

Temporal logics for multi-agent systems

Nicolas Markey

LSV, CNRS & ENS Cachan, France



Journées Nationales
Lyon, 21-22 January 2013

Verification of computerised systems

- Computers are everywhere



Verification of computerised systems

- Computers are everywhere

- Bugs are everywhere...

News

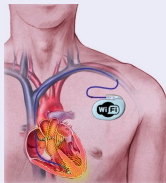
Toyota to recall Prius hybrids over ABS software

See video, below

By Martyr Williams

February 5, 2010 8:53 AM ET [Comments \(6\)](#) [Recommended \(10\)](#) [Like](#)

IDG News Service - Toyota plans to recall around 400,000 of its Prius hybrid cars to replace software that controls the antilock braking system (ABS), the auto maker said Tuesday.



Verification of computerised systems

- Computers are everywhere

- Bugs are everywhere...

News

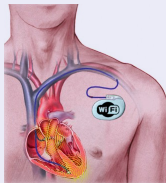
Toyota to recall Prius hybrids over ABS software

See video, below

By Marty Williams

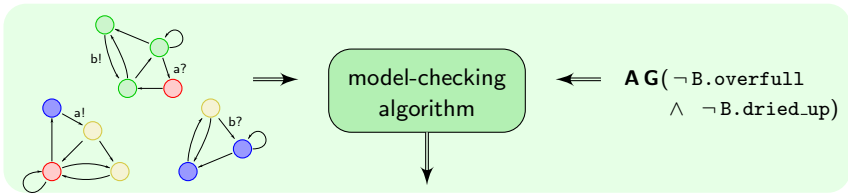
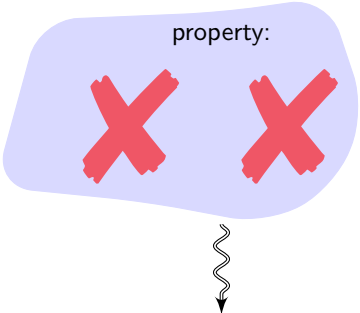
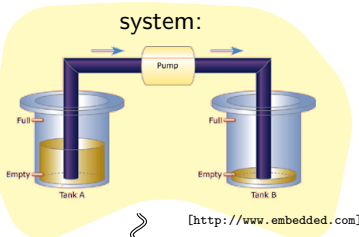
February 5, 2010 8:53 AM ET [Comments \(1\)](#) [Recommended \(15\)](#) [Like](#)

IDG News Service - Toyota plans to recall around 400,000 of its Prius hybrid cars to replace software that controls the antilock braking system (ABS), the auto maker said Tuesday.



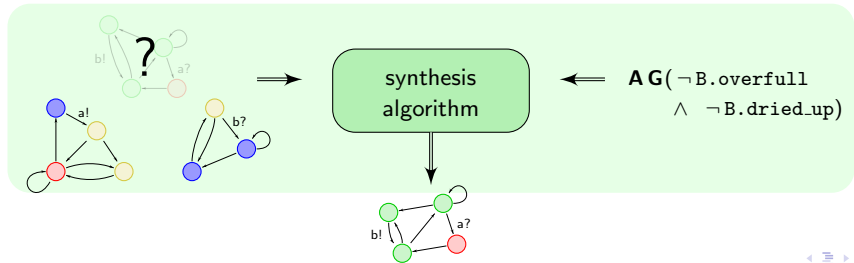
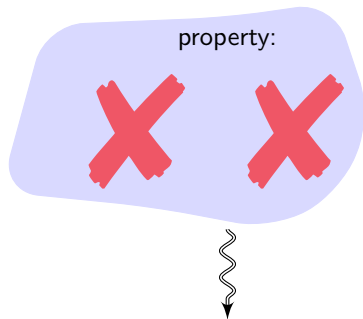
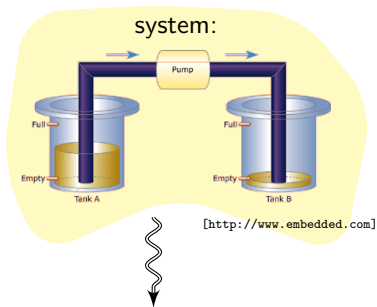
- Verification should be everywhere!

