Model-based statistical signal processing

and decision theoretic approaches to monitoring

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Contents

Monitoring and model-based statistical processing

Stochastic models (static, dynamic) $\longleftrightarrow$ uncertainties

Parameterized models (physical interpretation, diagnostics)

Fault in the system $\longleftrightarrow$ deviation in the parameter vector

(Fault, damage, deviation, change)

Problem

Parameterized nonlinear state-space models

$$\begin{cases} x_{k+1} = f(\theta, x_k, u_k, v_k) \\ y_k = h(\theta, x_k, u_k, v_k) + \epsilon_k \end{cases}$$

x_k, v_k : unknown states and inputs

u_k, y_k : measured inputs and outputs

θ : unknown parameters

Wanted:

- Detect and diagnose (small) deviations in θ.
- Not to detect events/features of no interest !

Which model(s) ?

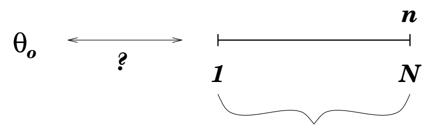
Physical models, black-box models

Neural networks, wavelet networks

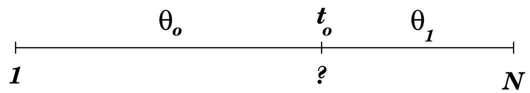
Approximate models

Three detection problems

– Model validation



– Off-line change detection



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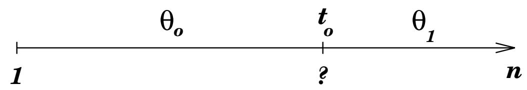


Simple hypotheses, composite hypotheses

Hypotheses	H_0	H_1	
Simple	θ_0	θ_1	Known parameter values
Composite	Θ_0	Θ_1	Unknown parameter values

Key concepts - Independent data

Likelihood $\quad p_\theta(y_i) \quad$ Likelihood ratio $\quad \dfrac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}$

Log-likelihood $\quad l_\theta(y_i) \triangleq \ln\, p_\theta(y_i)$

Log-likelihood ratio $\quad s_i \triangleq \ln \dfrac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)} = l_{\theta_1}(y_i) - l_{\theta_0}(y_i)$

$$\mathrm{E}_{\theta_0}(s_i) < 0$$

$$\mathrm{E}_{\theta_1}(s_i) > 0$$

Likelihood ratio $\quad \Lambda_N \triangleq \dfrac{p_{\theta_1}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} = \dfrac{\Pi_i\; p_{\theta_1}(y_i)}{\Pi_i\; p_{\theta_0}(y_i)}$

Log-likelihood ratio $\quad S_N \triangleq \ln\, \Lambda_N = \Sigma_{i=1}^N s_i$

Hypothesis testing: likelihood ratio, GLR

Simple hypotheses : **likelihood ratio** test

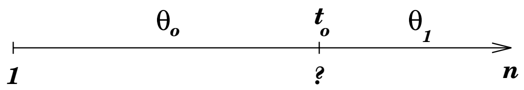
$$\begin{cases} \text{If } \Lambda_N < \lambda \ / \ S_N < h : \ \mathrm{H}_0 \text{ is chosen} \\ \text{If } \Lambda_N \geq \lambda \ / \ S_N \geq h : \ \mathrm{H}_1 \text{ is chosen} \end{cases}$$

Composite hypotheses : Generalized likelihood ratio (**GLR**)

Maximize the likelihoods / **unknown** values of θ_0 and θ_1:

$$\widehat{\Lambda}_N = \frac{\sup_{\theta_1 \in \Theta_1} \ p_{\theta_1}(\mathcal{Y}_1^N)}{\sup_{\theta_0 \in \Theta_0} \ p_{\theta_0}(\mathcal{Y}_1^N)} = \frac{p_{\widehat{\theta_1}}(\mathcal{Y}_1^N)}{p_{\widehat{\theta_0}}(\mathcal{Y}_1^N)}$$

On-line detection: CUSUM, GLR



Unknown onset time t_0; θ_0 assumed known

Alarm time t_a : $t_a = \min \{k \geq 1 : g_k \geq h\}$

Problem: design of the decision function g_k

Simple case (known θ_1): CUSUM

Composite case (unknown θ_1): modified CUSUM, GLR

Scalar parameter - Simple case: known θ_1

Hypothesis **H_0** $\theta = \theta_0 \quad (1 \leq i \leq k)$

Hypothesis **H_1** $\exists \ t_0$ s.t. $\theta = \begin{cases} \theta_0 & (1 \leq i < t_0) \\ \theta_1 & (t_0 \leq i \leq k) \end{cases}$

