In-operation subspace-based covariance-driven structural identification

Steelquake and Z24 bridge benchmarks

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Toolboxes: *LMS CADA-X*, *and Scilab*

ftp://ftp.inria.fr/INRIA/Projects/Meta2/Scilab/contrib/MODAL/

Problem :

In-operation modal identification of structures

- The excitation is typically:
  - natural, not controlled.

  - not measured:
    - buildings, bridges, offshore structures,
    - rotating machinery,
    - cars, trains, aircrafts.

  - nonstationary (e.g., turbulent).

- How to merge multiple measurements setups
  e.g. in case of moving sensors?

Identification and merging

- Output-only eigenstructure identification,
- In the presence of nonstationary excitation,
- Handling moving sensor pools, with some reference sensors.

Wanted:

- Avoid merging identification results from the different pools,
- Merge the data instead, and process them globally,
- Use a standard subspace algorithm.

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Modelling

FE model:

$$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \; , \quad \psi_\mu = L\Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \, \Phi_\lambda \; , \quad \varphi_\lambda \stackrel{\Delta}{=} H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \; , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}$$

Output-only covariance-based subspace identification

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k \, Y_{k-i}^T\right)}_{\text{ok if stationary !}} \; , \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H \; F^i \; G \; , \quad G \triangleq \mathbf{E}\left(X_k \; Y_k^T\right)$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \; , \; \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2G & \dots \end{pmatrix}$$

$$\mathcal{H} = \mathcal{O} \; \mathcal{C} \; , \; \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Implementation

$$\underbrace{\hat{R}_i \triangleq \frac{1}{N} \sum_{k=1}^{N} Y_k \; Y_{k-i}^T}_{\text{ok when nonstationary !}} \quad , \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = U \; \Delta \; W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T \; ; \quad \hat{\mathcal{O}} = U \; \Delta_1^{1/2}$$

$$\mathcal{O}_p^{\uparrow}(H, F) = \mathcal{O}_p(H, F) \; F$$

$$\det(F - \lambda \; I) = 0 \; , \quad F \; \Phi_\lambda = \lambda \; \Phi_\lambda, \quad \varphi_\lambda = H \; \Phi_\lambda$$

Merging multiple measurements setups

$$\underbrace{\begin{bmatrix} Y_k^{(0,1)} \\ Y_k^{(1)} \end{bmatrix}}_{\text{Record 1}} \quad \underbrace{\begin{bmatrix} Y_k^{(0,2)} \\ Y_k^{(2)} \end{bmatrix}}_{\text{Record 2}} \quad \cdots \quad \underbrace{\begin{bmatrix} Y_k^{(0,J)} \\ Y_k^{(J)} \end{bmatrix}}_{\text{Record J}}$$

$$\begin{cases} X_{k+1}^{(j)} = F\ X_k^{(j)} + V_k^{(j)} & \\ Y_k^{(0,j)} = H_0\ X_k^{(j)} & \text{(the reference)} \\ Y_k^{(j)} = H_j\ X_k^{(j)} & \text{(sensor pool n}^o\ j) \end{cases}$$

$$R_i^{0,j} \triangleq \mathrm{E}\ Y_k^{(0,j)}\ {Y_{k-i}^{(0,j)}}^T, \quad R_i^j \triangleq \mathrm{E}\ Y_k^{(j)}\ {Y_{k-i}^{(0,j)}}^T$$

$\mathrm{E}\ Y_k^{(j)}\ {Y_{k-i}^{(j)}}^T$ not used, $\mathrm{E}\ Y_k^{(j')}\ {Y_{k-i}^{(j)}}^T$ $(j \neq j')$ not available

Stationary excitation

$$\mathrm{cov} V_k^{(j)} = Q \;, \qquad G \triangleq \mathrm{E}\; X_k^{(j)}\; Y_k^{(0,j)^T}$$

$$R_i^{0,j} = H_0\; F^i\; G \;\triangleq\; R_i^0 \;, \qquad R_i^j = H_j\; F^i\; G$$

$$R_i^{\pi} \triangleq \begin{bmatrix} R_i^0 \\ R_i^1 \\ \vdots \\ R_i^J \end{bmatrix} = H\; F^i\; G \;, \qquad H \triangleq \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_J \end{bmatrix}$$

Nonstationary excitation

$$\mathrm{cov} V_k^{(j)} = Q_j \;, \qquad G_j \triangleq \mathrm{E}\; X_k^{(j)}\; Y_k^{(0,j)^T}$$

$$R_i^{0,j} = H_0\; F^i\; G_j \;, \qquad R_i^j = H_j\; F^i\; G_j$$

Hint: right renormalization of the covariances.

