Statistical fault detection and isolation for linear time-varying systems

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Problem and approaches

FDI for LTV systems

- Relevant approach to FDI of NL systems (linearization along the actual or nominal trajectory)
- LTV systems more general than widely used LPV systems

Three main approaches

- Detection filter, game theoretic approach to filter design, unknown input decoupled filter, UIO, finite horizon fault detection filter
  Keviczky, Edelmayer, Chung-Speyer, Chen-Patton, Hou-Muller, Zhong-Ding, ...
- Adaptive observers, set-valued observers, time domain solutions to different $\mathcal{H}_-/\mathcal{H}_\infty$ problems
  Zhang-Xu, Rosa-Shamma-Athans, Li-Zhou, ...
- Parity-based fault estimation Zhong-Ding

Model and assumptions

MIMO LTV system ($\mathbb{H}_0$)

$$\begin{cases} X_{k+1} = F_k X_k + G_k U_k + W_k \\ Y_k = H_k X_k + J_k U_k + V_k \end{cases}$$

F_k, G_k, H_k, J_k: bounded TV matrices
W_k, V_k: independent white Gaussian noises, TV cov. Q_k, R_k
(H_k, F_k) observable & $(F_k, Q_k^{1/2})$ controllable, both uniformly

Additive faults ($\mathbb{H}_1$)

$$\begin{cases} X_{k+1} = F_k X_k + G_k U_k + W_k + \Psi_k \theta \\ Y_k = H_k X_k + J_k U_k + V_k \end{cases}$$

θ: unknown fault vector
Ψ_k: known TV fault profile

Different fault cases

- Actuator bias: $U_k \to U_k + \theta$; then $\Psi_k = G_k$
- Actuator gain loss: $U_k \to (\mathbf{I} - \mathrm{diag}(\theta))U_k$;
  then $\Psi_k = -\, G_k\, \mathrm{diag}(U_k)$
- Sensor faults: use a similar term $\Psi_k\ \theta$ on the output equation (not treated here)
- Different fault occurrence speeds:
  ex: step change $\Psi_k(r) \triangleq \widetilde{\Psi}_k \times \mathbb{1}_{\{k \geq r\}}$ (and θ constant)

A particular case

- $\Psi_k = \delta_{r,k+1}\ \mathbf{I}$: investigated by Willsky-Jones, Gustafsson with F_k assumed exponentially stable

Fault effect on the Kalman filter innovation

State prediction error and innovation - Fault free case

$$\begin{aligned} \widetilde{X}_k &\triangleq X_k - \widehat{X}_{k|k-1} \\ \varepsilon_k &\triangleq Y_k - H_k\,\widehat{X}_{k|k-1} \end{aligned}$$

$$\begin{aligned} \widetilde{X}^0_{k+1} &= F_k\,(\mathbf{I} - K_k\,H_k)\,\widetilde{X}^0_k - F_k\,K_k\,V_k + W_k \\ \varepsilon^0_k &= H_k\,\widetilde{X}^0_k + V_k \end{aligned}$$

State prediction error and innovation - Faulty case

$$\begin{aligned} \widetilde{X}_{k+1} &= F_k\,(\mathbf{I} - K_k\,H_k)\,\widetilde{X}_k - F_k\,K_k\,V_k + W_k + \Psi_k\,\theta \\ \varepsilon_k &= H_k\,\widetilde{X}_k + V_k \end{aligned}$$

Introducing a matrix gain

$$\eta_k \triangleq \widetilde{X}_k - \Gamma_k \, \theta$$

$$\Gamma_{k+1} \triangleq F_k \, (\mathbf{I} - K_k \, H_k) \, \Gamma_k + \Psi_k \, , \quad \Gamma_0 \triangleq 0$$

$$\eta_{k+1} = F_k \, (\mathbf{I} - K_k \, H_k) \; \eta_k - F_k \; K_k \; V_k + W_k$$

$$\eta_k = \widetilde{X}_k^0$$

Additive fault effect

$$\varepsilon_k = \varepsilon_k^0 + H_k \, \Gamma_k \, \theta$$

Guaranteed properties of the recursive gain

- Γ_k depends on the fault gain Ψ_k, not on the fault vector θ.
- The matrix gain Γ_k computed from the bounded Ψ_k is bounded even when the system is not stable.
- The persistent excitation condition:
  $\sum_k \Gamma_k^T H_k^T \Sigma_k^{-1} H_k \Gamma_k$ *strictly positive definite*
  is satisfied even when the number of sensors is smaller than the number of faults.