Alarm time t_a : $t_a = \min \{k \geq 1 : g_k \geq h\}$

Estimated onset **time**: $\widehat{t_0}$

CUSUM algorithm

Ratio of likelihoods under H_0 and H_1:

$$\frac{\prod_{i=1}^{t_0-1} \ p_{\theta_0}(y_i) \ \cdot \ \prod_{i=t_0}^{k} \ p_{\theta_1}(y_i)}{\prod_{i=1}^{k} \ p_{\theta_0}(y_i)} = \frac{\prod_{i=t_0}^{k} \ p_{\theta_1}(y_i)}{\prod_{i=t_0}^{k} \ p_{\theta_0}(y_i)} = \Lambda_{t_0}^k$$

Maximize over the **unknown** onset time t_0:

$$\begin{aligned} (\widehat{t_0})_k &\triangleq \arg \max_{1 \leq j \leq k} \ \prod_{i=1}^{j-1} \ p_{\theta_0}(y_i) \ \cdot \ \prod_{i=j}^{k} \ p_{\theta_1}(y_i) \\ &= \arg \max_{1 \leq j \leq k} \ \Lambda_j^k \\ &= \arg \max_{1 \leq j \leq k} \ S_j^k, \qquad S_j^k = \ln \ \Lambda_j^k \end{aligned}$$

$$g_k \triangleq \max_{1 \leq j \leq k} S_j^k = \ln \Lambda_{\hat{t}_0}^k$$

CUSUM algorithm (Contd.)

$$g_k \triangleq \max_{1 \le j \le k} S_j^k$$
$$= S_1^k - \min_{1 \le j \le k} S_1^j = S_1^k - m_k, \qquad m_k \triangleq \min_{1 \le j \le k} S_1^j$$
$$t_a = \min \{k \ge 1 : g_k \ge h\}$$
$$t_a = \min \{k \ge 1 : S_1^k \ge m_k + h\}$$ Adaptative threshold

$$g_k = (g_{k-1} + s_k)^+$$
$$g_k = \left(S_{k-N_k+1}^k\right)^+, \qquad N_k \triangleq N_{k-1} \cdot I(g_{k-1}) + 1$$
$$(\widehat{t_0})_k = t_a - N_{t_a} + 1$$ Sliding window with adaptive size

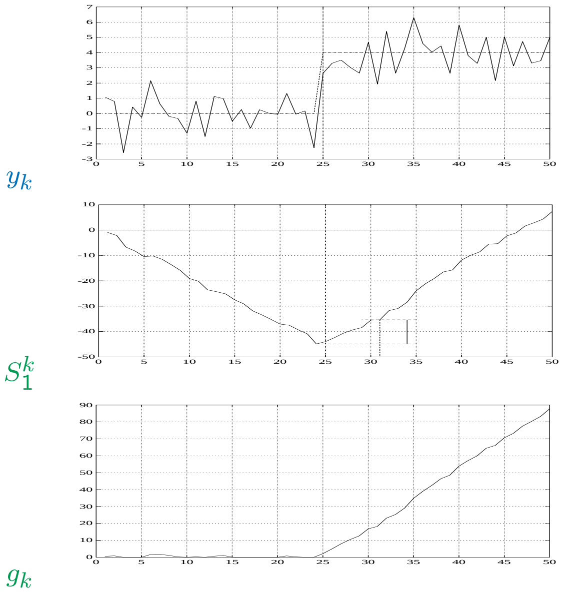
CUSUM algorithm (Contd.) - Gaussian example

$$\mathcal{N}(\mu, \sigma^2), \quad \theta \triangleq \mu, \quad p_\theta(y) \triangleq \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y_i-\mu)^2}{2\sigma^2}}$$

$$s_i = \ln \frac{p_{\mu_1}(y_i)}{p_{\mu_0}(y_i)}$$
$$= \frac{1}{2\,\sigma^2}\left((y_i - \mu_0)^2 - (y_i - \mu_1)^2\right)$$
$$= \frac{\nu}{\sigma^2}\left(y_i - \mu_0 - \frac{\nu}{2}\right), \qquad \nu = \mu_1 - \mu_0$$

S_1^k involves $\sum_{i=1}^{k} y_i$: Integrator (with adaptive threshold)

Log. likelihood ratio - CUSUM Algorithm



Composite case: unknown θ_1

Modified CUSUM algorithms

Minimum magnitude of change

Weighted CUSUM

GLR algorithm

Double maximization

$$g_k = \max_{1 \le j \le k} \sup_{\theta_1} S_j^k(\theta_1)$$

Gaussian case, additive faults: second maximization explicit.

