Satisfiability of ATL with strategy contexts

François Laroussinie and Nicolas Markey
LIAFA LSV

Paris, 18-21 September 2013

Outline of the presentation

1. Temporal logics for games: ATL and extensions
   - expressing properties of complex interacting systems
   - extensions to non-zero-sum games
2. From ATL with strategy contexts to QCTL
   - QCTL is CTL with propositional quantification
   - strategies encoded as propositions on the computation tree
3. Satisfiability of ATL with strategy contexts
   - QCTL satisfiability is decidable, but...
   - ATL_{sc} satisfiability is not, except for turn-based games

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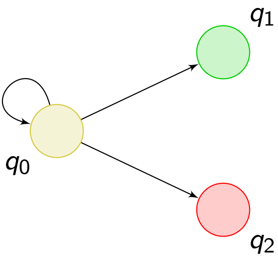
Reasoning about multi-agent systems

Concurrent games

A concurrent game is made of

- a transition system;
- a set of agents (or players);
- a table indicating the transition to be taken given the actions of the players.



		player 1		
				
player 2		q_0	q_2	q_1
		q_1	q_0	q_2
		q_2	q_1	q_0

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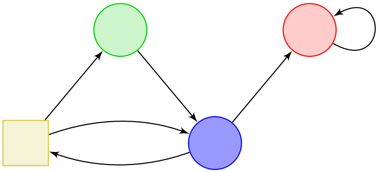
Turn-based games

A turn-based game is a game where only one agent plays at a time.



Reasoning about open systems

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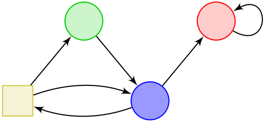

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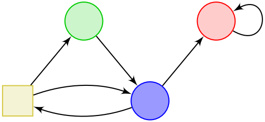

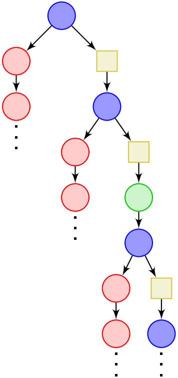

Temporal logics for games: ATL [AHK02]

ATL extends CTL with strategy quantifiers

$\langle\langle A \rangle\rangle\, \varphi$ expresses that A has a strategy to enforce φ.

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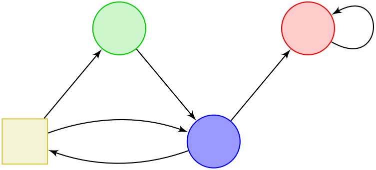

- $\langle\langle\square\rangle\rangle$ **F** 

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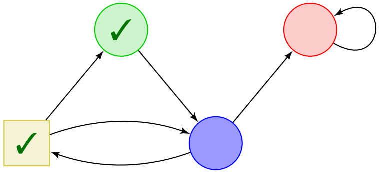

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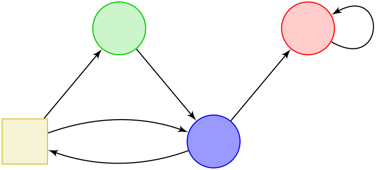

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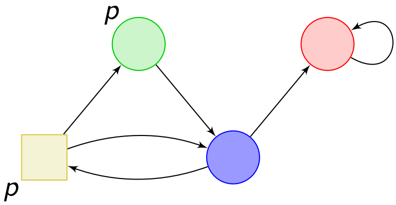

- $\langle\!\langle \square \rangle\!\rangle\ \mathbf{F}\ \bigcirc$
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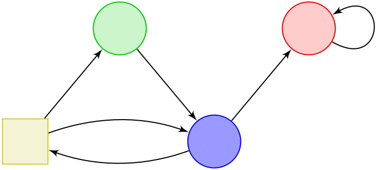
Another semantics: ATL with strategy contexts [BDLM09]



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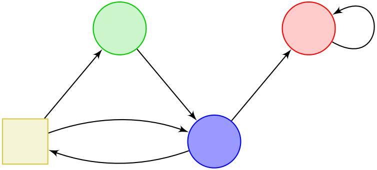

