Quantified CTL

Nicolas Markey
LSV – ENS Cachan

(joint work with François Laroussinie)

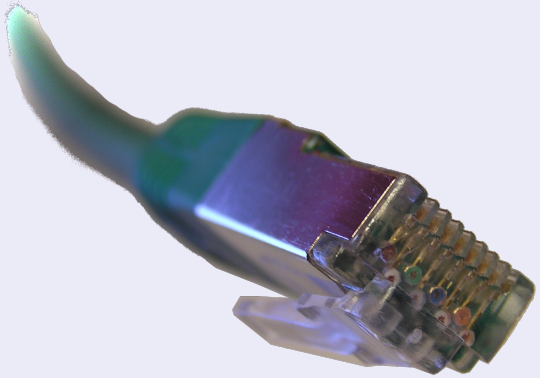
Nord-Pas-de-Calais – Belgium congress of mathematics

Valenciennes, 28 October 2013

Verification of computerised systems

- Computers are everywhere



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- Bugs are everywhere...


News

Toyota to recall Prius hybrids over ABS software

See video, below

By Martyn Williams

February 9, 2010 04:59 AM ET Comments (6) Recommended (15) Like

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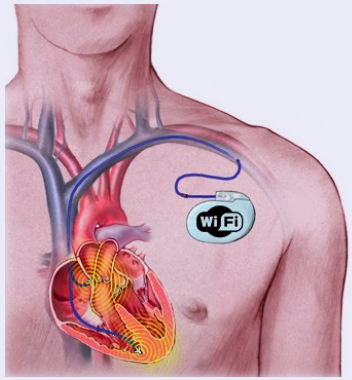

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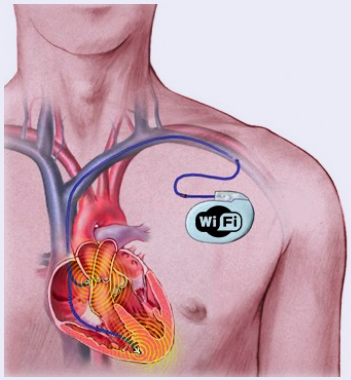
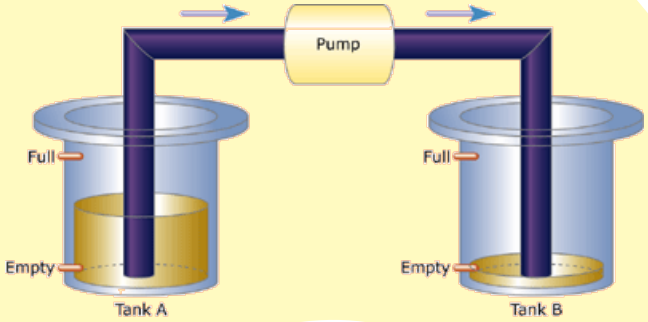


- Verification should be everywhere!

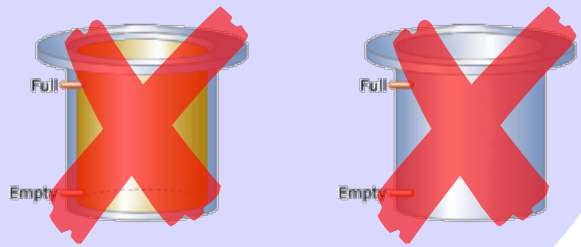
Model checking and synthesis

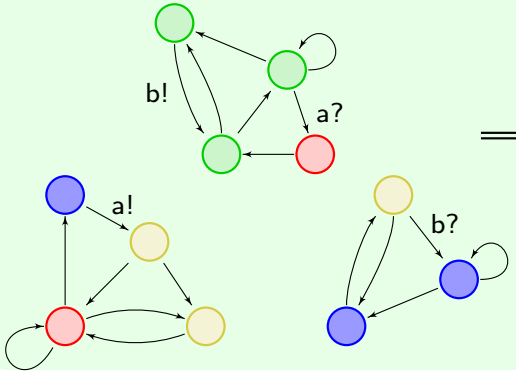
system:



[http://www.embedded.com]

property:



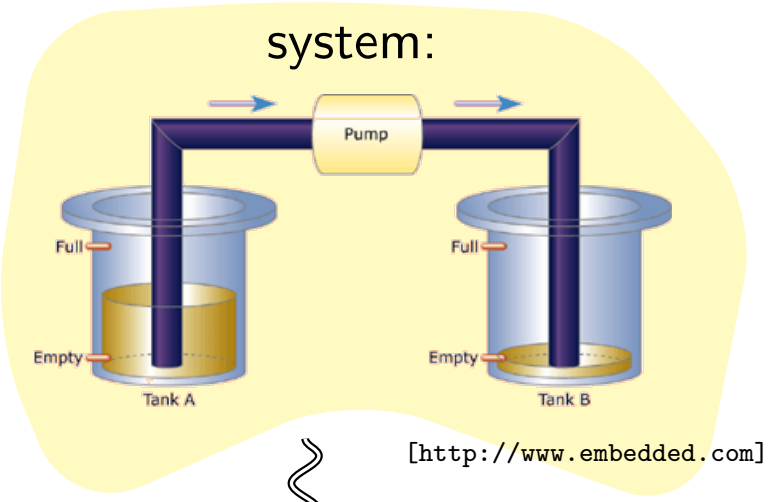


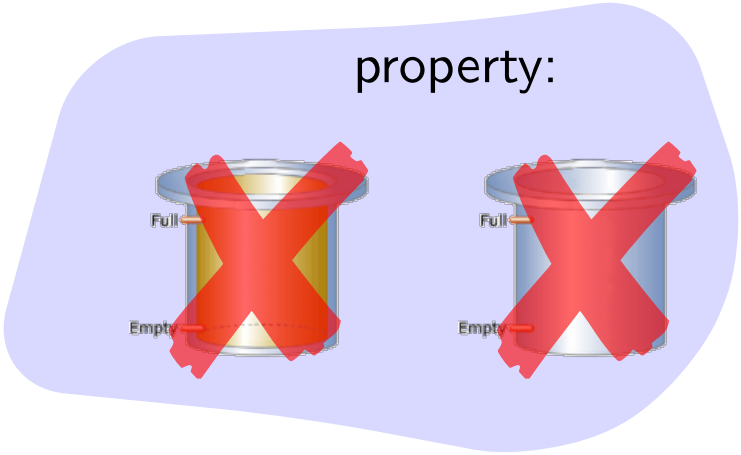
model-checking algorithm

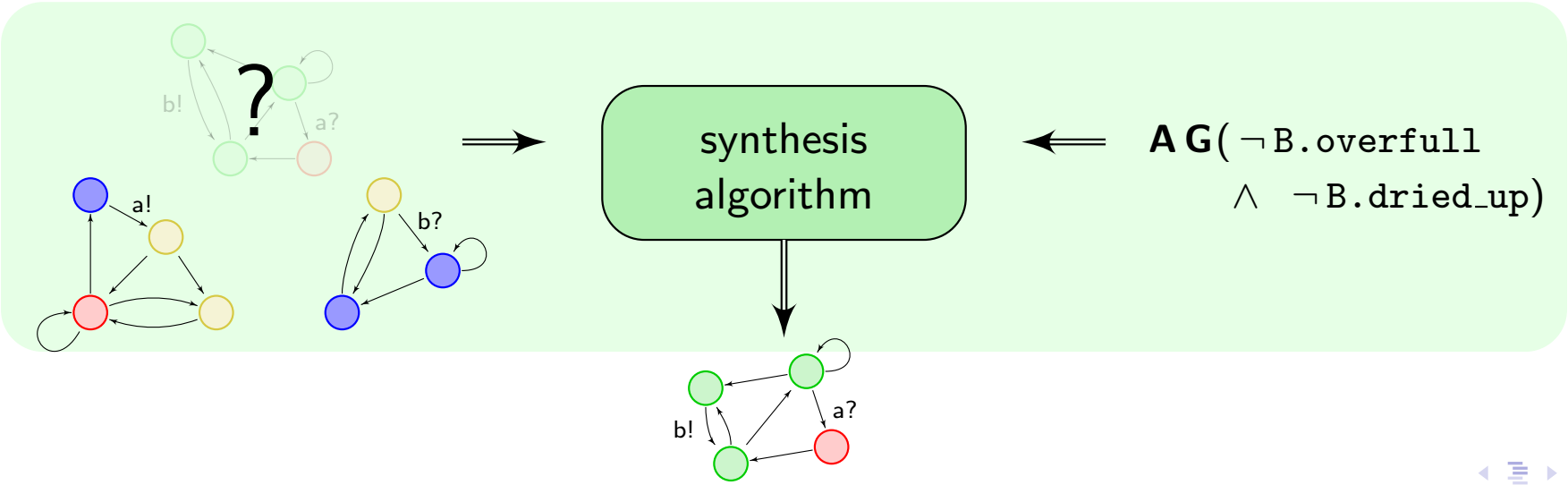
$\mathbf{A}\,\mathbf{G}(\neg\,\texttt{B.overfull} \wedge \neg\,\texttt{B.dried\_up})$

yes/no

Model checking and synthesis







Outline of the presentation

1. Basics about CTL
   - expressing properties of reactive systems
   - efficient verification algorithms
2. Quantified CTL
   - CTL with quantification over atomic propositions
   - model checking and satisfiability are mostly decidable
3. Temporal logics for games: ATL and extensions
   - expressing properties of complex interacting systems
   - QCTL-based decision procedures for ATL_{sc}

Computation-Tree Logic (CTL)

- **atomic propositions:**  ,  , ...

