Robustness issues in timed models

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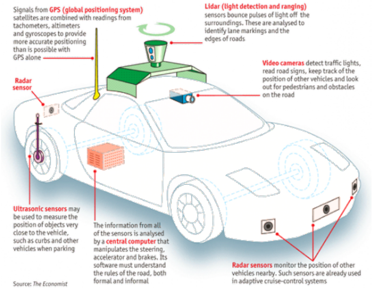
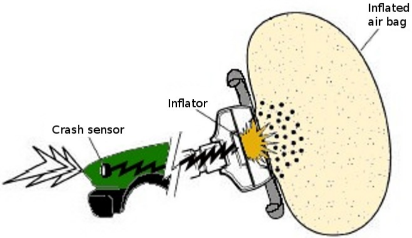
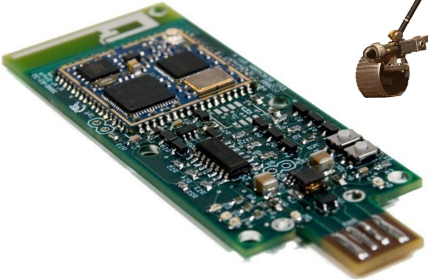
(based on joint works with Patricia Bouyer, Erwin Fang, Pierre-Alain Reynier, Ocan Sankur)
(also starring Martin De Wulf, Laurent Doyen, Jean-François Raskin)

QAPL'14 – Grenoble, France

Modelling real-time systems







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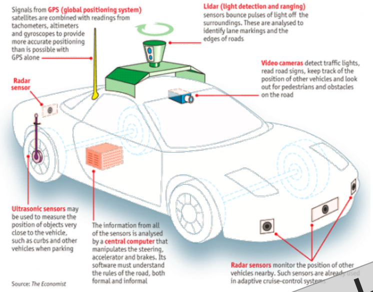
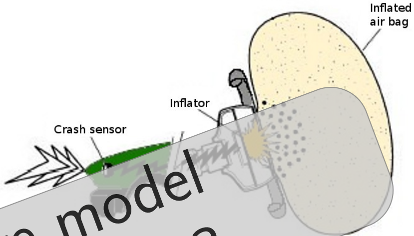
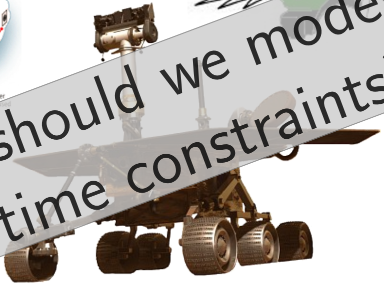
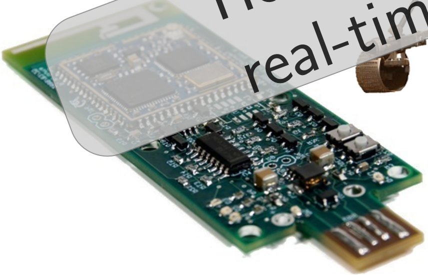




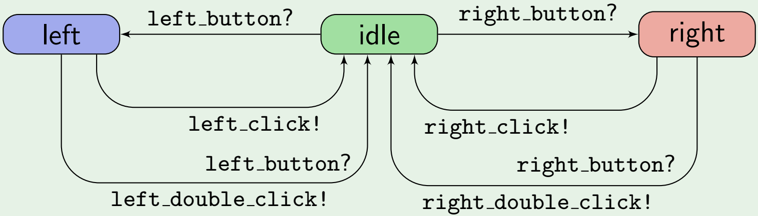




How should we model real-time constraints?

Reasoning about real-time systems

Example (A computer mouse)



Reasoning about real-time systems

Timed automata [AD90]

A timed automaton is made of

- a transition system,

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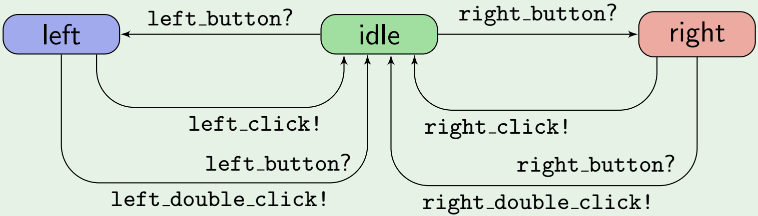

Reasoning about real-time systems

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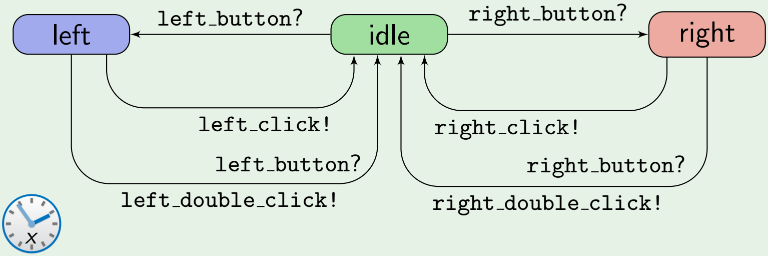


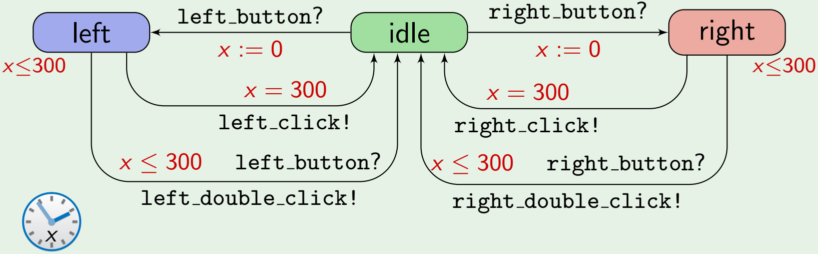
Reasoning about real-time systems

Timed automata [AD90]

A timed automaton is made of

- a transition system,
- a set of clocks,
- timing constraints on states and transitions.

Example (A computer mouse)



Discrete-time semantics

...because computers are digital!

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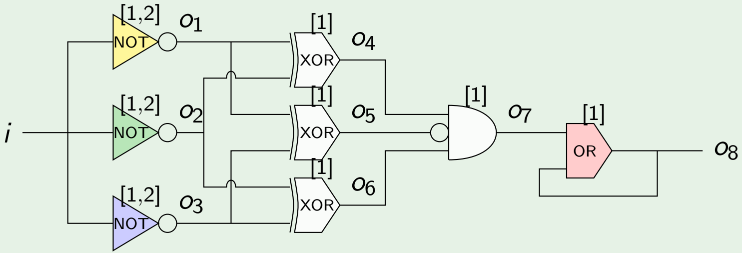
Example ([Alur91])



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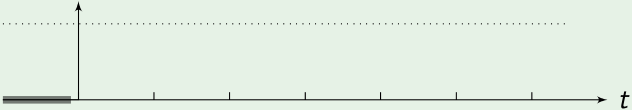