Robustness to nonstationary excitation

Time-varying excitation within each record

$$\mathrm{cov}\, V_k^{(j)} = Q_k$$

Approximate factorization of covariances

$$\hat{R}_i \;\approx\; H \; F^i \; \hat{G}$$

Consistency : $T^{-1}\ \hat{F}\ T \to F, \quad \hat{H} \to H$

Combination of:

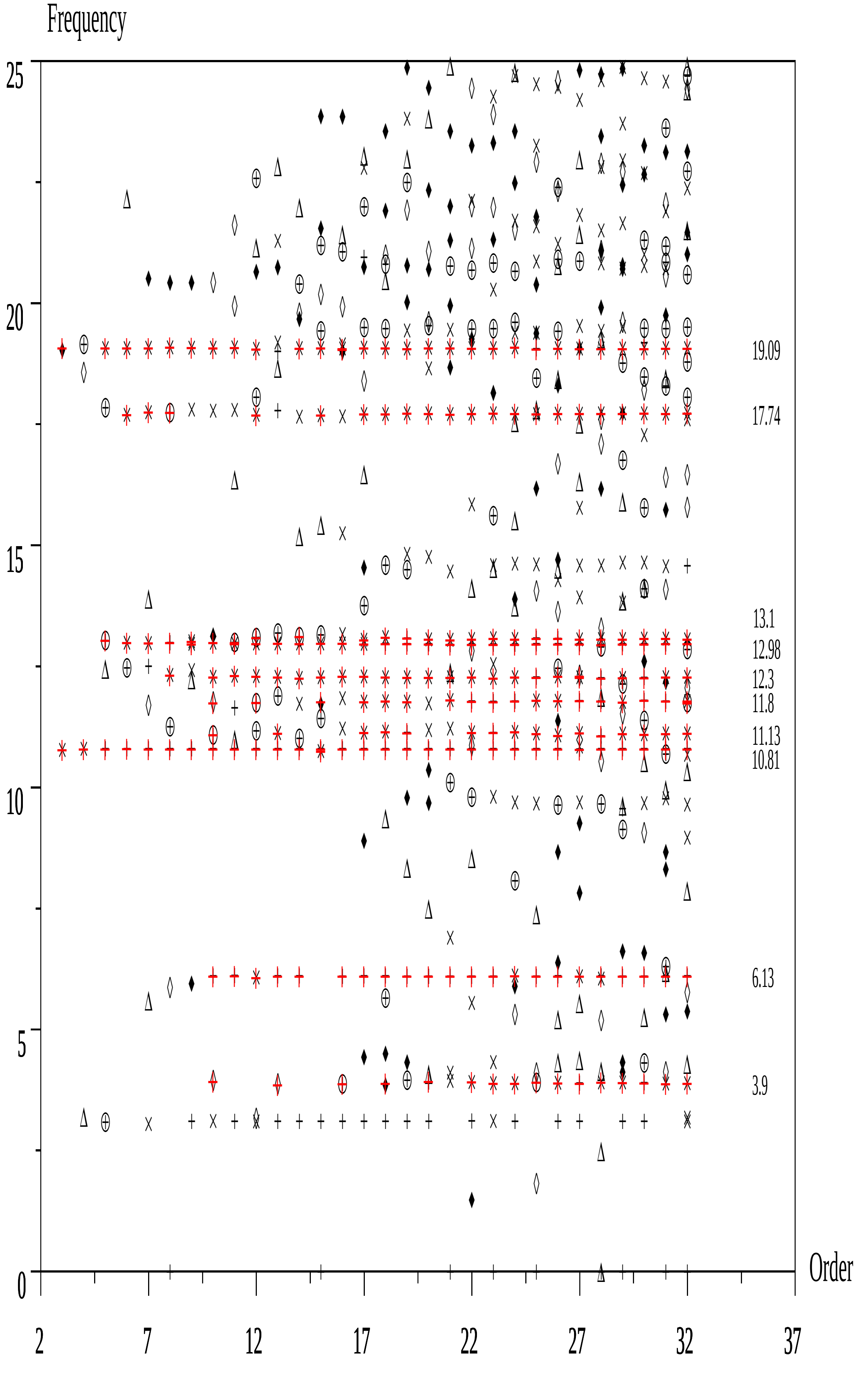
- the key factorization property of the covariances,
- the averaging operation underlying covariance computation,

allows to cancel out nonstationarities in the excitation.

