

Statistical Approaches

to Industrial Monitoring Problems -

Fault Detection and Isolation

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Three examples

Design of the algorithms : heuristics

Design of the algorithms : theory

Design of the algorithms : back to the examples

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Three examples

Structural vibration monitoring

Physical *linear dynamic* model.

Combustion set of gas turbines

Semi-physical *non-linear static* model.

Catalytic converter of an automobile

Semi-physical *non-linear dynamic* model.

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Introduction

Signal processing and on-board monitoring

Early detection of slight deviations

with respect to a characterization of the system

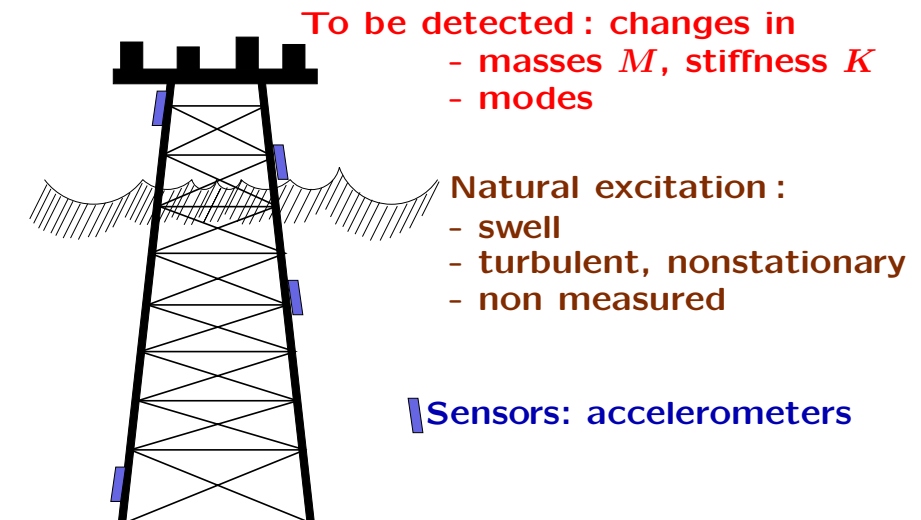
in usual working conditions



Condition-based maintenance, predictive maintenance

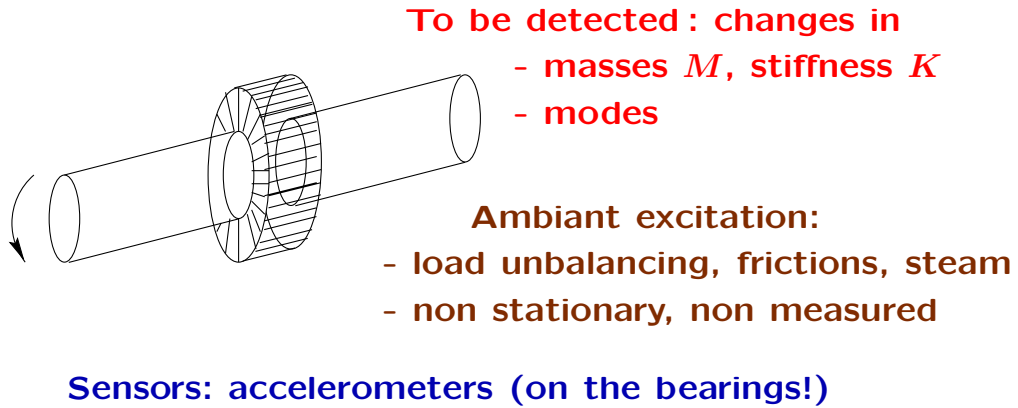
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VIBRATIONS : OFFSHORE STRUCTURES



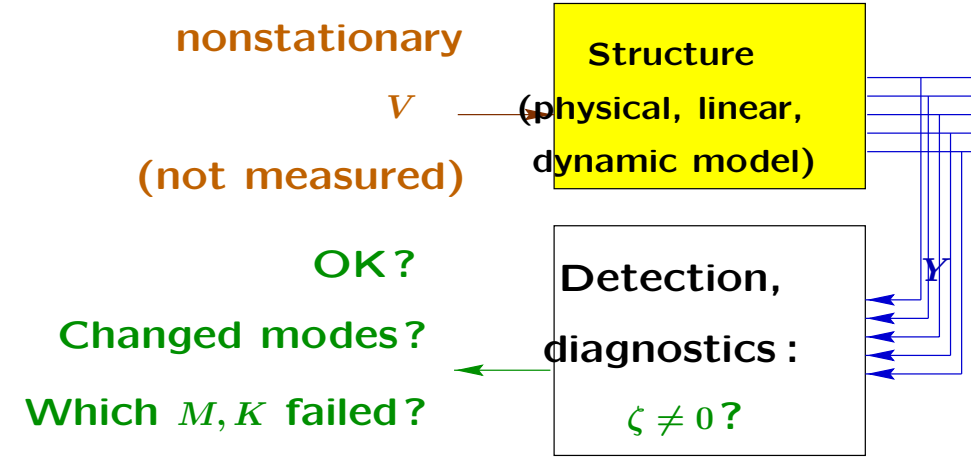
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VIBRATIONS : ROTATING MACHINES