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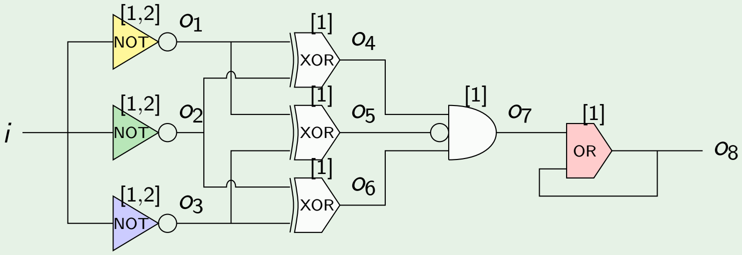

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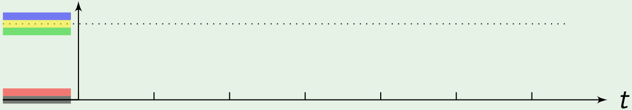

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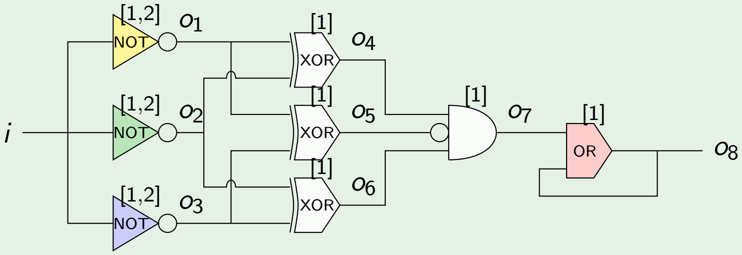

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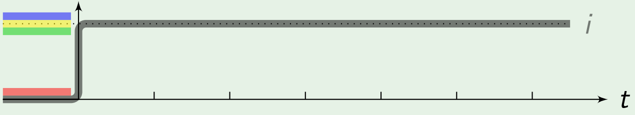

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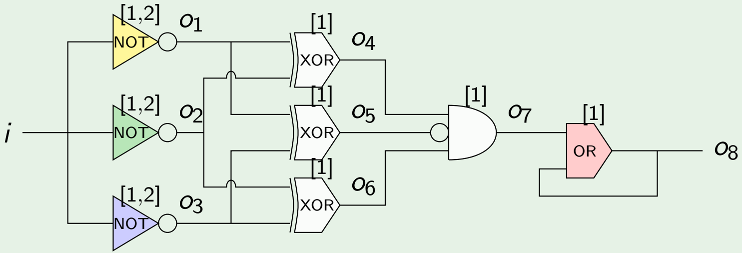

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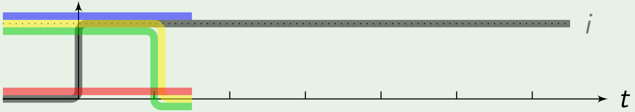

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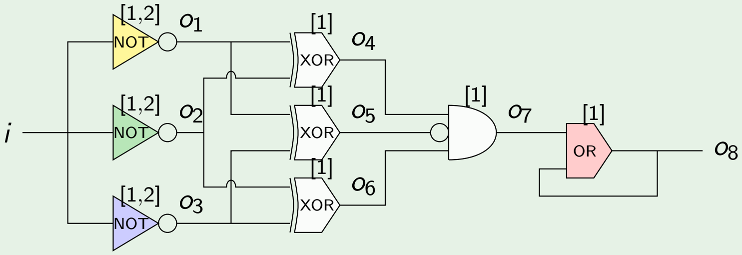

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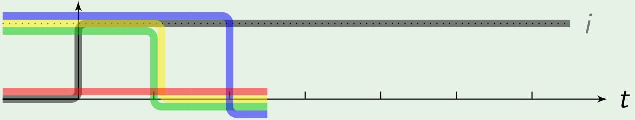

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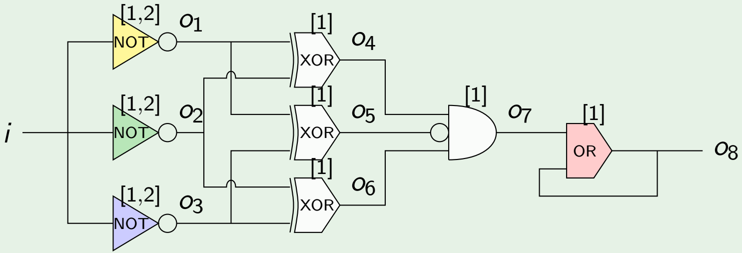

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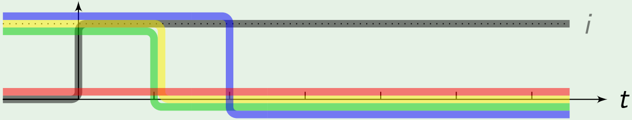

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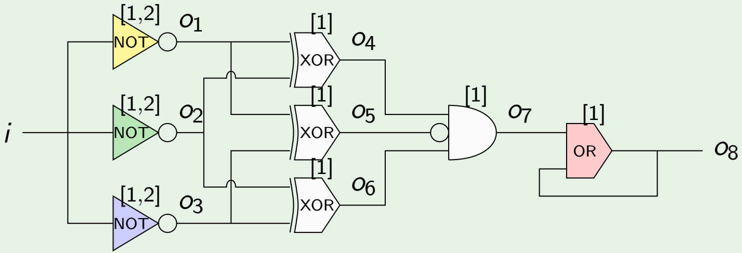
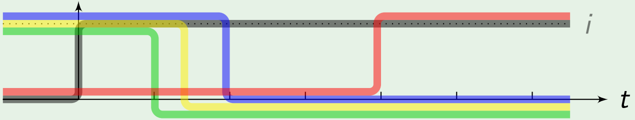


- under continuous-time, the output can change to 1:



Continuous-time semantics

...real-time models for real-time systems!

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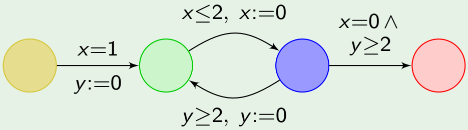

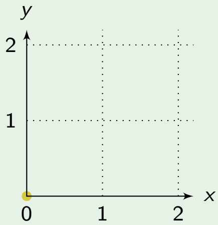

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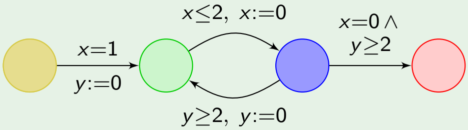

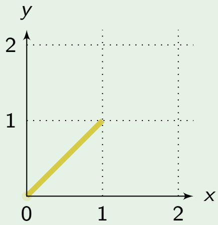

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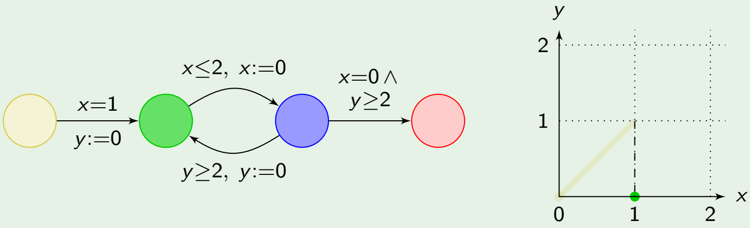

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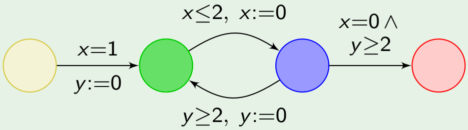