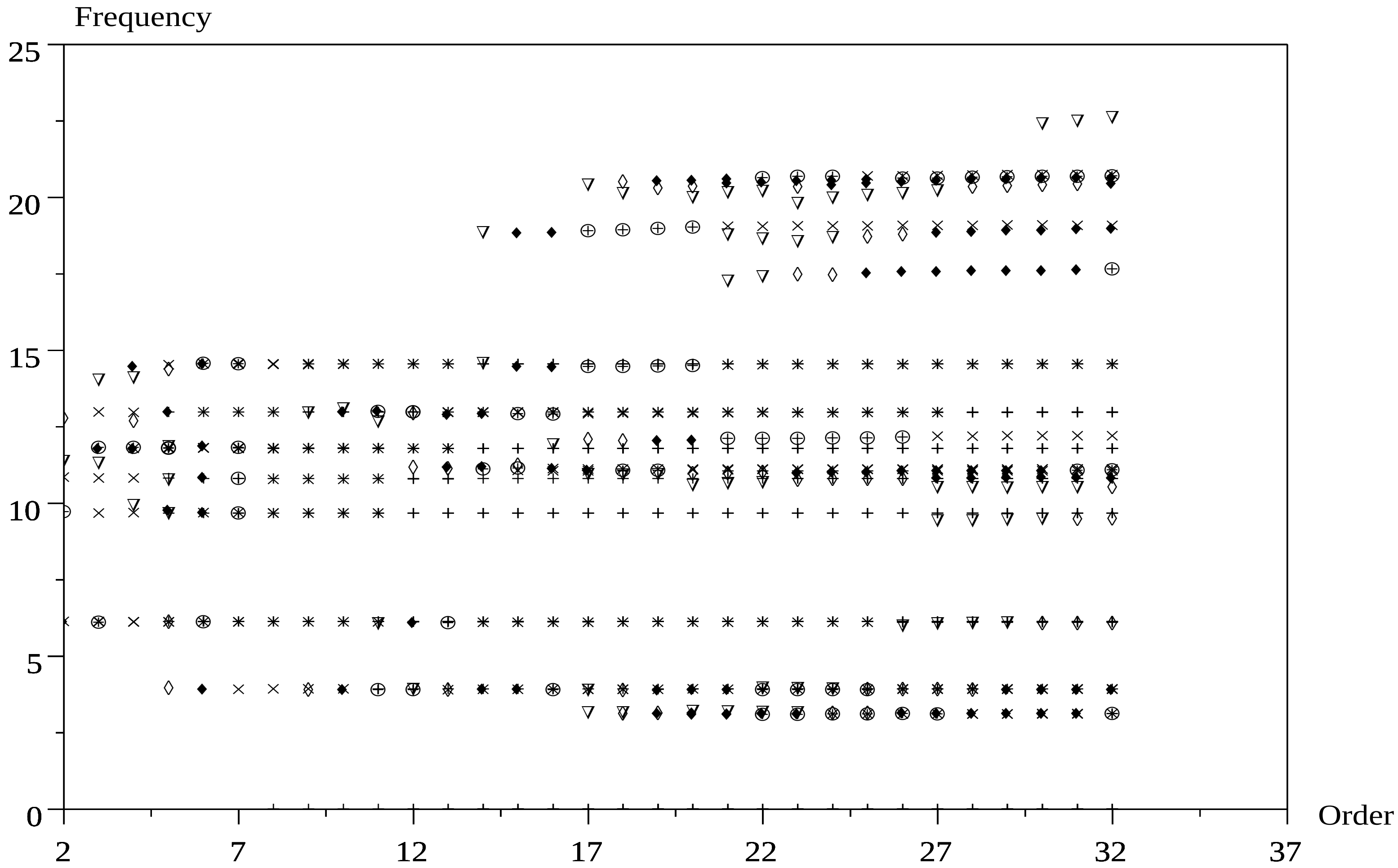
Numerical results - Steelquake

Estimated modes - Classical subspace identification

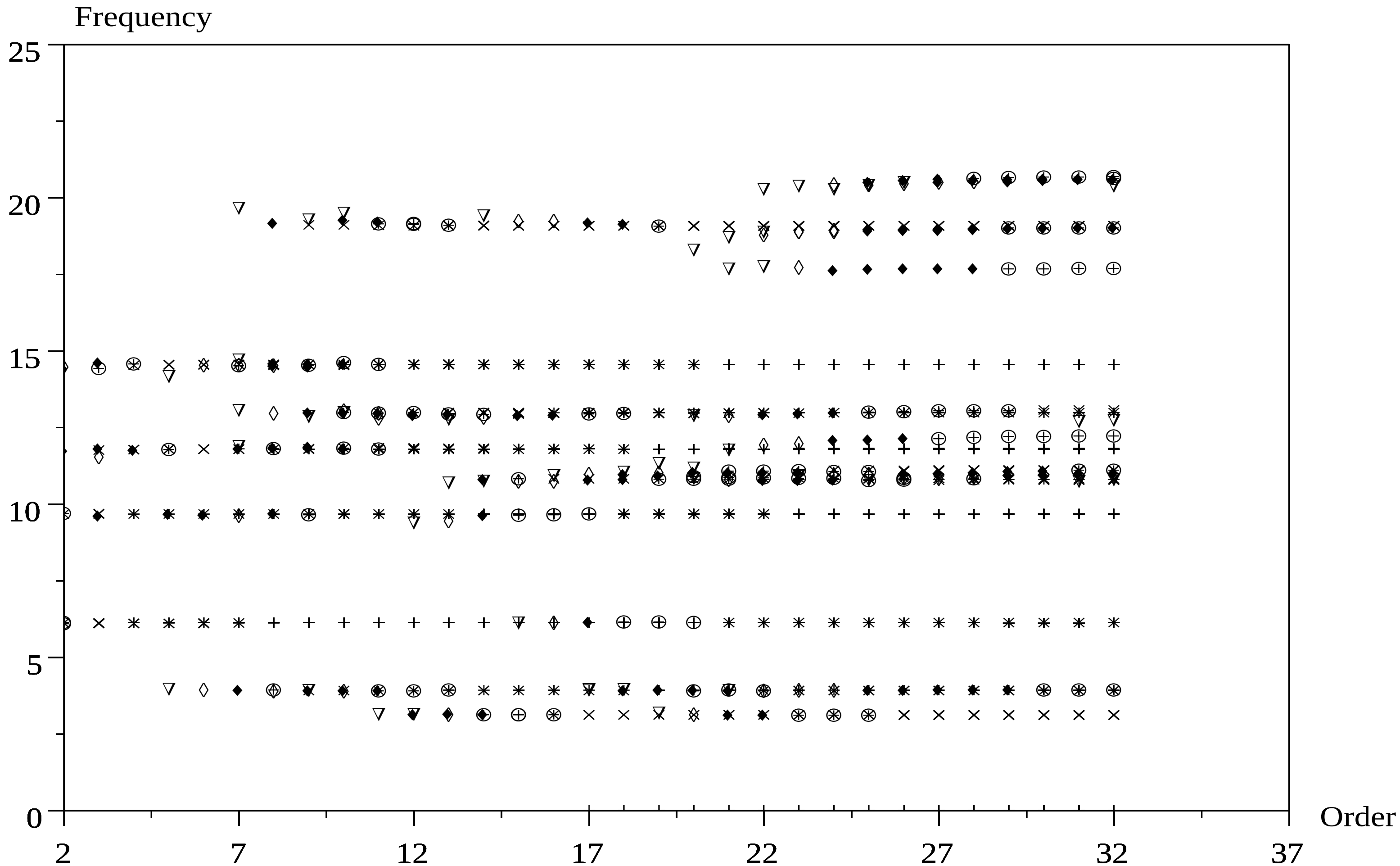
Mode	1	2	3	4	5	6	7	8
Freq.(Hz)	3.1	3.92	6.1	9.68	10.8	12.27	13.0	17.7
Dir.	X	Y	Y	Y	X	Z	Z	Z