Key concepts - Dependent data

Conditional likelihood	$p_\theta(y_i \mid \mathcal{Y}_1^{i-1})$		
Log-likelihood	$l_\theta(y_i \mid \mathcal{Y}_1^{i-1})$	$\triangleq$	$\ln\; p_\theta(y_i \mid \mathcal{Y}_1^{i-1})$
Log-likelihood ratio	s_i	$\triangleq$	$\ln \dfrac{p_{\theta_1}(y_i \mid \mathcal{Y}_1^{i-1})}{p_{\theta_0}(y_i \mid \mathcal{Y}_1^{i-1})}$
	$\mathrm{E}_{\theta_0}(s_i)$	$<$	0
	$\mathrm{E}_{\theta_1}(s_i)$	$>$	0
Likelihood ratio	Λ_N	$\triangleq$	$\dfrac{p_{\theta_1}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} = \dfrac{\Pi_i\; p_{\theta_1}(y_i \mid \mathcal{Y}_1^{i-1})}{\Pi_i\; p_{\theta_0}(y_i \mid \mathcal{Y}_1^{i-1})}$
Log-likelihood ratio	S_N	$\triangleq$	$\ln\; \Lambda_N = \Sigma_{i=1}^N\; s_i$

Key concepts - Dependent data (Contd.)

Efficient score

$$z_i^* \triangleq \left.\frac{\partial l_\theta(y_i \mid \mathcal{Y}_1^{i-1})}{\partial \theta}\right|_{\theta=\theta^*}$$

$$\zeta_N^* \triangleq \frac{1}{\sqrt{N}} \left.\frac{\partial l_\theta(\mathcal{Y}_1^N)}{\partial \theta}\right|_{\theta=\theta^*} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} z_i^*$$

$$\mathrm{E}_{\theta^*}(z_i^*) = 0$$

$$\mathrm{E}_{\theta_0}(z_i^*) < 0 \qquad (\theta_0 = \theta^* - \Upsilon)$$

$$\mathrm{E}_{\theta_1}(z_i^*) > 0 \qquad (\theta_1 = \theta^* + \Upsilon)$$

Fisher information

(Independent)

$$\mathrm{I}(\theta) = \mathrm{E}_\theta \left(\frac{\partial l_\theta(y_i)}{\partial \theta}\right)^2 = \mathrm{cov}_\theta \left(\frac{\partial l_\theta(y_i)}{\partial \theta}\right) = \mathrm{cov}_\theta(z_i)$$

$$= -\,\mathrm{E}_\theta \left(\frac{\partial^2 l_\theta(y_i)}{\partial \theta^2}\right)$$

(Dependent)

$$\mathrm{I}_N(\theta) = \frac{1}{N}\,\mathrm{cov}_\theta \left(\frac{\partial l_\theta(\mathcal{Y}_1^N)}{\partial \theta}\right)$$

$$= \mathrm{cov}_\theta\,(\mathcal{Z}_N)$$

$$\mathrm{I}(\theta) = \lim_{N\to\infty}\; \mathrm{I}_N(\theta)$$

Fisher information = inverse of the curvature of the likelihood

Residuals and sufficient statistics

$S(\theta, \mathcal{Y}_1^N)$ s.t. $\mathrm{P}_\theta(\mathcal{Y}_1^N \mid S)$ independent of θ.

Fisher information maintained.

- Likelihood ratio for both additive and non-additive faults,
- Innovation for additive faults,
- Efficient score for non-additive faults,
- Other estimating fonctions for non-additive faults.

Innovation not sufficient for monitoring the dynamics.

Fault detection

Local approach (small deviations)

θ_0 : reference parameter, known (or identified)

Y_k: N-size sample of new measurements

Build a residual ζ significantly non zero when fault

Test $\quad \mathrm{H}_0: \quad \theta = \theta_0 \quad$ against $\quad \mathrm{H}_1: \quad \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$

Dependent data - Small deviations - Likelihood

Second order Taylor expansion of the log-likelihood ratio

$$\theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$$

$$S_N(\theta_0, \theta) \triangleq \ln \frac{p_\theta(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} \approx \delta\theta^T \, \zeta_N(\theta_0) - \frac{1}{2}\, \delta\theta^T \, \mathrm{I}(\theta_0)\, \delta\theta$$

$$\mathrm{E}_{\theta_0}\, S_N \approx -\frac{1}{2}\, \delta\theta^T \, \mathrm{I}(\theta_0)\, \delta\theta$$

$$\mathrm{E}_{\theta}\, S_N \approx +\frac{1}{2}\, \delta\theta^T \, \mathrm{I}(\theta_0)\, \delta\theta \quad \approx -\mathrm{E}_{\theta_0}\, S_N$$

$$\mathrm{cov}_{\theta_0}\, S_N \approx \delta\theta^T \, \mathrm{I}(\theta_0)\, \delta\theta \quad \approx \mathrm{cov}_\theta\, S_N$$

First order Taylor expansion of the efficient score

$$\zeta_N(\theta) \approx \zeta_N(\theta_0) + \frac{1}{N} \left. \frac{\partial^2 \ln p_\theta(\mathcal{Y}_1^N)}{\partial \theta^2} \right|_{\theta=\theta_0} \delta\theta$$

$$\mathrm{E}_{\theta_0}\, \zeta_N(\theta) \approx -\, \mathrm{I}(\theta_0)\, \delta\theta$$

Efficient score $=$ ML estimating function

Characterized by: $\mathrm{E}_{\theta_0}\, \zeta_N(\theta) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$

Caution : Efficient score $\neq$ innovation !