Difference with the Willsky-Jones algorithm

- Computations based on recursive formulas involving F_k (thus required to be stable)

Known fault profile matrix

MLE of θ under $\mathbb{H}_1$

$$\mathbb{H}_0 : \varepsilon_k \sim \mathcal{N}\left(0, \Sigma_k\right), \quad \mathbb{H}_1 : \varepsilon_k \sim \mathcal{N}\left(H_k \, \Gamma_k \, \theta, \Sigma_k\right)$$

$$\widehat{\theta}_k = \arg\min_{\widetilde{\theta}} \sum_{j=1}^{k} (\varepsilon_j - H_j \, \Gamma_j \, \widetilde{\theta})^T \, \Sigma_j^{-1} \, (\varepsilon_j - H_j \, \Gamma_j \, \widetilde{\theta}) = C_k^{-1} \, d_k$$

$$C_k = C_{k-1} + \Gamma_k^T \, H_k^T \, \Sigma_k^{-1} \, H_k \, \Gamma_k$$

$$d_k = d_{k-1} + \Gamma_k^T \, H_k^T \, \Sigma_k^{-1} \, \varepsilon_k$$

GLR test

$$l_k \triangleq 2 \, \ln \, \frac{p(\varepsilon_1, \ldots, \varepsilon_k \mid \theta = \widehat{\theta}_k)}{p(\varepsilon_1, \ldots, \varepsilon_k \mid \theta = 0)} = d_k^T \, C_k^{-1} \, d_k$$

Unknown jump fault onset time

Handling the transient behavior after a jump

$$\Psi_k(r) = \widetilde{\Psi}_k \times \mathbb{1}_{\{k \geq r\}} \Longrightarrow \Gamma_{k+1}(r) = F_k\,(\mathbf{I} - K_k H_k)\,\Gamma_k(r) + \Psi_k(r)$$

Full treatment of the transient

$$\begin{aligned}
\widehat{\theta}_k(r) &= C_k^{-1}(r)\, d_k(r) \\
l_k &= \max_{1 \leq r \leq k} \; d_k^T(r)\, C_k^{-1}(r)\, d_k(r) \\
\hat{r}_k &= \arg\max_{1 \leq r \leq k} \; d_k^T(r)\, C_k^{-1}(r)\, d_k(r)
\end{aligned}$$

In practice

$\Gamma_k(r), C_k(r), d_k(r)$ computed for $r \in \{k-w+1, k-w+2, \ldots, k\}$

Locating non zero components

$$Z \triangleq \begin{bmatrix} \varepsilon_{\hat{r}+1} \\ \vdots \\ \varepsilon_k \end{bmatrix} \sim \mathcal{N}\left(M\,\theta, S\right)$$

$$M = \begin{bmatrix} H_{\hat{r}+1}\,\Gamma_{\hat{r}+1}(\hat{r}) \\ \vdots \\ H_k\,\Gamma_k(\hat{r}) \end{bmatrix}, \quad S = \mathrm{diag}(\Sigma_{\hat{r}+1}, \ldots, \Sigma_k)$$

$$\zeta \triangleq M^T S^{-1} Z\,, \quad \mathbb{F} \triangleq M^T S^{-1} M$$

$\theta = \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix}$ – $\theta_a = 0$ against $\theta_a \neq 0$, θ_b nuisance parameter