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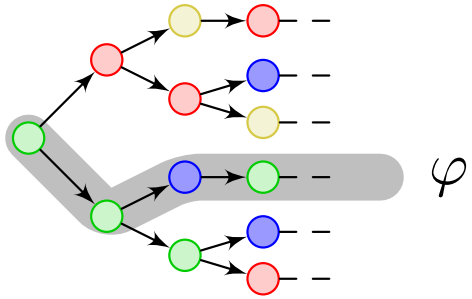
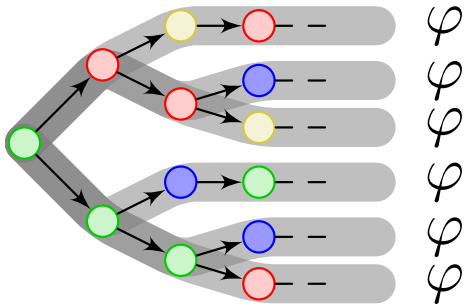
- **path quantifiers:**

Eφ



Aφ



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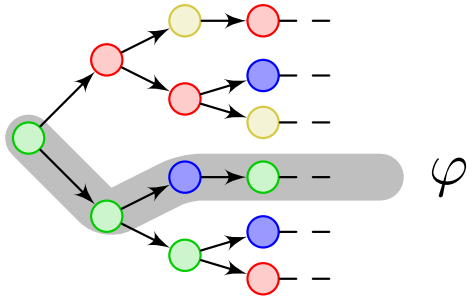

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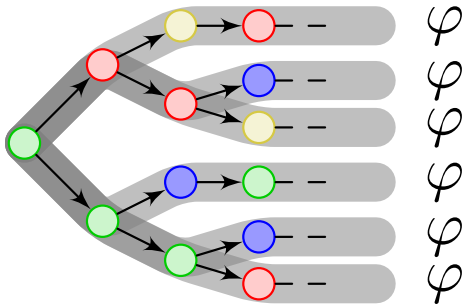
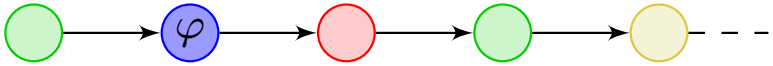


- **temporal modalities:**

X φ



"next φ"

φ **U** ψ

"φ until ψ"

Computation-Tree Logic (CTL)

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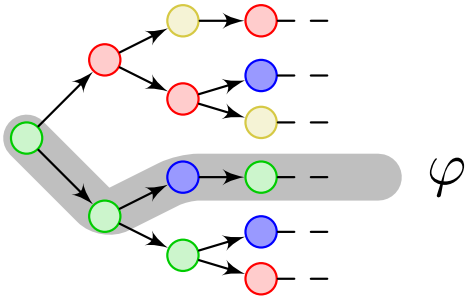
- **path quantifiers:**

$\mathbf{E}\varphi$



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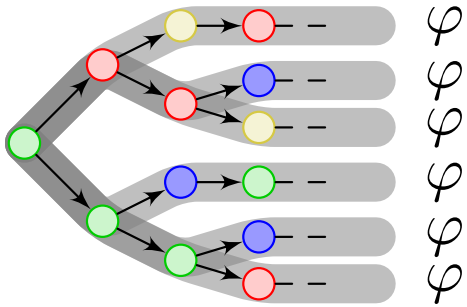
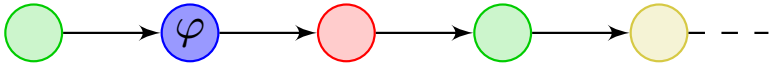
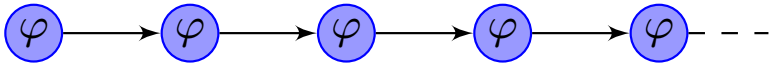


- **temporal modalities:**

$\mathbf{X}\,\varphi$		"next φ"
$\varphi\ \mathbf{U}\ \psi$		"φ until ψ"
$\texttt{true}\ \mathbf{U}\ \varphi \equiv \mathbf{F}\,\varphi$		"eventually φ"
$\neg\,\mathbf{F}\,\neg\varphi \equiv \mathbf{G}\,\varphi$		"always φ"

Examples of CTL formulas

In CTL, each temporal modality is in the immediate scope of a path quantifier.

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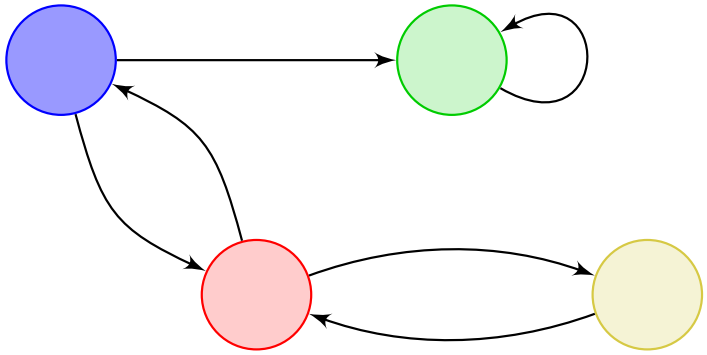
E F   is reachable



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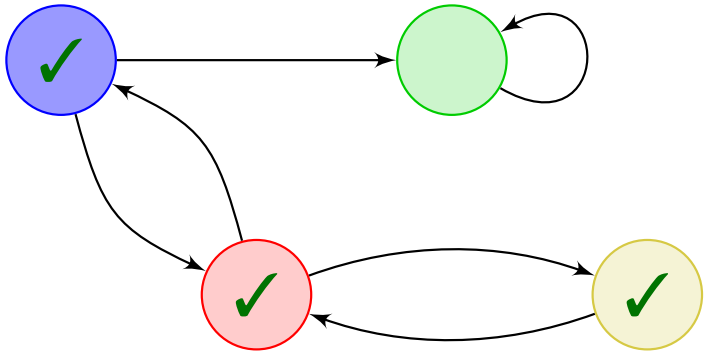
E F ● ● is reachable



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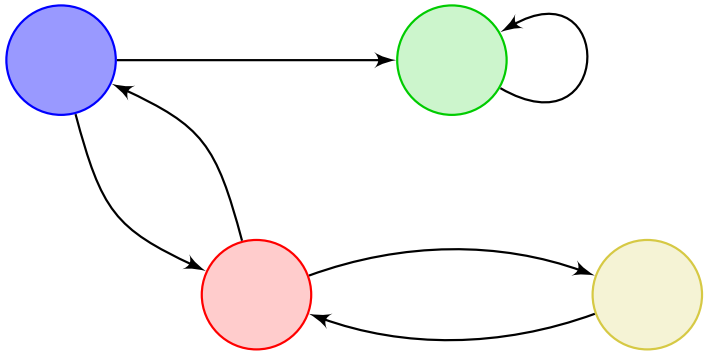
In CTL, each temporal modality is in the immediate scope of a path quantifier.

E G(**E F** ●) there is a path along which ● is always reachable



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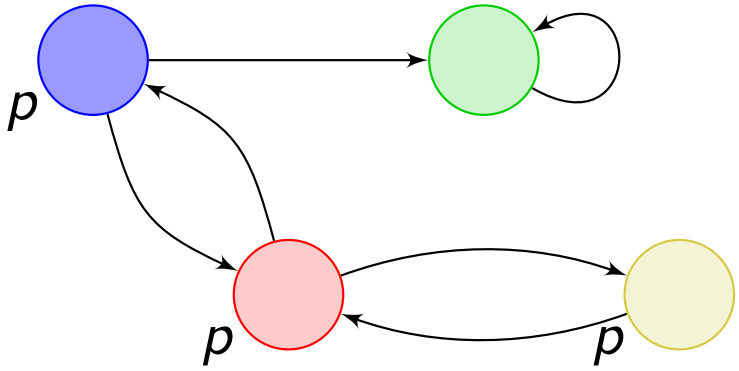
$\mathbf{E}\,\mathbf{G}(\underbrace{\mathbf{E}\,\mathbf{F}\,\bigcirc}_{p})$ there is a path along which $\bigcirc$ is always reachable



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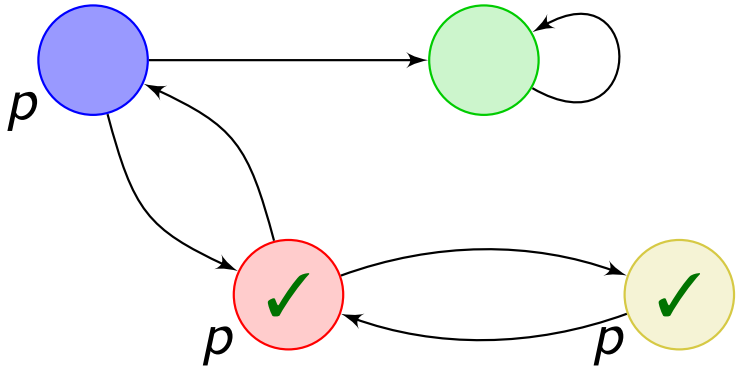

