

Subspace algorithms for data from varying sensor locations

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The problem

Output-only **eigenstructure identification**, based on **moving sensor pools**, with some reference sensors, the presence of **nonstationary excitation**.

Need:

Avoid merging identification results from the different pools, **merge the data** instead, and process them globally, **use a standard subspace algorithm**.

Context :

In-operation **modal identification** of large structures

- Use **moving sensors** to mimic a (much) larger set of sensors.
- The **excitation** is typically:
 - natural, **not controlled**.
 - **not measured**:
 - * buildings, bridges, offshore structures,
 - * rotating machinery,
 - * cars, trains, aircrafts.
 - **nonstationary** (e.g., turbulent).

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1. Modelling

State-space model:

$$\begin{cases} M\dot{Z}(s) + CZ(s) + KZ(s) = \nu(s) \\ Y(s) = LZ(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0, \quad \psi_\mu = L\Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu}}_{\text{modes}} \triangleq \lambda, \quad \underbrace{\psi_\mu}_{\text{mode shapes}} \triangleq \varphi_\lambda$$

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Implementation

$$\hat{R}_i \triangleq \frac{1}{N} \sum_{k=1}^N Y_k Y_{k-i}^T, \quad \hat{\mathcal{R}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

ok when nonstationary!

$$\hat{R}_i \approx H F^i \hat{G}, \quad \text{hence } \hat{\mathcal{R}} \approx \mathcal{O} \hat{C}$$

$\sqrt{D}(\hat{\mathcal{H}})$ + truncation $\longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\Lambda}, \hat{\varphi}_\lambda)$

$\hat{\mathcal{O}}$ robust against nonstationary excitation!

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2. Output-only subspace identification

$$R_i \triangleq \mathbb{E} \underbrace{\begin{pmatrix} Y_k \\ Y_{k-i} \end{pmatrix}^T}_{\text{ok if stationary!}}, \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$R_i = H F^i G, \quad G \triangleq \mathbb{E} \begin{pmatrix} X_k \\ Y_k^T \end{pmatrix}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2G & \dots \end{pmatrix}$$

$$\mathcal{H} = \mathcal{O} \mathcal{C}, \quad \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\Lambda, \varphi_\lambda)$$

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3. Polyreference subspace identification (1)

$$\begin{matrix} \text{Record 1} & \text{Record 2} & \dots & \text{Record } J \end{matrix}$$

$$\begin{bmatrix} Y_k^{(0,1)} \\ Y_k^{(1)} \\ Y_k^{(2)} \\ \vdots \end{bmatrix} \dots \begin{bmatrix} Y_k^{(0,J)} \\ Y_k^{(J)} \\ \vdots \end{bmatrix}$$

$$X_{k+1}^{(j)} = F X_k^{(j)} + V_k^{(j)}$$

$$Y_k^{(0,j)} = H_0 X_k^{(j)} \quad (\text{the reference})$$

$$Y_k^{(j)} = H_j X_k^{(j)} \quad (\text{sensor pool } n^o j)$$

$$R_{i_g}^{0,j} \triangleq \mathbb{E} Y_k^{(0,j)} Y_{k-i}^{(0,j)T}, \quad R_{i_g}^j \triangleq \mathbb{E} Y_k^{(j)} Y_{k-i}^{(j)T}$$

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onary excitation

$$\text{cov} V_k^{(j)} = Q, \quad G \triangleq E X_k^{(j)} Y_k^{(0,j)T}$$

$$R_i^{0,j} = H_0 F^i G \triangleq R_i^0, \quad R_i^j = H_j F^i G$$

$$R_i^T \triangleq \begin{bmatrix} R_i^0 \\ R_i^1 \\ \vdots \\ R_i^J \end{bmatrix} = H F^i G, \quad H \triangleq \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_J \end{bmatrix}$$

tionary excitation

$$\text{cov} V_k^{(j)} = Q_j, \quad G_j \triangleq E X_k^{(j)} Y_k^{(0,j)T}$$

$$R_i^{0,j} = H_0 F^i G_j, \quad R_i^j = H_j F^i G_j$$

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ng sensors: normalizing the data