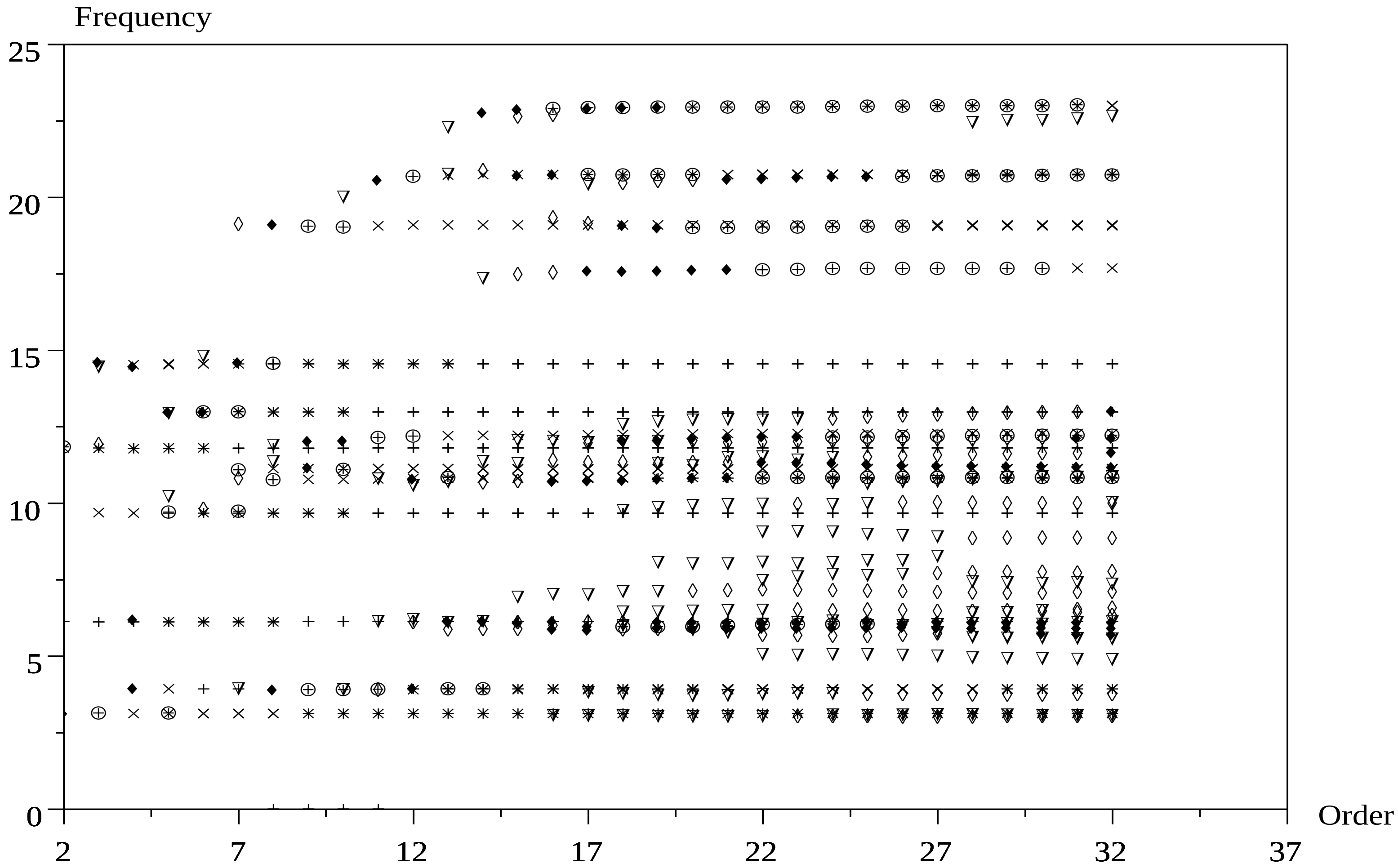
Twelve sensors (all except sensors 3, 7, 10) ($Q09 - 1$).