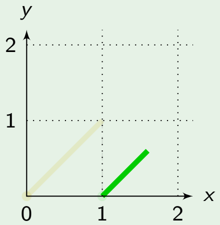

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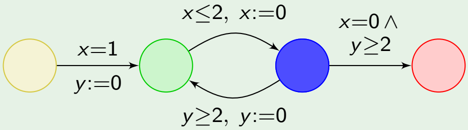

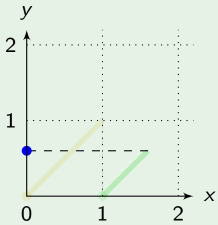

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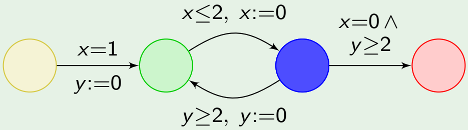

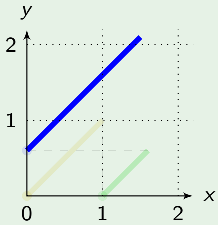

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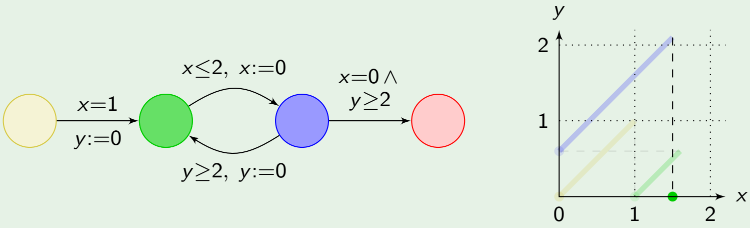

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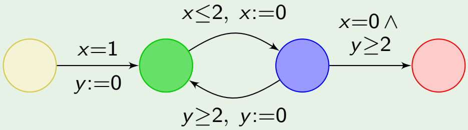

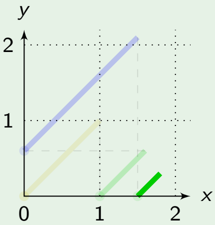

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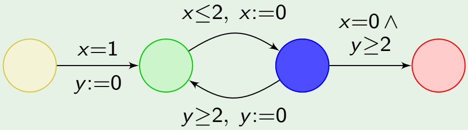

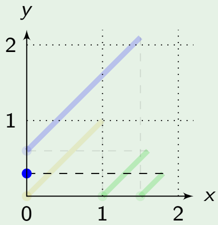

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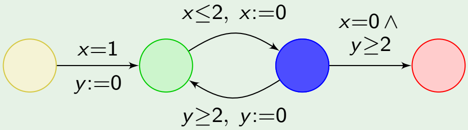

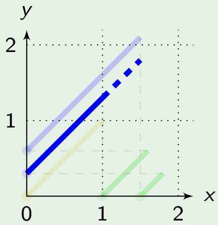

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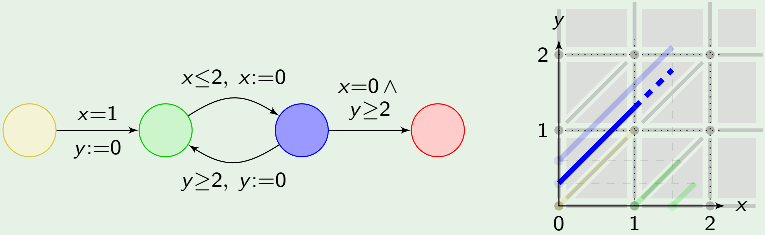

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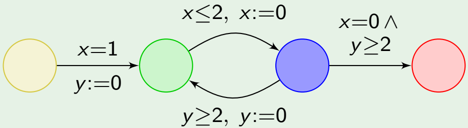
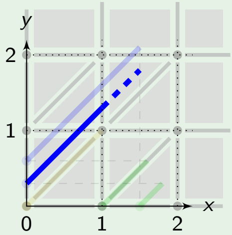
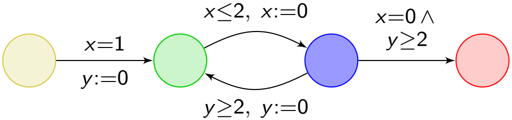




Theorem ([AD90,ACD93, ...])

Reachability in timed automata is decidable (as well as many other important properties).

Regions and zones



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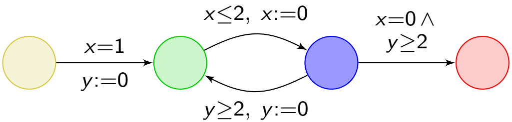
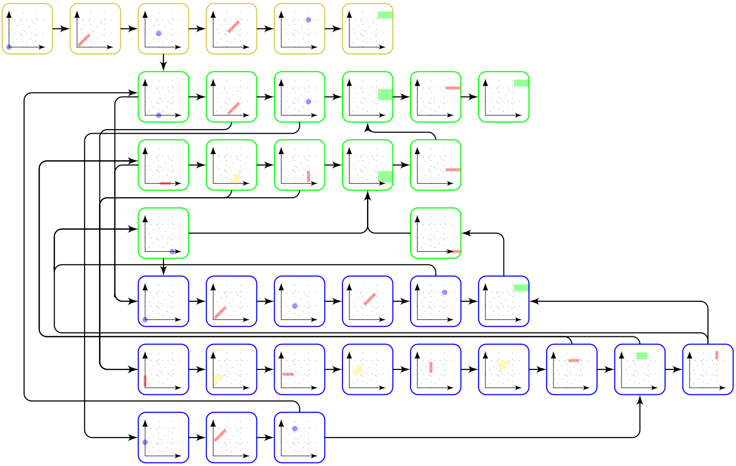


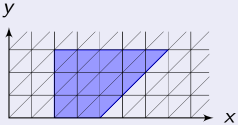


Regions and zones

Zones

Zones are a coarser abstraction:

$$(x \geq 2) \wedge (0 \leq y \leq 3) \wedge (x - y \leq 4)$$



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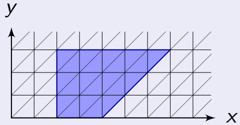
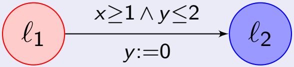


Representation as DBM:

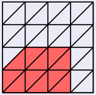
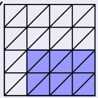
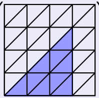
$$\begin{array}{c} \\ 0 \\ x \\ y \end{array}\begin{array}{c} \begin{array}{ccc} 0 & x & y \end{array} \\ \begin{pmatrix} 0 & -2 & 0 \\ +\infty & 0 & 4 \\ 3 & +\infty & 0 \end{pmatrix} \end{array} \equiv \begin{array}{c} \\ 0 \\ x \\ y \end{array}\begin{array}{c} \begin{array}{ccc} 0 & x & y \end{array} \\ \begin{pmatrix} 0 & -2 & 0 \\ 7 & 0 & 4 \\ 3 & 1 & 0 \end{pmatrix} \end{array}$$

