Subspace-based modal identification and monitoring of large structures: The MODAL toolbox within Scilab

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Scilab and MODAL toolbox in

`http://www.scilab.org` `http://www.scilab.org/contrib`

Framework: in-operation modal analysis

- **Excitation**: natural, not controlled, not measured, nonstationary (e.g. turbulent)
  - Output-only eigenstructure identification
- Merging data from moving sensor pools
- On-board detection and localization of small damages.
  - Avoid re-identification prior to detection
  - Avoid inverse problem solving prior to damage localization

Modelling - Eigenstructure problem

FE model:

$$\begin{cases} M\ \ddot{\mathcal{Z}}(s) + C\ \dot{\mathcal{Z}}(s) + K\ \mathcal{Z}(s) = \nu(s) \\ Y(s) = L\ \mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\ \Psi_\mu = 0\ , \quad \psi_\mu = L\ \Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases}$$

$$F\ \varphi_\lambda = \lambda\ \varphi_\lambda\ , \quad \phi_\lambda \triangleq H\ \varphi_\lambda$$

Parameter:

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}}\ , \quad \underbrace{\psi_\mu = \phi_\lambda}_{\text{mode shapes}}\ ; \ \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$$

Output-only covariance-based subspace identification

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k\ Y_{k-i}^T\right)}_{\text{ok if stationary !}}\ , \quad \mathcal{H} = \text{Hank}(R_i)\ , \qquad R_i = H\ F^i\ G$$

$$G \triangleq \mathbf{E}\left(X_k\ Y_k^T\right), \quad \mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq \left(G \quad FG \quad F^2G \quad \dots\right)$$

$$\mathcal{H} = \mathcal{O}\ \mathcal{C}\ , \quad \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Implementation:

$$\underbrace{\hat{R}_i \triangleq 1/n \sum_{k=1}^{n} Y_k\ Y_{k-i}^T}_{\text{ok when nonstationary !}}\ , \quad \hat{\mathcal{H}} = \text{Hank}(\hat{R}_i)$$

SVD($\hat{\mathcal{H}}$) + truncation $\longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$

Merging multiple measurements setups

$$\underbrace{\begin{bmatrix} Y_k^{(0,1)} \\ Y_k^{(1)} \end{bmatrix}}_{\text{Record 1}} \quad \underbrace{\begin{bmatrix} Y_k^{(0,2)} \\ Y_k^{(2)} \end{bmatrix}}_{\text{Record 2}} \quad \cdots \quad \underbrace{\begin{bmatrix} Y_k^{(0,J)} \\ Y_k^{(J)} \end{bmatrix}}_{\text{Record J}} \qquad \begin{matrix} \text{Fixed} \\ \text{Moving} \end{matrix}$$

$$R_i^{0,j} \triangleq \mathbf{E}\, Y_k^{(0,j)}\, Y_{k-i}^{(0,j)T}\,, \quad R_i^{j} \triangleq \mathbf{E}\, Y_k^{(j)}\, Y_{k-i}^{(0,j)T}\,, \quad G_j \triangleq \mathbf{E}\, X_k^{(j)}\, Y_k^{(0,j)T}$$

$$R_i^{0,j} = H_0\ F^i\ G_j\ , \qquad R_i^{j} = H_j\ F^i\ G_j$$

Hint: right renormalization of the covariances

$$R_i^{\pi} \triangleq \begin{bmatrix} R_i^{0} \\ R_i^{1} \\ \vdots \\ R_i^{J} \end{bmatrix} = H\ F^i\ G\ , \qquad H \triangleq \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_J \end{bmatrix}$$

Robustness to nonstationary excitation

The estimates are consistent.

Combination of:

- the key factorization property of the covariances,
- the averaging operation underlying covariance computation,

allows to cancel out nonstationarities in the excitation.

Eigenstructure monitoring

θ_0 : reference parameter, known (or identified)
Y_k: n-size sample of new measurements

System parameter characterization:

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel.

$$\exists U, \quad U^T\, U = I_s, \qquad U^T\, \mathcal{O}_{p+1}(\theta_0) = 0; \qquad \text{say } U(\theta_0)$$

$$\theta_0 \leftrightarrow (R_i^0)_i \quad \text{characterized by:} \quad U^T(\theta_0)\ \hat{\mathcal{H}}_{p+1,q}^0 = 0$$

Subspace-based residual for eigenstructure monitoring

$$\zeta_n(\theta_0) \triangleq \sqrt{n}\ \mathrm{vec}(U^T(\theta_0)\ \hat{\mathcal{H}}_{p+1,q})$$

The residual is asymptly Gaussian (local approach)