Model checking and synthesis



yes/no

Model checking and synthesis



Outline of the presentation

1 Introduction

↪ formal verification model checking and synthesis

2 Classical temporal logics: CTL and LTL

↪ expressing properties of “closed” systems

3 Temporal logics for games: ATL

↪ expressing properties of interacting systems
extensions to non-zero-sum games

Outline of the presentation

1 Introduction

↪ formal verification model checking and synthesis

2 Classical temporal logics: CTL and LTL

↪ expressing properties of “closed” systems



3 Temporal logics for games: ATL

↪ expressing properties of interacting systems
extensions to non-zero-sum games

Computation-Tree Logic (CTL*)

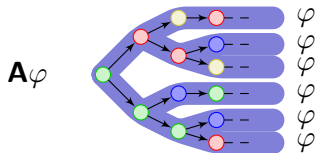
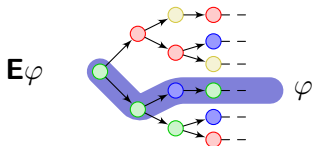
- **atomic propositions:** , , ...

Computation-Tree Logic (CTL*)

- **atomic propositions:** , , ...
- **boolean combinators:** $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...

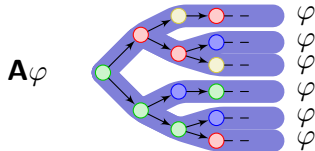
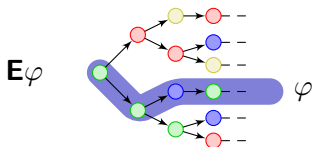
Computation-Tree Logic (CTL*)

- atomic propositions: \circ , \circ , ...
- boolean combinators: $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...
- path quantifiers:



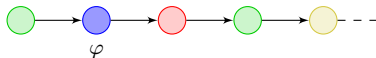
Computation-Tree Logic (CTL*)

- atomic propositions: \circ , \circ , ...
- boolean combinators: $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...
- path quantifiers:



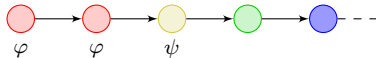
- temporal modalities:

$X\varphi$



“next φ ”

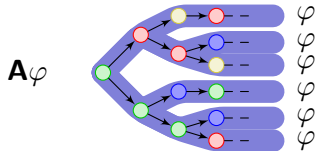
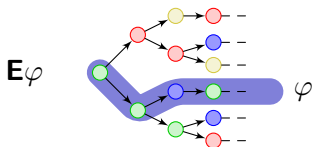
$\varphi U \psi$




“ φ until ψ ”


Computation-Tree Logic (CTL*)

- atomic propositions: \circ , \circ , ...
- boolean combinators: $\neg \varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...
- path quantifiers:

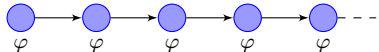


- temporal modalities:

$X\varphi$  "next φ "

$\varphi \mathbf{U} \psi$  " φ until ψ "

true $\mathbf{U} \varphi \equiv \mathbf{F} \varphi$  "eventually φ "

$\neg \mathbf{F} \neg \varphi \equiv \mathbf{G} \varphi$  "always φ "

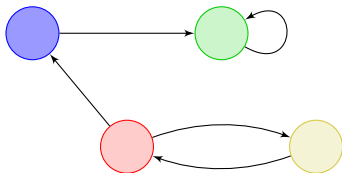
Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