Regions and zones

Zones



The predecessors of $(\ell_2, x \leq 3 \wedge y - x \leq 0)$ are computed as

 $= \text{Pre}_{\text{time}} \Big($  $\cap\ \text{Unreset}_y \big($  $\big)\Big)$

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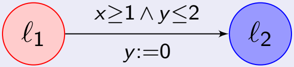

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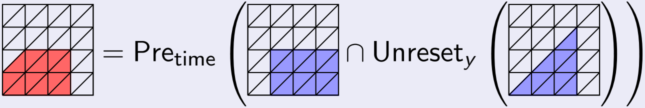
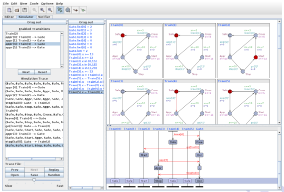


$\rightsquigarrow$ efficient implementations



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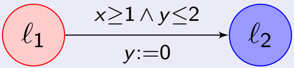

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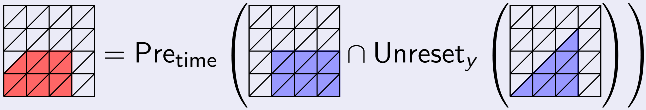

- $\rightsquigarrow$ efficient implementations
- $\rightsquigarrow$ successful applications

BANG & OLUFSEN

Outline of the talk

1. Discrete time vs. dense time
2. From models to implementations
3. Checking robust safety
   - Enlarging clock constraints
   - Shrinking clock constraints
4. Checking robust controllability
   - Parametrized perturbations
   - Permissive strategies
5. Conclusions and future works

Outline of the talk

From models to implementations

Example: Patriot anti-ballistic-missile failure

25 February 1991, during Gulf war.

28 soldiers died.

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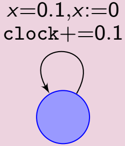
Problem: clock drift

Internal clock incremented by 1/10 every 1/10 s.



From models to implementations

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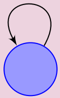
Clock stored in 24-bit register:

$$\frac{1}{10} - \left\langle \frac{1}{10} \right\rangle_{24\text{ bit}} \simeq 10^{-7}$$



After 100 hours, the total drift was 0.34 seconds.
The incoming missile could not be destroyed.

From models to implementations

the continuous-time semantics is a mathematical idealization

- it assumes zero-delay transitions;
- it assumes infinite precision of the clocks;
- it assumes immediate communication between systems.

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- it assumes infinite precision of the clocks;
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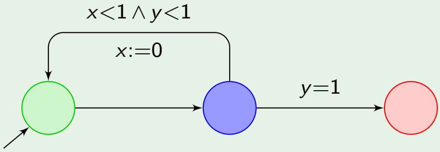
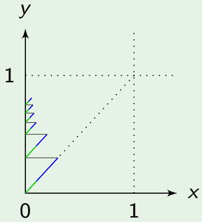
Example (Zeno behaviors)





From models to implementations

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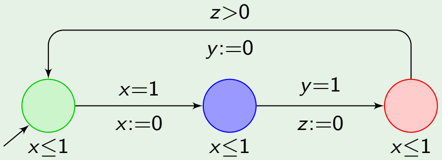

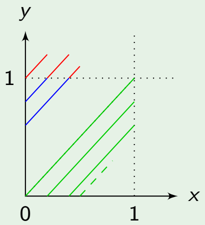

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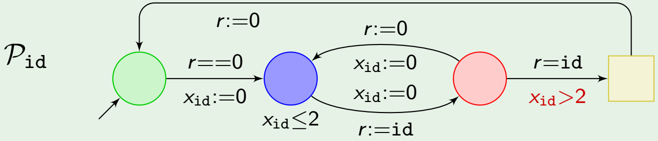
Example (Strict timing constraints)



When P_1 and P_2 run in parallel (sharing variable r), the state where both of them are in ▢ is not reachable.

This property is lost when $x_{\mathtt{id}} > 2$ is replaced with $x_{\mathtt{id}} \geq 2$.

From models to implementations

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- parametrized discrete-time semantics:

  Does there exists a time step δ (*sampling rate*) under which the system behaves correctly?

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- parametrized continuous-time semantics:

  Does the system behave correctly under continuous-time semantics with imprecisions up to some δ?

Outline of the talk

Enlarged semantics for timed automata

a transition can be taken at any time in $[t - \delta; t + \delta]$.

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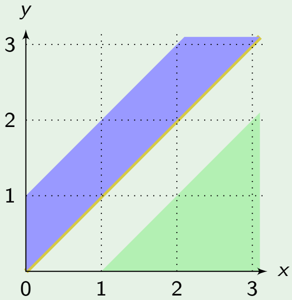

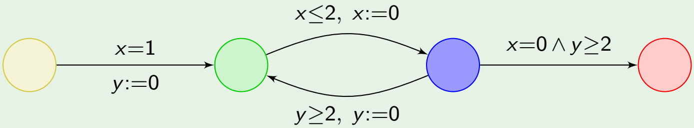

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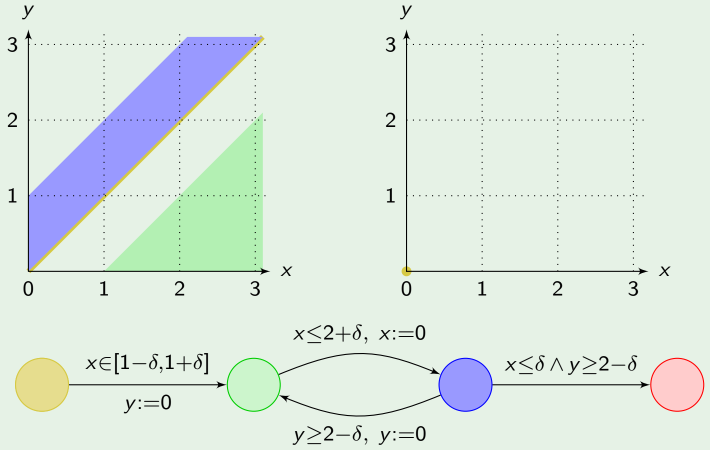

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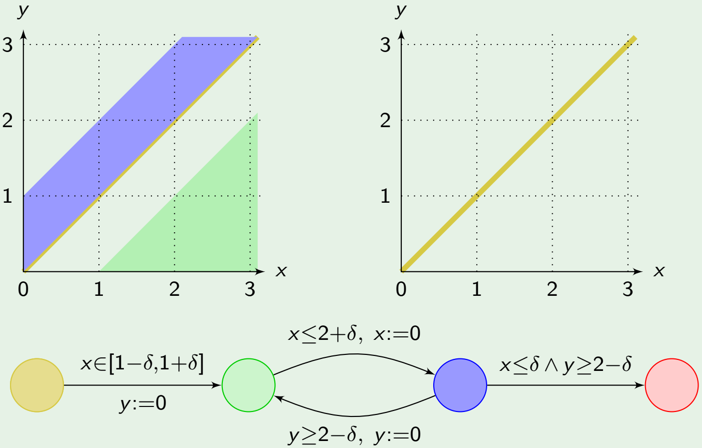

