Handling parametric and non-parametric additive faults in LTV Systems

Qinghua Zhang & Michèle Basseville

INRIA & CNRS-IRISA, Rennes, France

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Problem and approaches

FDI for LTV systems

- Relevant approach to FDI of NL systems (linearization along the actual or nominal trajectory)
- LTV systems more general than widely used LPV systems

Three main approaches

- Detection filter, game theoretic approach to filter design, unknown input decoupled filter, UIO, finite horizon fault detection filter
  Keviczky, Edelmayer, Chung-Speyer, Chen-Patton, Hou-Muller, Zhong-Ding, ...
- Adaptive observers, set-valued observers, time domain solutions to different $\mathcal{H}_-/\mathcal{H}_\infty$ problems
  Zhang-Xu, Rosa-Shamma-Athans, Li-Zhou, ...
- Parity-based fault estimation Zhong-Ding

Different fault types

- Parametric fault : (rare) changes in a parameter vector
- Non-parametric fault : arbitrary unknown function of time
- Most FDI methods for LTV systems address the non-parametric fault case, or the parametric one

Contribution

- A statistical approach exists for constant parametric faults
- Extension to both TV parametric and non-parametric faults

Two solutions :

- Assuming (piecewise) constant parametric fault and rejecting the non-parametric fault
- Adapting to the TV parametric fault

Model and assumptions

MIMO LTV system ($\mathbb{H}_0$)

$$\begin{cases} X_{k+1} = F_k\, X_k + G_k\, U_k + W_k \\ Y_k = H_k\, X_k + J_k\, U_k + V_k \end{cases}$$

F_k, G_k, H_k, J_k: bounded TV matrices
W_k, V_k: independent white Gaussian noises, TV cov. Q_k, R_k
(H_k, F_k) observable & $(F_k, Q_k^{1/2})$ controllable, both uniformly

Additive faults ($\mathbb{H}_1$)

$$\begin{cases} X_{k+1} = F_k\, X_k + G_k\, U_k + W_k + \Psi_k\, \theta_k + E_k\, f_k \\ Y_k = H_k\, X_k + J_k\, U_k + V_k \end{cases}$$

known fault profile matrix Ψ_k, unknown fault vector θ_k
known fault incidence matrix E_k, unknown fault profile vector f_k

Different fault cases

- $E_k f_k$ and $\Psi_k \theta_k$ typically represent actuator faults
- f_k : no *a priori* information; θ_k : constant or slowly varying
- Parametric faults in both state and output equations (sensor faults) can be handled
- This modeling framework encompasses multiple faults
- Non-additive faults are not handled

Particular cases

- Actuator bias: $U_k \rightarrow U_k + \theta$; then $\Psi_k = G_k$
- Actuator gain loss: $U_k \rightarrow (\mathbf{I} - \mathrm{diag}(\theta))U_k$; then $\Psi_k = -G_k \,\mathrm{diag}(U_k)$
- $\Psi_k = \delta_{r,k+1}\,\mathbf{I}$: investigated by Willsky-Jones, Gustafsson with F_k assumed exponentially stable

Fault effect on the innovation of a linear filter

State prediction error and innovation - Fault free case

$$\begin{aligned} \widetilde{X}_k &\triangleq X_k - \widehat{X}_{k|k-1} \\ \varepsilon_k &\triangleq Y_k - J_k\, U_k - H_k\, \widehat{X}_{k|k-1} \end{aligned}$$

$$\begin{aligned} \widetilde{X}^0_{k+1} &= F_k(\mathbf{I} - \mathcal{K}_k\, H_k)\widetilde{X}^0_k - F_k\, \mathcal{K}_k\, V_k + W_k \\ \varepsilon^0_k &= H_k\, \widetilde{X}^0_k + V_k \end{aligned}$$

State prediction error and innovation - Faulty case

$$\begin{aligned} \widetilde{X}_{k+1} &= F_k(\mathbf{I} - \mathcal{K}_k\, H_k)\widetilde{X}_k - F_k\, \mathcal{K}_k\, V_k + W_k + \Psi_k\, \theta_k + E_k\, f_k \\ \varepsilon_k &= H_k\, \widetilde{X}_k + V_k \end{aligned}$$

