5 picoseconds
$\times$	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts



Example: task graph scheduling

Compute $D \times (C \times (A + B)) + (A + B) + (C \times D)$ using two processors:

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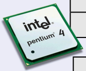

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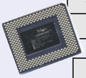

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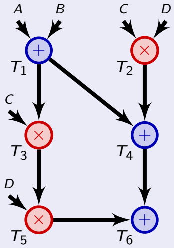

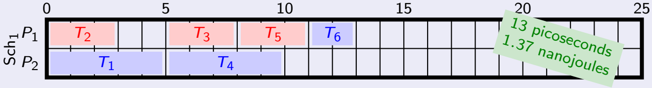

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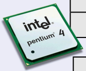

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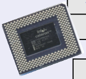

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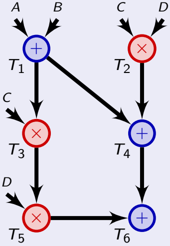

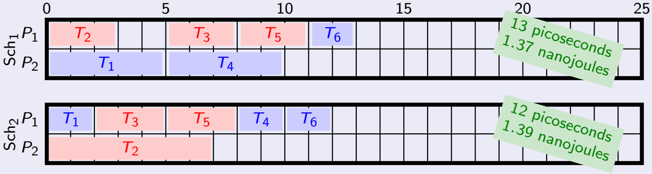

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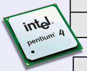

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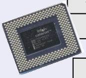

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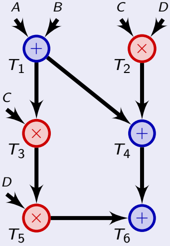
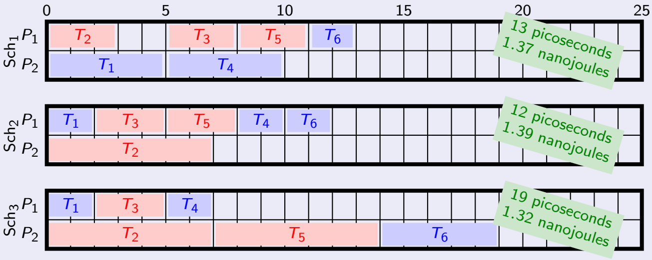




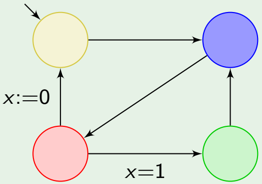
Priced timed automata

Definition ([KPSY99,ALP01,BFH$^+$01])

A priced timed automaton is made of

- a timed automaton;

Example



[KPSY99] Kesten, Pnueli, Sifakis, Yovine. Decidable Integration Graphs. Inf. & Comp., 1999.
[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata. HSCC, 2001.
[BFH$^+$01] Behrmann *et al.* Minimum-cost reachability in priced timed automata. HSCC, 2001.

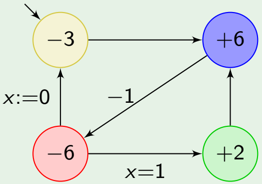
Priced timed automata

Definition ([KPSY99,ALP01,BFH+01])

A priced timed automaton is made of

- a timed automaton;
- the price of each transition and location.

Example



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Priced timed automata

Definition ([KPSY99,ALP01,BFH$^+$01])

A priced timed automaton is made of

- a timed automaton;
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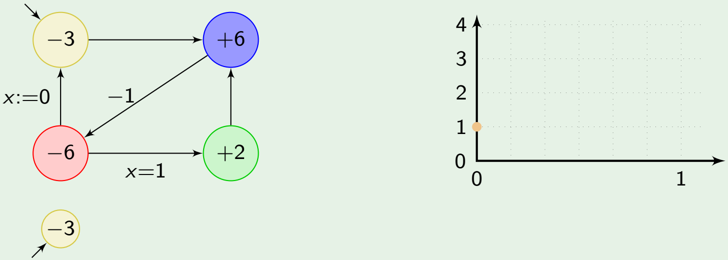
Example



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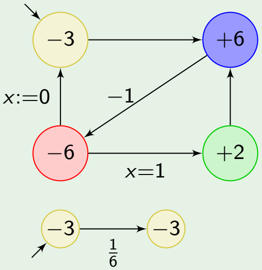

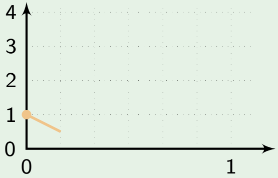

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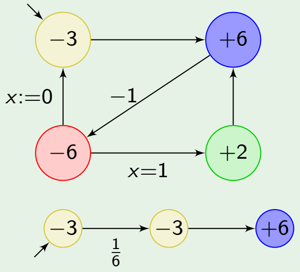

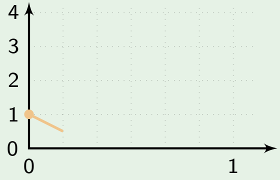

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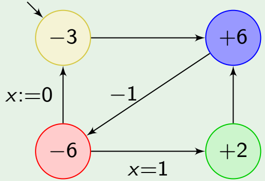

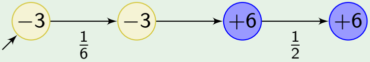

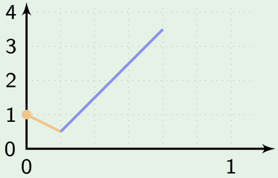

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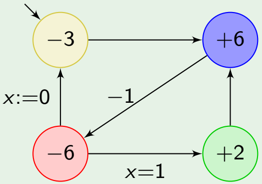

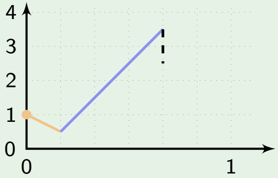

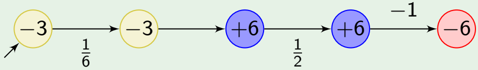

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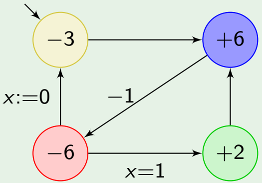

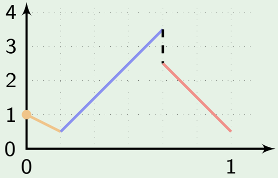

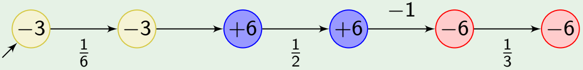

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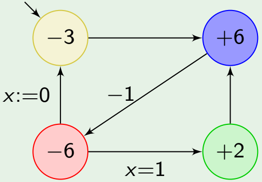