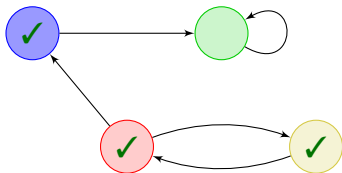
EF ● ● is reachable



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

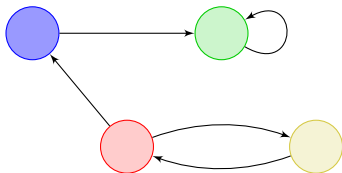
EF ● ● is reachable



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

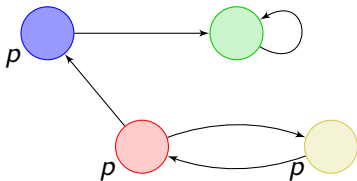
EG(EF ●) there is a path along which ● is always reachable



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

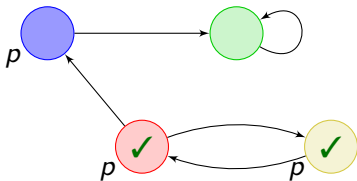
$\mathbf{EG}(\underbrace{\mathbf{EF}}_p \bullet)$ there is a path along which \bullet is always reachable



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

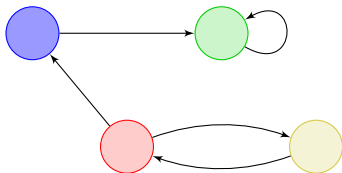
$\mathbf{EG}(\underbrace{\mathbf{EF}}_p \bullet)$ there is a path along which \bullet is always reachable



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

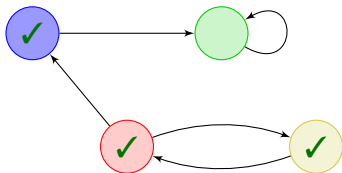
$\neg \mathbf{E}(\neg \text{blue}) \mathbf{U} \text{green}$ in order to reach green, we have to visit blue



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

$\neg \mathbf{E}(\neg \text{blue}) \mathbf{U} \text{green}$ in order to reach green, we have to visit blue



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

Theorem

CTL model checking is PTIME-complete.

*CTL **symbolic** model checking is PSPACE-complete.*

Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

Theorem

CTL model checking is PTIME-complete.

*CTL **symbolic** model checking is PSPACE-complete.*

- LTL: $\mathbf{E}\varphi$ or $\mathbf{A}\varphi$, where φ has no path quantifier.

Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

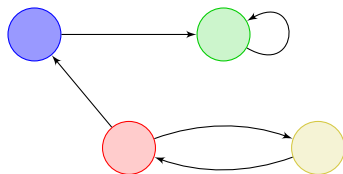
Theorem

CTL model checking is PTIME-complete.

CTL *symbolic* model checking is PSPACE-complete.

- LTL: $\mathbf{E}\varphi$ or $\mathbf{A}\varphi$, where φ has no path quantifier.

$\mathbf{E}(\mathbf{G} \text{F } \bigcirc)$ there is a path visiting \bigcirc infinitely many times



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

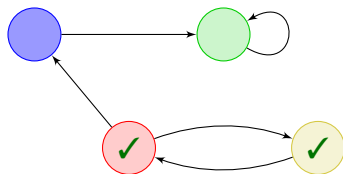
Theorem

CTL model checking is PTIME-complete.

CTL *symbolic* model checking is PSPACE-complete.

- LTL: $\mathbf{E}\varphi$ or $\mathbf{A}\varphi$, where φ has no path quantifier.

$\mathbf{E}(\mathbf{G} \text{F } \bigcirc)$ there is a path visiting \bigcirc infinitely many times



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

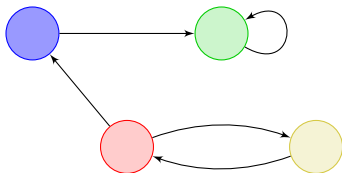
Theorem

CTL model checking is PTIME-complete.

CTL *symbolic* model checking is PSPACE-complete.

- LTL: $\mathbf{E}\varphi$ or $\mathbf{A}\varphi$, where φ has no path quantifier.