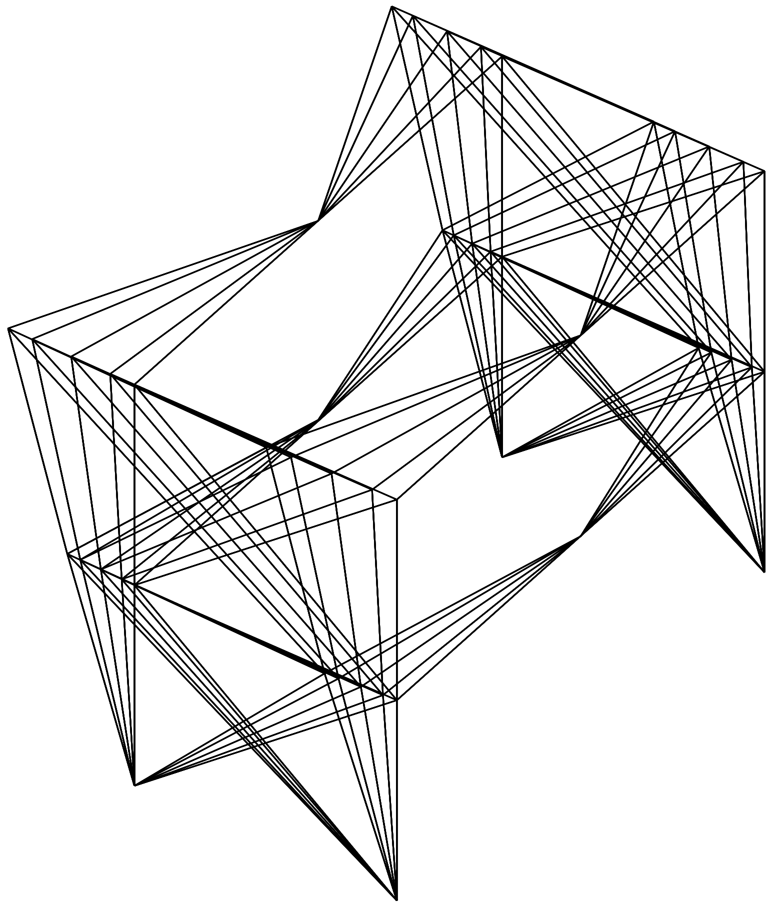
Reference sensors: 6/8; moving sensors: 11/14 ($Q09-1$), 9/15 ($Q10-2$), 1/4 ($Q10-3$), 2/13 ($Q09-4$), 5/12 ($Q10-5$).



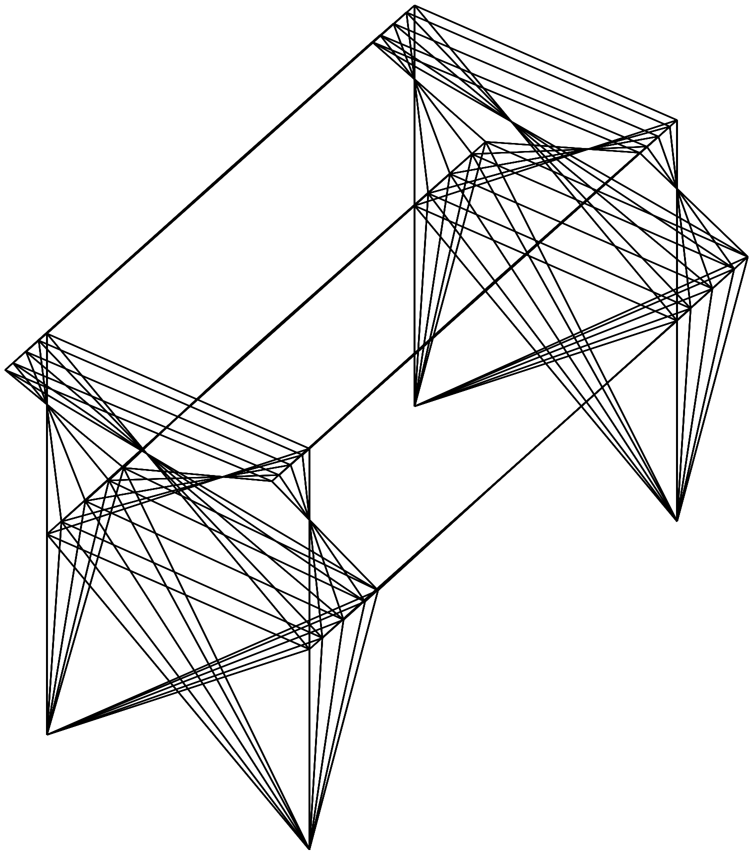
Reference sensors: 1/6/7; moving sensors: 4/5 ($Q09-1$), 8/13 ($Q10-2$), 12/15 ($Q10-3$), 11/14 ($Q09-4$), 2/10 ($Q09-5$).