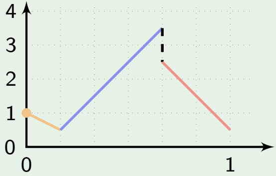

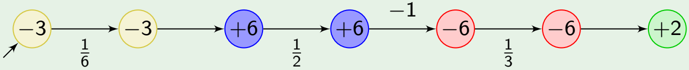

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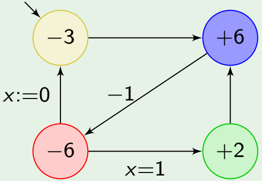

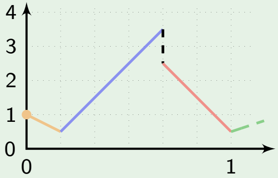

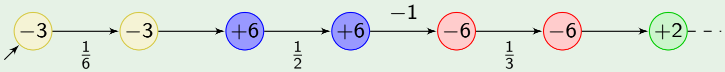

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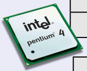
Example: task graph scheduling

Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

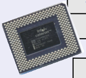
P_1 (fast):



time	
+	2 picoseconds
×	3 picoseconds

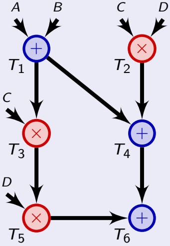
energy	
idle	10 Watt
in use	90 Watts

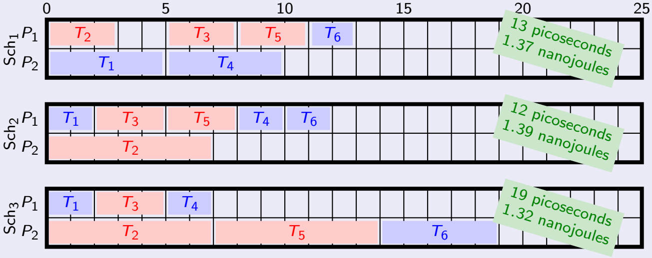
P_2 (slow):



time	
+	5 picoseconds
×	7 picoseconds

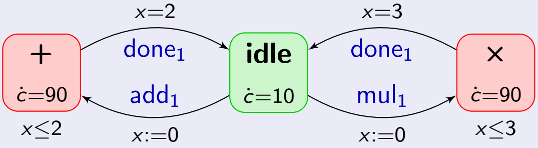
energy	
idle	20 Watts
in use	30 Watts





Modelling the task graph scheduling problem

Processors:



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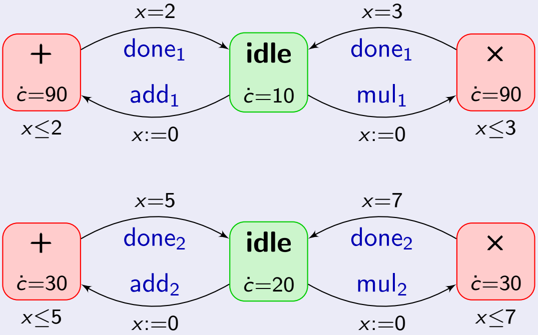

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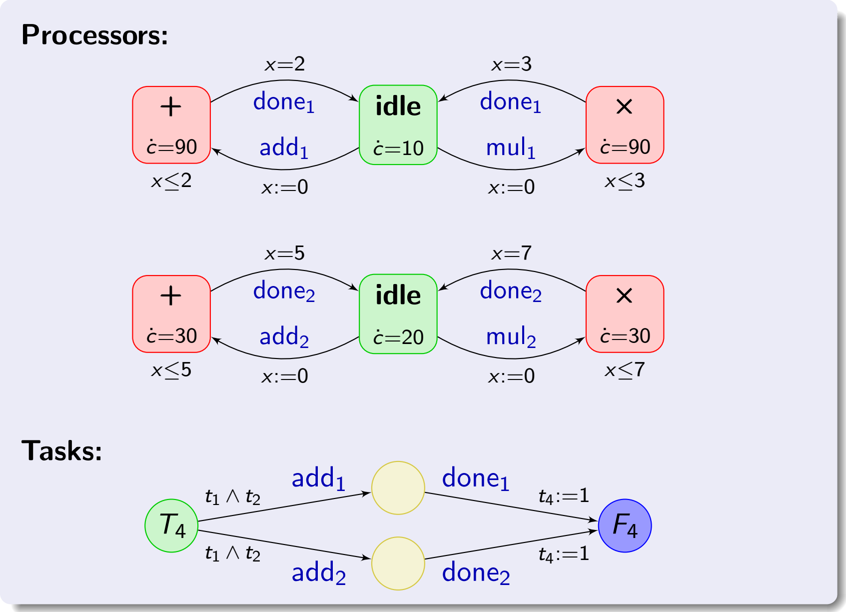
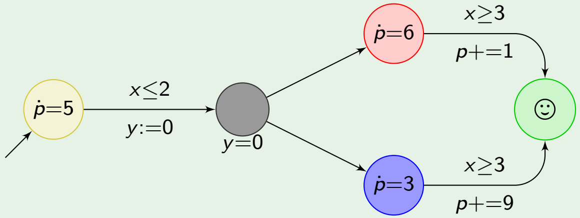


Outline of the talk

Cost-optimal reachability in priced timed automata

Example



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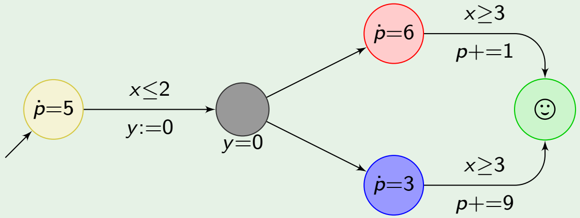

Minimal cost for reaching ☺:

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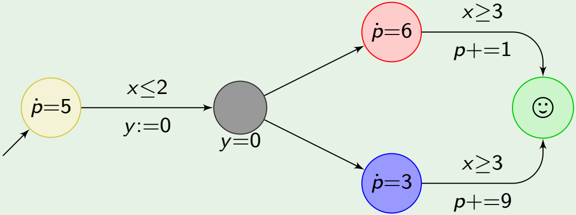
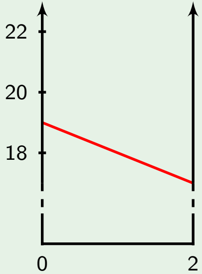


Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$



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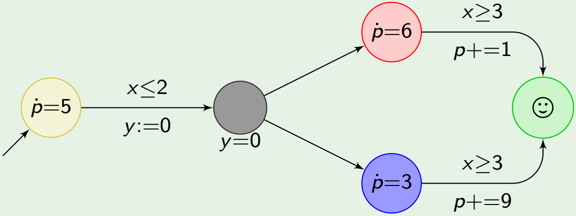
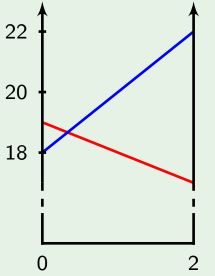


Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$
$$5t + 3(3 - t) + 9$$



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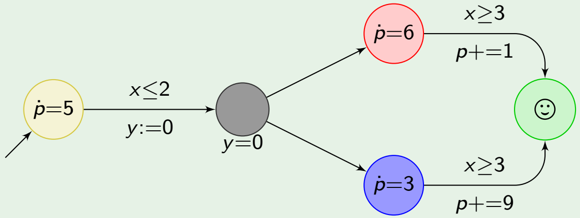
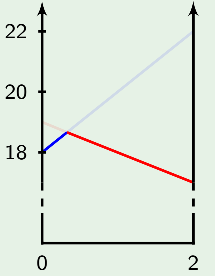


Minimal cost for reaching ☺:

$$\min \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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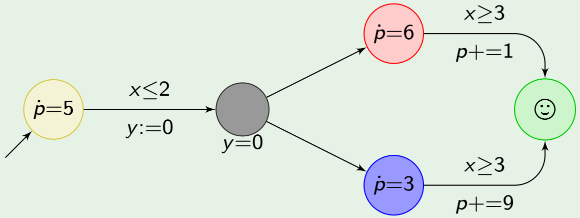
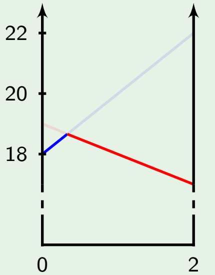


Minimal cost for reaching ☺:

$$\inf_{0\leq t\leq 2} \min \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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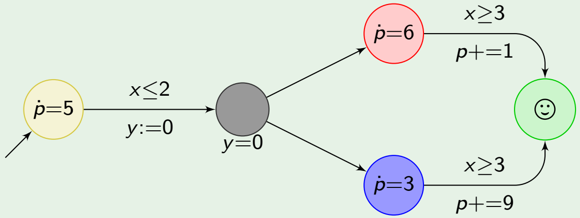

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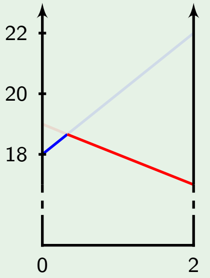
$$\inf_{0\leq t\leq 2} \min \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix} = 17$$



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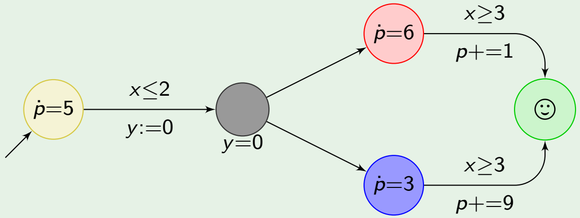

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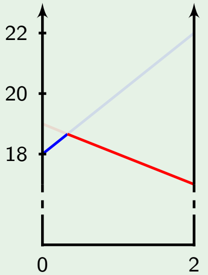

The optimal *schedule* consists in

- waiting 2 time units in ◯;
- going through ◯.

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Theorem ([BBBR07])

Optimal reachability in priced timed automata is PSPACE-*complete.*

[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem. FMSD, 2007.

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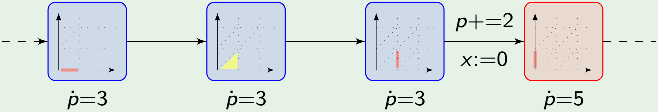

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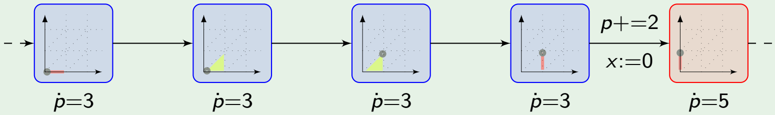

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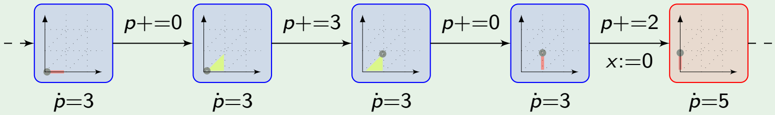

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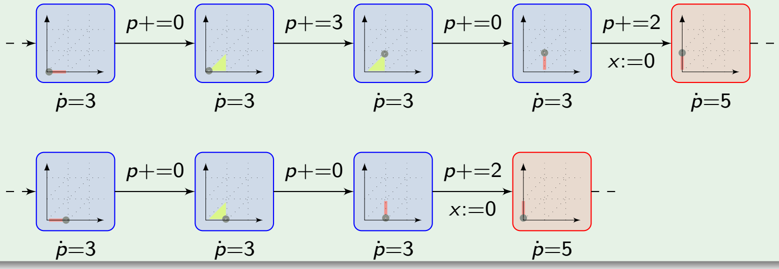

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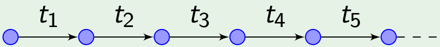

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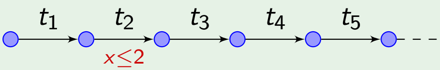

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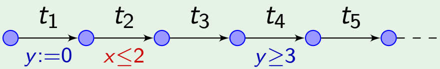

$$t_1 + t_2 \leq 2$$
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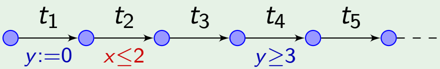

Minimize
$\sum_i c_i \cdot t_i + C_{\mathrm{disc}}$

$t_1 + t_2 \leq 2$

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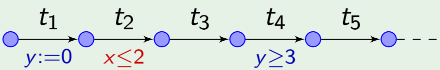

Minimize
$\sum_i c_i \cdot t_i + C_{\text{disc}}$

$t_1 + t_2 \leq 2$

$t_2 + t_3 + t_4 \geq 3$

$\rightsquigarrow$ infimum over bounded zone reached at a point on the frontier, with integer coordinates.

[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem. FMSD, 2007.

Cost-optimal reachability in priced timed automata

Theorem ([BBBR07])

Optimal reachability in priced timed automata is PSPACE-*complete.*

Proof

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$$\forall \pi.\ \exists \pi_{cp}.\ cost(\pi_{cp}) \leq cost(\pi).$$

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$$\forall\pi.\ \exists\pi_{cp}.\ \mathit{cost}(\pi_{cp}) \leq \mathit{cost}(\pi).$$

- approximate path in corner-point abstraction by a real run:

$$\forall\pi_{cp}.\ \exists\pi.\ \mathit{cost}(\pi) \leq \mathit{cost}(\pi_{cp}) + \epsilon.$$

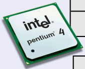
[BBBR07] Bouyer, Brihaye, Bruyère, Raskin. On the optimal reachability problem. FMSD, 2007.