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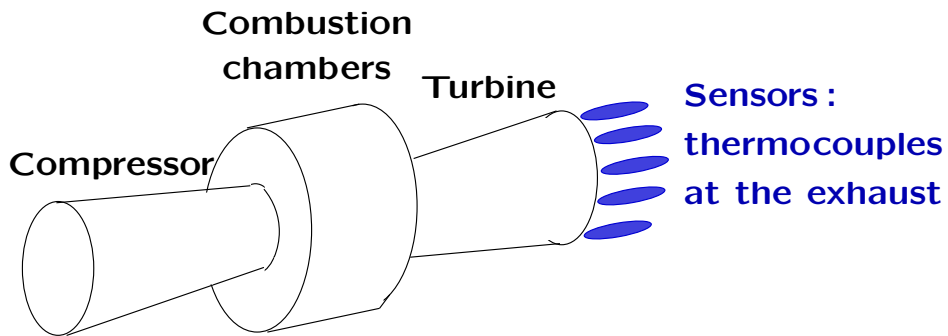
VIBRATIONS: MONITORING SCHEME



ζ shows if the model still fits the data

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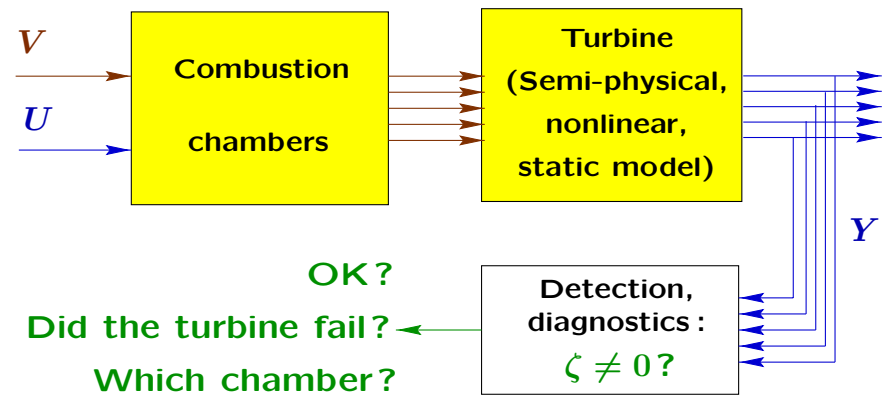
COMBUSTION : GAS TURBINES



To be detected : changes in burners and turbine

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COMBUSTION : MONITORING SCHEME