$\mathbf{A}[(\mathbf{F} \text{blue}) \Rightarrow (\mathbf{F} \mathbf{G} \neg \text{yellow})]$ any path that visits blue
visits yellow finitely many times



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

Theorem

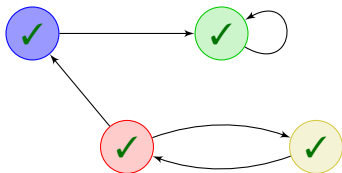
CTL model checking is PTIME-complete.

CTL *symbolic* model checking is PSPACE-complete.

- LTL: $\mathbf{E}\varphi$ or $\mathbf{A}\varphi$, where φ has no path quantifier.

$$\mathbf{A}[(\mathbf{F} \text{blue}) \Rightarrow (\mathbf{F} \mathbf{G} \neg \text{yellow})]$$

any path that visits blue
visits yellow finitely many times



Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

Theorem

CTL model checking is PTIME-complete.

*CTL **symbolic** model checking is PSPACE-complete.*

- LTL: $\mathbf{E}\varphi$ or $\mathbf{A}\varphi$, where φ has no path quantifier.

Theorem

*LTL (**symbolic**) model checking is PSPACE-complete.*

Fragments of CTL*

- CTL: each temporal modality is in the immediate scope of a path quantifier.

Theorem

CTL model checking is PTIME-complete.

*CTL **symbolic** model checking is PSPACE-complete.*

- LTL: $\mathbf{E}\varphi$ or $\mathbf{A}\varphi$, where φ has no path quantifier.

Theorem

*LTL (**symbolic**) model checking is PSPACE-complete.*

Theorem

CTL (**symbolic**) model checking is PSPACE-complete.*

Outline of the presentation

1 Introduction

~> formal verification model checking and synthesis

2 Classical temporal logics: CTL and LTL

~> expressing properties of “closed” systems

3 Temporal logics for games: ATL

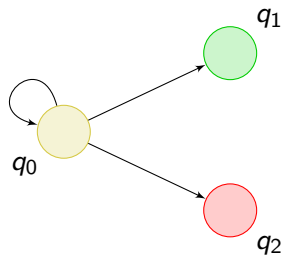
~> expressing properties of interacting systems
extensions to non-zero-sum games

Reasoning about multi-agent systems

Concurrent games

A **concurrent game** is made of

- a transition system;

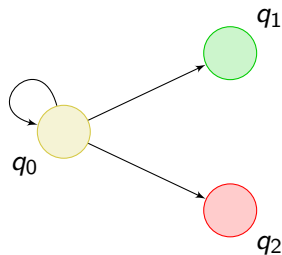


Reasoning about multi-agent systems

Concurrent games

A **concurrent game** is made of

- a transition system;
- a set of **agents** (or **players**);

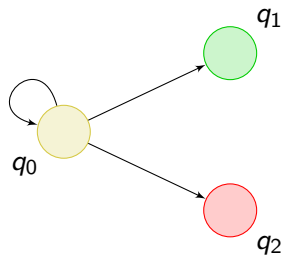


Reasoning about multi-agent systems

Concurrent games

A **concurrent game** is made of

- a transition system;
- a set of **agents** (or **players**);
- a table indicating the transition to be taken given the actions of the players.



		player 1		
				
player 2				
				
				

Reasoning about multi-agent systems

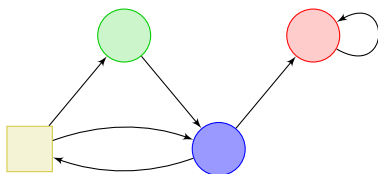
Concurrent games

A **concurrent game** is made of

- a transition system;
- a set of **agents** (or **players**);
- a table indicating the transition to be taken given the actions of the players.

Turn-based games

A **turn-based game** is a game where only one agent plays at a time.