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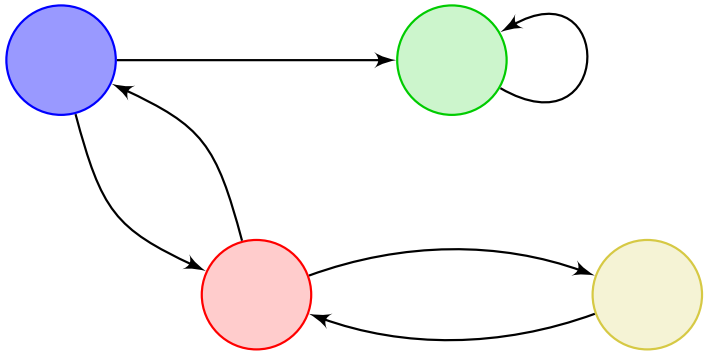
$\neg\, \mathbf{E}(\neg$ ● $)\ \mathbf{U}$ ● in order to reach ●, we have to visit ●



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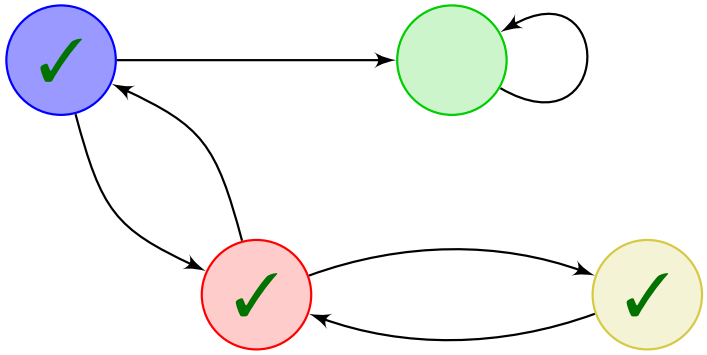

Examples of CTL formulas

In CTL, each temporal modality is in the immediate scope of a path quantifier.

Theorem ([CE81,QS82])

CTL model checking is PTIME*-complete.*

[CE81] Clarke, Emerson. Design and Synthesis of Synchronization Skeletons using Branching-Time Temporal Logic. LOP'81.
[QS82] Queille, Sifakis. Specification and verification of concurrent systems in CESAR. SOP'82.

Examples of CTL formulas

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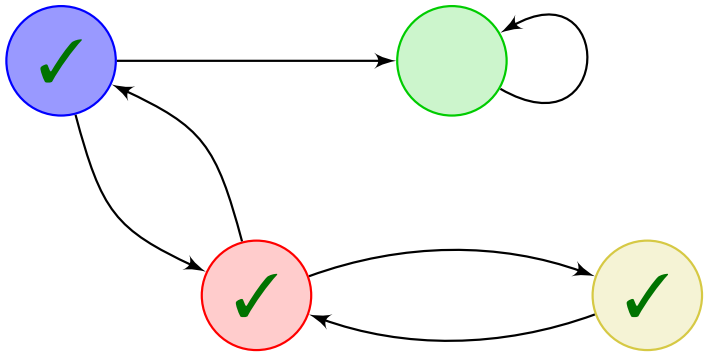
E G F ● there is a path visiting ● infinitely many times



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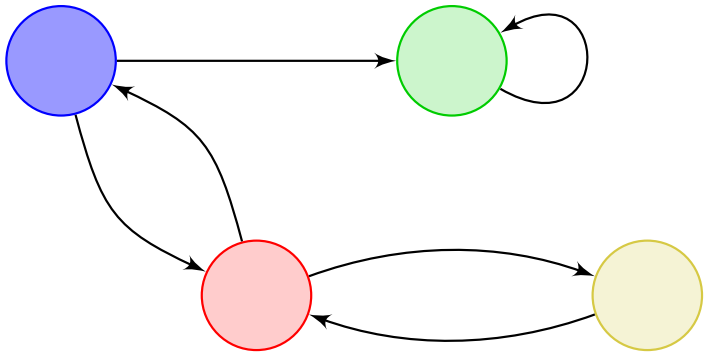
In CTL*, we have no restriction on modalities and quantifiers.

$\mathbf{A}(\mathbf{G}\,\mathbf{F}\,\bigcirc \Rightarrow \mathbf{G}\,\mathbf{F}\,\bigcirc)$ any path that visits $\bigcirc$ infinitely many times, also visits $\bigcirc$ infinitely many times



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In CTL*, we have no restriction on modalities and quantifiers.

Theorem ([EH86])

CTL* model checking is PSPACE*-complete.*

[EH86] Emerson, Halpern. "Sometimes" and "Not Never" Revisited: On Branching versus Linear Time Temporal Logic. J.ACM, 1986.

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Quantified CTL

[Kup95,Fre01]

QCTL extends CTL with propositional quantifiers

$\exists p.\ \varphi$ means that there exists a labelling of the model with p under which φ holds.

[Kup95] Kupferman. Augmenting Branching Temporal Logics with Existential Quantification over Atomic Propositions. CAV, 1995.
[Fre01] French. Decidability of Quantifed Propositional Branching Time Logics. AJCAI, 2001.

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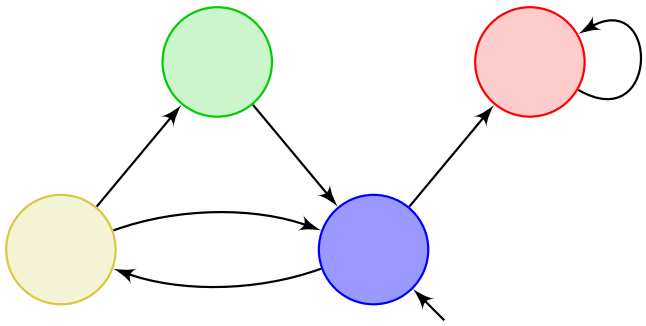

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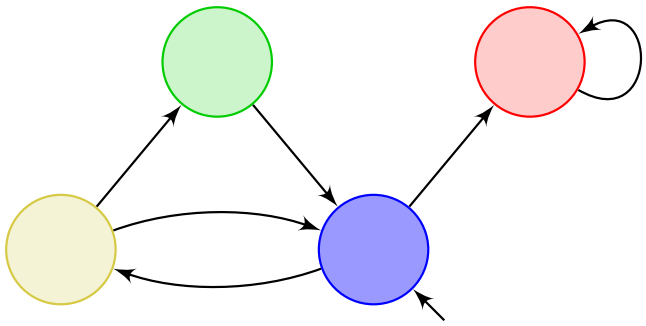

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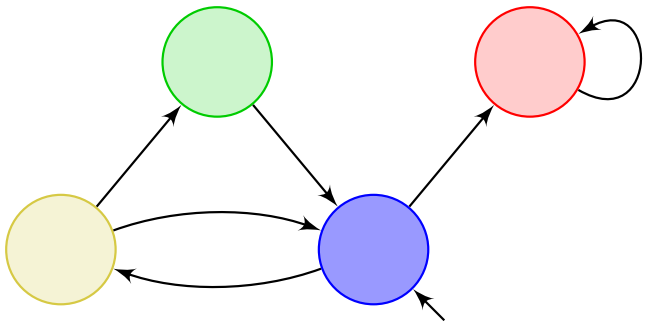

$\rightsquigarrow$ true if we label the Kripke structure;

$\rightsquigarrow$ false if we label the computation tree;

[Kup95] Kupferman. Augmenting Branching Temporal Logics with Existential Quantification over Atomic Propositions. CAV, 1995.

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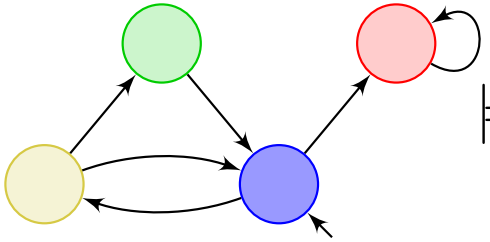
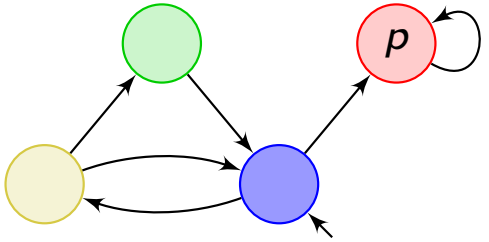
Semantics of QCTL

- structure semantics:



$\models_s \exists p.\varphi \quad \Leftrightarrow$



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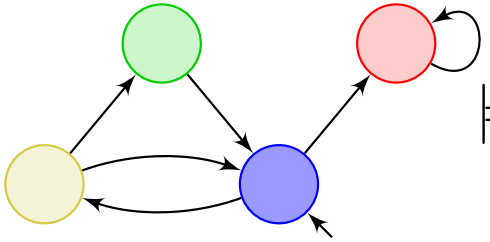

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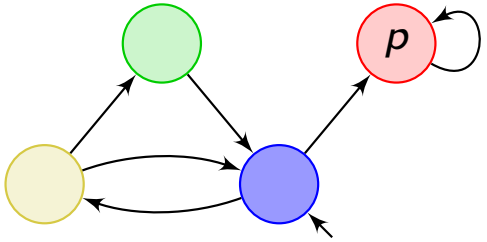
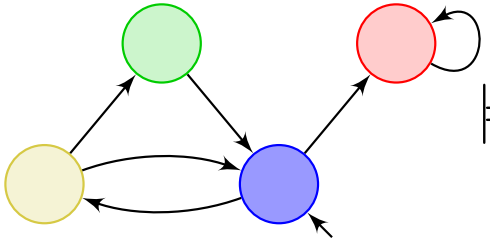