Minmax test

$$I_a \triangleq {\zeta_a^*}^T \, {\mathbb{F}_a^*}^{-1} \, \zeta_a^*$$

$$\zeta_a^* \triangleq \zeta_a - \mathbb{F}_{ab} \, \mathbb{F}_{bb}^{-1} \, \zeta_b \, , \quad \mathbb{F}_a^* \triangleq \mathbb{F}_{aa} - \mathbb{F}_{ab} \, \mathbb{F}_{bb}^{-1} \, \mathbb{F}_{ba}$$

$$I_a \sim \chi^2(\dim(\theta_a))$$

Leakage detection in gas transportation networks

Thanks to Paulo Lopes dos Santos et al., IEEE CST, Jan. 2011

Gas dynamics as a LPV model

Hyperbolic model linking edge pressure drop and mass flow

Discrete time LPV model

$$\begin{cases} X_{k+1} = (F_0 + F_p\, p_k)\, X_k + (G_0 + G_p\, p_k)\, U_k + K_k\, e_k \\ Y_k = (H_0 + H_p\, p_k)\, X_k + (J_0 + J_p\, p_k)\, U_k + e_k \end{cases}$$

$U_k \in \mathbb{R}$: input mass flow, $Y_k \in \mathbb{R}$: output mass flow
$X_k \in \mathbb{R}^2$: mass flow and pressure drop within the first section
$p_k \in \mathbb{R}$: scheduling parameter (pressure pattern)

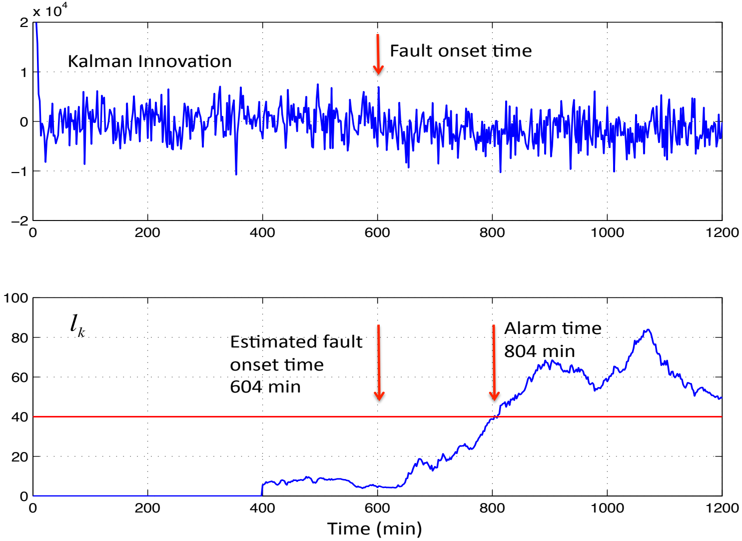
Simulated leakage

- Additive changes on $G_0, G_p \in \mathbb{R}^2$ (actuator gain loss) - Monitoring the first component
- Nominal values:
  $G_0(1) = -7.8297e-4$
  $G_p(1) = +3.8290e-5$
- Changed values:
  $G_0(1) + 1.6e-5$ Fault 1
  $G_p(1) + 9.5e-6$ Fault 2

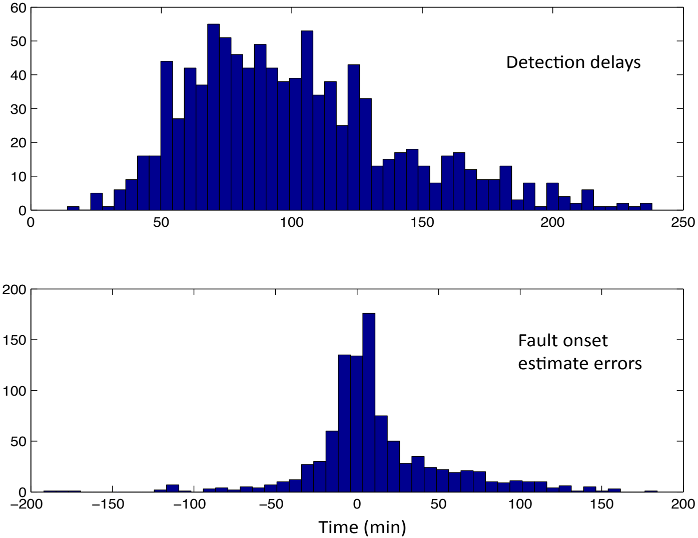
Available data, simulated data

- U_k provided by P. Lopes dos Santos *et al.*
- Y_k simulated using the LPV model