Outline of the talk

Example: task graph scheduling

Compute $D \times (C \times (A + B)) + (A + B) + (C \times D)$ using two processors:

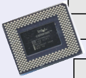
P_1 (fast):



	time
+	2 picoseconds
×	3 picoseconds

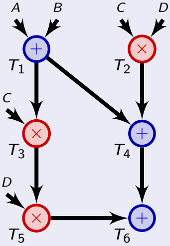
energy	
idle	10 Watt
in use	90 Watts

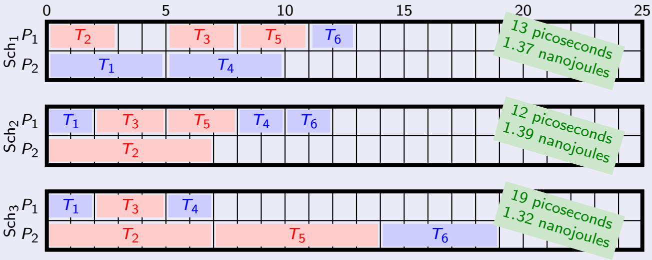
P_2 (slow):



	time
+	5 picoseconds
×	7 picoseconds

energy	
idle	20 Watts
in use	30 Watts

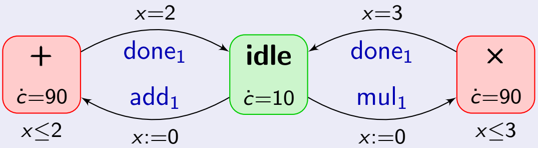




Cost-optimal reachability in priced timed games

Using games to model uncertainty over delays

Processors with exact delays:



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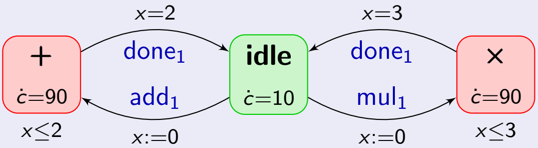
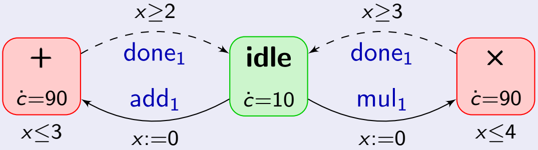
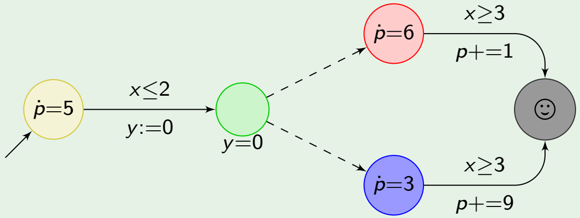


Processors with approximate delays:



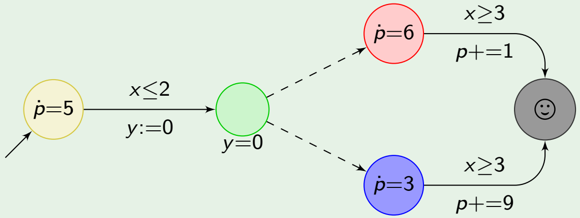
Cost-optimal reachability in priced timed games

Example



Cost-optimal reachability in priced timed games

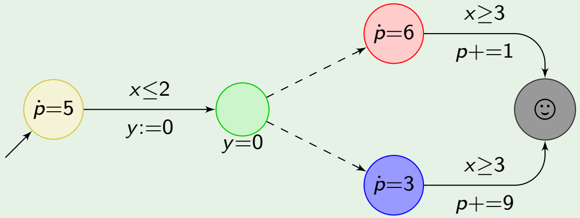
Example



Minimal cost for reaching ☺:

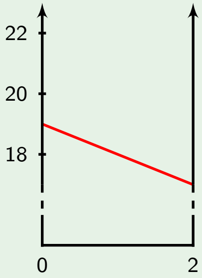
Cost-optimal reachability in priced timed games

Example



Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$



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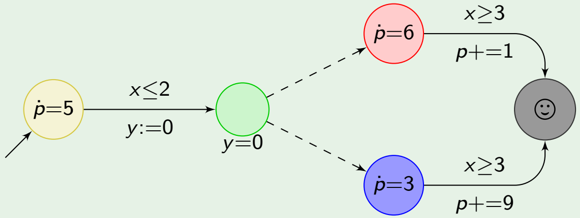
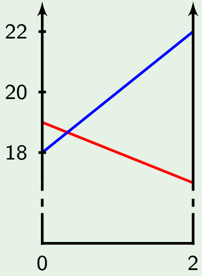


Minimal cost for reaching ☺:

$$5t + 6(3 - t) + 1$$
$$5t + 3(3 - t) + 9$$



Cost-optimal reachability in priced timed games

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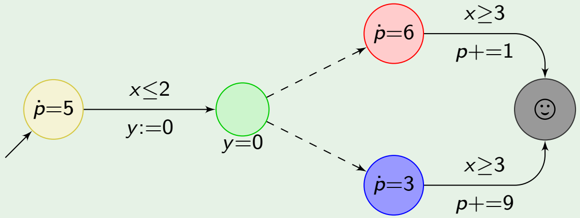
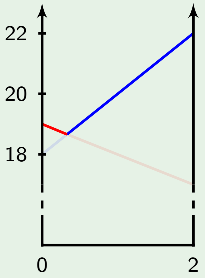


Minimal cost for reaching ☺:

$$\max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



Cost-optimal reachability in priced timed games

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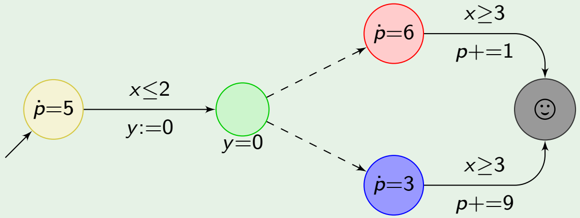
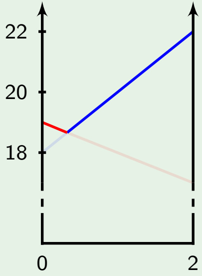


Minimal cost for reaching ☺:

$$\inf_{0\leq t\leq 2} \max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix}$$



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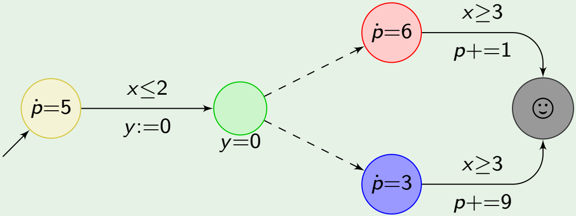
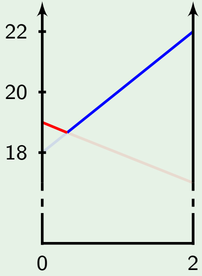
Example