Reasoning about open systems




Strategies

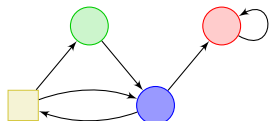
A **strategy** for a given player is a function telling what to play depending on what has happened previously.

Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.




Strategy for player :
alternately go to  and .

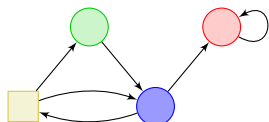


Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.




Strategy for player :
alternately go to  and .

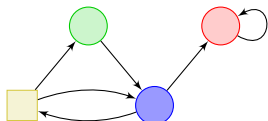
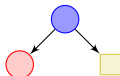


Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.




Strategy for player :
alternately go to  and .

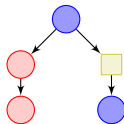
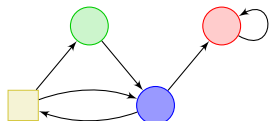


Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.




Strategy for player :
alternately go to  and .

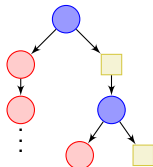
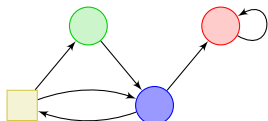


Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.




Strategy for player :
alternately go to  and .

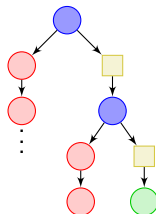
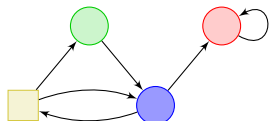


Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.




Strategy for player 
alternately go to  and .

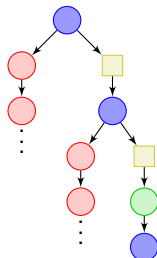
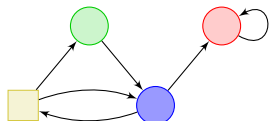


Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.




Strategy for player :
alternately go to  and .

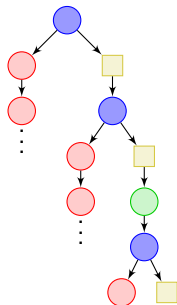
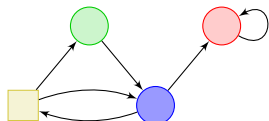


Reasoning about open systems

Strategies

A **strategy** for a given player is a function telling what to play depending on what has happened previously.

Strategy for player 
alternately go to  and .



Alternating-time Temporal Logic

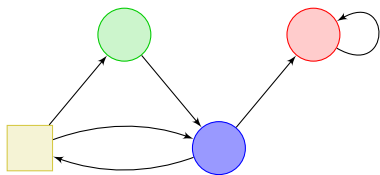
ATL formulas are built inductively using atomic propositions, Boolean combinations, temporal modalities **X** and **U**, and strategy quantifiers:


$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .

Alternating-time Temporal Logic

ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities X and U** , and **strategy quantifiers**:

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .

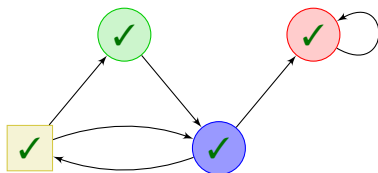



• $\langle\langle O \rangle\rangle F$ 

Alternating-time Temporal Logic

ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities X and U** , and **strategy quantifiers**:

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .

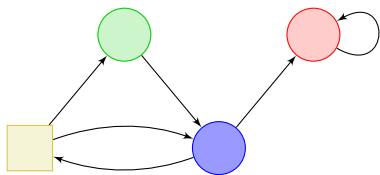


• $\langle\langle O \rangle\rangle F$ 

Alternating-time Temporal Logic

ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities X and U** , and **strategy quantifiers**:

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .



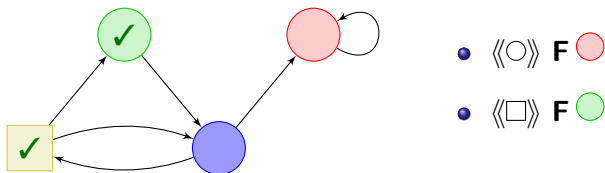
• $\langle\langle \circ \rangle\rangle F$ (Red Circle)

• $\langle\langle \square \rangle\rangle F$ (Green Circle)

Alternating-time Temporal Logic

ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities X and U** , and **strategy quantifiers**:

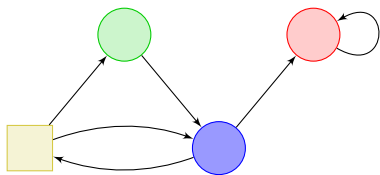
$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .






Alternating-time Temporal Logic

ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities X and U** , and **strategy quantifiers**:

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .

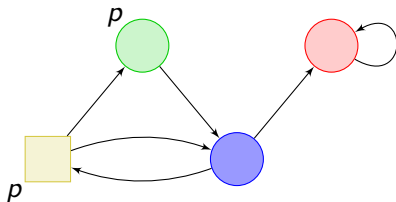


- $\langle\langle \circ \rangle\rangle F$ 
- $\langle\langle \square \rangle\rangle F$ 
- $\langle\langle \circ \rangle\rangle G(\langle\langle \square \rangle\rangle F \text{ })$

Alternating-time Temporal Logic

ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities \mathbf{X} and \mathbf{U}** , and **strategy quantifiers**:

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .



- $\langle\langle \circ \rangle\rangle \mathbf{F}$ (red circle)

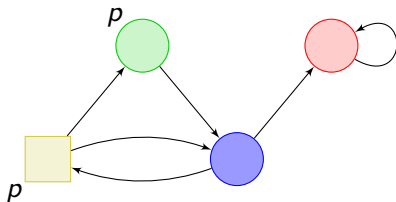
- $\langle\langle \square \rangle\rangle \mathbf{F}$ (green circle)


- $\langle\langle \circ \rangle\rangle \mathbf{G}(\underbrace{\langle\langle \square \rangle\rangle \mathbf{F}}_p)$ (green circle) $\equiv \langle\langle \circ \rangle\rangle \mathbf{G} p$


Alternating-time Temporal Logic


ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities **X** and **U****, and **strategy quantifiers**:

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .



- $\langle\langle \circ \rangle\rangle \mathbf{F}$ 

- $\langle\langle \square \rangle\rangle \mathbf{F}$ 

- $\langle\langle \circ \rangle\rangle \mathbf{G}(\underbrace{\langle\langle \square \rangle\rangle \mathbf{F}}_p \text{ }) \equiv \langle\langle \circ \rangle\rangle \mathbf{G} p$

Alternating-time Temporal Logic

ATL formulas are built inductively using **atomic propositions**, **Boolean combinations**, **temporal modalities X and U**, and **strategy quantifiers**:

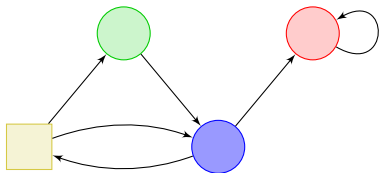
$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ .

Theorem

ATL model checking is PTIME-complete.

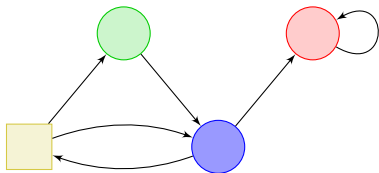
*ATL **symbolic** model checking is EXPTIME-complete.*

Another semantics: ATL with strategy contexts

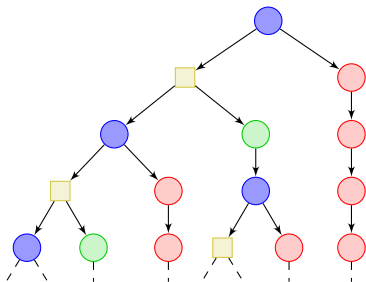


$\langle\langle \circ \rangle\rangle \mathbf{G}(\langle\langle \square \rangle\rangle \mathbf{F} \circ)$