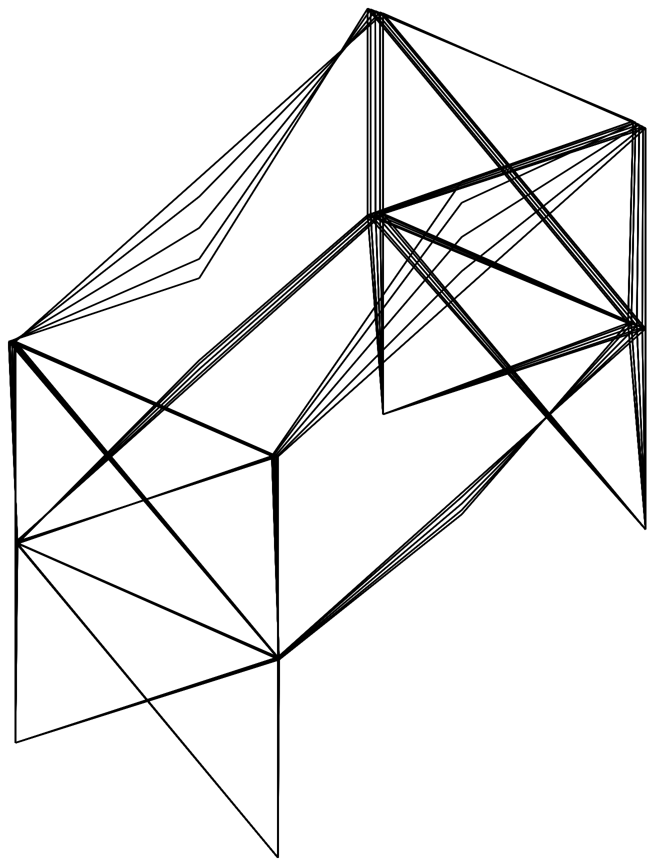
Reference sensors: 4/5/12; moving sensors: 1/9/15 ($Q10-1$), 2/6/8/10/13 ($Q10-2$), 3/7/11/14 ($Q10-3$).



Bending mode in Y direction at frequency 3.92 Hz.



Bending mode in X direction at frequency 10.8 Hz.