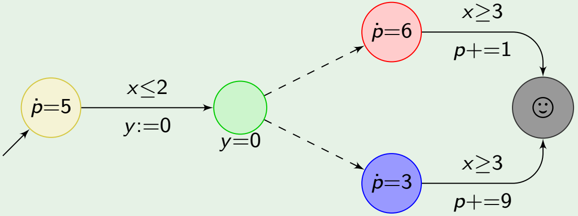
Minimal cost for reaching ☺:

$$\inf_{0\le t\le 2} \max \begin{pmatrix} 5t + 6(3-t) + 1 \\ 5t + 3(3-t) + 9 \end{pmatrix} = 18.66$$



Cost-optimal reachability in priced timed games

Example



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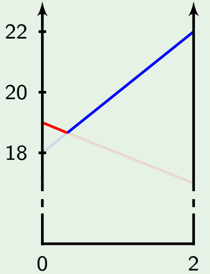
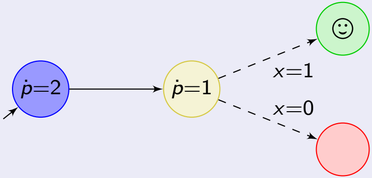
$$(\text{with } t_{\text{opt}} = \frac{1}{3})$$



Looking for optimal strategies...

Optimal strategies need not exist...



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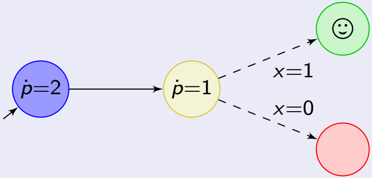
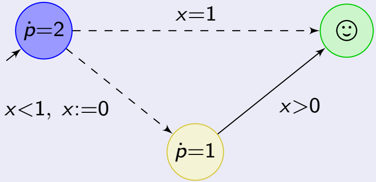


Optimal strategies may need memory...



Cost-optimal reachability in priced timed games

Theorem ([BBR05,BBM06])

Optimal reachability in priced timed games is undecidable.

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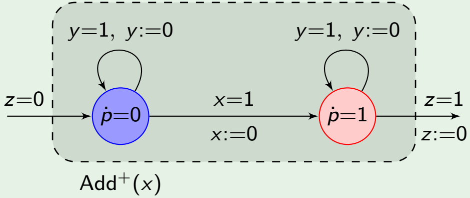
- add the value of clock x to the accumulated cost



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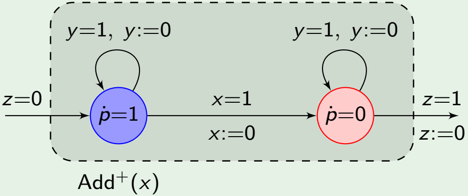
- add $1 - x$ to the accumulated cost



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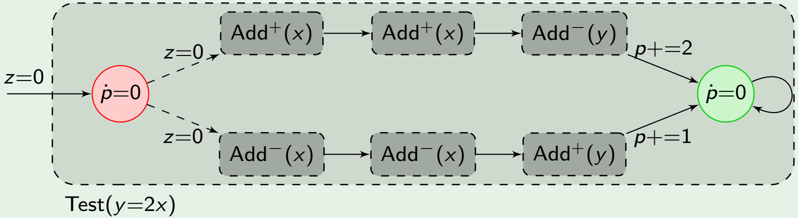
- check that $y = 2x$



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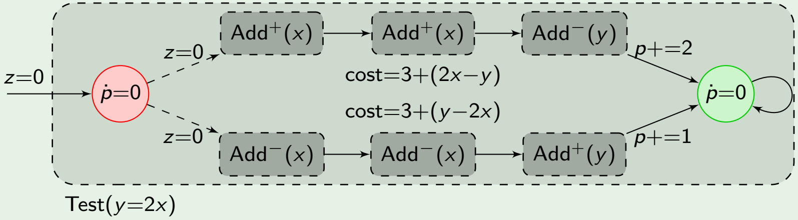

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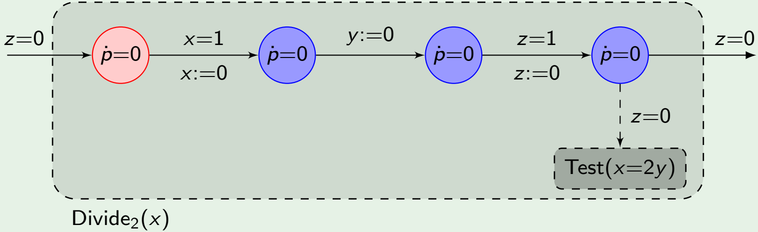
- check that $y = 2x$
- divide clock x by 2



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$\rightsquigarrow$ We can use the following encoding:

$$x_1 = \frac{1}{2^{c_1}} \qquad x_2 = \frac{1}{2^{c_2}}$$

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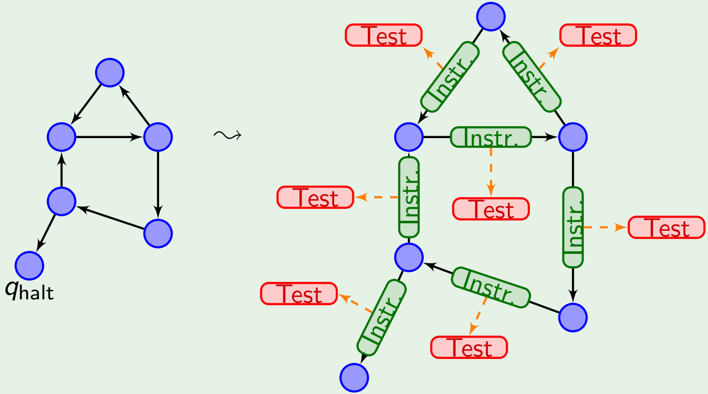

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reach terminal location with
total weight at most 3

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Optimal cost in a priced timed game:

$\text{optcost}_{\mathcal{G}} = \inf\{\text{cost}(\sigma) \mid \sigma \text{ winning strategy in } \mathcal{G}\}$

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Optimal cost in a priced timed game:

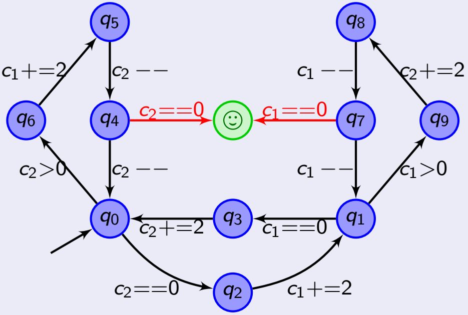
$\text{optcost}_{\mathcal{G}} = \inf\{\text{cost}(\sigma) \mid \sigma \text{ winning strategy in } \mathcal{G}\}$

The existence of a strategy with cost less than k is undecidable.