Example : Gaussian scalar AR process

$$Y_k = \sum_{i=1}^{p} a_i\, Y_{k-i} + E_k, \qquad \theta^T = (\, a_1 \;\; \dots \;\; a_p \,)$$

$$\zeta_N(\theta) = \frac{1}{\sqrt{N}} \frac{1}{\sigma^2} \sum_{k=1}^{N} \mathcal{Y}_{k-1,p}^{-}\, \varepsilon_k(\theta)$$

$$\mathrm{I}(\theta) = \frac{1}{\sigma^2}\, \mathrm{T}_p$$

Efficient score ζ : vector-valued function

Innovation ε : scalar function

Hypotheses testing - Asymptotic Gaussianity

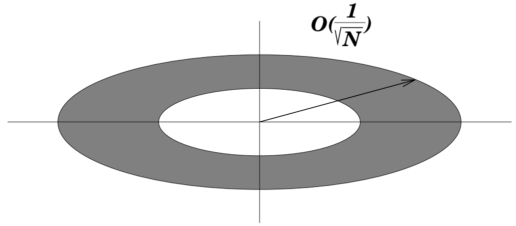
For i.i.d. variables, stationary Gaussian processes, stationary Markov processes, ...:

$$S_N(\theta_0,\theta) \;\to\; \begin{cases} \mathcal{N}(-\frac{1}{2}\,\delta\theta^T\,\mathrm{I}(\theta_0)\,\delta\theta,\quad \delta\theta^T\,\mathrm{I}(\theta_0)\,\delta\theta) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(+\frac{1}{2}\,\delta\theta^T\,\mathrm{I}(\theta_0)\,\delta\theta,\quad \delta\theta^T\,\mathrm{I}(\theta_0)\,\delta\theta) & \text{under} \quad \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

$$\zeta_N(\theta_0) \;\to\; \begin{cases} \mathcal{N}(0, \quad \mathrm{I}(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\mathrm{I}(\theta_0)\,\delta\theta, \quad \mathrm{I}(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

The efficient score ζ_N is asymptotically a sufficient statistics.

Asymptotically optimum and equivalent tests for composite hypotheses



$$\textbf{GLR} \qquad \frac{\sup_{\theta\in\Theta_1}\, p_\theta(\mathcal{Y}_1^N)}{\sup_{\theta\in\Theta_0}\, p_\theta(\mathcal{Y}_1^N)} \;\geq\; \lambda$$

$$\frac{p_{\hat\theta}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} \;\geq\; \lambda$$

$$N\;(\hat\theta-\theta_0)^T\;\mathrm{I}(\theta_0)\;(\hat\theta-\theta_0) \;\geq\; \lambda$$

$$\zeta_N{}^T(\theta_0)\;\mathrm{I}^{-1}(\theta_0)\;\zeta_N(\theta_0) \;\geq\; \lambda$$

Small deviations - Other estimating functions

Residual $\leftrightarrow$ Estimating function

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \mathcal{K}(\theta_0, Y_k)$$

Characterized by: $\mathrm{E}_{\theta_0}\,\mathcal{K}(\theta, Y_k) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$

Mean sensitivity (Jacobian) and covariance

$$\mathcal{J}(\theta_0) \triangleq -\mathrm{E}_{\theta_0}\frac{\partial\mathcal{K}(\theta_0, Y_k)}{\partial\theta}, \qquad \Sigma(\theta_0) \triangleq \lim_{N\to\infty} \mathrm{E}_{\theta_0}\;\zeta_N(\theta_0)\zeta_N^T(\theta_0)$$

First order Taylor expansion of this residual

$$\theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$$

$$\zeta_N(\theta) \;\approx\; \underbrace{\zeta_N(\theta_0)} \;+\; \sqrt{N}\; \underbrace{\frac{1}{N}\left(\sum_{k=1}^{N} \frac{\partial}{\partial\theta}\mathcal{K}(\theta, Y_k)\Big|_{\theta=\theta_0}\right)} \; \frac{\delta\theta}{\sqrt{N}}$$

CLT, under θ_0 $\downarrow$ $\qquad$ LLN, under θ_0 $\downarrow$

$$\mathcal{N}(0, \Sigma(\theta_0)) \qquad\qquad \underbrace{\mathrm{E}_{\theta_0}\frac{\partial}{\partial\theta}\mathcal{K}(\theta, Y_k)\Big|_{\theta=\theta_0}}$$

$$= \;-\;\mathcal{J}(\theta_0)$$

The residual is asymptotically Gaussian

$$\zeta_N(\theta_0) \;\to\; \begin{cases} \mathcal{N}(\quad 0,\ \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\ \mathcal{J}(\theta_0)\ \delta\theta,\ \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

(On-board) χ^2-test for composite hypotheses

$$\zeta_N^T\ \underbrace{\Sigma^{-1}\ \mathcal{J}\ (\mathcal{J}^T\ \Sigma^{-1}\ \mathcal{J})^{-1}\ \mathcal{J}^T\ \Sigma^{-1}}_{\mathrm{I}^{-1}}\ \zeta_N \ \geq h$$

Invariant / pre-multiplication of ζ with invertible gain.