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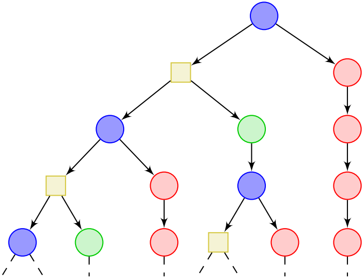

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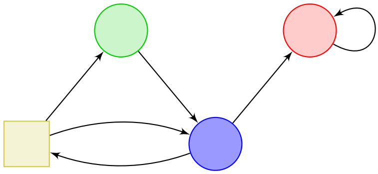

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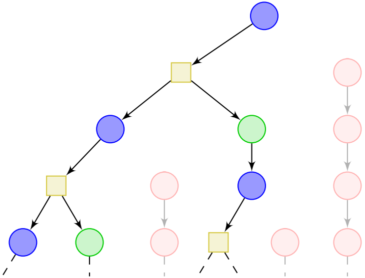

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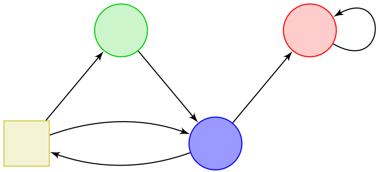

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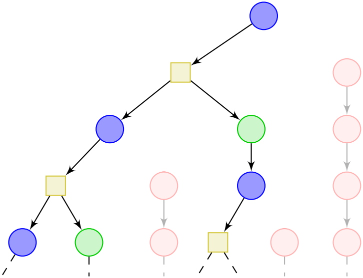

- consider the following strategy of Player $\bigcirc$: "always go to $\square$";
- in the remaining tree, Player $\square$ can always enforce a visit to $\bigcirc$.

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- Client-server interactions for accessing a shared resource:

$$\langle\cdot \mathrm{Server} \cdot\rangle\ \mathbf{G} \left[\begin{array}{ll} & \displaystyle\bigwedge_{c \in \mathrm{Clients}} \langle\cdot c \cdot\rangle\ \mathbf{F}\, \mathrm{access}_c \\ \wedge & \\ & \neg \displaystyle\bigwedge_{c \neq c'} \mathrm{access}_c \wedge \mathrm{access}_{c'} \end{array} \right]$$

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- Existence of Nash equilibria:

$$\langle\!\cdot A_1,\ldots,A_n\cdot\!\rangle \bigwedge_i \left(\langle\!\cdot A_i\cdot\!\rangle\,\varphi_{A_i} \Rightarrow \varphi_{A_i}\right)$$

- Existence of dominating strategy:

$$\langle\!\cdot A\cdot\!\rangle\ [\!\cdot B\cdot\!]\,(\neg\varphi \Rightarrow [\!\cdot A\cdot\!]\,\neg\varphi)$$

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Quantified CTL [Kup95,Fre01]

QCTL extends CTL with propositional quantifiers

$\exists p.\ \varphi$ means that there exists a labelling of the model with p under which φ holds.

[Kup95] Kupferman. Augmenting Branching Temporal Logics with Existential Quantification over Atomic Propositions. CAV, 1995.
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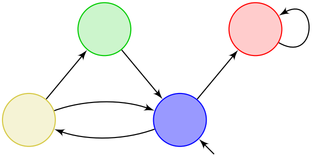

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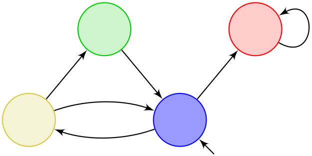

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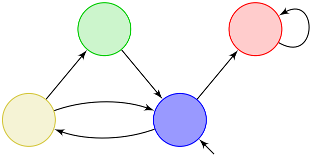

- $\mathbf{EF}\,\bigcirc \wedge \forall p.\ \left[\mathbf{EF}(p \wedge \bigcirc) \Rightarrow \mathbf{AG}(\bigcirc \Rightarrow p)\right] \equiv \mathrm{uniq}(\bigcirc)$

  $\rightsquigarrow$ true if we label the Kripke structure;
  $\rightsquigarrow$ false if we label the computation tree;

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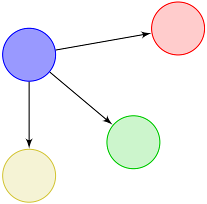
Translating ATL$_{sc}$ into QCTL



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(), A, m_i^A) of states that can be reached from  when player A plays m_i^A.