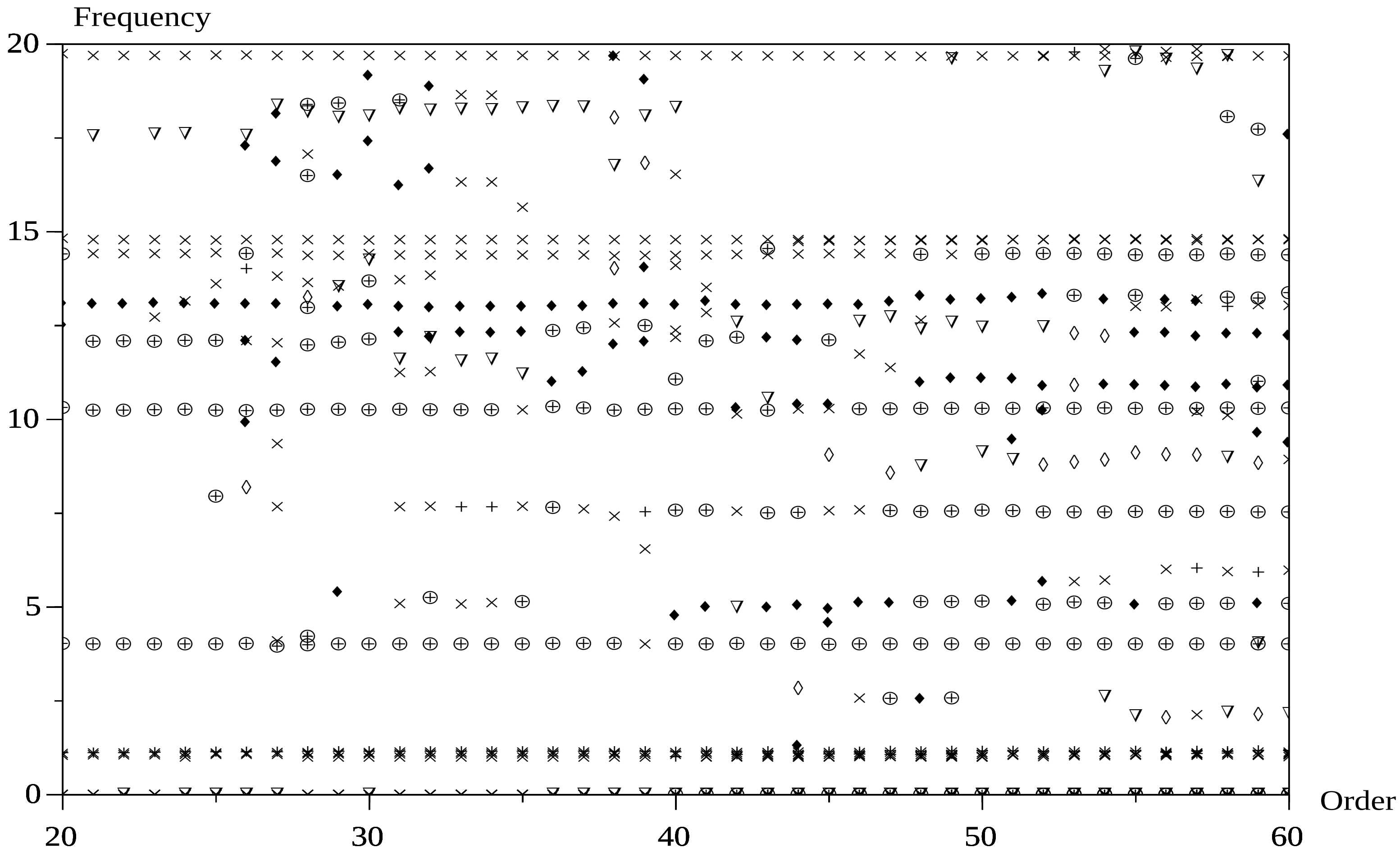


Slab torsion in Z direction at frequency 17.7 Hz.