Another semantics: ATL with strategy contexts



$\langle\langle \circ \rangle\rangle \mathbf{G}(\langle\langle \square \rangle\rangle \mathbf{F} \circ)$



- consider the following strategy of Player \circ : “always go to \square ”;

ATL with strategy contexts

Definition

ATL_{sc} has two new strategy quantifiers: $\langle \cdot A \cdot \rangle \varphi$ and $\langle\langle A \rangle\rangle \varphi$.

- $\langle \cdot A \cdot \rangle$ is similar to $\langle\langle A \rangle\rangle$ but **assigns** the corresponding strategy to A for evaluating φ ;
- $\langle\langle A \rangle\rangle$ drops the assigned strategies for A .

ATL with strategy contexts

Theorem

ATL_{sc} is strictly more expressive than ATL.

ATL with strategy contexts

Theorem

ATL_{sc} is strictly more expressive than ATL.

Proof

$$\langle\langle A \rangle\rangle \varphi \equiv \langle\langle \text{Agt} \rangle\rangle \langle \cdot A \cdot \rangle \hat{\varphi}$$

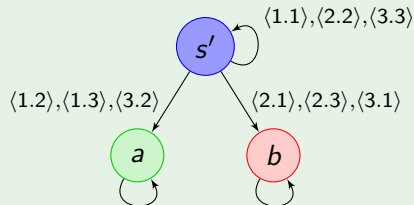
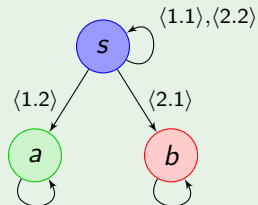
ATL with strategy contexts

Theorem

ATL_{sc} is strictly more expressive than ATL.

Proof

$\langle 1 \rangle (\langle 2 \rangle \mathbf{X} a \wedge \langle 2 \rangle \mathbf{X} b)$ is only true in the second game.
But ATL cannot distinguish between these two games.



What ATL_{sc} can express

- All ATL^* properties;

What ATL_{SC} can express

- All ATL^* properties;
- Client-server interactions for accessing a shared resource:

$$\langle \text{Server} \rangle \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \text{Clients}} \langle \cdot c \rangle \mathbf{F} \text{ access}_c \\ \neg \bigwedge_{c \neq c'} \text{access}_c \wedge \text{access}_{c'} \end{array} \right]$$

What ATL_{sc} can express

- All ATL^* properties;
- Client-server interactions for accessing a shared resource:

$$\langle \text{Server} \rangle \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \text{Clients}} \langle c \rangle \mathbf{F} \text{ access}_c \\ \neg \bigwedge_{c \neq c'} \text{access}_c \wedge \text{access}_{c'} \end{array} \right]$$

- Existence of Nash equilibria:

$$\langle A_1, \dots, A_n \rangle \bigwedge_i (\langle A_i \rangle \varphi_{A_i} \Rightarrow \varphi_{A_i})$$

What ATL_{sc} can express

- All ATL^* properties;
- Client-server interactions for accessing a shared resource:

$$\langle \cdot \text{Server} \cdot \rangle \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \text{Clients}} \langle \cdot c \cdot \rangle \mathbf{F} \text{ access}_c \\ \neg \bigwedge_{c \neq c'} \text{access}_c \wedge \text{access}_{c'} \end{array} \right]$$

- Existence of Nash equilibria:

$$\langle A_1, \dots, A_n \rangle \bigwedge_i (\langle A_i \cdot \rangle \varphi_{A_i} \Rightarrow \varphi_{A_i})$$

- Existence of dominating strategy:

$$\langle A \cdot \rangle [B] (\neg \varphi \Rightarrow [A] \neg \varphi)$$

Model checking ATL_{sc}

Theorem

Given a CGS \mathcal{C} , a state l_0 and an ATL_{sc} formula φ , we can build an alternating parity tree automaton \mathcal{A} s.t.

$$\mathcal{L}(\mathcal{A}) \neq \emptyset \quad \Leftrightarrow \quad \mathcal{C}, l_0 \models_{\emptyset} \varphi.$$

\mathcal{A} has size d -exponential, where d is the maximal number of nested quantifiers.