$\models \varphi$

- tree semantics:



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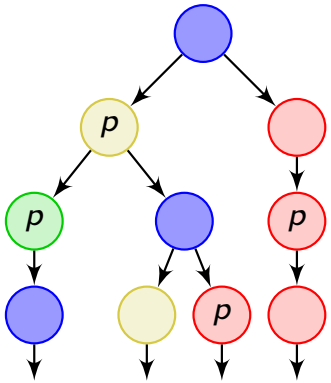

$\models \varphi$

Expressiveness of QCTL

- QCTL can "count":

$$\mathbf{E}\,\mathbf{X}_1\,\varphi \equiv \mathbf{E}\,\mathbf{X}\,\varphi \wedge \forall p.\ [\mathbf{E}\,\mathbf{X}(p \wedge \varphi) \Rightarrow \mathbf{A}\,\mathbf{X}(\varphi \Rightarrow p)]$$
$$\mathbf{E}\,\mathbf{X}_2\,\varphi \equiv \exists q.\ [\mathbf{E}\,\mathbf{X}_1(\varphi \wedge q) \wedge \mathbf{E}\,\mathbf{X}_1(\varphi \wedge \neg q)]$$

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

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- QCTL can express (least or greatest) fixpoints:

$$\mu T.\varphi(T) \equiv \exists t.\ [\mathbf{A}\,\mathbf{G}(t \iff \varphi(t)) \wedge (\forall t.'(\mathbf{A}\,\mathbf{G}(t' \iff \varphi(t')) \Rightarrow \mathbf{A}\,\mathbf{G}(t \Rightarrow t')))]$$

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Theorem

For both semantics, QCTL, QCTL* and MSO are equally expressive.

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

QCTL with structure semantics

Theorem

Model checking QCTL for the structure semantics is PSPACE-*complete.*

Proof

Membership:

Iteratively
- guess a labelling,
- check the subformula.

Hardness:

QBF is a special case (without even using temporal modalities).

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

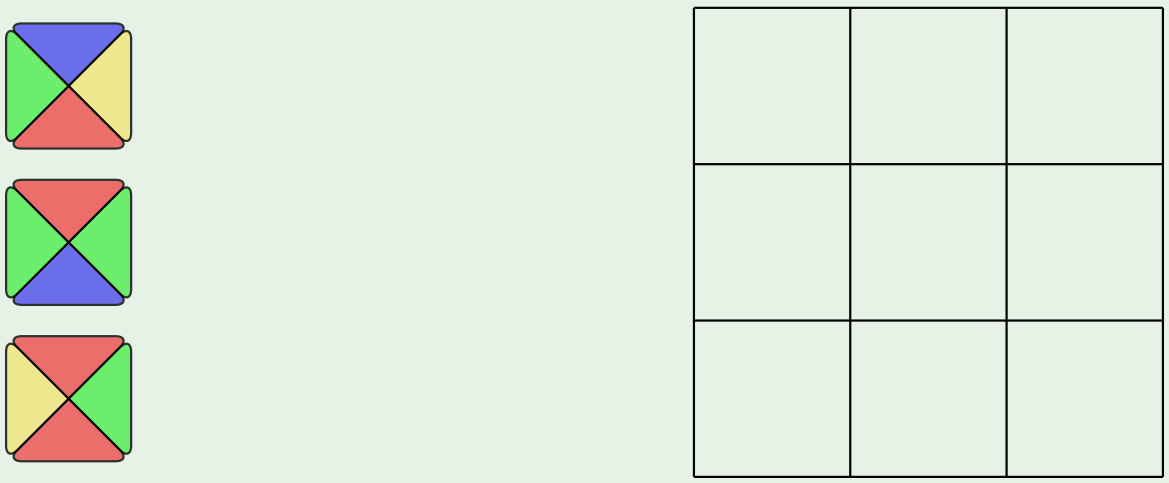
QCTL with structure semantics

Theorem

QCTL satisfiability for the structure semantics is undecidable.

Proof

Encode the problem of tiling finite square grids.



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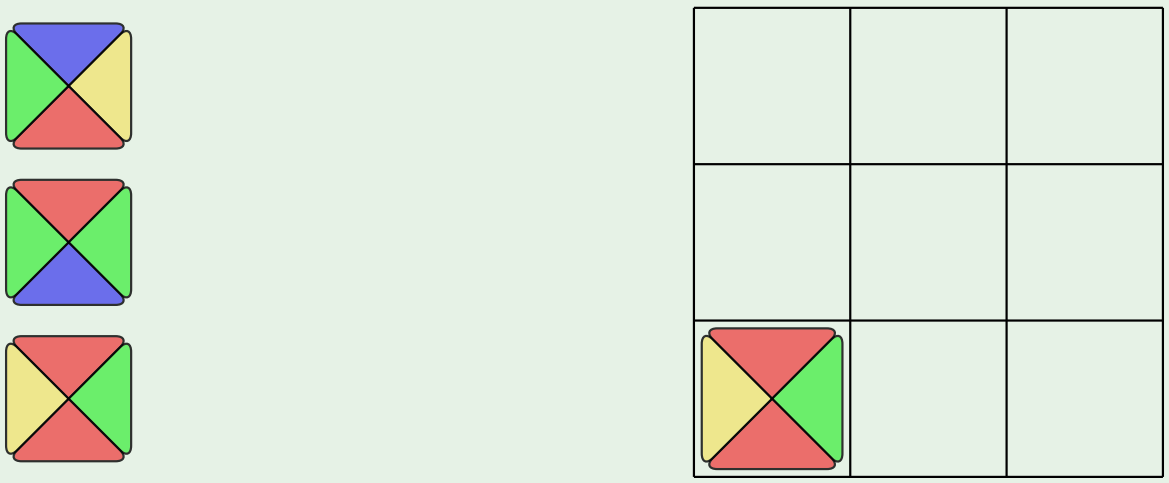

QCTL with structure semantics

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QCTL satisfiability for the structure semantics is undecidable.

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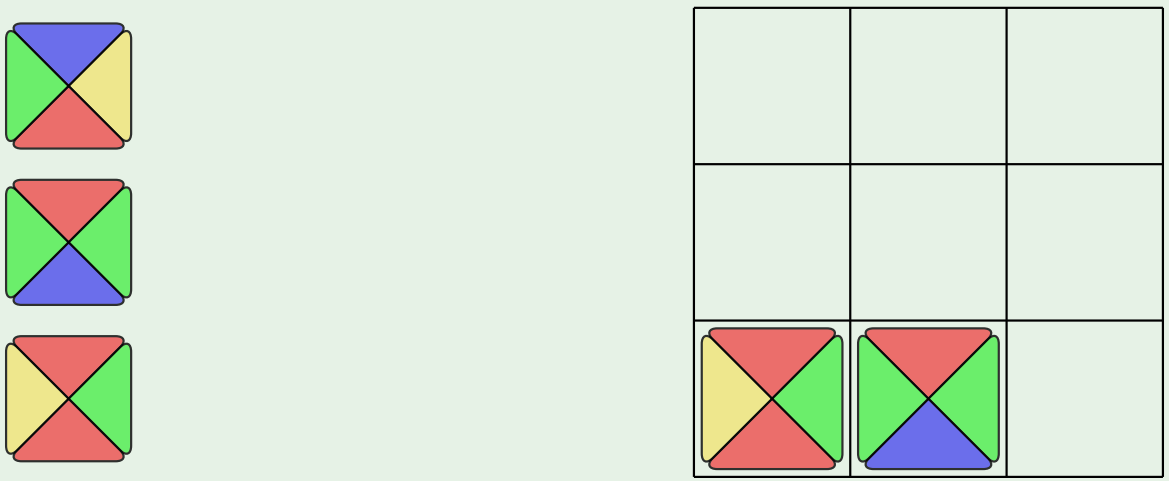

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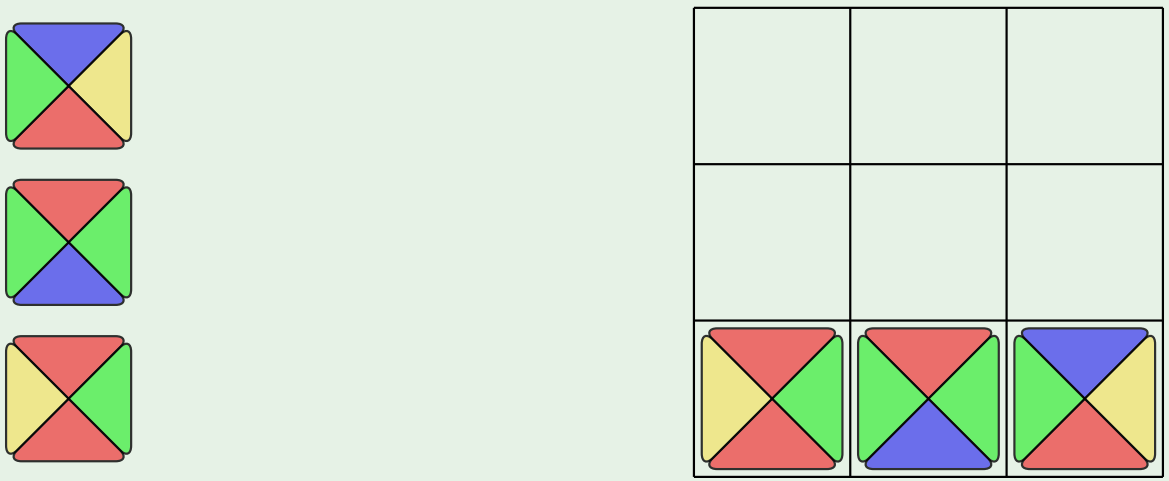