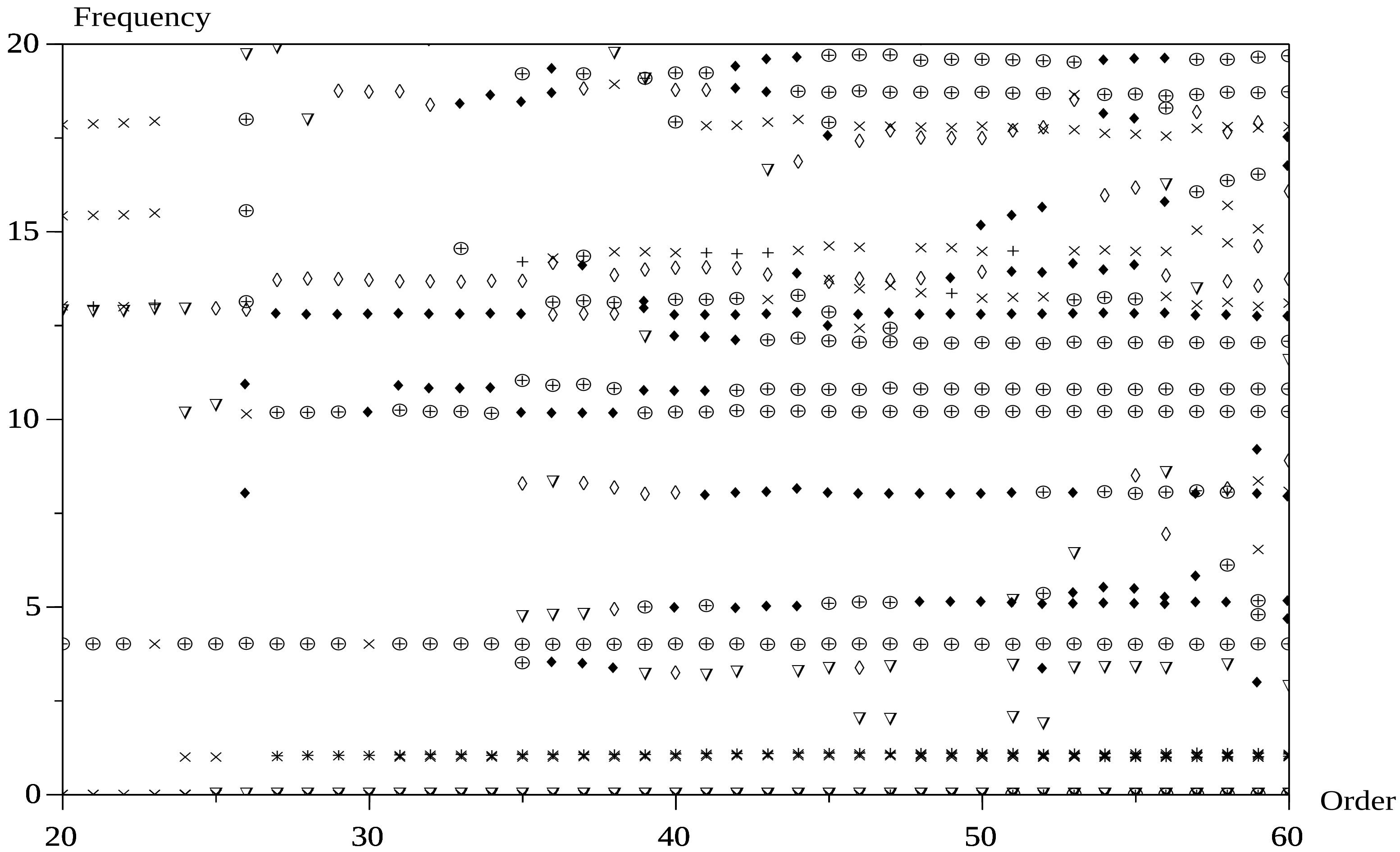
Numerical results - Z24 bridge

Estimated modes - Classical subspace identification

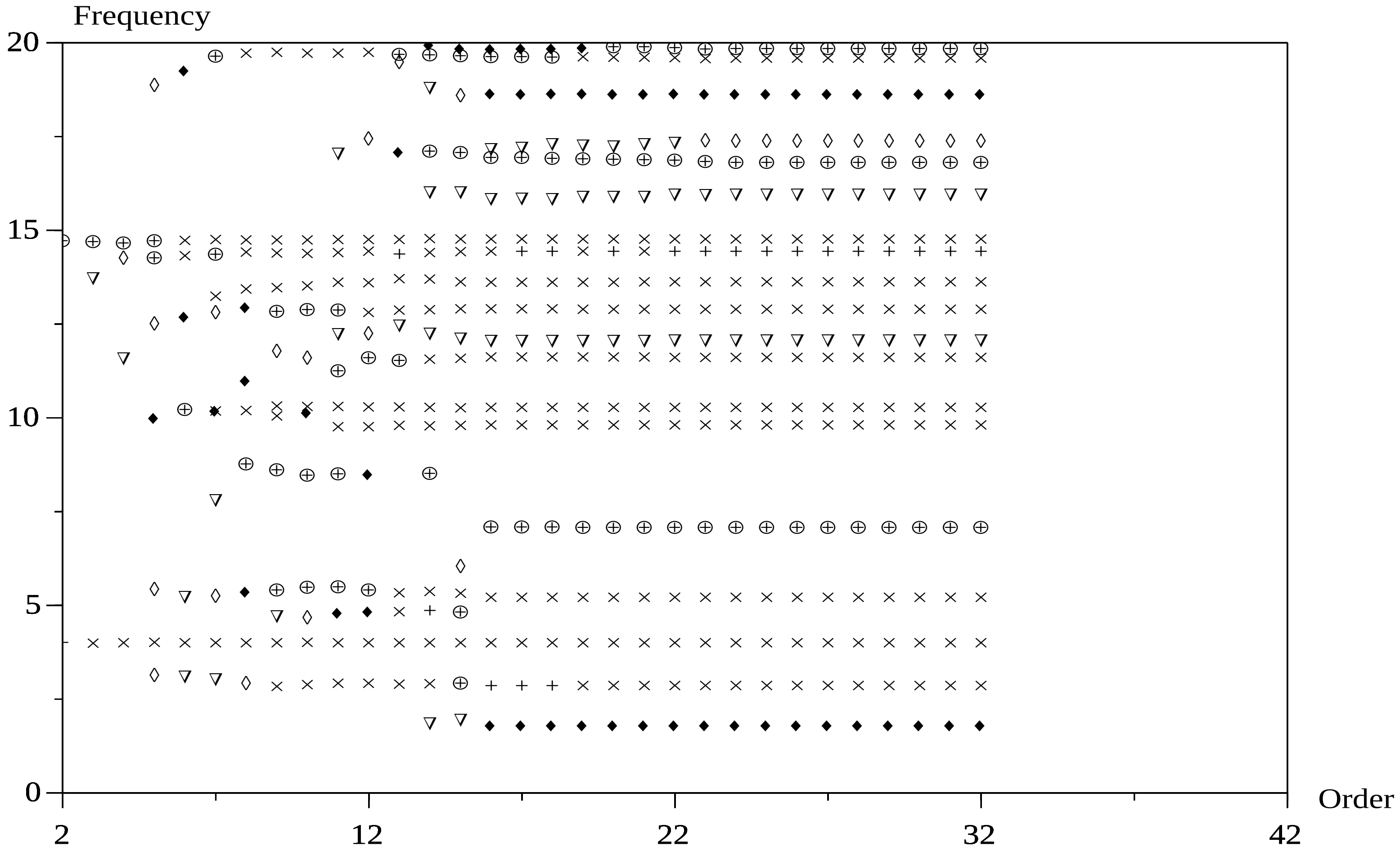
Mode	1	2	3	4	5
Frequency (Hz)	4	5.3	9.8	10.3	12



Classical subspace identification: first record (3 sensors).



Classical subspace identification: second record (2 sensors).



Polyreference subspace identification with the two records.