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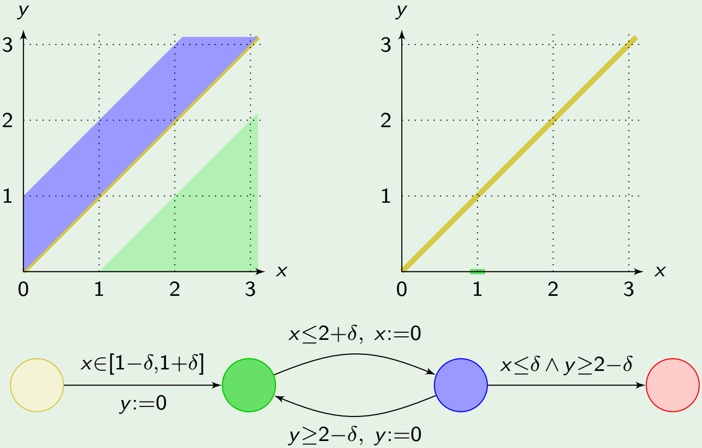

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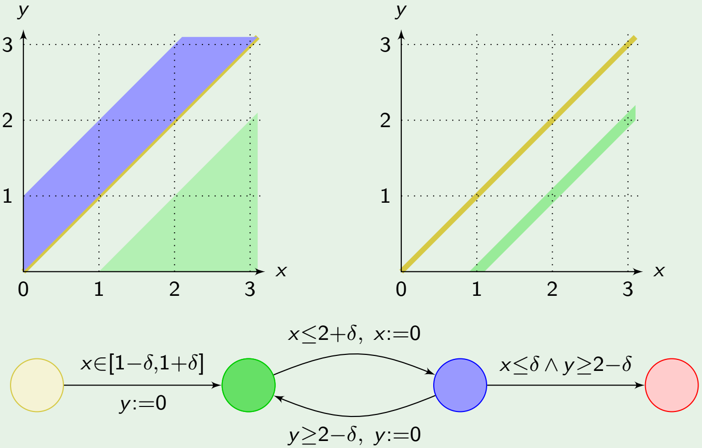

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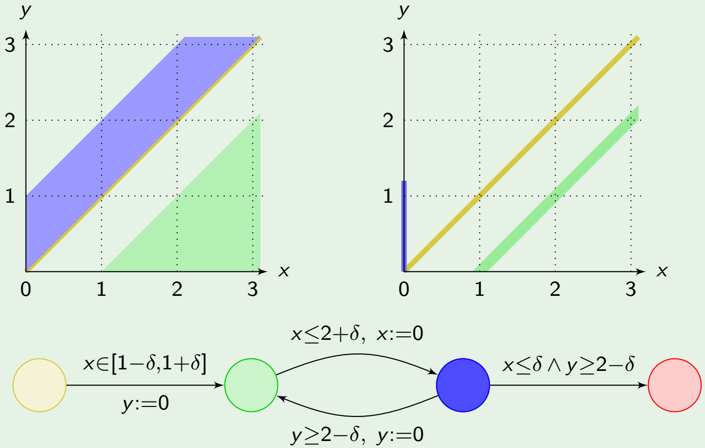

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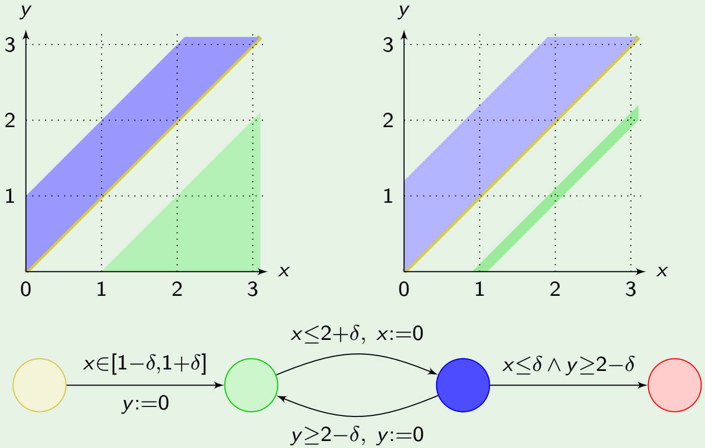

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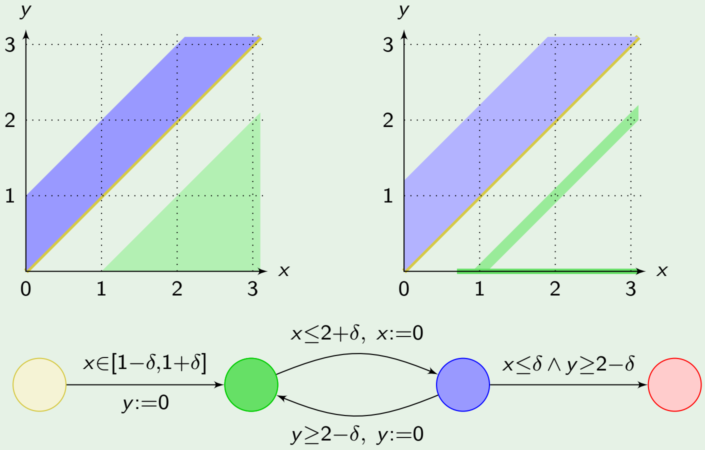

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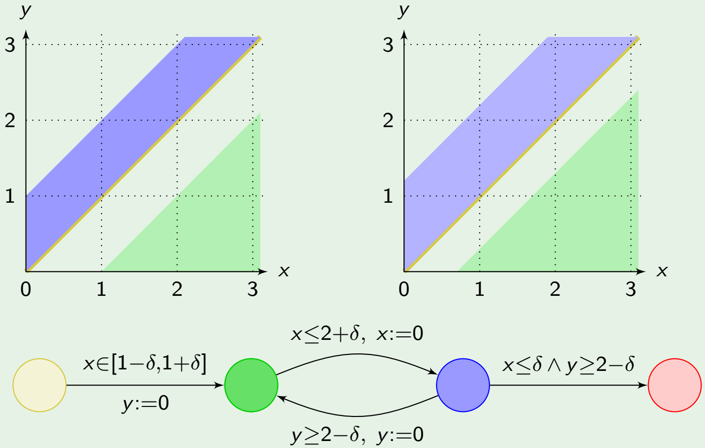

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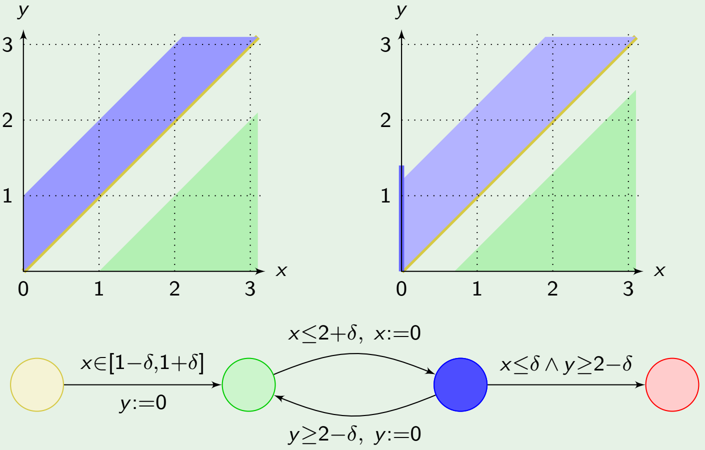