Introducing a matrix gain

$$\eta_k \triangleq \widetilde{X}_k - \Gamma_k \, \theta_k$$

$$\Gamma_{k+1} \triangleq F_k \, (\mathbf{I} - \mathcal{K}_k \, H_k) \, \Gamma_k + \Psi_k \; , \quad \Gamma_0 \triangleq 0$$

$$\eta_{k+1} = F_k \, (\mathbf{I} - \mathcal{K}_k \, H_k) \; \eta_k - F_k \, \mathcal{K}_k \; V_k + W_k$$

$$- \, \Gamma_{k+1} \, (\theta_{k+1} - \theta_k) + \; E_k \; f_k$$

Distinguishing two cases for the parametric fault vector

- Constant θ
- TV θ_k

Constant parametric fault vector

$$\theta_k \triangleq \theta$$

Let $\zeta_{k+1} \triangleq F_k \, (\mathbf{I} - \mathcal{K}_k \, H_k) \, \zeta_k + E_k \, f_k \, , \quad \zeta_0 = 0$

cf. $\Gamma_{k+1} \triangleq F_k \, (\mathbf{I} - \mathcal{K}_k \, H_k) \, \Gamma_k + \Psi_k \, , \quad \Gamma_0 \triangleq 0$

$$\eta_{k+1} = F_k \, (\mathbf{I} - \mathcal{K}_k \, H_k) \, \eta_k - F_k \, \mathcal{K}_k \, V_k + W_k + E_k \, f_k$$

$$\eta_k = \widetilde{X}_k^0 + \zeta_k$$

Additive fault effect

$$\varepsilon_k = \varepsilon_k^0 + H_k \, \Gamma_k \, \theta + H_k \, \zeta_k$$

Guaranteed properties of the recursive Γ_k and ζ_k

- Γ_k depends on the fault gain Ψ_k, not on the fault vector θ.
- The matrix gain Γ_k computed from the bounded Ψ_k is bounded even when the system is not stable.
- Similarly, if f_k is bounded, ζ_k is bounded.
- The persistent excitation condition:
  $\sum_k \Gamma_k^T H_k^T \Sigma_k^{-1} H_k \Gamma_k$ *strictly positive definite*
  is satisfied even when the number of sensors is smaller than the number of faults.

Difference with the Willsky-Jones algorithm

- Computations based on recursive formulas involving F_k (thus required to be stable)

TV parametric fault vector

$$\theta_{k+1} = \theta_k + e_k \,, \quad |e_k| \leq \delta$$

$$\text{Let} \quad \delta_{k+1} \triangleq F_k\,(\mathbf{I} - \mathcal{K}_k\, H_k)\,\delta_k - \Gamma_{k+1}\, e_k, \quad \delta_0 \triangleq 0$$

$$\eta_k = \widetilde{X}_k^0 + \delta_k + \zeta_k$$

Additive fault effect

$$\varepsilon_k = \varepsilon_k^0 + H_k\,\Gamma_k\,\theta_k + H_k\,\delta_k + H_k\,\zeta_k$$

Γ_k is bounded

$F_k\,(\mathbf{I} - \mathcal{K}_k\, H_k)$ defines an exponentially stable LTV system

Kitanidis filter (UI-KF) for rejecting $E_k\ f_k$

$$\begin{cases} X_{k+1} = F_k\, X_k + G_k\, U_k + W_k + E_k\, f_k \\ Y_k = H_k\, X_k + J_k\, U_k + V_k \end{cases}$$

$$\widehat{X}_{k+1} = F_k \widehat{X}_k + G_k U_k + F_k\, L_k\, (Y_k - J_k U_k - H_k \widehat{X}_k)$$

$$L_k = K_k + (I - K_k H_k) E_{k-1} (E_{k-1}^T H_k^T \Sigma_k^{-1} H_k E_{k-1})^{-1} E_{k-1}^T H_k^T \Sigma_k^{-1}$$

$$K_k = P_k\, H_k^T\, \Sigma_k^{-1}$$

$$P_{k+1} = F_k (I - L_k H_k) P_k\, (I - L_k H_k)^T F_k^T + F_k L_k R_k L_k^T F_k^T + Q_k$$

$$\Sigma_k = H_k\, P_k\, H_k^T + R_k$$

$$\mathcal{K}_k \triangleq L_k$$

Monitoring a constant parametric fault

Fault effect on the Kitanidis filter innovation

$$\varepsilon_k = \varepsilon_k^0 + H_k \, \Delta_k \, \theta$$

$$\Delta_{k+1} = F_k \, (\mathbf{I} - L_k \, H_k) \Delta_k + \Psi_k \, , \quad \Delta_0 = 0$$

The Kitanidis filter innovation is white

Proof in the notes.