Mean sensitivity (Jacobian) $\mathcal{J}(\theta_0)$ and covariance $\Sigma(\theta_0)$

$$\zeta_n(\theta_0) \ \rightarrow \begin{cases} \mathcal{N}(\qquad\qquad 0,\ \Sigma(\theta_0)) & \text{under} \quad \mathbf{P}_{\theta_0} \\ \mathcal{N}(\ \mathcal{J}(\theta_0)\ \delta\theta,\ \Sigma(\theta_0)) & \text{under} \quad \mathbf{P}_{\theta_0+\frac{\delta\theta}{\sqrt{n}}} \end{cases}$$

(GLR) χ^2-test for modal monitoring

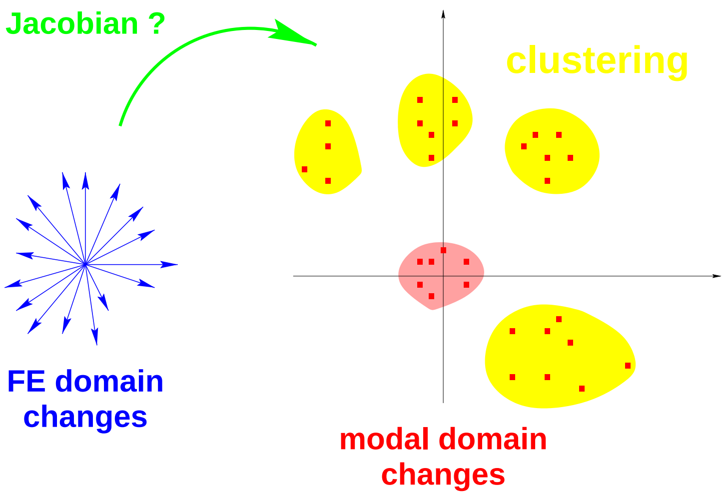
$$\zeta_n^T\ \Sigma^{-1}\ \mathcal{J}\ (\mathcal{J}^T\, \Sigma^{-1}\ \mathcal{J})^{-1}\ \mathcal{J}^T\ \Sigma^{-1}\ \zeta_n \ \ \geq h$$

(GLR) Directional χ^2-test for modal diagnosis

$$\zeta_n^T\ \Sigma^{-1}\ \mathcal{J}_i\ (\mathcal{J}_i^T\, \Sigma^{-1}\ \mathcal{J}_i)^{-1}\ \mathcal{J}_i^T\ \Sigma^{-1}\ \zeta_n \ \ \geq h$$

On-board damage diagnostics

projecting changes and (local) sensitivity approach



The MODAL toolbox

- Developped under Scilab (free)
- Completely driven by mouse/menu/parameter boxes
- Interactive graphical interface
- Automated and fully featured, easy to use
- Identification and damage detection/localization modules
- Based on covariance-driven subspace method

  output-only and input-output versions
- Localization: interfaced with the FE Scilab toolbox

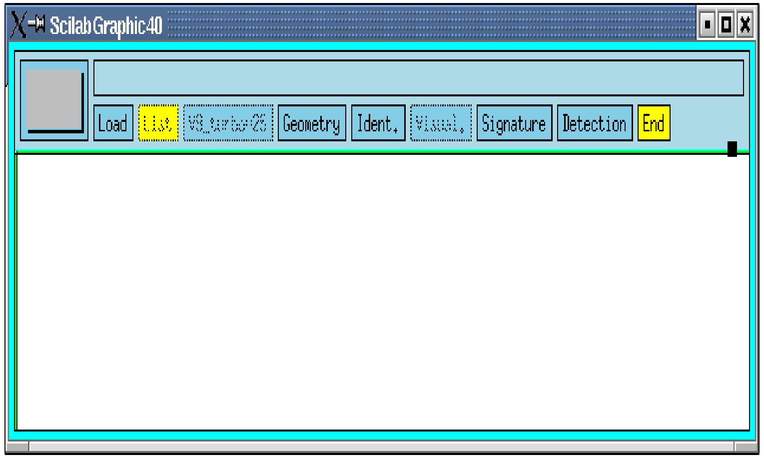
Some key features

- Processing:
  Offline / Online (specified response time) / Very fast
- Huge model orders (several hundreds)
- Fixed order methods of no help
- AIC/BIC/... order selection criteria fail
- Model selection performed by heuristics,
  using certain properties of subspace methods