What about deciding if $\text{optcost}_{\mathcal{G}} \leq k$?

Undecidability of the value problem

Trying to reuse the previous reduction...



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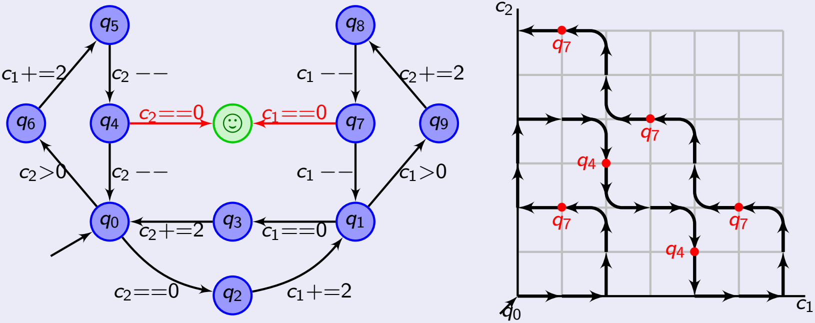

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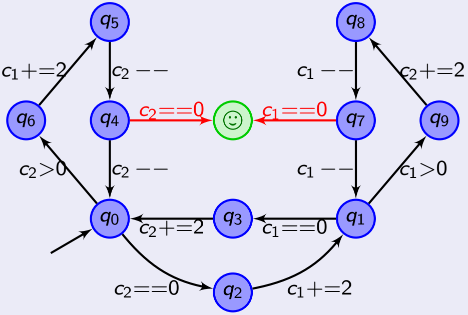

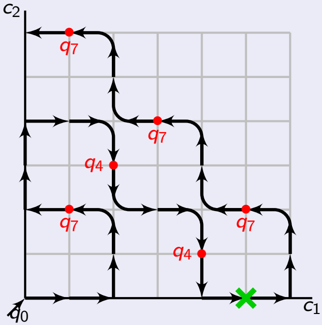

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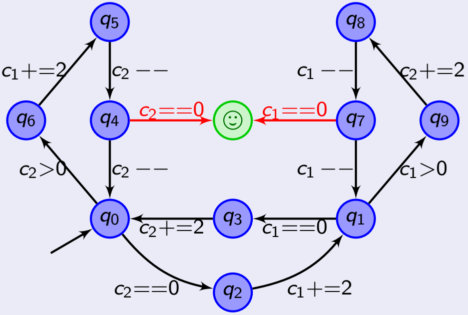

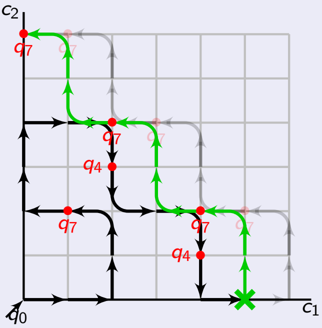

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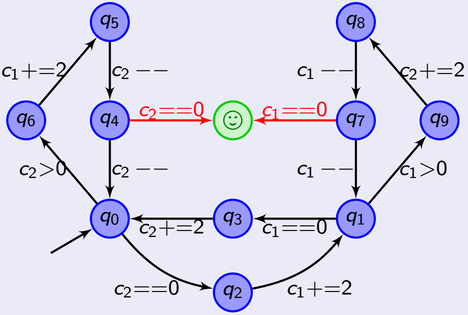
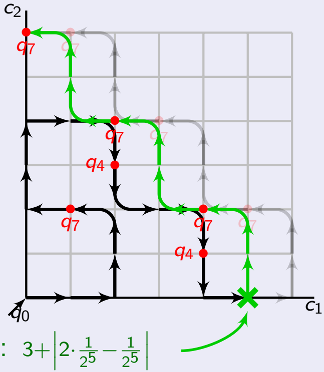
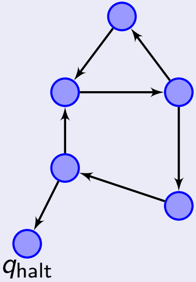




The value of the game is 3, but there is no optimal strategy...

Undecidability of the value problem

Adapting the previous reduction...



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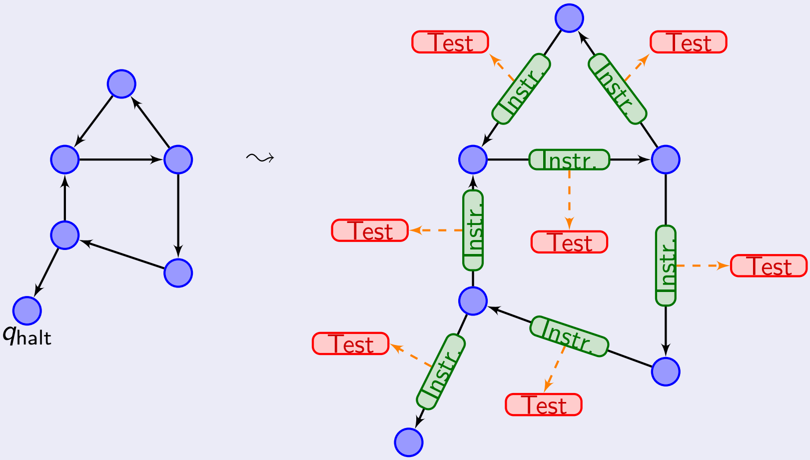

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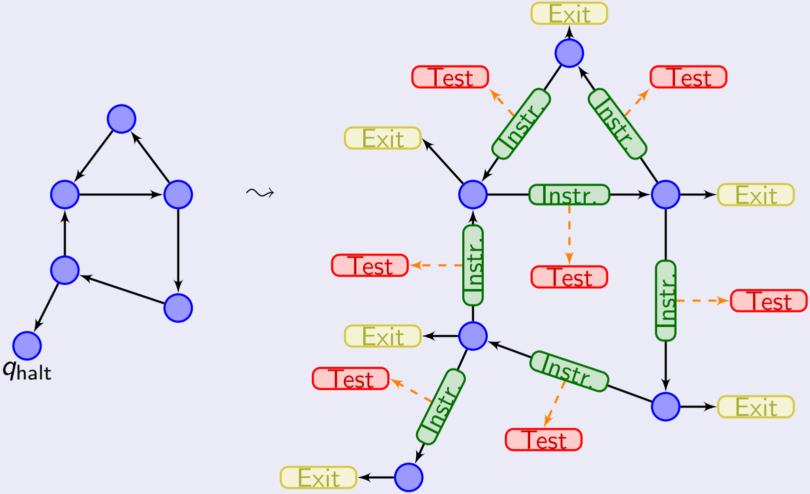
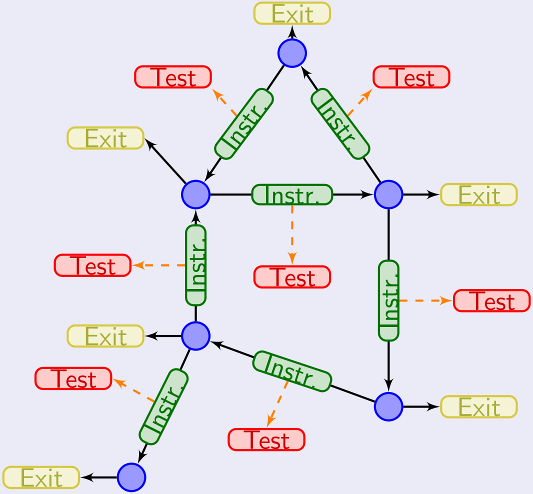