Use the GLR algorithm

MLE of θ under $\mathbb{H}_1$ - Known fault profile matrix

$$\mathbb{H}_0 : \varepsilon_k \sim \mathcal{N}(0, \Sigma_k), \quad \mathbb{H}_1 : \varepsilon_k \sim \mathcal{N}(H_k \, \Gamma_k \, \theta, \Sigma_k)$$

$$\widehat{\theta}_k = \arg\min_{\widetilde{\theta}} \sum_{j=1}^{k} (\varepsilon_j - H_j \, \Gamma_j \, \widetilde{\theta})^T \, \Sigma_j^{-1} \, (\varepsilon_j - H_j \, \Gamma_j \, \widetilde{\theta}) = C_k^{-1} \, d_k$$

$$C_k = C_{k-1} + \Gamma_k^T \, H_k^T \, \Sigma_k^{-1} \, H_k \, \Gamma_k$$

$$d_k = d_{k-1} + \Gamma_k^T \, H_k^T \, \Sigma_k^{-1} \, \varepsilon_k$$

GLR test

$$l_k \triangleq 2 \, \ln \, \frac{p(\varepsilon_1, \ldots, \varepsilon_k \mid \theta = \widehat{\theta}_k)}{p(\varepsilon_1, \ldots, \varepsilon_k \mid \theta = 0)} = d_k^T \, C_k^{-1} \, d_k$$

Monitoring the non-parametric fault

While the GLR does not detect anything

Run a Kalman filter based on the fault-free model

$$
\begin{aligned}
\widehat{X}_{k+1} &= F_k \widehat{X}_k + G_k U_k + F_k K_k (Y_k - J_k U_k - H_k \widehat{X}_k) \\
K_k &= P_k \, H_k^T \, \Sigma_k^{-1} \\
P_{k+1} &= F_k (I - K_k \, H_k) P_k \, F_k^T + Q_k \\
\Sigma_k &= H_k \, P_k \, H_k^T + R_k
\end{aligned}
$$

Monitor its energy

OK when $\dim(f_k) \geq \dim(Y_k)$, i.e. testing a Gaussian white noise against an arbitrary signal.
More sophisticated tests might be considered in the case where $\dim(f_k) < \dim(Y_k)$.

Tracking a slowly time-varying θ_k

$$\varepsilon_k = \varepsilon_k^0 + H_k \, \Gamma_k \, \theta_k + H_k \, \delta_k + H_k \, \zeta_k$$

RLS

$$\begin{aligned}
\widehat{\theta}_k &= \widehat{\theta}_{k-1} + \mathcal{L}_k \left(\varepsilon_k - H_k \, \Gamma_k \, \widehat{\theta}_{k-1} \right), \quad \widehat{\theta}_0 \triangleq 0 \\
S_k &= \left(\lambda \, \Sigma_k + H_k \, \Gamma_k \, \mathcal{P}_{k-1} \, \Gamma_k^T \, H_k^T \right)^{-1} \\
\mathcal{L}_k &= \mathcal{P}_{k-1} \, \Gamma_k^T \, H_k^T \, S_k \\
\mathcal{P}_k &= \lambda^{-1} \left(\mathcal{P}_{k-1} - \mathcal{P}_{k-1} \, \Gamma_k^T \, H_k^T \, S_k \, H_k \, \Gamma_k \, \mathcal{P}_{k-1} \right), \quad \mathcal{P}_0 \triangleq \mathbf{I}
\end{aligned}$$

$\mathcal{E}_k \triangleq \varepsilon_k - H_k \, \Gamma_k \, \widehat{\theta}_k$; monitor its energy to detect $E_k \, f_k$

FDI for LTV systems with TV additive faults

- Constant parametric faults
  - Combining a recursive and stable filter that cancels out the fault dynamics and a GLR test
  - Handling additive parametric faults with weaker assumptions than usual on the system stability and the number of required sensors
- Handling both TV parametric and non-parametric faults
  Two solutions
  - Assuming constant parametric fault and rejecting the non-parametric fault
  - Adapting to the TV parametric fault