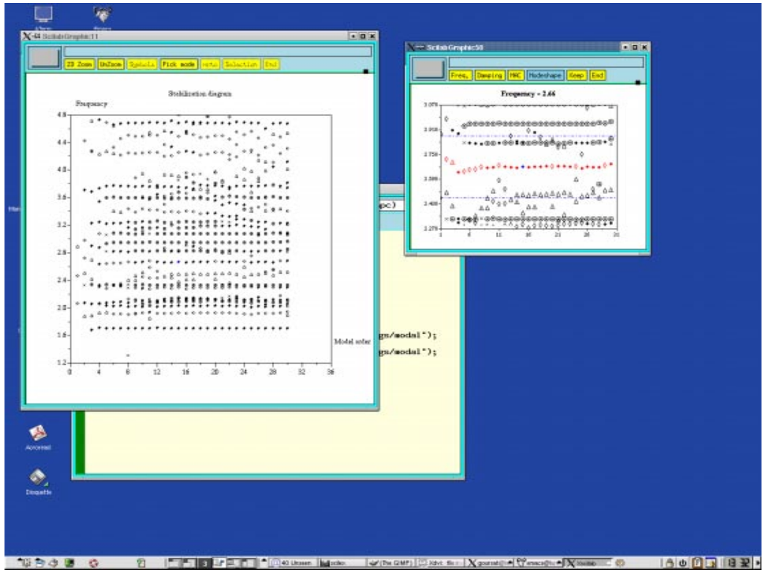
Numerical results

- Identification
  - Military aircraft (Dassault Aviation)

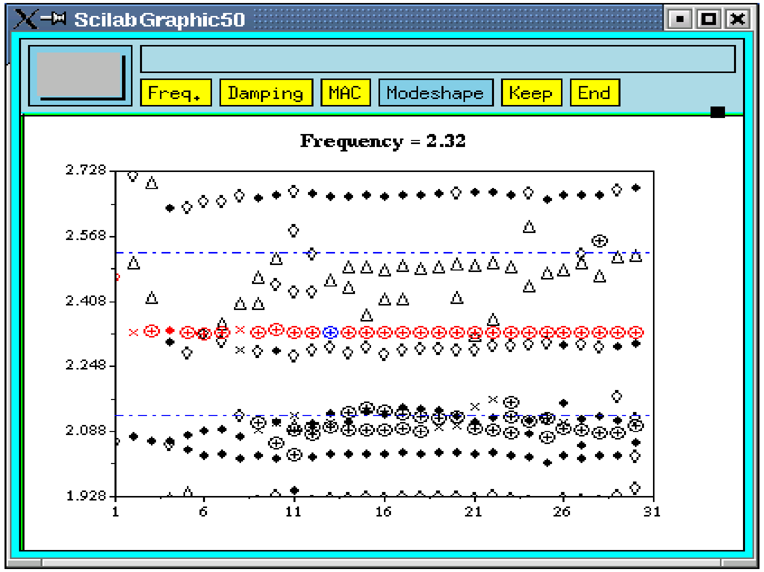
    In-flight: decreasing fuel level $\longrightarrow$ increasing frequencies
- Damage localization
  - Steelquake benchmark (COST F3)