Noises and uncertainty on θ_0 taken into account.

Nonlinear dynamic systems - LS residuals

State estimation: (full order) observers

$$\begin{cases} \dot{\hat{x}} &= \hat{f}(\theta, \hat{x}, u, y) \\ \hat{y}_k(\theta) &= h(\theta, \hat{x}(k\tau), u(k\tau)) \end{cases}$$

$$\hat{f}(\theta, \hat{x}, u, y) \triangleq f(\theta, \hat{x}, u) + K_o\ (y - h(\theta, \hat{x}, u))$$

$$\zeta_N \ : \quad \mathcal{K}(\theta_0, Y_k) = \left(\left.\frac{\partial \hat{y}_k(\theta)}{\partial \theta}\right|_{\theta=\theta_0}\right)^T (y_k - \hat{y}_k(\theta_0))$$

$$\begin{cases} \dfrac{\partial \dot{\hat{x}}}{\partial \theta} &= \dfrac{\partial}{\partial \hat{x}}\ \hat{f}(\theta_0, \hat{x}, u, y)\ \dfrac{\partial \hat{x}}{\partial \theta} \quad + \dfrac{\partial}{\partial \theta}\ \hat{f}(\theta_0, \hat{x}, u, y) \\ \dfrac{\partial \hat{y}_k(\theta)}{\partial \theta} &= \dfrac{\partial}{\partial \hat{x}}\ h(\theta_0, \hat{x}(k\tau), u(k\tau))\ \dfrac{\partial \hat{x}_k}{\partial \theta} + \dfrac{\partial}{\partial \theta}\ h(\theta_0, \hat{x}(k\tau), u(k\tau)) \end{cases}$$

Nonlinear dynamic systems - Simulation methods

Monte Carlo sequential simulation

Particle filters

Numerical approximation of the efficient score

Isolation (1) - Nuisance approach

Which components of θ?

$$\zeta \sim \mathcal{N}(\mathcal{J}\ \theta,\ \ \Sigma), \quad \theta = \begin{pmatrix} \theta_a \\ \theta_b \end{pmatrix}, \quad \mathcal{J} = (\ \mathcal{J}_a \ \ \mathcal{J}_b\ ), \quad p_{\theta_a,\theta_b}(\zeta)$$

Decide between $\theta_a = 0$ and $\theta_a \neq 0$; θ_b unknown

$$\mathrm{I} \triangleq \mathcal{J}^T\ \Sigma^{-1}\ \mathcal{J} \triangleq \begin{pmatrix} \mathrm{I}_{aa} & \mathrm{I}_{ab} \\ \mathrm{I}_{ba} & \mathrm{I}_{bb} \end{pmatrix}$$

I_a^{*-1} : upper-left block of I^{-1} ; $\quad \mathrm{I}_a^* = \mathrm{I}_{aa} - \mathrm{I}_{ab}\ \mathrm{I}_{bb}^{-1}\ \mathrm{I}_{ba}$

Statistical projection (sensitivity)

$$2 \ \ln \frac{\max_{\theta_a} \ p_{\theta_a,0}(\zeta)}{p_{0,0}(\zeta)} = \zeta_a^T \ \mathrm{I}_{aa}^{-1} \ \zeta_a \ , \qquad \zeta_a \ : \text{partial score}$$

Statistical rejection (minmax)

$$2 \ \ln \frac{\max_{\theta_a,\theta_b} \ p_{\theta_a,\theta_b}(\zeta)}{\max_{\theta_b} \ p_{0,\theta_b}(\zeta)} = \zeta_a^{*T} \ \mathrm{I}_a^{*\,-1} \ \zeta_a^* \ , \qquad \zeta_a^* \ : \text{effective score}$$

$$\zeta_a^* = \zeta_a - \mathrm{I}_{ab} \ \mathrm{I}_{bb}^{-1} \ \zeta_b$$

Regression of informative score over nuisance score (Neyman,1954).

Dealing with nuisance parameters - Contd.