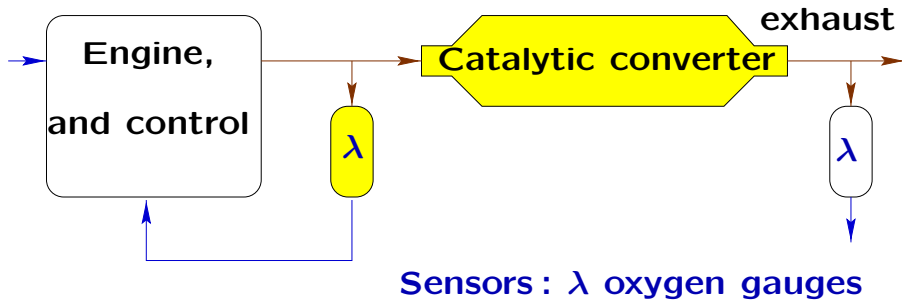


ζ shows if the model still fits the data

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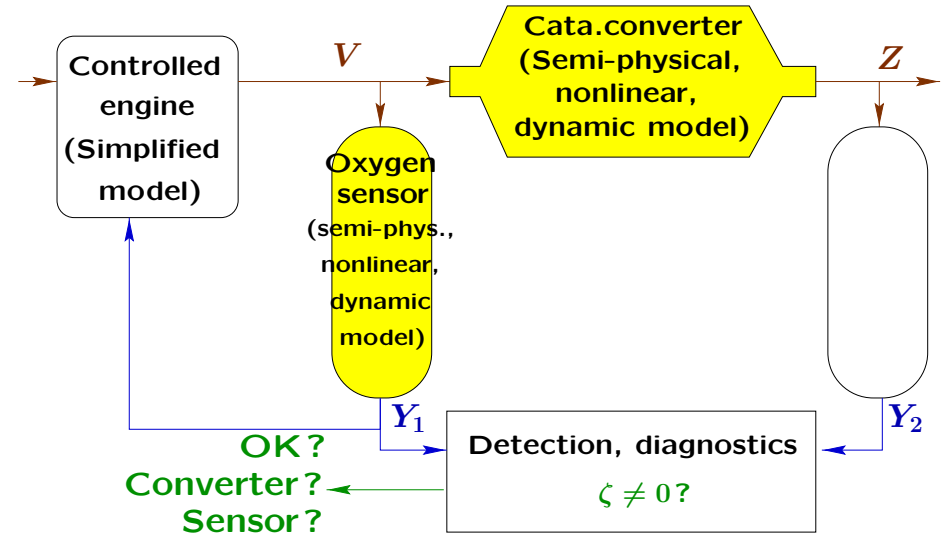
AUTOMOBILE : DEPOLLUTION SYSTEM

To be diagnosed (OBD2 norm):
catalytic converter, front λ oxygen sensors



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DEPOLLUTION : MONITORING SCHEME



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Design of the algorithms : heuristics

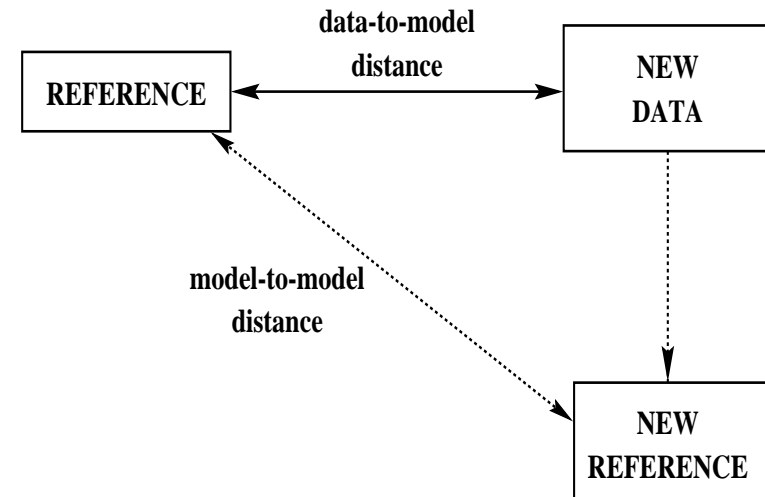
(Model validation)

Modeling and identification

Monitoring

Reduction to a universal problem

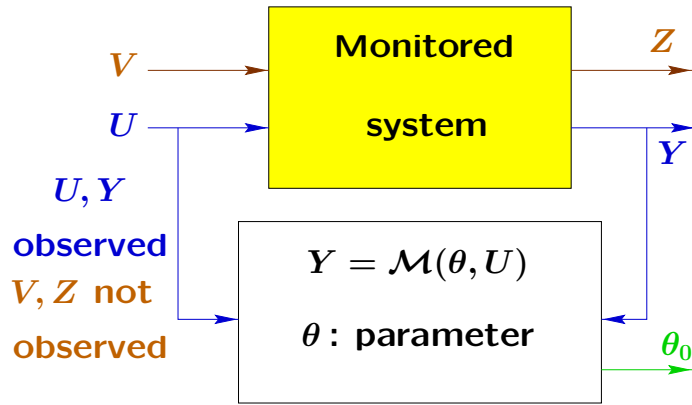
On-Board Detection : don't re-identify!



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Modeling and identification

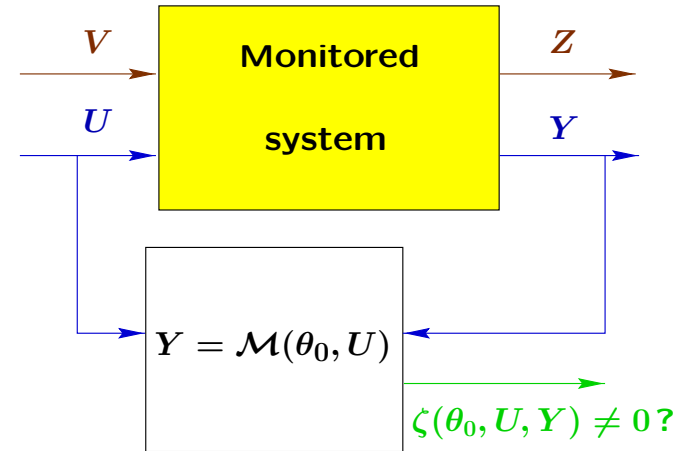


$$\theta_0 = \arg \min_{\theta} \sum_k (Y_k - \mathcal{M}(\theta, U_k))^2$$

or $\theta : \sum_k \mathcal{C}(Y_k, \mathcal{M}(\theta_0, U_k)) = 0$

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Monitoring



$$\zeta(\theta, U, Y) \triangleq \frac{\partial}{\partial \theta} \sum_k (Y_k - \mathcal{M}(\theta, U_k))^2$$

or: $\zeta(\theta, U, Y) \triangleq \sum_k \mathcal{C}(Y_k, \mathcal{M}(\theta, U_k))$