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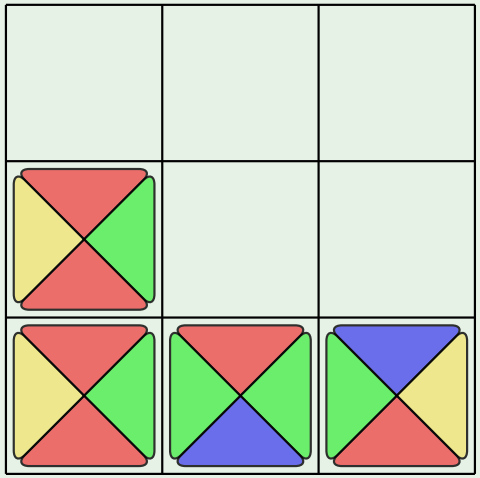

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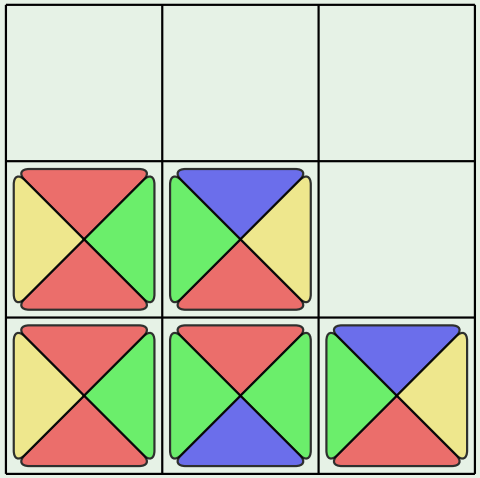

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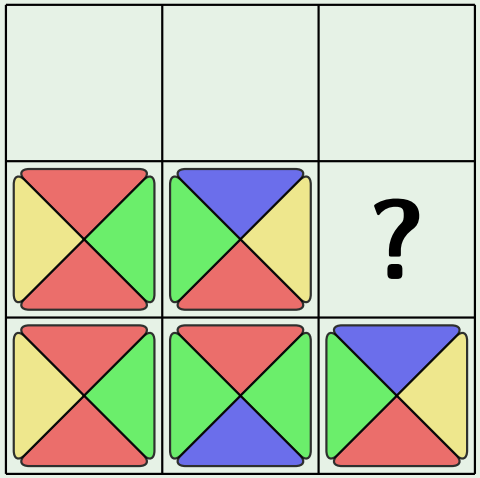

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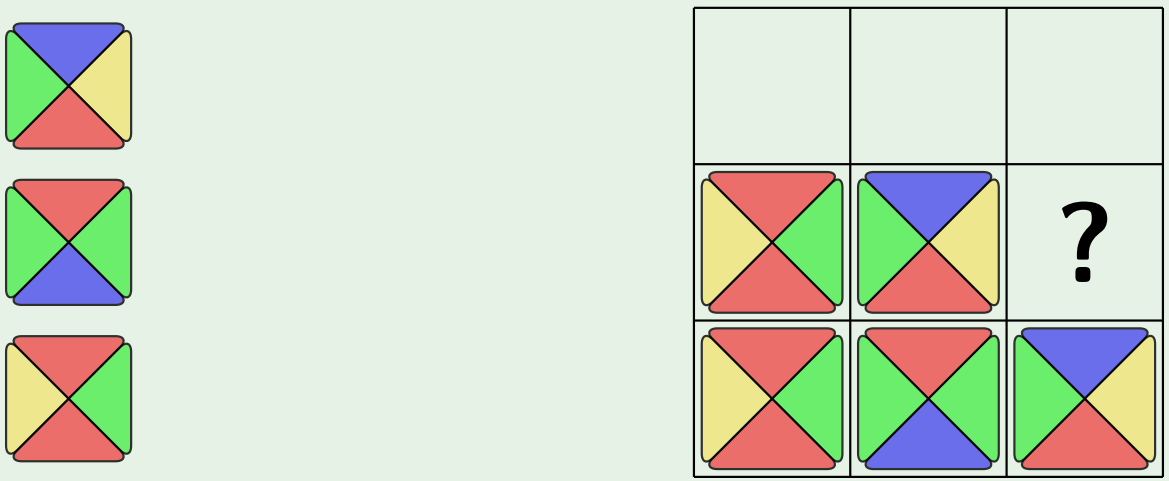

Given a set of tiles, whether any square grid can be tiled is undecidable.

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Reduction: is there a finite Kripke structure such that

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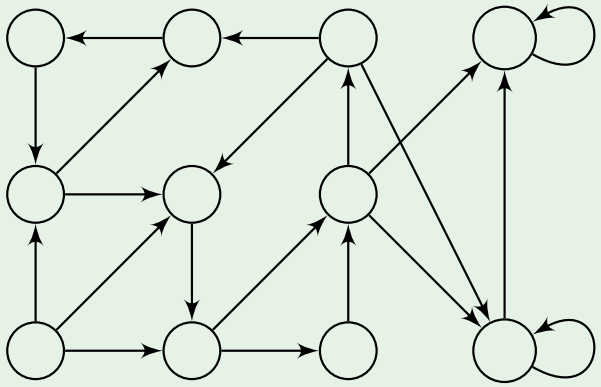

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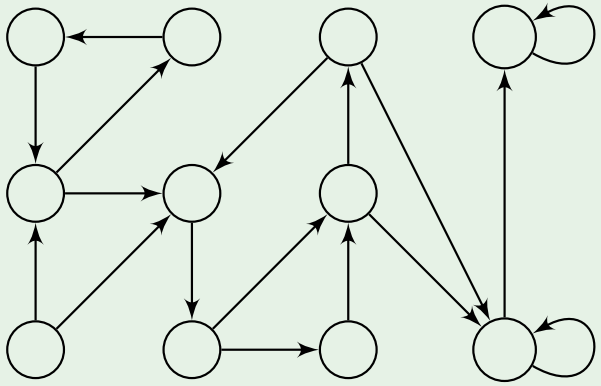

- each state has one or two successors

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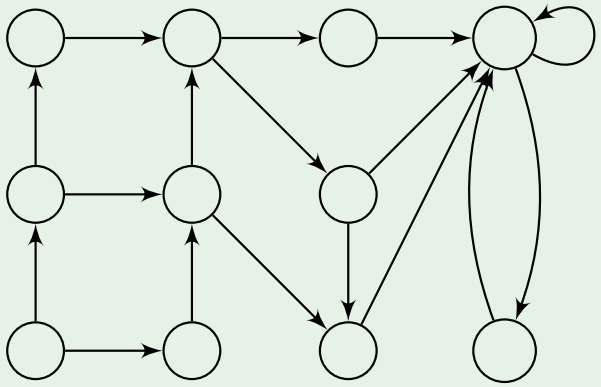

- two successors of the same state have a common successor

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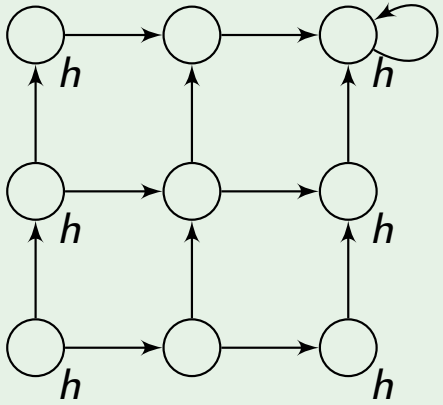

- [... many more conditions ...]

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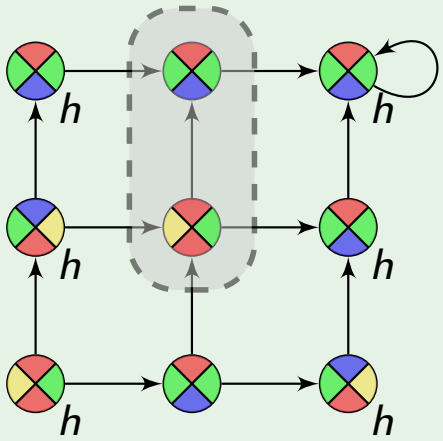


- for any tiling, there is a position where the neighbouring tiles do not match

QCTL with tree semantics

Theorem

- *Model checking QCTL with k quantifiers in the tree semantics is* k-EXPTIME-*complete.*
- *Satisfiability of QCTL with k quantifiers in the tree semantics is* (k+1)-EXPTIME-*complete.*

Proof

Using alternating tree automata:

- polynomial-size automata for CTL;
- boolean combinators can be handled easily;
- quantification is handled by projection, which requires alternation removal (exponential blowup).

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.
[LM13a] Laroussinie, M. Quantified CTL: expressiveness and complexity. Submitted, 2013.