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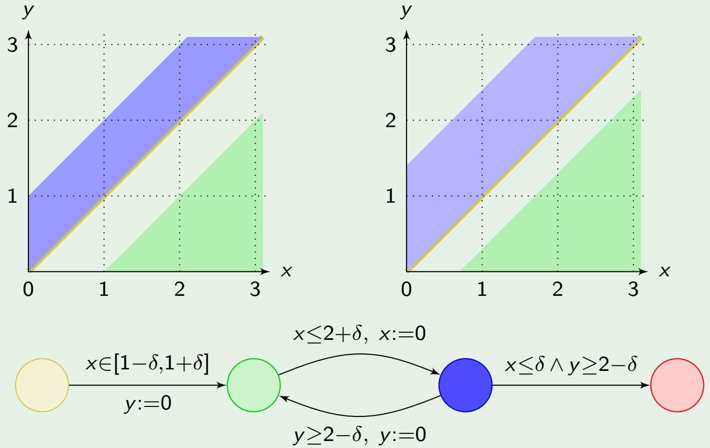

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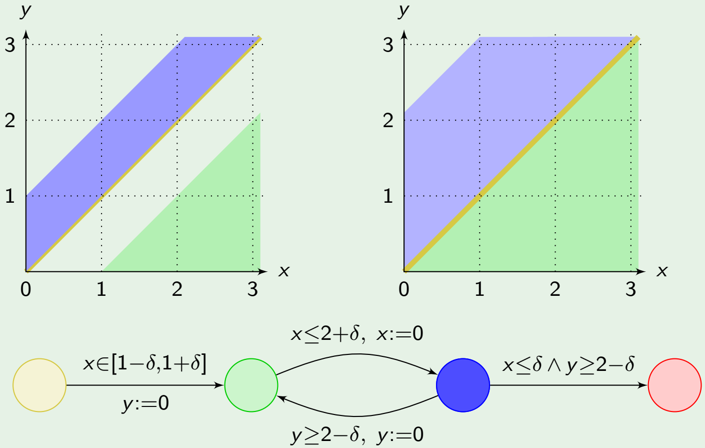

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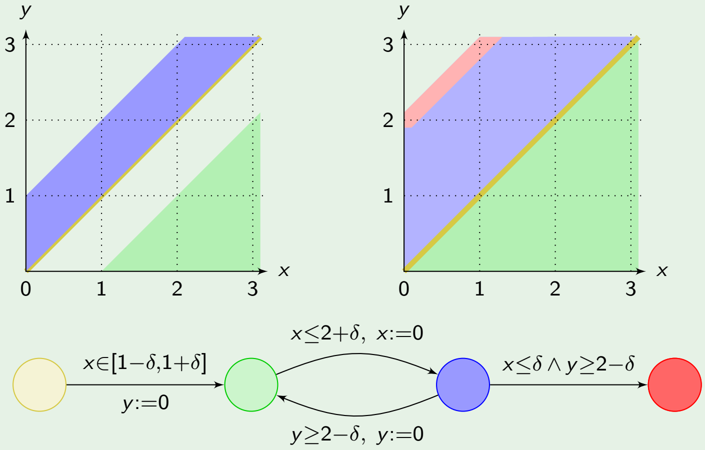

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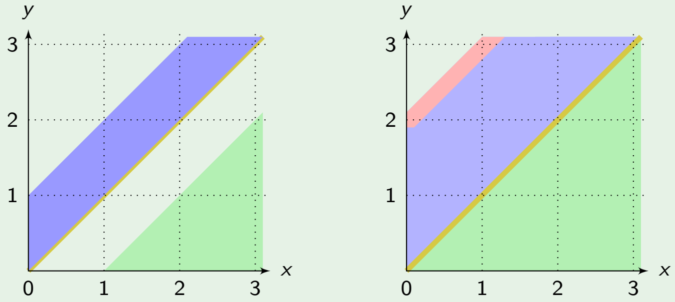


Theorem ([Pur98,DDMR04])

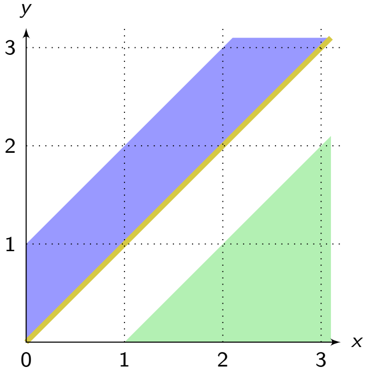
Parametrized robust safety is decidable.

Extended region automaton

For any location ℓ and any two regions r and r', if

- $\overline{r} \cap \overline{r'} \neq \varnothing$ and
- (ℓ, r') belongs to an SCC of $\mathcal{R}(\mathcal{A})$,

then we add a transition $(\ell, r) \xrightarrow{\gamma} (\ell, r')$.



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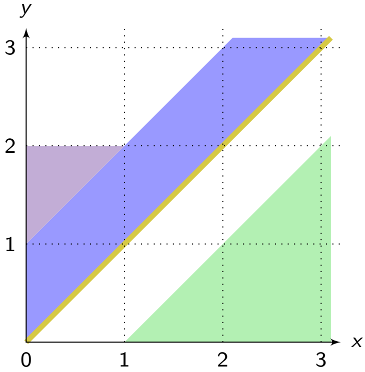

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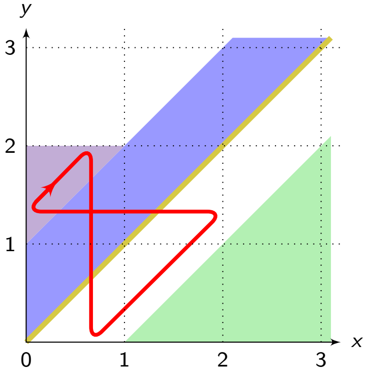

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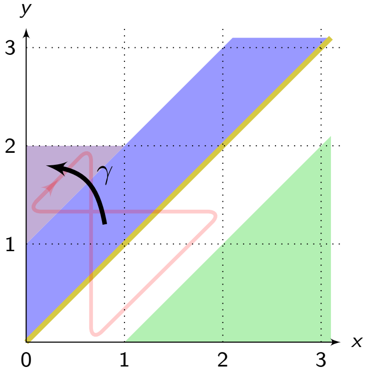

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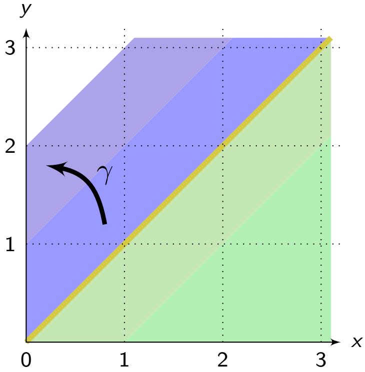

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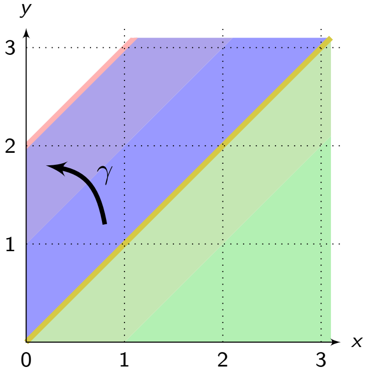

Shrinking timing constraints

Counteracting guard enlargement

Shrinking turns constraints $[a, b]$ into $[a + \delta, b - \delta]$.