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Reduction to a universal problem

$$\zeta_N \triangleq \zeta(\theta_0, \mathcal{Y}_1^N) \neq 0 ?$$

Local approach

Test $\mathcal{H}_0 : \theta = \theta_0$ against $\mathcal{H}_1 : \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$

$$\zeta_N \sim \mathcal{N}(0, \Sigma(\theta_0)) \quad \zeta_N \sim \mathcal{N}(M(\theta_0) \delta\theta, \Sigma(\theta_0))$$

$$\chi^2 \text{ in } \zeta_N$$

Noises and uncertainty on θ_0 taken into account.

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Design of the algorithms : theory

Local approach : likelihood

Local approach : other estimating/monitoring functions

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Local approach : likelihood

Log-likelihood function

$$\ln p_{\theta}(\mathcal{Y}_1^N) = \sum_{k=1}^N \ln p_{\theta}(Y_k | \mathcal{Y}_1^{k-1})$$

Efficient score

$$\zeta_N(\theta_0) \triangleq \frac{1}{\sqrt{N}} \left. \frac{\partial \ln p_{\theta}(\mathcal{Y}_1^N)}{\partial \theta} \right|_{\theta=\theta_0} = \frac{1}{\sqrt{N}} \sum_{k=1}^N \dots$$

Fisher information matrix

$$\mathcal{I}(\theta) \triangleq \lim_{N \rightarrow \infty} \mathcal{I}_N(\theta), \quad \mathcal{I}_N(\theta) \triangleq \text{cov} \zeta_N(\theta) = -\frac{1}{N} \mathbb{E}_{\theta} \frac{\partial^2 \ln p_{\theta}(\mathcal{Y}_1^N)}{\partial \theta^2}$$

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First order Taylor expansion of the efficient score

$$\zeta_N(\theta) \approx \zeta_N(\theta_0) + \frac{1}{N} \left. \frac{\partial^2 \ln p_{\theta}(\mathcal{Y}_1^N)}{\partial \theta^2} \right|_{\theta=\theta_0} \delta \theta$$

$$\mathbb{E}_{\theta_0} \zeta_N(\theta) \approx -\mathcal{I}(\theta_0) \delta \theta$$

Efficient score = ML estimating function

$$\mathbb{E}_{\theta_0} \zeta_N(\theta) = 0 \iff \theta = \theta_0$$

Caution : Efficient score \neq innovation !

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Second order Taylor expansion of the log-likelihood ratio

$$\theta = \theta_0 + \frac{\delta \theta}{\sqrt{N}}$$

$$S_N(\theta_0, \theta) \triangleq \ln \frac{p_{\theta}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} \approx \delta \theta^T \zeta_N(\theta_0) - \frac{1}{2} \delta \theta^T \mathcal{I}(\theta_0) \delta \theta$$

$$\mathbb{E}_{\theta_0} S_N \approx -\frac{1}{2} \delta \theta^T \mathcal{I}(\theta_0) \delta \theta$$

$$\mathbb{E}_{\theta} S_N \approx +\frac{1}{2} \delta \theta^T \mathcal{I}(\theta_0) \delta \theta \approx -\mathbb{E}_{\theta_0} S_N$$

$$\text{cov}_{\theta_0} S_N \approx \delta \theta^T \mathcal{I}(\theta_0) \delta \theta \approx \text{cov}_{\theta} S_N$$

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Example : Gaussian scalar AR process

$$Y_k = \sum_{i=1}^p a_i Y_{k-i} + E_k, \quad \theta^T = (a_1 \dots a_p)$$

$$\zeta_N(\theta) = \frac{1}{\sqrt{N}} \frac{1}{\sigma^2} \sum_{k=1}^N \mathcal{Y}_{k-1,p}^- \varepsilon_k(\theta)$$

$$\mathcal{I}(\theta) = \frac{1}{\sigma^2} \mathbb{T}_p$$

Efficient score ζ : vector-valued function

Innovation ε : scalar function

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Hypotheses testing - CLT

Locally Asymptotic Normal (LAN) family (Le Cam, 1960)

$$\begin{aligned}
 S_N(\theta_0, \theta_N) &\triangleq \ln \frac{p_{\theta_N}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} \\
 &\approx \delta\theta^T \zeta_N(\theta_0) - \frac{1}{2} \delta\theta^T \mathcal{I}_N(\theta_0) \delta\theta + \alpha_N(\theta_0, \mathcal{Y}_1^N, \delta\theta) \\
 \zeta_N(\theta_0) &\rightarrow \mathcal{N}(0, \mathcal{I}(\theta_0)) \\
 \alpha_N &\rightarrow 0 \text{ a.s. under } \mathcal{H}_0
 \end{aligned}$$

Examples :

i.i.d. variables, stationary Gaussian processes, stationary Markov processes.