exit nodes: cost $3 + \frac{1}{2^n}$
(n = length of path)

Undecidability of the value problem

Adapting the previous reduction...



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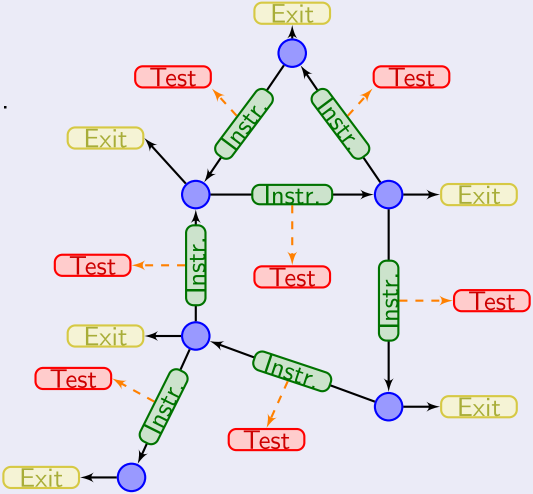
Adapting the previous reduction...

- **if $\mathcal{M}$ does not halt:** Player 1 simulates correctly until $2^n > \frac{1}{\epsilon}$.

  $\rightsquigarrow$ $\text{cost}(\sigma) \leq 3 + \epsilon$



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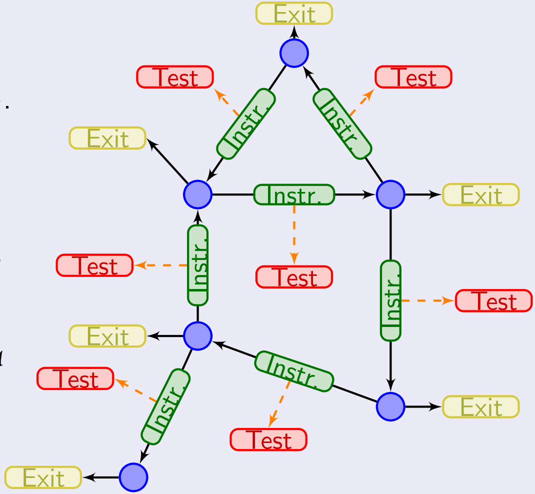
- **if $\mathcal{M}$ halts:** correct simulation for finite duration.

  $\rightsquigarrow$ $\text{cost}(\sigma) \geq 3 + \alpha_{\mathcal{M}}$ for all σ



exit nodes: cost $3 + \frac{1}{2^n}$
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Undecidability of the value problem

Theorem ([BJM15])

The value problem is undecidable in priced timed games.

[BJM15] Bouyer, Jaziri, Markey. On the Value Problem in Weighted Timed Games. CONCUR, 2015.

Undecidability of the value problem

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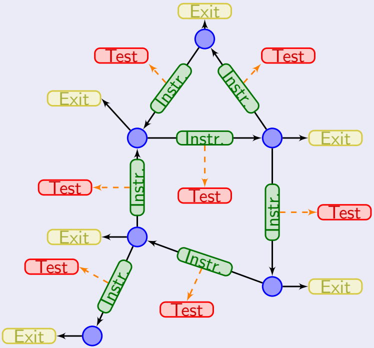
Remark

- blue nodes and intermediary instruction modules have cost zero everywhere;
- positive weights only occur in acyclic parts.



[BJM15] Bouyer, Jaziri, Markey. On the Value Problem in Weighted Timed Games. CONCUR, 2015.

Approximation of the optimal cost

Definition

A priced timed game $\mathcal{G}$ is almost-strongly non-Zeno if there exists $\kappa > 0$ for any run ρ that starts and ends in the same region:

$$\mathsf{cost}(\rho) \geq \kappa \qquad \text{or} \qquad \mathsf{cost}(\rho) = 0$$

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Theorem ([BJM15])

The optimal cost of almost-strongly non-Zeno priced timed automata can be approximated: for every $\epsilon > 0$, we can compute

- *values v_ϵ^+ and v_ϵ^- such that*

$$|v_\epsilon^+ - v_\epsilon^-| < \epsilon \qquad v_\epsilon^- \leq optcost_{\mathcal{G}} \leq v_\epsilon^+$$

- *a strategy σ_ϵ such that*

$$optcost_{\mathcal{G}} \leq cost(\sigma_\epsilon) \leq optcost_{\mathcal{G}} + \epsilon.$$

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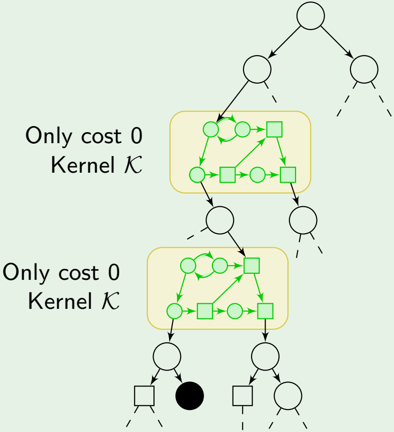
Approximation of the optimal cost

Proof

- **semi-unfolding** of region automaton (seen as a timed game)



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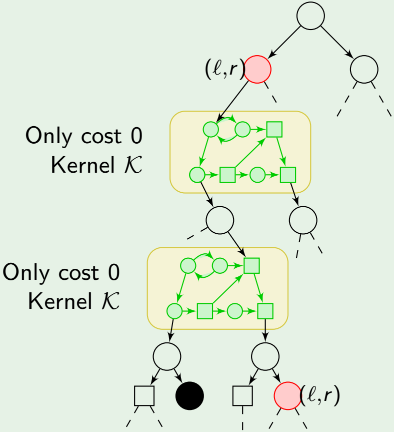