Other approaches :

- Reparameterization : generalization of sensitivity approach
- Invariant tests
- Minimax tests

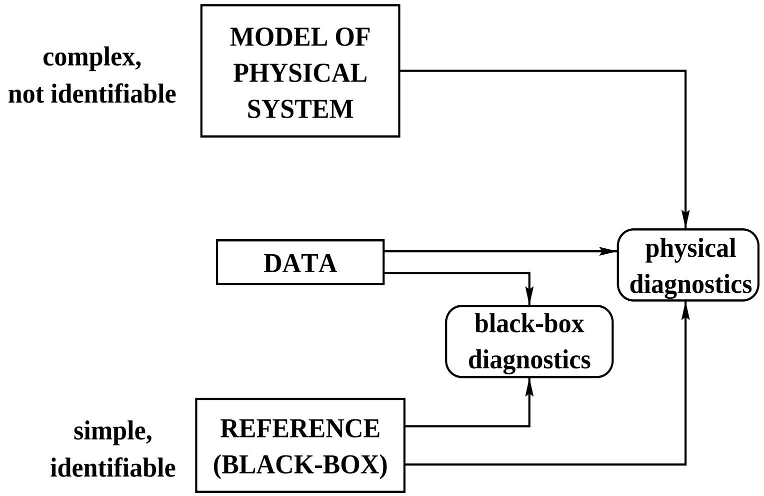
Isolation (2) - Multiple hypotheses approach

$$\mathrm{H}_0 \ : \quad \theta \in \Theta_0$$

$$\mathrm{H}_i \ : \quad \theta \in \Theta_i, \ \ (i = 1 : m)$$

- Bayesian approach
- Invariant tests

(On-Board) Diagnostics/localization : don't solve the inverse problem!



Optimal sensor location for monitoring

Maximize the power of the detection algorithms :

$$\text{Trace}\ (\mathcal{J}^T\ \Sigma^{-1}\ \mathcal{J})$$

Physical model necessary,
'compensate' for the number of d.o.f. of χ^2 tests.

Two possible uses :

– For a given set of faults : how many sensors, and where ?

– For a given sensor pool : which faults are detectable ?

Fault detectability

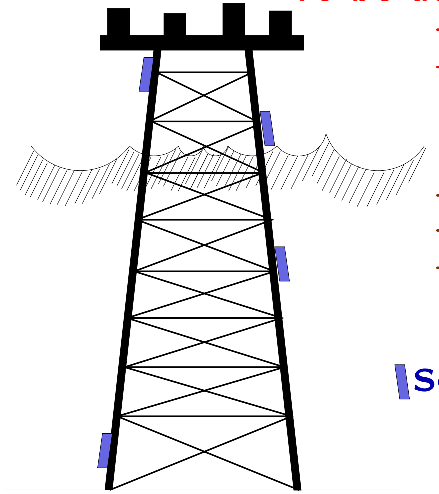
EXAMPLE

Structural health monitoring

With Laurent Mével, Maurice Goursat, Albert Benveniste

Toolboxes: (free) Scilab; LMS CADA-X

VIBRATIONS : OFFSHORE STRUCTURES

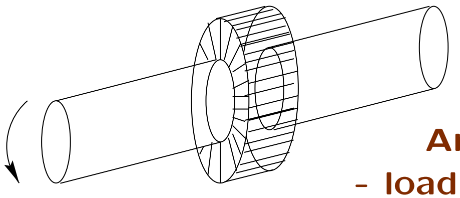


To be detected : changes in
- masses M, stiffness K
- modes

Natural excitation :
- swell
- turbulent, nonstationary
- non measured

Sensors: accelerometers

VIBRATIONS : ROTATING MACHINES



To be detected : changes in
- masses M, stiffness K
- modes

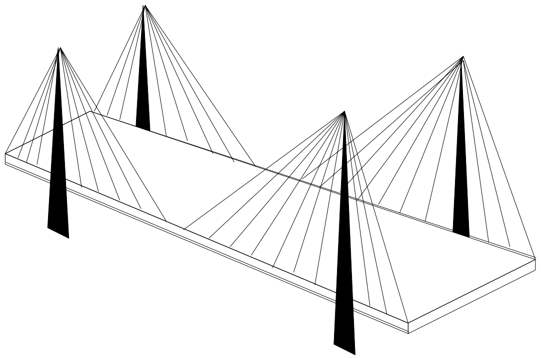
Ambiant excitation:
- load unbalancing, frictions, steam
- non stationary, non measured

Sensors: accelerometers (on the bearings!)