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Asymptotically optimum tests for composite hypotheses

Asymptotically equivalent problems :

$$(1) : \mathcal{P} = \{P_\theta\}_{\theta \in \Theta} \text{ LAN}$$

$$\mathcal{Y}_1^N, N \rightarrow \infty$$

$$\mathcal{H}_0 = \{\theta \in \Theta_0\}, \quad \mathcal{H}_1 = \{\theta \in \Theta_1\}, \quad \Theta_i = \theta_0 + \frac{\Gamma_i}{\sqrt{N}}$$

$$(2) : \zeta \sim \mathcal{N}(\Upsilon, \mathcal{I}^{-1}(\theta_0))$$

$$\mathcal{H}_0 = \{\Upsilon \in \Gamma_0\}, \quad \mathcal{H}_1 = \{\Upsilon \in \Gamma_1\}$$

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CLT in LAN families

(i.i.d. variables, stationary Gaussian processes, stationary Markov processes)

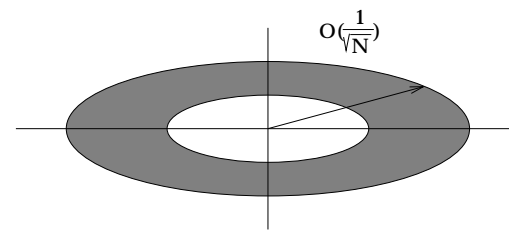
$$\begin{aligned}
 S_N(\theta_0, \theta) &\rightarrow \begin{cases} \mathcal{N}\left(-\frac{1}{2} \delta\theta^T \mathcal{I}(\theta_0) \delta\theta, \delta\theta^T \mathcal{I}(\theta_0) \delta\theta\right) & \text{under } P_{\theta_0} \\ \mathcal{N}\left(+\frac{1}{2} \delta\theta^T \mathcal{I}(\theta_0) \delta\theta, \delta\theta^T \mathcal{I}(\theta_0) \delta\theta\right) & \text{under } P_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{cases} \\
 \zeta_N(\theta_0) &\rightarrow \begin{cases} \mathcal{N}(0, \mathcal{I}(\theta_0)) & \text{under } P_{\theta_0} \\ \mathcal{N}(\mathcal{I}(\theta_0) \delta\theta, \mathcal{I}(\theta_0)) & \text{under } P_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{cases}
 \end{aligned}$$

ζ_N is asymptotically a sufficient statistics.

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Asymptotically equivalent and UMP tests

(Kushnir-Pinski, 1971; Nikiforov, 1982)



$$\frac{\sup_{\theta \in \Theta_1} p_\theta(\mathcal{Y}_1^N)}{\sup_{\theta \in \Theta_0} p_\theta(\mathcal{Y}_1^N)} \geq \lambda$$

$$\frac{p_{\hat{\theta}}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} \geq \lambda$$

$$N (\hat{\theta} - \theta_0)^T \mathcal{I}(\theta_0) (\hat{\theta} - \theta_0) \geq \lambda$$

$$\zeta_N^T(\theta_0) \mathcal{I}^{-1}(\theta_0) \zeta_N(\theta_0) \geq \lambda$$

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Local approach : other estimating functions

Quasi-score

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^N H(\theta_0, Y_k)$$

Estimating function

$$E_{\theta_0} H(\theta, Y_k) = 0 \iff \theta = \theta_0$$

Mean deviation

$$M(\theta_0) \triangleq - E_{\theta_0} \left. \frac{\partial}{\partial \theta} H(\theta, Y_k) \right|_{\theta=\theta_0}$$

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Computation of the mean deviation

$$M(\theta_0) \triangleq - E_{\theta_0} \left. \frac{\partial}{\partial \theta} H(\theta, Y_k) \right|_{\theta=\theta_0}$$

$$= - \left. \frac{\partial}{\partial \theta} E_{\theta_0} H(\theta, Y_k) \right|_{\theta=\theta_0}$$

$$= + \left. \frac{\partial}{\partial \theta} E_{\theta} H(\theta_0, Y_k) \right|_{\theta=\theta_0}$$

$$= \text{cov}_{\theta_0}(\zeta_N(\theta_0), \zeta_N^{ML}(\theta_0)) \quad (\text{i.i.d. case})$$