Model checking ATL_{sc}

Theorem

Given a CGS \mathcal{C} , a state l_0 and an ATL_{sc} formula φ , we can build an alternating parity tree automaton \mathcal{A} s.t.

$$\mathcal{L}(\mathcal{A}) \neq \emptyset \quad \Leftrightarrow \quad \mathcal{C}, l_0 \models_{\emptyset} \varphi.$$

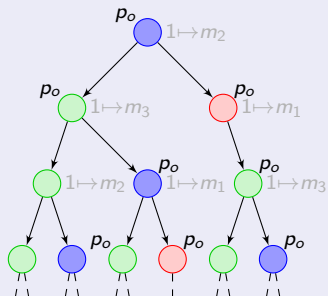
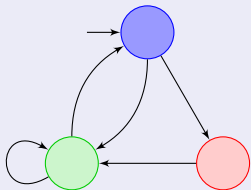
\mathcal{A} has size d -exponential, where d is the maximal number of nested quantifiers.

Theorem

Model checking ATL_{sc} is **d-EXPTIME**-complete.

Model checking ATL_{sc}

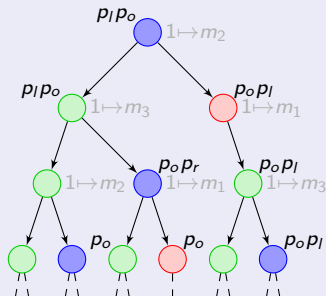
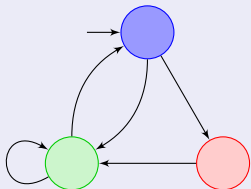
Tree-automata approach



- We can mark outcomes corresponding to selected strategies;

Model checking ATL_{sc}

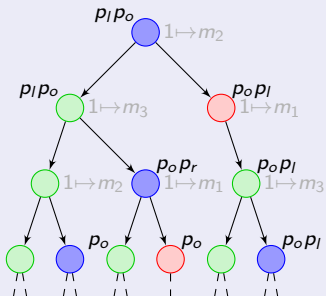
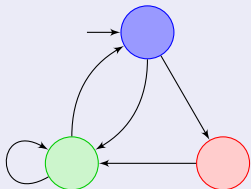
Tree-automata approach



- We mark the tree with extra propositions p_l and p_r , and require that it satisfies $\mathbf{A}(\mathbf{G} p_o \Rightarrow p_l \mathbf{U} p_r)$;

Model checking ATL_{sc}

Tree-automata approach



- We require that subtrees rooted at a p_l or p_r node is accepted by the automaton for φ or φ' , respectively;

Conclusions

- Our results on ATL_{SC} :
 - ATL_{SC} is a **natural semantic extension** of the popular ATL;
 - ATL_{SC} is **much more expressive**: equilibria, client-server interactions... Well-suited for non-zero-sum objectives;
 - There is a price for this expressiveness: **high complexity** of the model-checking algorithm.

Conclusions

- Our results on ATL_{SC} :
 - ATL_{SC} is a **natural semantic extension** of the popular ATL;
 - ATL_{SC} is **much more expressive**: equilibria, client-server interactions... Well-suited for non-zero-sum objectives;
 - There is a price for this expressiveness: **high complexity** of the model-checking algorithm.

- Future works:
 - study satisfiability of ATL_{SC} ;
 - behavioural equivalence for ATL_{SC} .
 - handle stochastic strategies, partial observation, ...