Outline of the presentation

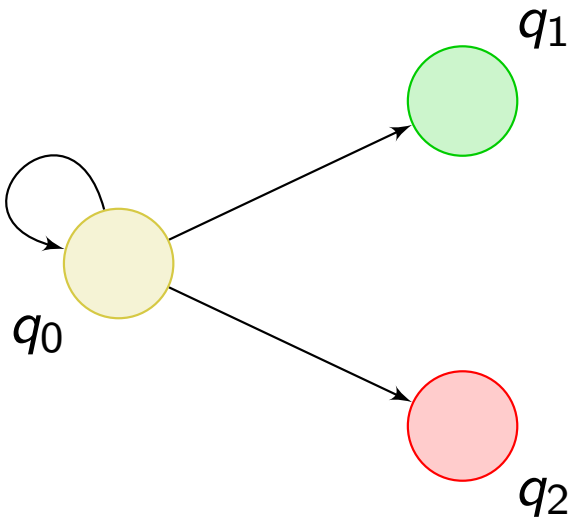
1. Basics about CTL
   - expressing properties of reactive systems
   - efficient verification algorithms
2. Quantified CTL
   - CTL with quantification over atomic propositions
   - model checking and satisfiability are mostly decidable
3. Temporal logics for games: ATL and extensions
   - expressing properties of complex interacting systems
   - QCTL-based decision procedures for ATL_{sc}

Reasoning about multi-agent systems

Concurrent games

A concurrent game is made of

- a transition system;



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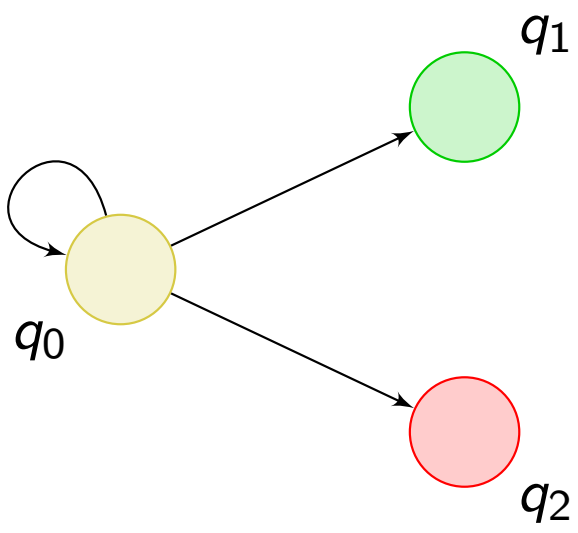

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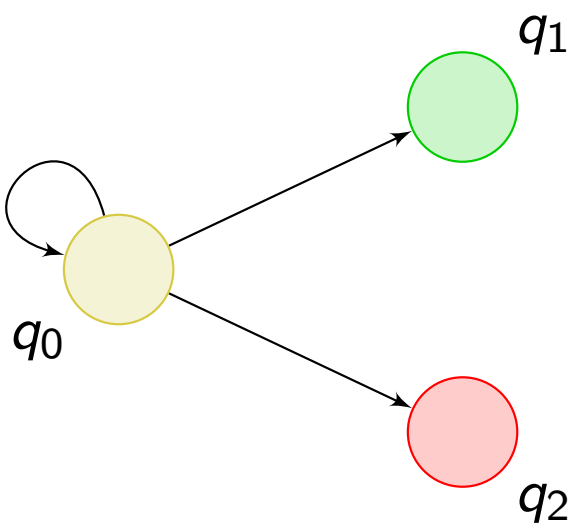

		player 1		
				
player 2		q_0	q_2	q_1
		q_1	q_0	q_2
		q_2	q_1	q_0

Reasoning about multi-agent systems

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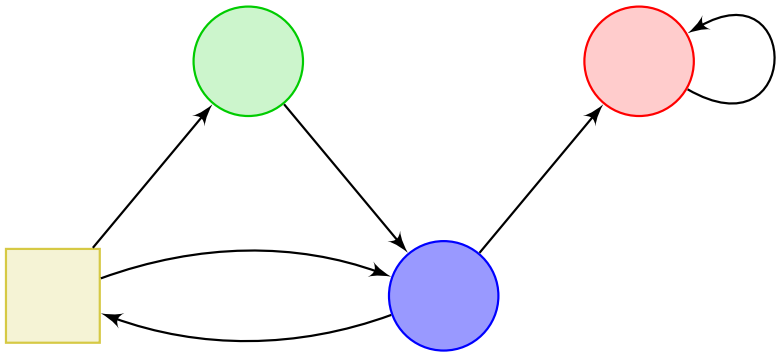
Turn-based games

A turn-based game is a game where only one agent plays at a time.



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Strategies

A strategy for a given player is a function telling what to play depending on what has happened previously.

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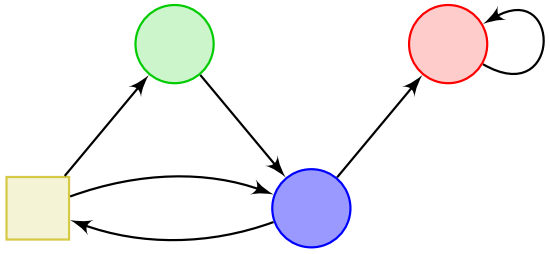

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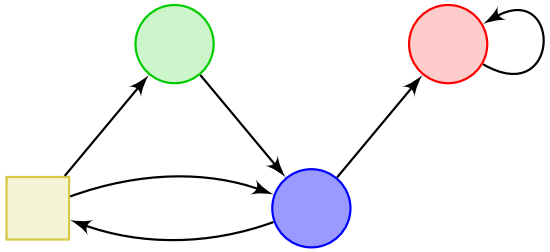

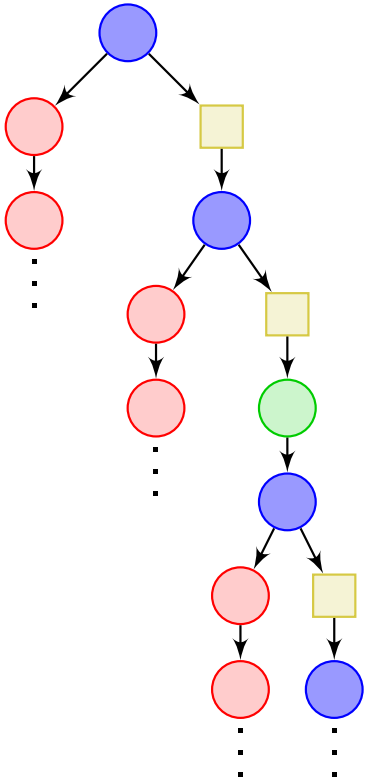

Temporal logics for games: ATL

ATL extends CTL with strategy quantifiers

$\langle\langle A \rangle\rangle\, \varphi$ expresses that A has a strategy to enforce φ.

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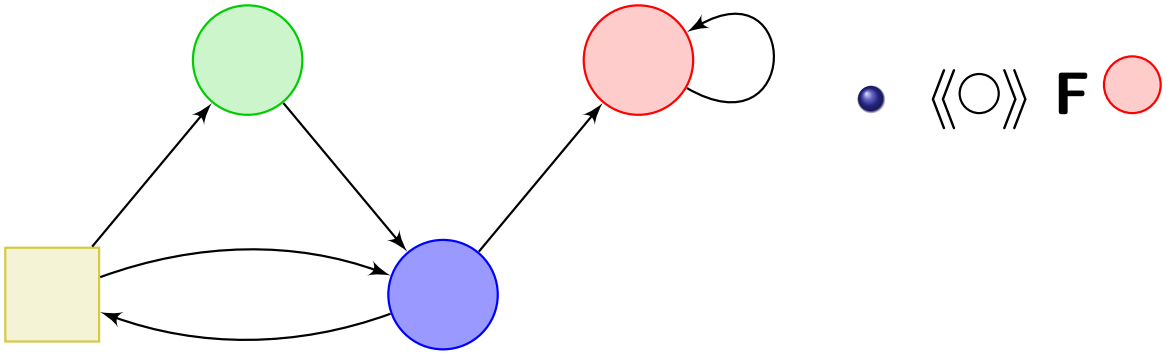

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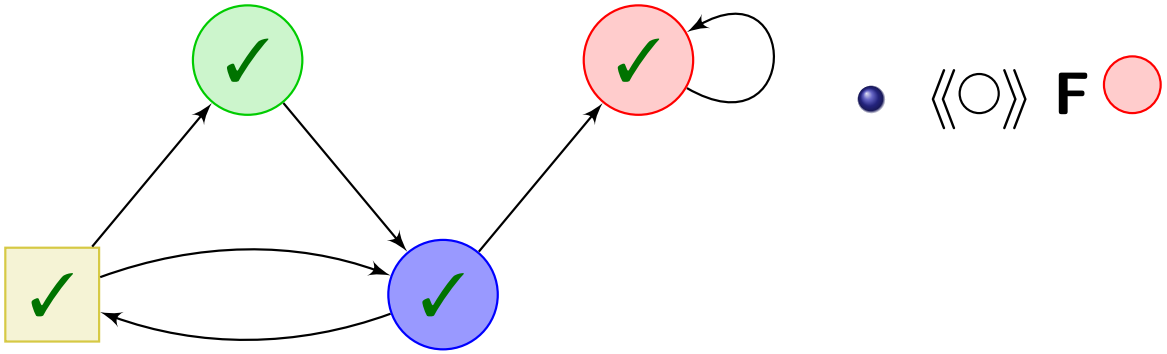