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First order Taylor expansion of a quasi-score

$$\theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$$

$$\zeta_N(\theta) \approx \underbrace{\zeta_N(\theta_0)}_{\text{CLT}} + \sqrt{N} \underbrace{\frac{1}{N} \left(\sum_{k=1}^N \left. \frac{\partial}{\partial \theta} H(\theta, Y_k) \right|_{\theta=\theta_0} \right)}_{\text{LLN}} \frac{\delta\theta}{\sqrt{N}}$$

CLT \downarrow under θ_0

$\mathcal{N}(0, \Sigma(\theta_0))$

LLN \downarrow under θ_0

$$\underbrace{E_{\theta_0} \left. \frac{\partial}{\partial \theta} H(\theta, Y_k) \right|_{\theta=\theta_0}}$$

$$= - M(\theta_0)$$

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Estimation efficiency and covariance of estimating fct

$$\sqrt{N} (\hat{\theta}_N - \theta_0) \approx \underbrace{- \left(E_{\theta_0} \left. \frac{\partial}{\partial \theta} H(\theta, Y_k) \right|_{\theta=\theta_0} \right)^{-1}}_{M(\theta_0)^{-1}} \zeta_N(\theta_0)$$

Hence

$$\text{cov}(\hat{\theta}_N - \theta_0) = (M^T(\theta_0) \Sigma^{-1}(\theta_0) M(\theta_0))^{-1}$$

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Quasi score - CLT

(McLeish, 1977; Heyde, 1989; Delyon and Juditsky, 1997)

$$\zeta_N(\theta_0) \rightarrow \begin{cases} \mathcal{N}(0, \Sigma(\theta_0)) & \text{under } \mathbf{P}_{\theta_0} \\ \mathcal{N}(M(\theta_0) \delta\theta, \Sigma(\theta_0)) & \text{under } \mathbf{P}_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

Tests for composite hypotheses

(Basawa, 1985; Benveniste et al., 1987)

$$\zeta_N^T \underbrace{\Sigma^{-1} M (M^T \Sigma^{-1} M)^{-1} M^T \Sigma^{-1}}_{\mathcal{I}^{-1}} \zeta_N \geq \lambda$$

χ^2 -test invariant w.r.t. pre-multiplication of H
(and thus ζ) by an invertible matrix gain.

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Structural vibration monitoring

$$\begin{cases} M\ddot{\mathbf{Z}}_t + C\dot{\mathbf{Z}}_t + K\mathbf{Z}_t = \mathbf{E}_t & (M\mu^2 + C\mu + K)\Psi_\mu = 0 \\ \mathbf{Y}_t = L\mathbf{Z}_t & \psi_\mu = L\Psi_\mu \end{cases}$$

$$\begin{cases} \mathbf{X}_{k+1} = F\mathbf{X}_k + W_k & F\varphi_\lambda = \lambda\varphi_\lambda \\ \mathbf{Y}_k = H\mathbf{X}_k & e^{\tau\mu} = \lambda, \quad \psi_\mu = H\varphi_\lambda \end{cases}$$

$$\mathbf{Y}_k = \sum_{i=1}^p A_i \mathbf{Y}_{k-i} + \sum_{j=0}^q B_j(k) V_{k-j} \quad H F^p = \sum_{i=1}^p A_i H F^{p-i}$$

Monitor AR part, with nonstationary MA part.

Modal changes not visible on spectra (1%).

Likelihood : no hope !

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Design of the algorithms : back to the examples

Vibrations

Gas turbines

Catalytic converter

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Eigenstructure monitoring

$$\begin{cases} \mathbf{X}_{k+1} = F\mathbf{X}_k + W_k & F\varphi_\lambda = \lambda\varphi_\lambda \\ \mathbf{Y}_k = H\mathbf{X}_k & \Phi_\lambda \triangleq H\varphi_\lambda \end{cases}$$

Canonical system parameter

$$\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}, \quad \mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$$

System parameter characterization

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel space

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1. **Input-output (IV-based) monitoring functions :**
(Rougée, 1985)

$$(A^T \quad -I_r) \bar{\mathcal{H}}_{p+1,q}^0 = 0$$

$$\left\{ \begin{array}{l} A^T \triangleq (A_p \quad \dots \quad A_1), \quad \bar{\mathcal{H}}_{p+1,q} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \dots & \hat{R}_{q-1} \\ \hat{R}_1 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \hat{R}_p & \dots & \dots & \hat{R}_{p+q-1} \end{pmatrix} \\ (A^T(\theta_0) \quad -I_r) \mathcal{O}_{p+1}(\theta_0) = 0 \end{array} \right.$$

$$\zeta_N(\theta_0) = \sqrt{N} \text{vec}((A^T(\theta_0) \quad -I_r) \bar{\mathcal{H}}_{p+1,q})$$

$$\bar{\mathcal{H}}_{p+1,q} = \frac{1}{N-p-q+1} \sum_{k=q}^{N-p} \mathcal{Y}_{k,p+1}^+ \mathcal{Y}_{k,q}^{-T}$$