In particular, punctual constraints become empty.

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Theorem ([SBM11])

Shrinkability is decidable in EXPTIME.

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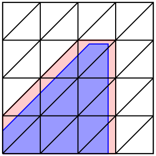
Main tools: parametrized shrunk DBMs
max-plus fixpoint equations



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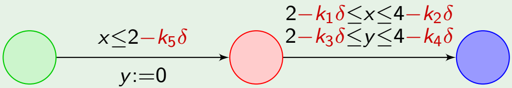
$\rightsquigarrow$ prototype tool:

`http://www.lsv.ens-cachan.fr/Software/shrinktech/`

Shrinking timing constraints

Example



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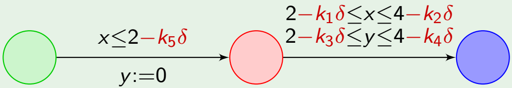

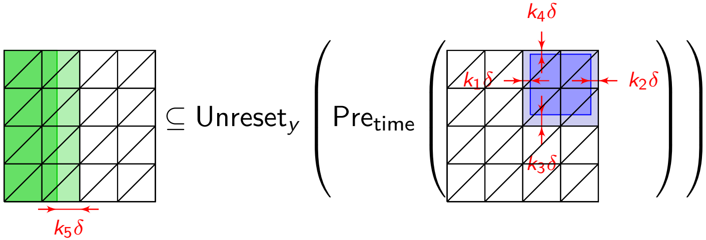

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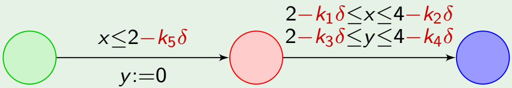

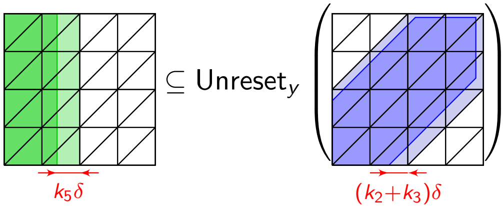

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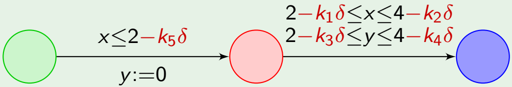

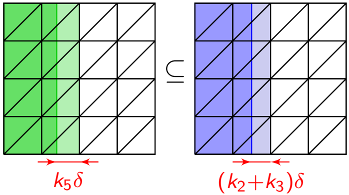

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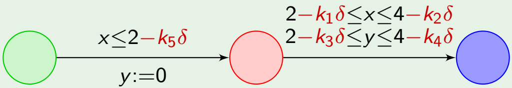

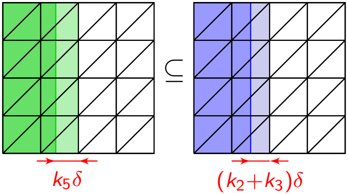

$$\rightsquigarrow \qquad k_5 = \max(k_5, k_2 + k_3)$$

Outline of the talk

Game-based approach to robustness

Solving robust reachability

- Player 1 proposes a delay d and a transition t;
- transition t is taken after some delay in $[d-\delta, d+\delta]$ chosen by Player 2.

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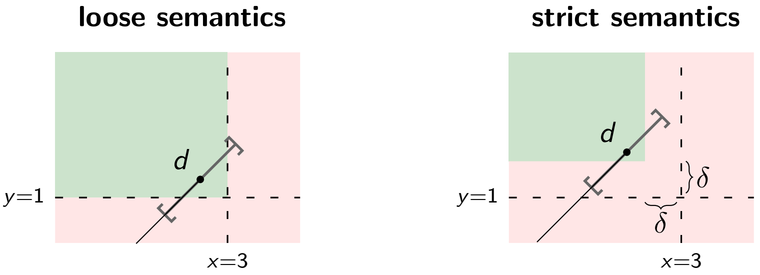
Consider a transition with guard $x \leq 3 \wedge y \geq 1$:



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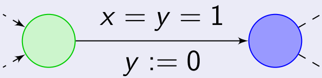
Theorem ([BMS12,SBMR13])

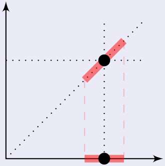
Robust reachability is EXPTIME*-complete in the loose semantics.*

Robust reachability and repeated reachability are PSPACE*-complete in the strict semantics.*

Shrunk DBMs for the loose semantics

Extend the region automaton into a 2-player turn-based game





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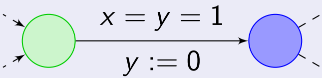
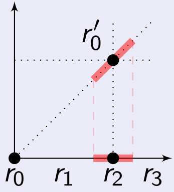
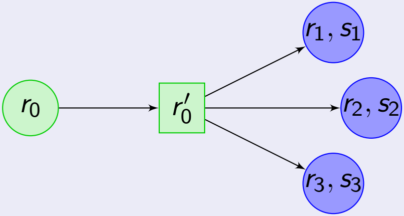
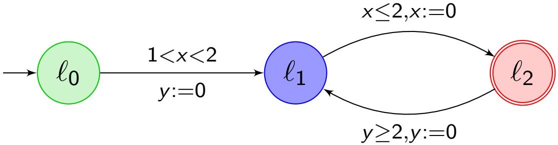
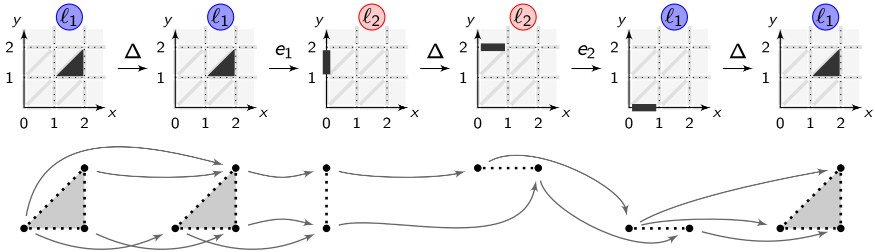






Orbit graphs for the strict semantics





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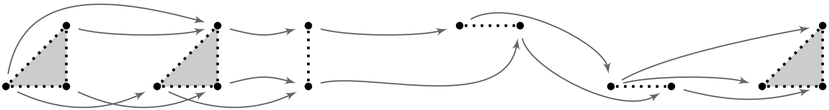

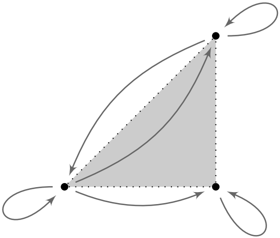

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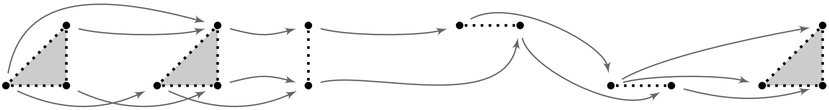

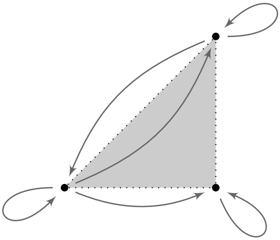

Definition

A cycle π is forgetful if its orbit graph is strongly connected.