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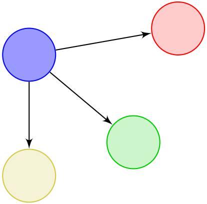

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$\langle\cdot A\cdot\rangle\, \varphi$ can be encoded as follows:

$\exists m_1^A.\ \exists m_2^A \ldots \exists m_n^A.$

- this corresponds to a strategy: $\quad \mathbf{A}\,\mathbf{G}(m_i^A \Leftrightarrow \bigwedge \neg\, m_j^A)$;
- the outcomes all satisfy φ:

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^A \Rightarrow \mathbf{X}\,\mathrm{Next}(q, A, m_i^A)) \Rightarrow \varphi\big].$$

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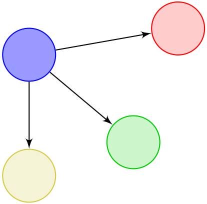

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Theorem (DLM12)

QCTL model checking is decidable (in the tree semantics).

Corollary

ATL$_{sc}$ model checking is decidable.

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

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What about satisfiability?

Theorem (LM13a)

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Theorem (LM13a)

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Theorem (TW12)

ATL_{sc} satisfiability is undecidable.

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[TW12] Troquard, Walther. On Satisfiability in ATL with Strategy Contexts. JELIA, 2012.

What about satisfiability?

Theorem (LM13a)

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Theorem (TW12)

ATL_{sc} satisfiability is undecidable.

Why?

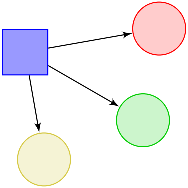
The translation from ATL_{sc} to QCTL assumes that the game structure is fixed!

[LM13a] Laroussinie, M. Quantified CTL: expressiveness and complexity. Submitted, 2013.
[TW12] Troquard, Walther. On Satisfiability in ATL with Strategy Contexts. JELIA, 2012.

Satisfiability for turn-based games

Theorem (LM13b)

When restricted to turn-based games, ATL_{sc} satisfiability is decidable.



- player $\square$ has moves (green circle), (yellow circle) and (red circle).
- a strategy can be encoded by marking some of the nodes of the tree with proposition mov_A.

$\langle\cdot A\cdot\rangle\, \varphi$ can be encoded as follows:

$\exists mov_A.$

- it corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(turn_A \Rightarrow \mathbf{E}\,\mathbf{X}_1\, mov_A)$;
- the outcomes all satisfy φ: $\mathbf{A}\big[\mathbf{G}(turn_A \wedge \mathbf{X}\, mov_A) \Rightarrow \varphi\big]$.

[LM13b] Laroussinie, M. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

Restricting to memoryless strategies

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One move in each state of the structure (not of its execution tree).

Our reduction to QCTL is still valid!
(but we now label the structure)

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Theorem

Model checking ATL_{sc} with only memoryless quantification is PSPACE*-complete.*

However:

Theorem

Satisfiability of ATL_{sc} with memoryless quantification is undecidable (even on turn-based structures).

What about Strategy Logic [CHP07,MMV10]?

Strategy logic

Explicit quantification over strategies + strategy assignement

Strategy logic can also be translated into QCTL.

Theorem

- *Strategy-logic satisfiability is decidable when restricted to turn-based games.*
- *Memoryless strategy-logic satisfiability is undecidable.*

[CHP07] Chatterjee, Henzinger, Piterman. Strategy Logic. CONCUR, 2007.
[MMV10] Mogavero, Murano, Vardi. Reasoning about strategies. FSTTCS, 2010.

Conclusions and future works

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- ATL_{sc} is a very powerful logic for reasoning about games.
- QCTL is a nice tool to understand such logics.
- Satisfiability is undecidable, except when looking for turn-based games (or when fixing the set of moves).
- Restricting to memoryless strategies does not help (actually, it is even worse).

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Future directions

- Defining interesting (expressive yet tractable) fragments of those logics;
- Obtaining practicable algorithms.
- Considering randomised strategies.