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2. **Subspace-based monitoring functions :**
(Abdelghani et al., 1996)

$$\text{SVD of } W_1 \bar{\mathcal{H}}_{p+1,q}^0 W_2^T$$

$$\left\{ \begin{array}{l} S^T(\theta) W_1 \bar{\mathcal{H}}_{p+1,q}^0 W_2^T = 0 \\ S^T S = I_s, \quad S^T W_1 \mathcal{O}_{p+1}(\theta) = 0 \end{array} \right.$$

$$\zeta_N(\theta_0) = \sqrt{N} \text{vec}(S^T(\theta_0) W_1 \bar{\mathcal{H}}_{p+1,q} W_2^T)$$

$$\bar{\mathcal{H}}_{p+1,q} = \frac{1}{N-p-q+1} \sum_{k=q}^{N-p} \mathcal{Y}_{k,p+1}^+ \mathcal{Y}_{k,q}^{-T}$$

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Experimental results

- **Simulated and real data** (offshore platform, alternator):
- **Detect < 1% frequency changes**
- **Detect changes mainly in modal shapes**
- **In the LMS modal analysis software environment:**
current experiments on several laboratory experimental setups and international benchmarks

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Combustion set of gas turbines

$$y_j = f(U) \sum_{i=1}^{10} \alpha_i g(\phi_j - h(U) - \beta) \quad (1 \leq j \leq 18)$$

$$\theta \triangleq (\alpha_1 \quad \dots \quad \alpha_{10} \quad \beta)^T$$

$$Y_k(\theta) \triangleq (y_1 \quad \dots \quad y_{18})^T(k)$$

Noisy outputs Y and inputs U : biased reference θ_0 .

Identification bias is not an obstacle to monitoring!
(Zhang, 1992)

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$$H(\theta, Y_k, U_k) = - \left(\frac{\partial \epsilon_k(\theta)}{\partial \theta} \right)^T \epsilon_k(\theta)$$

$$\epsilon_k(\theta) = Y_k - Y_k(\theta)$$

Bias-adjusted estimating function

$$\hat{h}_0 = \frac{1}{N} \sum_{k=1}^N H(\theta_0, Y_k^0, U_k^0)$$

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^N (H(\theta_0, Y_k, U_k) - \hat{h}_0)$$

Robustness w.r.t. operating conditions :

shape of the hypotheses! (Mathis, 1994)

Experimental results

- **Simulated and real data**
(a turbine operating during several months)
- **Two algorithms tuned and implemented differently :**
one on-line: for large sudden drops in temperatures,
the other off-line: for smaller and slower drops
- **Robustness improved using confidence ellipsoids**
- **Correct diagnostics**

Catalytic converter and oxygen sensor of an automobile

$$\begin{cases} \dot{X} = f(\theta, X, U) \\ Y = g(\theta, X, U) \end{cases}$$

$$\begin{cases} \delta X_k = f(\theta, X_k, U_k) \\ Y_k = g(\theta, X_k, U_k) + \epsilon_k^{(Y)} \end{cases}$$

Unknown input U ; highly nonlinear functions f, g .

1. Full state observer and LS-prediction-score :

$$H(\theta, \mathcal{Y}_1^k, \mathcal{U}_1^k) = \left(\frac{\partial \hat{Y}_{k|k-1}(\theta)}{\partial \theta} \right)^T (Y_k - \hat{Y}_{k|k-1}(\theta))$$

$$\begin{cases} \hat{X}_{k|k-1} = \hat{f}(\theta, \hat{X}_{k-1|k-2}, Y_{k-1}, U_{k-1}) \\ \hat{Y}_{k|k-1}(\theta) = g(\theta, \hat{X}_{k|k-1}, U_{k-1}) \end{cases}$$

$$\frac{\partial \hat{Y}_{k|k-1}(\theta)}{\partial \theta} \text{ solution of a differential system.}$$

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2. Input-output description and LS-score : (Zhang, 1996)

$$F(\theta, Y_k, U_k, \delta) = \varepsilon_k^{(F)}$$

$$H(\theta, Y_k, U_k, \delta) = - \left(\frac{\partial F(\theta, Y_k, U_k, \delta)}{\partial \theta} \right)^T F(\theta, Y_k, U_k, \delta)$$

$$F(\theta, Y_k, U_k, \delta) = P(Y_k, U_k, \delta) \theta - Q(Y_k, U_k, \delta)$$

$$\begin{aligned} 2 H(\theta, Y_k, U_k, \delta) = & - P^T(Y_k, U_k, \delta) P(Y_k, U_k, \delta) \theta \\ & + P^T(Y_k, U_k, \delta) Q(Y_k, U_k, \delta) \end{aligned}$$