A cycle π is aperiodic if π^k is forgetful, for all k.

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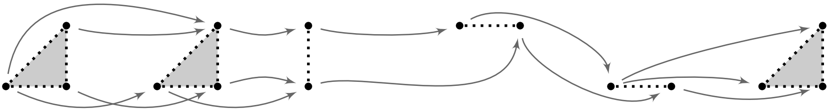

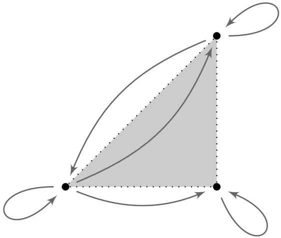

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Theorem

The automaton is robustly controllable if, and only if, it has a reachable aperiodic cycle.

Synthesizing permissive strategies

Permissive strategies

Permissive strategies can propose several moves rather than a single one.

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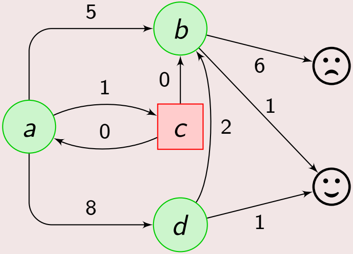

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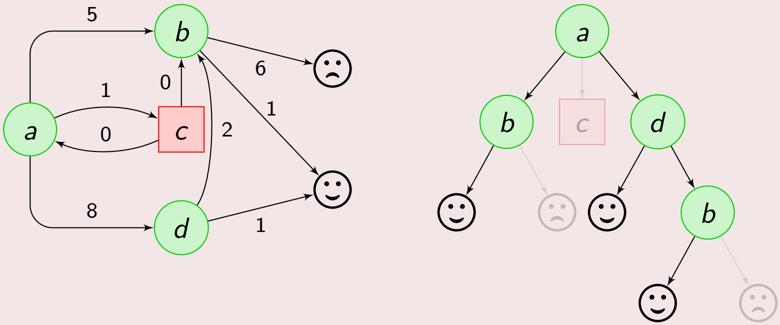
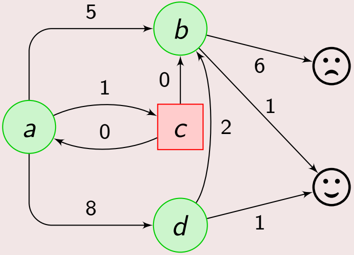


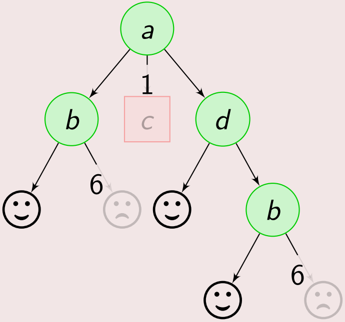
Synthesizing permissive strategies

Permissive strategies

Permissive strategies can propose several moves rather than a single one.

In the untimed setting... [BDMR09, BMOU11]





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In the timed setting...

Permissive strategies propose intervals of delays.

Our setting:

the penalty assigned to interval $[a, b]$ is $1/(b - a)$.

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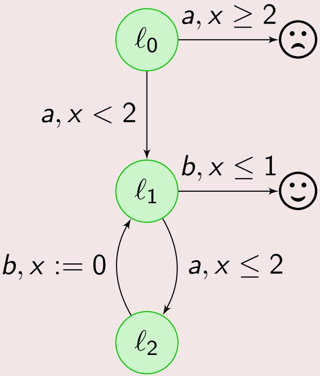

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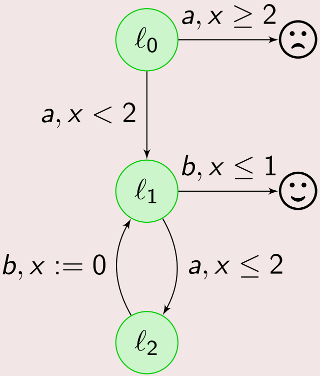

Possible (memoryless) strategy:

- in ℓ_0, play $(a, [0, 2))$;
- in ℓ_1:
  - if $x \leq 1$, play $(b, [0, 1 - x])$;
  - otherwise, play $(a, [0, 2 - x])$;
- in ℓ_2, play $(b, [0, +\infty))$

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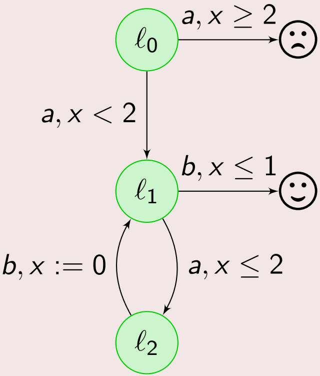

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$\rightsquigarrow$ **penalty** $= +\infty$

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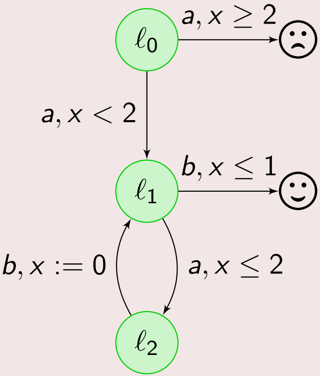

Possible (memoryless) strategy:

- in ℓ_0, play $(a, [0, 1])$;
- in ℓ_1:
  - if $x = 0$, play $(b, [0, 1])$;
  - otherwise, play $(a, [0, 2 - x])$;
- in ℓ_2, play $(b, [0, +\infty))$

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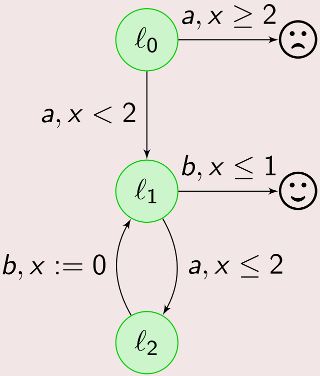

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$\rightsquigarrow$ **penalty = 1**

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In the timed setting...

Theorem

For one-clock timed games:

- *Memoryless optimal-penalty strategies exist.*
- *They can be computed in polynomial time.*

Outline of the talk

1. Discrete time vs. dense time
2. From models to implementations
3. Checking robust safety
   - Enlarging clock constraints
   - Shrinking clock constraints
4. Checking robust controllability
   - Parametrized perturbations
   - Permissive strategies
5. Conclusions and future works

Conclusion and challenges

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Robustness issues identified long ago...

Several attempts, but no satisfactory solution yet!

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Several attempts, but no satisfactory solution yet!

Challenges and open questions

- symbolic algorithms;
- measuring robustness, using distances between automata;
  $\rightsquigarrow$ link between "syntactic distance" and "semantic distance"
- probabilistic approach to robustness;
  $\rightsquigarrow$ evaluate expected time before a new state is visited.
- investigate robustness in weighted timed automata;
  $\rightsquigarrow$ energy constraints;
  $\rightsquigarrow$ imprecision on cost rates;
- synthesis of robust strategies.