In-operation structural health monitoring:

a statistical approach

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Toolboxes: LMS CADA-X, and free Scilab

http://www.irisa.fr/sigma2/constructif/modal.htm

Problems : In-operation modal identification and damage detection and localization

- **The excitation is typically:**
  - **natural, not controlled.**
  - **not measured:**
    - **buildings, bridges, offshore structures,**
    - **rotating machinery (e.g. steam flowing),**
    - **cars, trains, aircrafts.**
  - **nonstationary (e.g., turbulent).**
- How to merge multiple measurements setups e.g. in case of moving sensors?
- **How to detect and localize small damages?**

Identification and merging

- **Output-only eigenstructure identification,**
- **In the presence of nonstationary excitation,**
- Handling moving sensor pools, with some reference sensors : avoid merging identification results from the different pools, merge the data instead, and process them globally, using a standard subspace algorithm.

Damage detection and localization

- **Output-only damage detection and localization,**
- **In the presence of nonstationary excitation,**
- **On-board handling of small damages.**

Wanted:

- **Early warning and interpretation of damages,**
- **Avoid re-identification prior to detection,**
- **Avoid inverse problem solving prior to damage localization.**

Contents

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- Output-only covariance-driven subspace identification
- Robustness to nonstationary excitation
- Damage detection
- Damage diagnostics
- Examples

Modelling

FE model:

$$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) &= \nu(s) \\ Y(s) &= L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \ , \quad \psi_\mu = L\Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} &= FX_k + V_k \\ Y_k &= HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \ \Phi_\lambda \ , \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}$$

Output-only covariance-driven subspace identification

$$\begin{cases} X_{k+1} &= F \ X_k \ + \ V_k \\ Y_k &= H \ X_k \end{cases}$$

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k \ Y_{k-i}^T\right)}_{\text{ok if stationary !}} \ , \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \ , \quad \begin{matrix} \mathcal{C} \triangleq (\ G \quad FG \quad F^2G \quad \dots \ ) \\ G \triangleq \mathbf{E}\left(X_k \ Y_k^T\right) \end{matrix}$$

$$R_i = H \ F^i \ G \Longrightarrow \mathcal{H} = \mathcal{O} \ \mathcal{C}$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Implementation

$$\underbrace{\hat{R}_i \triangleq 1/N \sum_{k=1}^{N} Y_k \ Y_{k-i}^T}_{\text{ok when nonstationary !}} \ , \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\hat{\mathcal{H}} \approx \hat{\mathcal{O}} \ \hat{\mathcal{C}}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = U \ \Delta \ W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T \ ; \quad \hat{\mathcal{O}} = U \ \Delta_1^{1/2}$$

$$\mathcal{O}_p^{\uparrow}(H, F) = \mathcal{O}_p(H, F) \ F$$

$$\det(F - \lambda \ I) = 0 \ , \quad F \ \Phi_\lambda = \lambda \ \Phi_\lambda, \quad \varphi_\lambda = H \ \Phi_\lambda$$

Robustness to nonstationary excitation

Approximate factorization of covariances : $\hat{R}_i \approx H\, F^i\, \hat{G}$

Consistency : $T^{-1}\, \hat{F}\, T \to F,\ \hat{H} \to H; \quad (\hat{\lambda}, \hat{\varphi}_\lambda) \to (\lambda, \varphi_\lambda)$

Theory and experience show that the combination of:

- the key factorization property of the covariances,
- the averaging operation in their computation,

allows to cancel out nonstationarities in the excitation.

Structural monitoring : Eigenstructure monitoring

$$\begin{cases} X_{k+1} = F\, X_k + V_k \\ Y_k = H\, X_k \end{cases} \qquad \begin{matrix} F\, \Phi_\lambda = \lambda\, \Phi_\lambda \\ \varphi_\lambda \triangleq H\, \Phi_\lambda \end{matrix}$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\, \Phi \end{pmatrix}$ modes / mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

Damage detection

θ_0 : reference parameter, known (or identified)

Y_k: N-size sample of new measurements

Build a residual ζ significantly non zero when damage

Local approach (small deviations)

Test $\quad \mathcal{H}_0 : \ \theta = \theta_0 \quad$ against $\quad \mathcal{H}_1 : \ \theta = \theta_0 + \delta\theta/\sqrt{N}$

Subspace model/data correlation (1)

Fresh data $\longrightarrow \hat{R}_i \longrightarrow \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$

Nominal model : $\mathcal{O}(\theta_0) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \Phi\Delta^2 \\ \vdots \end{pmatrix}$ Observability in modal basis

$! \ \mathcal{H} = \mathcal{O}\ \mathcal{C} \ ! \qquad \ker \hat{\mathcal{H}}^T \overset{?}{=} \ker \mathcal{O}^T(\theta_0)$

Subspace model/data correlation (2)

$$\exists S, \quad S^T\, S = I_s, \qquad S^T\, \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } S(\theta_0)$$

Check if : $\quad S^T(\theta_0)\, \hat{\mathcal{H}} \;\approx\; 0$

Residual for structural damage monitoring

$$\zeta_N(\theta_0) \triangleq \text{vec}(\; S^T(\theta_0)\, \hat{\mathcal{H}} \;)$$

? How to assess the significance of : $S^T(\theta_0)\, \hat{\mathcal{H}} \approx 0$?