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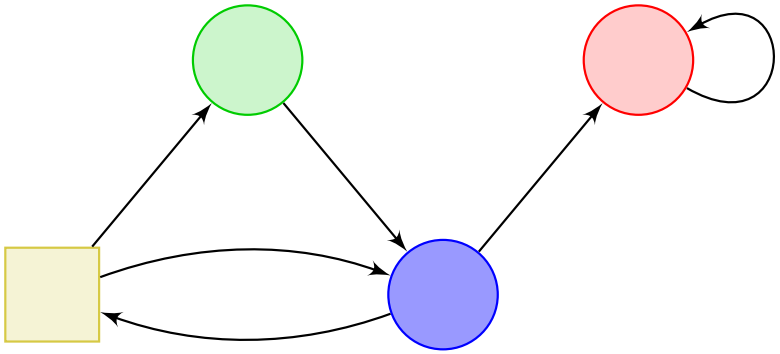

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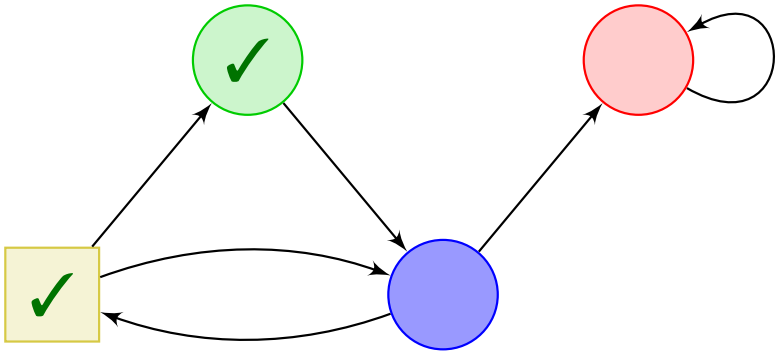

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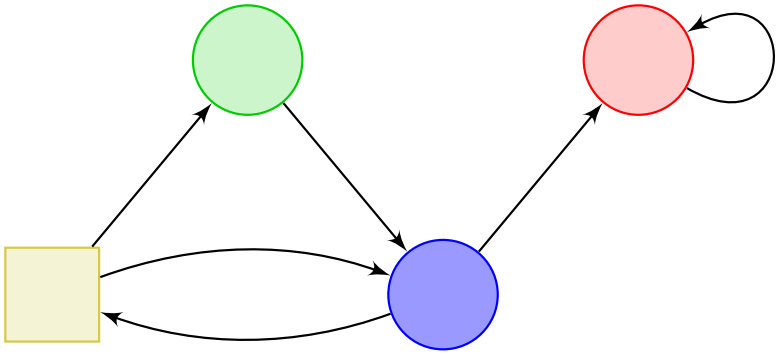

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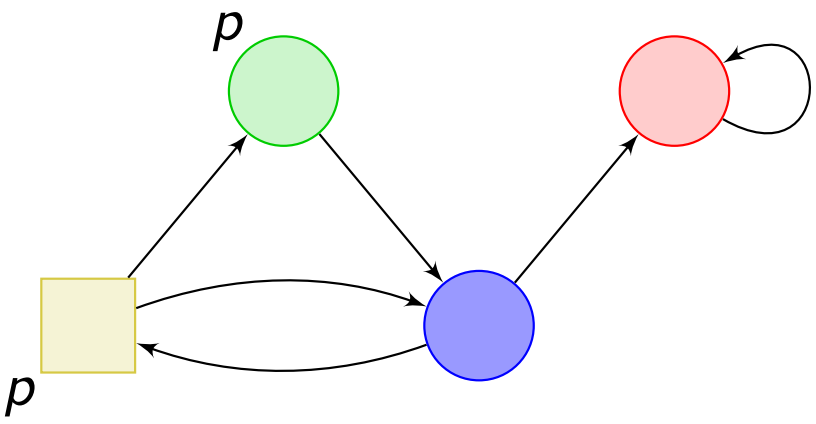

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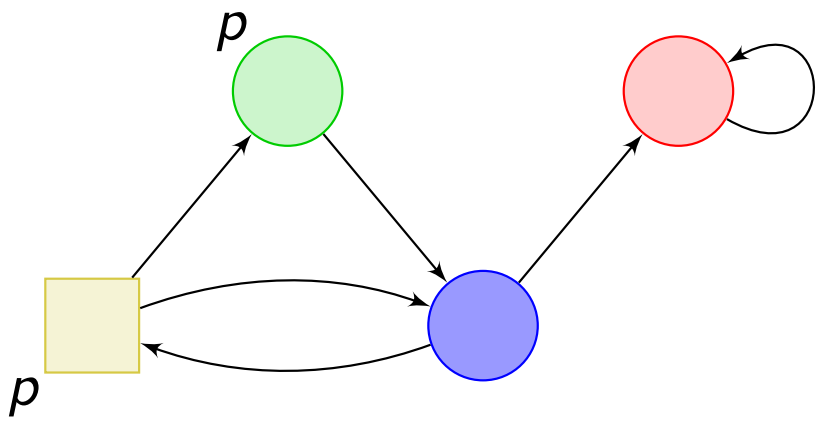

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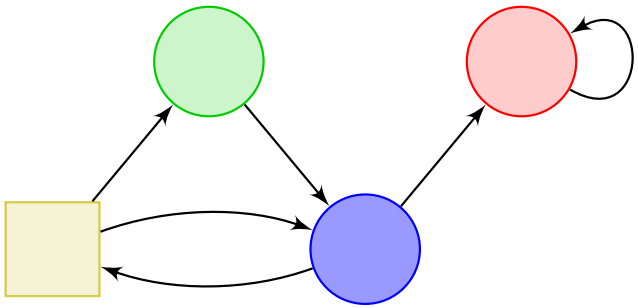
Theorem

Model checking ATL is PTIME-complete.

[AHK02] Alur, Henzinger, Kupferman. Alternating-time Temporal Logic. J. ACM, 2002.

ATL with strategy contexts

[BDLM09]



$\langle\langle \bigcirc \rangle\rangle\ \mathbf{G}(\ \langle\langle \Box \rangle\rangle\ \mathbf{F}$ ●$)$

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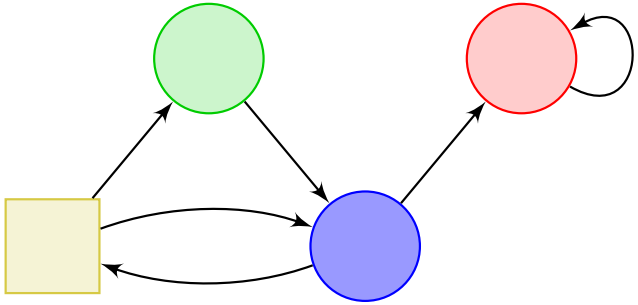

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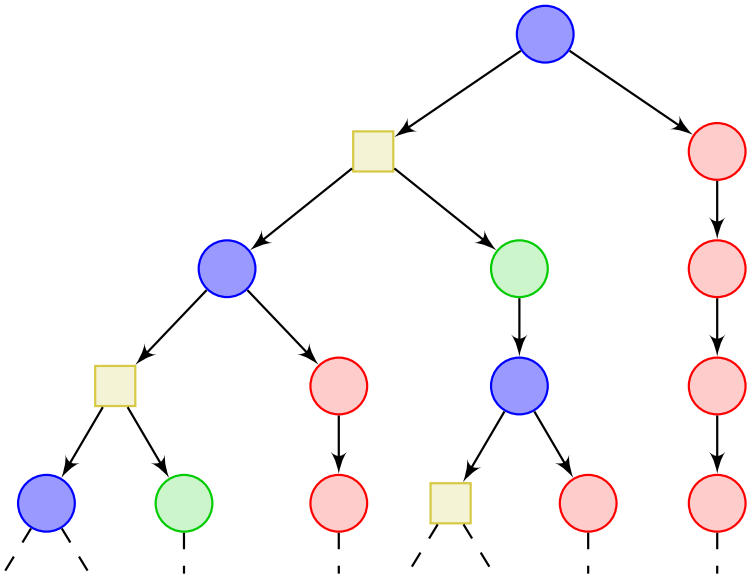

- consider the following strategy of Player ●: "always go to □";

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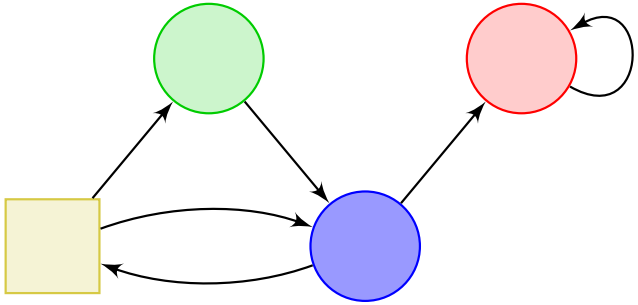