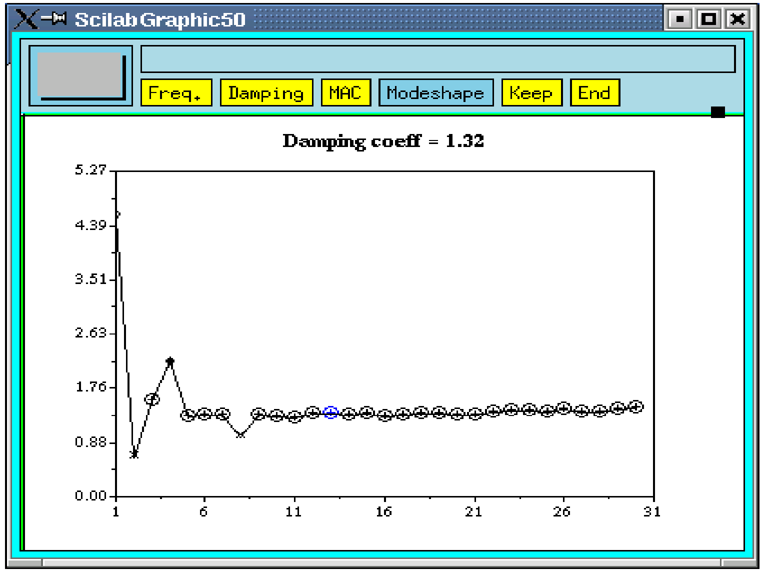
Main window



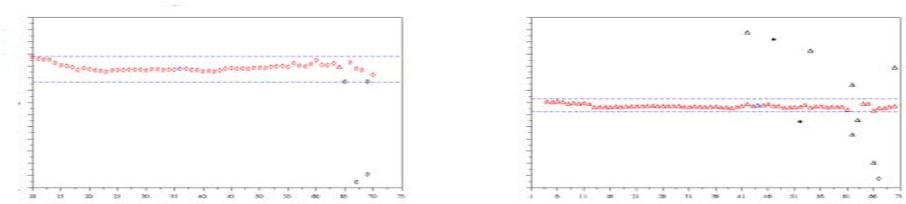
Identification window



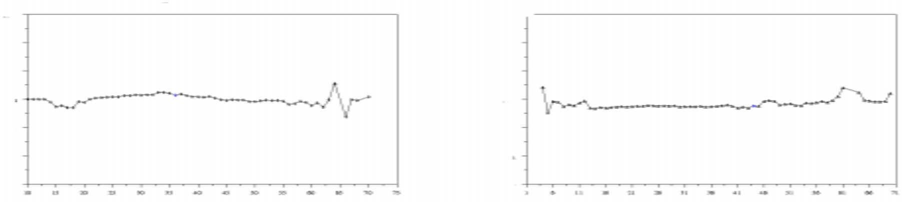
Stabilization diagram focussed on one frequency.



The corresponding damping values.

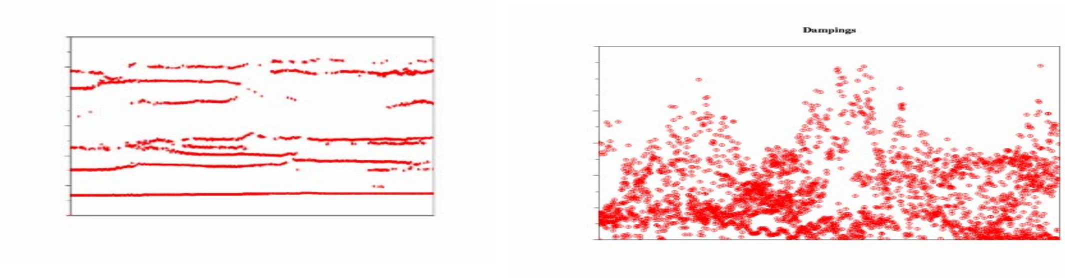


One eigenfrequency

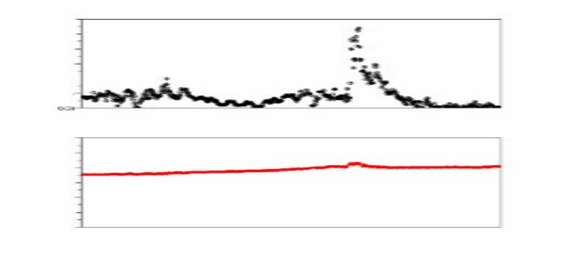


The corresponding damping coefficient

Flight beginning (left), less fuel in tanks (right).

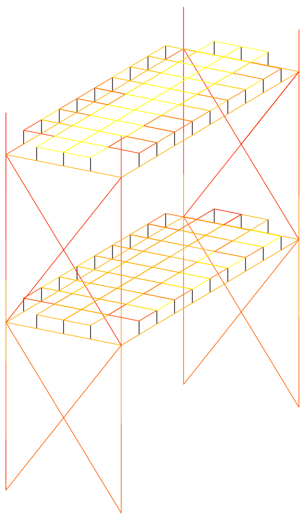


Automated identification: frequency (left) and damping (right) w.r.t. time.



Zoom on one mode: damping (top) and frequency (bottom).

Damage localization example - Steelquake benchmark



Under current investigation

- Handling more eigenfrequencies, hard model selection issue
  - new huge commercial aircrafts
- Designing a fast test for monitoring damping coefficients
  - aircrafts flight flutter issue
- Handling environmental effects (e.g. temperature)
  - civil engineering structures (bridges)

Identification (1) : offline

- Plot stabilization diagram for increasing orders
- Using the mouse, pick any mode for analysis
- Extract the corresponding alignement (frequency/damping/MAC)
- Simultaneous display of freq./damping/MAC w.r.t. order
- Plot and animate modeshapes
- Select and save good modes
- Save modal parameters for later use (identification/detection)

Identification (2) : online

- Embedded and automated modal analysis
- Modes selection from histograms of frequency/damping/MAC
- Store "best" frequencies and dampings with their alignements
- Plot the evolution of the frequency and damping over time
- Pause for screening the extracted modes alignements
- Zoom on any mode for online monitoring

Damage detection and localization

- Offline: Preprocessing, using safe data for estimating
  the reference model ($\theta_0, \mathcal{J}(\theta_0), \Sigma(\theta_0)$)

- Online:
  - Monitoring: one index result per experiment
  - Localization: one index per FE plotted on the structure
  - Fast monitoring: one index result for each new sample
  - Displays: monitoring indexes on a time axis /
    localization results on the structure
  - Localization: plot the localized macro-failures