VIBRATIONS : BRIDGES

Ambiant excitation:
- wind
- traffic UNDER
- traffic ON

To be detected: changes in
- masses M, stiffness K
- modes



Nuisance:

- traffic ON
- temperature

Sensors: accelerometers, laser

Problems : In-operation modal identification and damage detection and localization

- The excitation is typically:
  - natural, not controlled.
  - not measured:
    - ∗ buildings, bridges, offshore structures,
    - ∗ rotating machinery,
    - ∗ cars, trains, aircrafts.
  - nonstationary (e.g., turbulent).
- How to detect and localize small damages ? Early ? On-board ? (without re-identification)
- Output-only damage detection and localization

Modelling

FE model:
$$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \ , \quad \psi_\mu = L\Psi_\mu$$

State space:
$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda\ \Phi_\lambda \ , \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}$$

Output-only covariance-based subspace identification

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k\, Y_{k-i}^T\right)}_{\text{ok if stationary !}} \ , \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\ F^i\ G \ , \quad G \triangleq \mathbf{E}\left(X_k\ Y_k^T\right)$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \ , \ \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2G & \dots \end{pmatrix}$$

$$\mathcal{H} = \mathcal{O}\ \mathcal{C} \ , \ \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Implementation

$$\underbrace{\hat{R}_i \triangleq \frac{1}{N}\sum_{k=1}^{N} Y_k\, Y_{k-i}^T}_{\text{ok when nonstationary !}} \quad , \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = U\ \Delta\ W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T \ ; \quad \hat{\mathcal{O}} = U\ \Delta_1^{1/2}$$

$$\mathcal{O}_p^{\uparrow}(H,F) = \mathcal{O}_p(H,F)\ F$$

$$\det(F - \lambda\ I) = 0\ , \quad F\ \Phi_\lambda = \lambda\ \Phi_\lambda, \quad \varphi_\lambda = H\ \Phi_\lambda$$

Robustness to nonstationary excitation

The estimates are consistent.

Combination of:

- the key factorization property of the covariances,
- the averaging operation underlying covariance computation,

allows to cancel out nonstationarities in the excitation.

Introducing the parameter vector

FE model: $$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0\ , \quad \psi_\mu = L\Psi_\mu$$

State space: $$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda\ \Phi_\lambda\ , \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

Parameter: $$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}}\ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}\ ; \quad \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$$

Eigenstructure monitoring

$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases} \qquad \begin{matrix} F\ \varphi_\lambda = \lambda\ \varphi_\lambda \\ \Phi_\lambda \triangleq H\ \varphi_\lambda \end{matrix}$$

Canonical parametrization: $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$

Observability in modal basis: $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

System parameter characterization:

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel.

Back to structural subspace identification

$$\exists S, \quad S^T\, S = I_s, \qquad S^T\, \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } S(\theta_0)$$

$\theta_0 \leftrightarrow (R_i^0)_i$ characterized by: $S^T(\theta_0)\, \hat{\mathcal{H}}^0_{p+1,q} = 0$

Residual for structural damage monitoring

$$\zeta_N(\theta_0) \triangleq \mathrm{vec}(\; S^T(\theta_0)\, \hat{\mathcal{H}}_{p+1,q} \;)$$

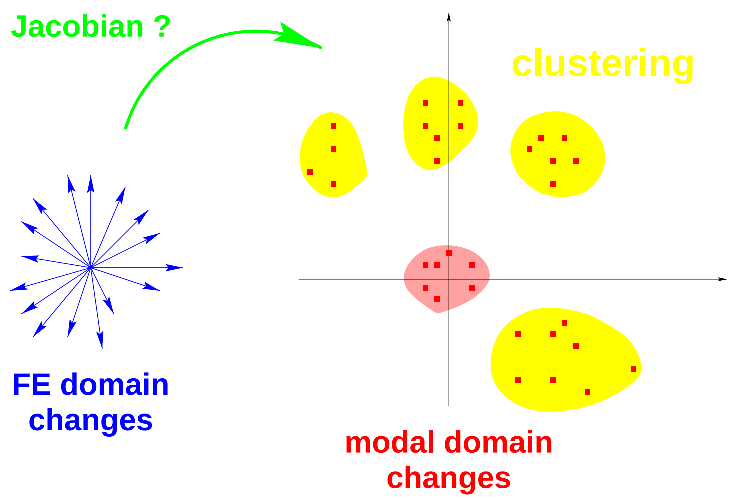
Relation to parity space

$$\zeta_{\text{parity}} = \mathcal{G}\, \mathcal{Y}^+_{k,p+1}, \qquad \mathcal{G}\, \mathcal{O}_{p+1} = 0$$

$$\zeta_{\text{subspace}} = S^T\, \hat{\mathcal{H}}_{p+1,q}, \qquad S^T\, \mathcal{O}_{p+1} = 0$$

First order statistics $\longleftrightarrow$ Second order statistics

On-board damage diagnostics: projecting changes



Damage diagnostics: (local) sensitivity approach

$$\zeta \sim \mathcal{N}(\mathcal{J}\ \delta\theta, \quad \Sigma), \qquad \delta\theta = \mathcal{I}\ \mathcal{J}_{(M_0^\star, K_0^\star)} \begin{pmatrix} \delta M \\ \delta K \end{pmatrix}$$

$(M_0^\star, K_0^\star)$: design model

Jacobian : $(\delta M, \delta K) \xrightarrow{\mathcal{J}_{(M_0^\star, K_0^\star)}} (\delta\mu, \delta\psi_\mu)$