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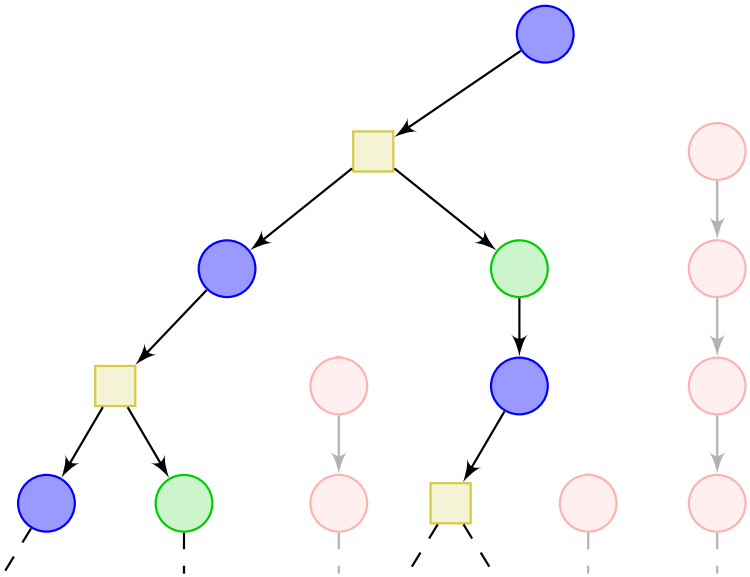

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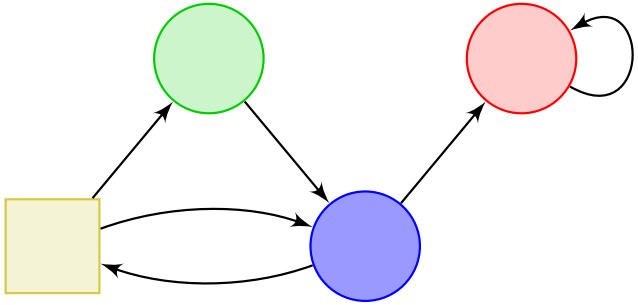

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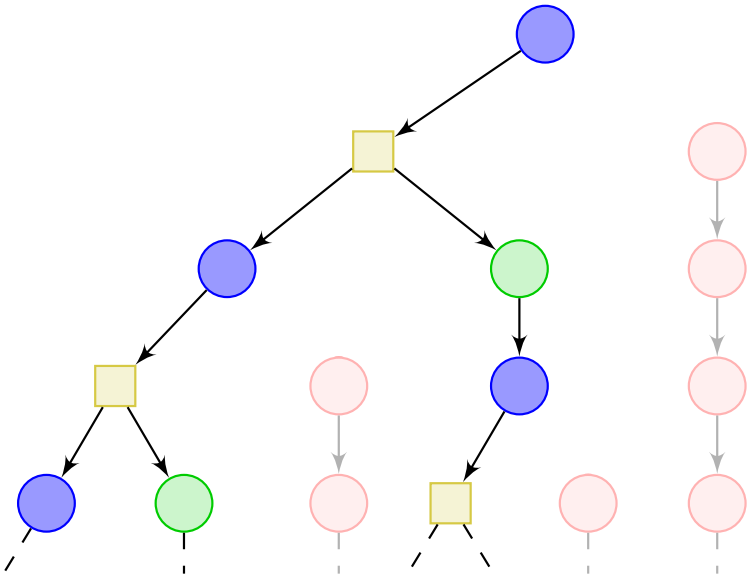

- consider the following strategy of Player ●: "always go to □";
- in the remaining tree, Player □ can always enforce a visit to ●.

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What ATL_{sc} can express

- Client-server interactions for accessing a shared resource:

$$\langle\cdot \mathrm{Server} \cdot\rangle\ \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \mathrm{Clients}} \langle\cdot c \cdot\rangle\ \mathbf{F}\ \mathrm{access}_c \\ \wedge \\ \neg \bigwedge_{c \neq c'} \mathrm{access}_c \wedge \mathrm{access}_{c'} \end{array} \right]$$

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- Existence of Nash equilibria:

$$\langle\cdot A_1, \ldots, A_n \cdot\rangle \bigwedge_i \left(\langle\cdot A_i \cdot\rangle\, \varphi_{A_i} \Rightarrow \varphi_{A_i} \right)$$

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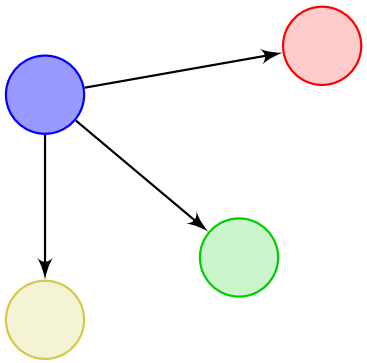
- Existence of dominating strategy:

$$\langle\cdot A \cdot\rangle\ [\cdot B \cdot]\, (\neg \varphi \Rightarrow [\cdot A \cdot]\, \neg \varphi)$$

Translating ATL_{sc} into QCTL



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(●), A, m_i^A) of states that can be reached from ● when player A plays m_i^A.

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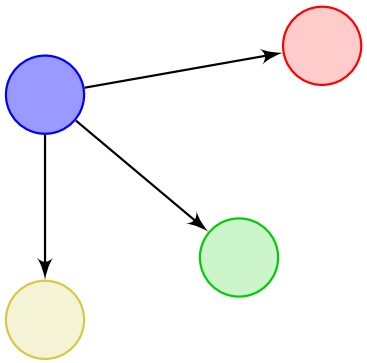

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$\langle \cdot A \cdot \rangle\, \varphi$ can be encoded as follows:

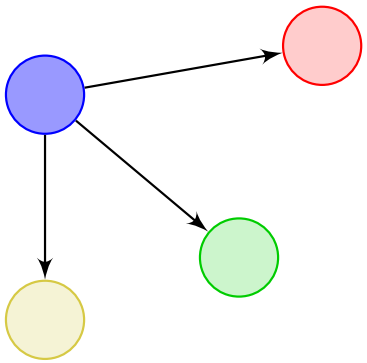
$\exists m_1^A.\ \exists m_2^A \ldots \exists m_n^A.$

- this corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(m_i^A \Leftrightarrow \bigwedge \neg\, m_j^A)$;
- the outcomes all satisfy φ:

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^A \Rightarrow \mathbf{X}\,\mathrm{Next}(q, A, m_i^A)) \Rightarrow \varphi\big].$$

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

Translating ATL_{sc} into QCTL



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(●), A, m_i^A) of states that can be reached from ● when player A plays m_i^A.

Corollary

ATL_{sc} model checking is decidable.

Corollary

ATL_{sc}^0 (memoryless quantification) model checking is decidable.

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

What about satisfiability?

Theorem

QCTL satisfiability is decidable.

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Theorem (TW12)

ATL_{sc} satisfiability is undecidable.

[TW12] Troquard, Walther. On Satisfiability in ATL with Strategy Contexts. JELIA, 2012.

What about satisfiability?

Theorem

QCTL satisfiability is decidable.

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Theorem (TW12)

ATL_{sc} satisfiability is undecidable.

Why?

The translation from ATL_{sc} to QCTL assumes that the game structure is fixed!

[TW12] Troquard, Walther. On Satisfiability in ATL with Strategy Contexts. JELIA, 2012.

Satisfiability for turn-based games

Theorem (LM13b)

When restricted to turn-based games, ATL_{sc} satisfiability is decidable.

[LM13b] Laroussinie, M. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

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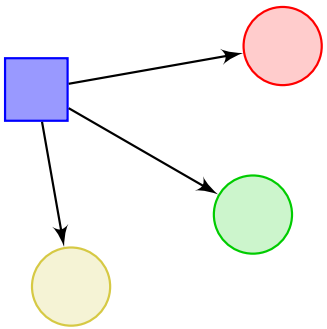

- player □ has moves ●, ● and ●.
- a strategy can be encoded by marking some of the nodes of the tree with proposition mov_A.

$\langle\cdot A\cdot\rangle\,\varphi$ can be encoded as follows:

$\exists \mathrm{mov}_A.$

- it corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(\mathrm{turn}_A \Rightarrow \mathbf{E}\,\mathbf{X}_1\,\mathrm{mov}_A)$;
- the outcomes all satisfy φ: $\mathbf{A}\big[\mathbf{G}(\mathrm{turn}_A \wedge \mathbf{X}\,\mathrm{mov}_A) \Rightarrow \varphi\big]$.

[LM13b] Laroussinie, M. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

Satisfiability for turn-based games

Theorem (LM13b)

When restricted to turn-based games, ATL_{sc} satisfiability is decidable.

Theorem

Model checking ATL_{sc} with only memoryless quantification is PSPACE-*complete.*

[LM13b] Laroussinie, M. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

What about Strategy Logic? [CHP07,MMV10]

Strategy logic

Explicit quantification over strategies + strategy assignement

Example

$$\langle \cdot A \cdot \rangle \, \varphi \equiv \exists \sigma_1 . \mathsf{assign}(\sigma_1, A) . \varphi$$

Strategy logic can also be translated into QCTL.

Theorem

- *Strategy-logic satisfiability is decidable when restricted to turn-based games.*
- *Memoryless strategy-logic satisfiability is undecidable.*

[CHP07] Chatterjee, Henzinger, Piterman. Strategy Logic. CONCUR, 2007.
[MMV10] Mogavero, Murano, Vardi. Reasoning about strategies. FSTTCS, 2010.

Conclusions and future works

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- QCTL is a powerful extension of CTL;
- it is equivalent to MSO over finite graphs and regular trees;
- it is a nice tool to understand temporal logics for games;

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- QCTL is a powerful extension of CTL;
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Future directions

- Defining interesting (expressive yet tractable) fragments of those logics;
- Obtaining practicable algorithms.
- Considering randomised strategies.