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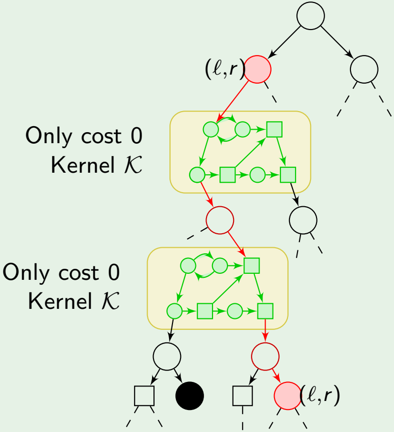

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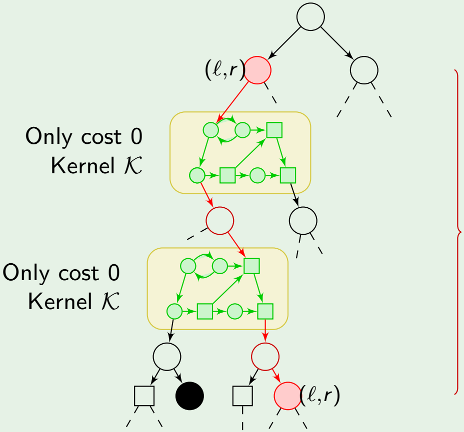

Hypothesis:

cost > 0

$\downarrow$

cost $\geq \kappa$

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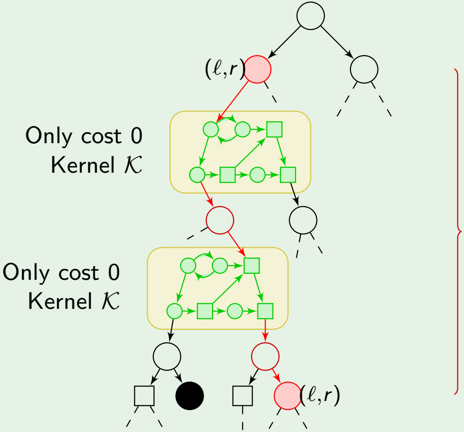

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$\rightsquigarrow$ bounded depth

Approximation of the optimal cost

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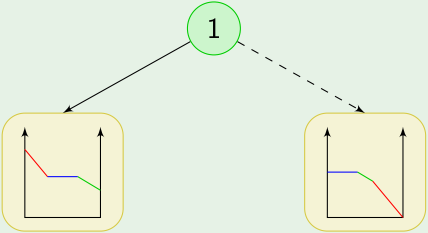
- **semi-unfolding** of region automaton (seen as a timed game)
- compute **exact** optimal cost in tree-like parts



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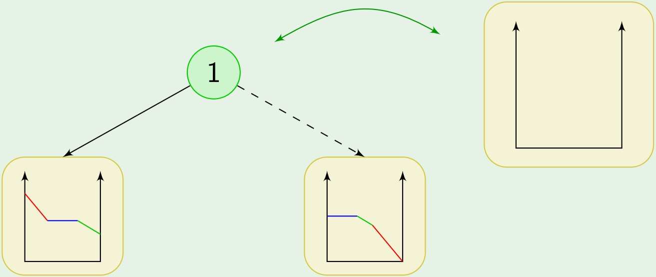

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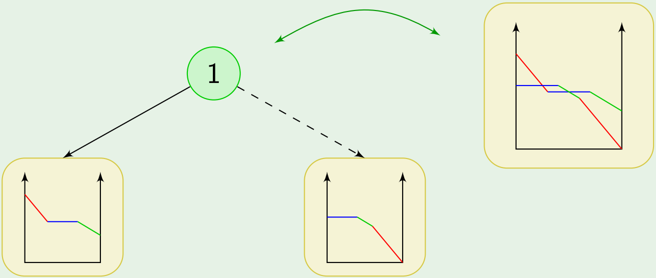

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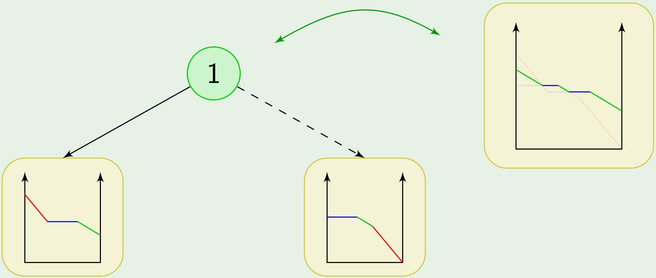

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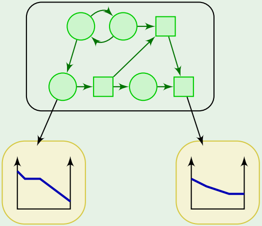

Output cost functions f

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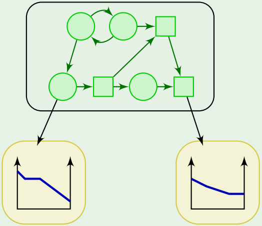
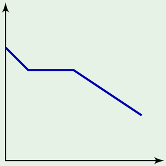
Under- and over-approximate by piecewise constant functions f_ϵ^- and f_ϵ^+





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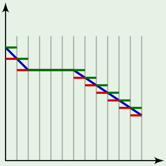

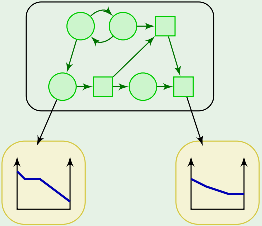

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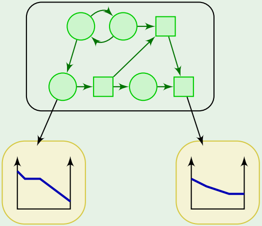

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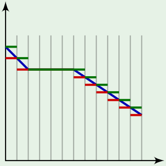

$\rightsquigarrow$ reachability timed game in small regions

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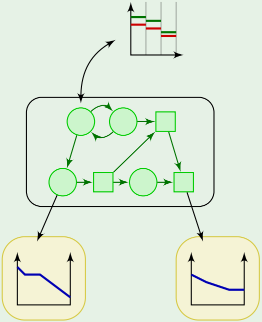

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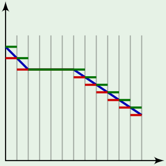

$\leadsto$ reachability timed game in small regions

Outline of the talk

Conclusions and future directions

Priced timed automata and games

- convenient for modelling resources;
- 1-player setting remains tractable (sort of);
- 2-player setting undecidable, but approximable.
- approximation algorithms are a convenient trade-off.

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- 2-player setting undecidable, but approximable.
- approximation algorithms are a convenient trade-off.

Future work

- improve approximation technique (in terms of complexity);
- extend results to whole class of priced timed games;
- average energy and energy constraints;
- robust analysis of priced timed games;
- develop a tool.