Subspace model/data correlation (3)

The residual is asymptotically Gaussian

$$\zeta_N(\theta_0) \;\rightarrow\; \begin{cases} \mathcal{N}(\quad 0, \; \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\; \mathcal{M}(\theta_0)\, \delta\theta, \; \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0+\delta\theta/\sqrt{N}} \end{cases}$$

$\mathcal{M}(\theta_0)$: mean sensitivity (Jacobian) of residual ζ w.r.t. modal changes

(On-board) χ^2-test in the residual

$$\zeta_N^T \;\Sigma^{-1}\, \mathcal{M}\; (\mathcal{M}^T\, \Sigma^{-1}\, \mathcal{M})^{-1}\, \mathcal{M}^T\, \Sigma^{-1}\; \zeta_N \;\; \geq h$$

(On-board) modal χ^2-test

$$\zeta_N^T \;\Sigma^{-1}\, \mathcal{M}_i\; (\mathcal{M}_i^T\, \Sigma^{-1}\, \mathcal{M}_i)^{-1}\, \mathcal{M}_i^T\, \Sigma^{-1}\; \zeta_N \;\; \geq h$$

Model/data correlation - Generalization

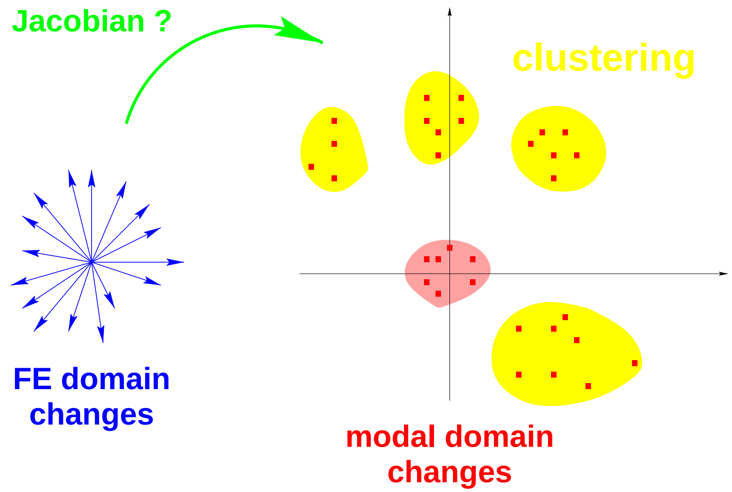
Any estimating function can play the role of a residual

Warning:

The prediction error is OK for sensor faults,

NOT for structural damages !

On-board damage diagnostics: projecting changes



Damage diagnostics: (local) sensitivity approach

$$\zeta \sim \mathcal{N}(\mathcal{M}\ \delta\theta,\ \ \Sigma), \qquad \delta\theta = \mathcal{I}\ \mathcal{J}_{(M_0^\star, K_0^\star)} \begin{pmatrix} \delta M \\ \delta K \end{pmatrix}$$

$(M_0^\star, K_0^\star)$: design model

Jacobian : $(\delta M, \delta K) \xrightarrow{\mathcal{J}_{(M_0^\star, K_0^\star)}} (\delta\mu, \delta\psi_\mu)$

Reduction: $\mathcal{I}$ matching computed/identified modes

Problem : $\dim \begin{pmatrix} M \\ K \end{pmatrix} \gg \dim \theta$

Hint: Cluster the vectors $(\delta\mu, \delta\psi_\mu)$ using the χ^2-metric

Examples

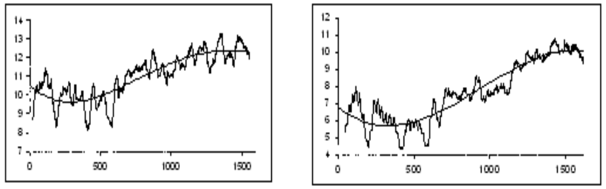
- Sports car
- **Z24 bridge**
- Reticular structure
- Slat track
- **Aircraft flutter monitoring**

Z24 bridge

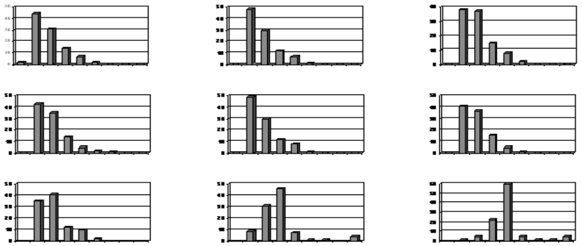
- A benchmark of the BRITE/EURAM project SIMCES and of the European COST action F3
- Response to traffic excitation under the bridge measured over one year in 139 points
- Two damage scenarios (DS1 and DS2): pier settlements of 20mm and 80mm.

Identified first four natural frequencies / Test values
(Results with four sensors)

	Mode	1	2	3	4	χ^2
Undamaged	Freq.(Hz)	3.88	5.01	9.80	10.30	$8.80 \cdot 10e2$
Damaged (1)	Freq.(Hz)	3.87	5.06	9.79	10.32	$8.00 \cdot 10e5$
Damaged (2)	Freq.(Hz)	3.76	4.93	9.74	10.25	$3.96 \cdot 10e6$



Evolution of the test values over three months on a log-scale amplitude. Two different sets of sensors (Left and Right).



Distribution of the test values for each of the nine months.

Aircraft flutter monitoring

- Aero-elastic flutter: critical instability phenomenon
- Flight flutter testing procedure
- Objective : on-line in-flight exploitation of test data
- On-line flight flutter monitoring problem: monitoring some specific damping coefficient
- Using a different computation of the residual ζ, introducing a minimum magnitude of change, and using the CUSUM test

Test for $\rho_c = \rho_c^{(1)}$.

Test for $\rho_c = \rho_c^{(2)} < \rho_c^{(1)}$.

Bottom: $-g_n^-$ reflects $\rho < \rho_c$. Top: g_n^+ reflects $\rho > \rho_c$.