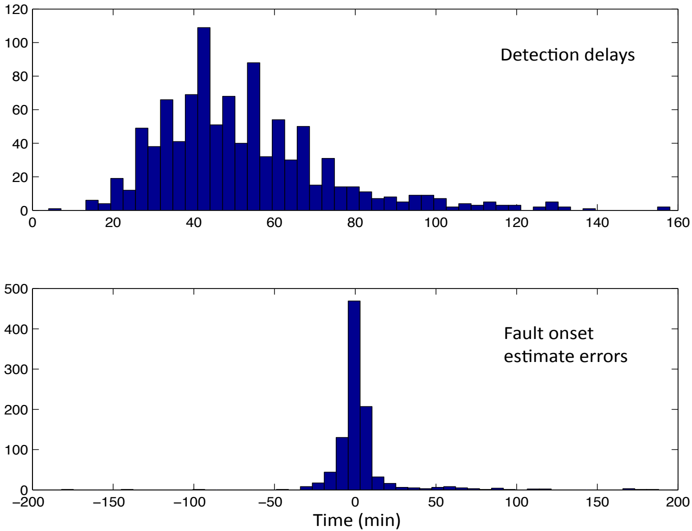
Fault detection - Fault 1



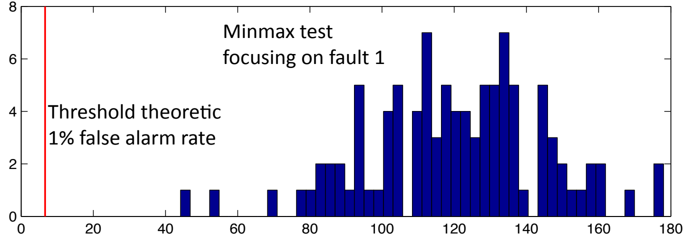
Detection delay and onset time estimate - Fault 1

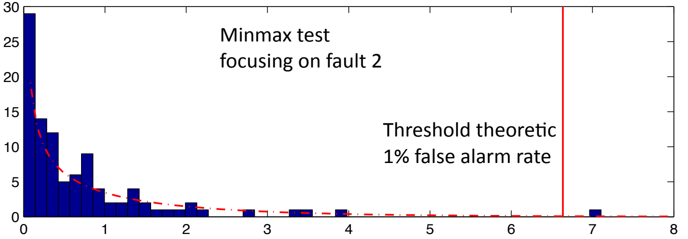


Detection delay and onset time estimate - Fault 2

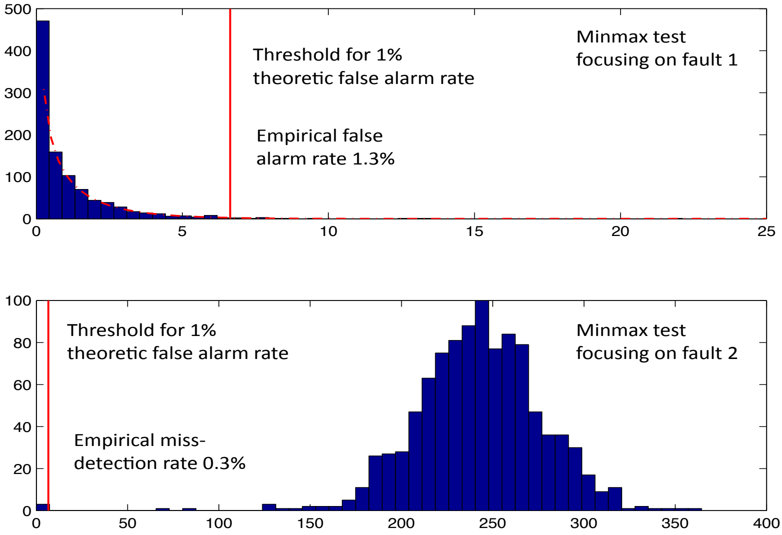


Fault isolation with the minmax tests - Fault 1





Fault isolation with the minmax tests - Fault 2



FDI for LTV systems with TV additive faults

- Combining a recursive and stable filter that cancels out the fault dynamics and a GLR test
- Handling additive parametric faults with weaker assumptions than usual on the system stability and the number of required sensors
- Simulations confirm the stability of the proposed filter and suggest the relevance of the proposed approach
- Future investigations include numerical experiments on simulated and real cases to assess the quality of the full and simplified treatments of the transient