Reduction: $\mathcal{I}$ matching computed/identified modes

Problem : $\dim \begin{pmatrix} M \\ K \end{pmatrix} \gg \dim \theta$

Computing Jacobian

1. $(\delta M, \delta K) \xrightarrow[\text{mode selection}]{\mathcal{IJ}_{(M_0^\star, K_0^\star)}} (\delta\mu, \delta\psi_\mu)$

2. Apply $\mathcal{IJ}$ to unit vectors $(\delta M, \delta K)$

3. Truncate small vectors $(\delta\mu, \delta\psi_\mu)$

4. Cluster the remaining vectors $(\delta\mu, \delta\psi_\mu)$, using the χ^2-metric.

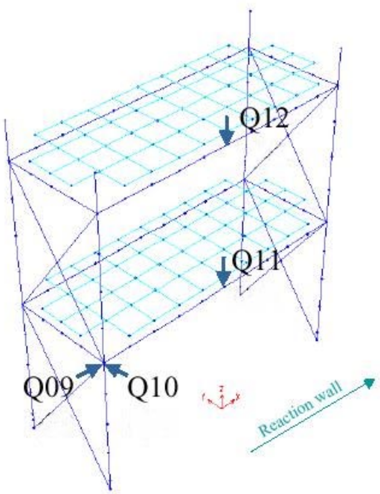
Results on real data

Steelquake

Z24 bridge

Flutter monitoring

Steelquake

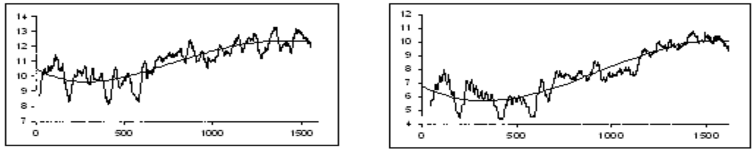


Scenario	Undamaged	Damaged
Q09 /39	$2.81 \cdot 10e2$	$3.78 \cdot 10e6$
Q10 /40	$1.53 \cdot 10e2$	$2.20 \cdot 10e7$
Q11 /41	$6.75 \cdot 10e2$	$2.18 \cdot 10e4$
Q12 /42	$2.88 \cdot 10e2$	$1.62 \cdot 10e4$

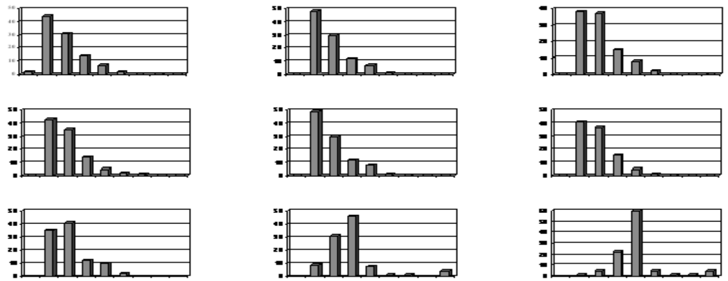
Z24 bridge

	Mode	1	2	3	4	χ^2
Undamaged	Freq.(Hz)	3.88	5.01	9.80	10.30	$8.80 \cdot 10e2$
Damaged (1)	Freq.(Hz)	3.87	5.06	9.79	10.32	$8.00 \cdot 10e5$
Damaged (2)	Freq.(Hz)	3.76	4.93	9.74	10.25	$3.96 \cdot 10e6$

Z24 bridge (Contd.)

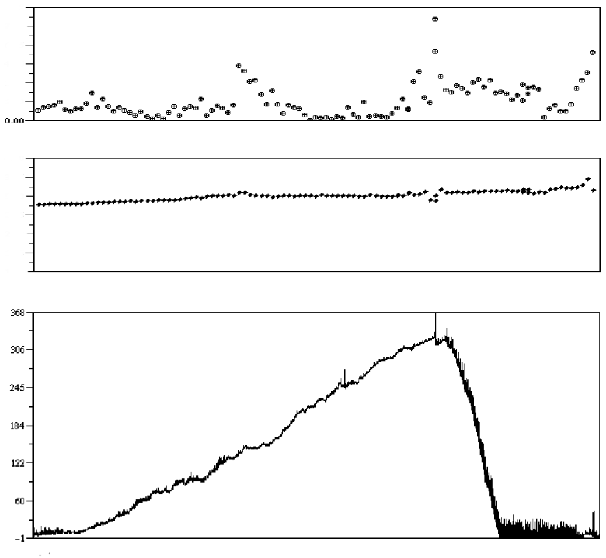


The test values over three months, log-scale. Two sensors sets.



Distribution of the test values for each of the nine months.

Flutter monitoring



Damping coefficient (top), frequency (middle), CUSUM test (bottom).

Conclusion

A statistical framework

enlightens the meaning and increases the power

of a number of familiar operations