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Statistical approaches to isolation

$$\zeta \sim \mathcal{N}(M \eta, \bar{\Sigma} \bar{\Sigma}^T), \quad \eta = \begin{pmatrix} \eta_a \\ \eta_b \end{pmatrix}, \quad M = \begin{pmatrix} M_a & M_b \end{pmatrix}, \quad p_{\eta_a, \eta_b}(\zeta)$$

Decide between $\eta_a = 0$ and $\eta_a \neq 0$; η_b unknown

$$\mathcal{I} = M^T \Sigma^{-1} M \triangleq \begin{pmatrix} \mathcal{I}_{aa} & \mathcal{I}_{ab} \\ \mathcal{I}_{ba} & \mathcal{I}_{bb} \end{pmatrix}$$

$$\mathcal{I}_a^{*-1} : \text{upper-left term of } \mathcal{I}^{-1}; \quad \mathcal{I}_a^* = \mathcal{I}_{aa} - \mathcal{I}_{ab} \mathcal{I}_{bb}^{-1} \mathcal{I}_{ba}$$

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Experimental results

– Combined LS-score and observer :

Simulated and real data

- catalytic converter and front oxygen sensor;
- some subsystems of a nuclear power plant;
- some subsystems of a thermal plant.

– Combined LS-score and input-output equations :

- some subsystems of a nuclear power plant.

Statistical projection (sensitivity)

$$2 \ln \frac{\max_{\eta_a} p_{\eta_a,0}(\zeta)}{p_{0,0}(\zeta)} = \zeta_a^T \mathcal{I}_{aa}^{-1} \zeta_a, \quad \zeta_a : \text{partial score}$$

Statistical rejection (minmax)

$$2 \ln \frac{\max_{\eta_a, \eta_b} p_{\eta_a, \eta_b}(\zeta)}{\max_{\eta_b} p_{0, \eta_b}(\zeta)} = \zeta_a^{*T} \mathcal{I}_a^{*-1} \zeta_a^*, \quad \zeta_a^* : \text{effective score}$$

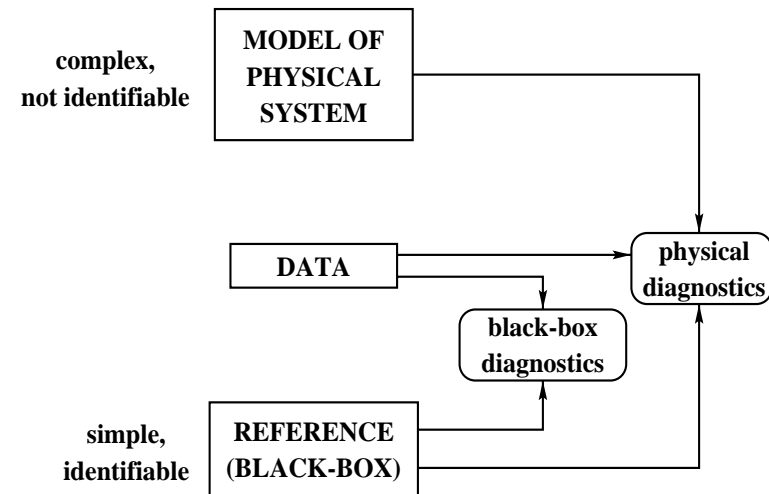
$$\zeta_a^* = \zeta_a - \mathcal{I}_{ab} \mathcal{I}_{bb}^{-1} \zeta_b$$

regression of partial score on nuisance score (Neyman, 1954).

Optimality (Spjøtvoll, 1971; Mathis, 1994).

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On-Board Diagnostics : don't solve the inverse problem!



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'Black-box' diagnostics : sensitivity

$$\delta\theta_i = \mathcal{J}_i \delta\phi_i, \quad \dim \phi = \dim \theta$$

'Physical' diagnostics : sensitivity with model reduction

$$\delta\theta_i = \mathcal{J}_i \delta\Phi_i, \quad \dim \Phi \gg \dim \theta$$

Aggregate the $\delta\theta_i$ with the metric of the χ^2 -test.

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Optimal sensor location

Maximize the power of the detection algorithms :

$$\text{Trace} (M^T \Sigma^{-1} M)$$

Physical model necessary,
'compensate' for the number of d.o.f. of χ^2 tests.

Two possible uses :

- For a given sensor pool : which faults are detectable?
- For a given set of faults : how many sensors, and where?

Fault detectability

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Conclusion

Actual and potential benefits of statistical local approach

Question: quasi-scores for (complex) state-space models?