$$= H_j F^i G_j, \quad \mathcal{H}_j \triangleq \begin{pmatrix} R_0^j & R_1^j & R_2^j & \dots \\ R_1^j & R_2^j & R_3^j & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} = \mathcal{O}(H_j, F) \mathcal{C}(F, G_j)$$

$$\triangleq \mathcal{H}_j \left(\mathcal{C}^T(F, G_j) \left(\mathcal{C}(F, G_j) \mathcal{C}^T(F, G_j) \right)^{-1} \mathcal{C}(F, G_1) \right)$$

$$\tilde{\mathcal{H}}_j \leftrightarrow \tilde{R}_i^j \triangleq H_j F^i G_1 \quad (1 \leq j \leq J)$$

$$\tilde{R}_i \triangleq \begin{pmatrix} \tilde{R}_i^0 \\ \tilde{R}_i^1 \\ \vdots \\ \tilde{R}_i^J \end{pmatrix}, \quad \tilde{R}_i^0 \triangleq R_i^{0,1} = H_0 F^i G_1$$

$$\tilde{\mathcal{H}} \triangleq \begin{pmatrix} \tilde{R}_0 & \tilde{R}_1 & \tilde{R}_2 & \dots \\ \tilde{R}_1 & \tilde{R}_2 & \tilde{R}_3 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} = \mathcal{O}(H, F) \mathcal{C}(F, G_1)$$

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4. Nonstationary excitation: data renormalization

Reference sensors: using the redundancy

$$R_i^{0,j} = H_0 F^i G_j, \quad G \triangleq (G_1 \ G_2 \ \dots \ G_J)$$

$$R_i^0 \triangleq (R_i^{0,1} \ R_i^{0,2} \ \dots \ R_i^{0,J}) = H_0 F^i G$$

$$\mathcal{H}_0 \triangleq \begin{pmatrix} R_0^0 & R_1^0 & R_2^0 & \dots \\ R_1^0 & R_2^0 & R_3^0 & \dots \\ R_2^0 & R_3^0 & R_4^0 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} = \mathcal{O}(H_0, F) \mathcal{C}(F, G) \longrightarrow \mathcal{C}(F, G_j)$$

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5. Polyreference subspace identification (2)

- **Row-stack** the $R_i^{(0,j)}$ into R_i^0
- Build \mathcal{H}_0 on R_i^0
- **SVD** (\mathcal{H}_0) + truncation $\longrightarrow \mathcal{C}(F, G) \triangleq \mathcal{C}$
- Partition $\mathcal{C} = (\mathcal{C}_1 \ \mathcal{C}_2 \ \dots \ \mathcal{C}_J)$
- Compute $(\mathcal{C}_j^T (\mathcal{C}_j \mathcal{C}_j^T)^{-1} \mathcal{C}_1)$
- **Renormalize** $\tilde{\mathcal{H}}_j = \mathcal{H}_j (\mathcal{C}_j^T (\mathcal{C}_j \mathcal{C}_j^T)^{-1} \mathcal{C}_1)$
- Deduce \tilde{R}_i^j
- **Column-stack** the \tilde{R}_i^j into \tilde{R}_i
- Build $\tilde{\mathcal{H}}$ on \tilde{R}_i
- We are back to the standard problem.

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- Nonstationary excitation **within** the records

$$\begin{cases} \mathbf{X}_{k+1}^{(j)} &= F \mathbf{X}_k^{(j)} + V_k & \text{cov } V_k = Q_k \\ Y_k^{(0,j)} &= H_0 \mathbf{X}_k^{(j)} & \text{(the reference)} \\ Y_k^{(j)} &= H_j \mathbf{X}_k^{(j)} & \text{(sensor pool } n^{o_j}) \end{cases}$$

$$A_j(K_j) = \frac{1}{k_j} \sum_{k \in \mathcal{I}_j} \left(\|Y_k^{(0,j)}\|^2 + \|Y_k^j\|^2 \right)$$

$$R_i^j(K_j) = \frac{1}{A(K_j)} \sum_{k \in \mathcal{I}_j} Y_k^j Y_{k-i}^{(0,j)T}$$

$$R_i^{0,j}(K_j) = H_0 F^i G_j(K_j) + o(K_j)$$

$$R_i^j(K_j) = H_j F^i G_j(K_j) + o(K_j)$$