

# In-operation **damage detection** and **localization**

## **Steelquake** and **Z24 bridge** benchmarks

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*Toolboxes: LMS **CADA-X**, and **Scilab***

`ftp://ftp.inria.fr/INRIA/Projects/Meta2/Scilab/contrib/MODAL/`

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## Modelling

FE model: 
$$\begin{cases} M\ddot{\mathbf{Z}}(s) + C\dot{\mathbf{Z}}(s) + K\mathbf{Z}(s) = \nu(s) \\ Y(s) = L\mathbf{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0, \quad \psi_\mu = L\Psi_\mu$$

State space: 
$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

Parameter: 
$$\underbrace{e^{\delta\mu}}_{\text{modes}} = \lambda, \quad \underbrace{\psi_\mu}_{\text{mode shapes}} = \varphi_\lambda; \quad \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$$

# Damage **detection**

Local approach (**small** deviations)

$\theta_0$ : reference parameter, known (or identified)

$Y_k$ :  $N$ -size sample of new measurements

Build a **residual**  $\zeta$  **significantly non zero** when damage

Test  $\mathcal{H}_0 : \theta = \theta_0$  against  $\mathcal{H}_1 : \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$

Residual  $\leftrightarrow$  **Estimating function**

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^N K(\theta_0, Y_k)$$

Characterized by:  $E_{\theta_0} K(\theta, Y_k) = 0 \iff \theta = \theta_0$

**Warning:** Prediction error for sensor faults **ONLY!**

Mean **sensitivity** (Jacobian) and covariance

$$\mathcal{M}(\theta_0) \triangleq -E_{\theta_0} \frac{\partial K(\theta_0, Y_k)}{\partial \theta}, \quad \Sigma(\theta_0) \triangleq \lim_{N \rightarrow \infty} E_{\theta_0} \zeta_N(\theta_0) \zeta_N^T(\theta_0)$$

The **residual** is asymptotically **Gaussian**

$$\zeta_N(\theta_0) \rightarrow \begin{cases} \mathcal{N}(\mathbf{0}, \Sigma(\theta_0)) & \text{under } \mathbb{P}_{\theta_0} \\ \mathcal{N}(\mathcal{M}(\theta_0) \delta\theta, \Sigma(\theta_0)) & \text{under } \mathbb{P}_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

**(On-board)  $\chi^2$ -test**

$$\zeta_N^T \Sigma^{-1} \mathcal{M} (\mathcal{M}^T \Sigma^{-1} \mathcal{M})^{-1} \mathcal{M}^T \Sigma^{-1} \zeta_N \geq h$$

**Invariant** / pre-multiplication of  $\zeta$  with invertible gain

## Back to **eigenstructure** monitoring

$$\begin{cases} X_{k+1} = F X_k + V_k & F \varphi_\lambda = \lambda \varphi_\lambda \\ Y_k = H X_k & \Phi_\lambda \triangleq H \varphi_\lambda \end{cases}$$

**Canonical** parametrization:  $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$

Observability in modal basis:  $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$

**System parameter characterization:**

$\mathcal{H}_{p+1,q}$  and  $\mathcal{O}_{p+1}(\theta)$  have the **same left kernel**.

## Back to structural **subspace** identification

$$\exists S, \quad S^T S = I_s, \quad S^T \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } S(\theta_0)$$

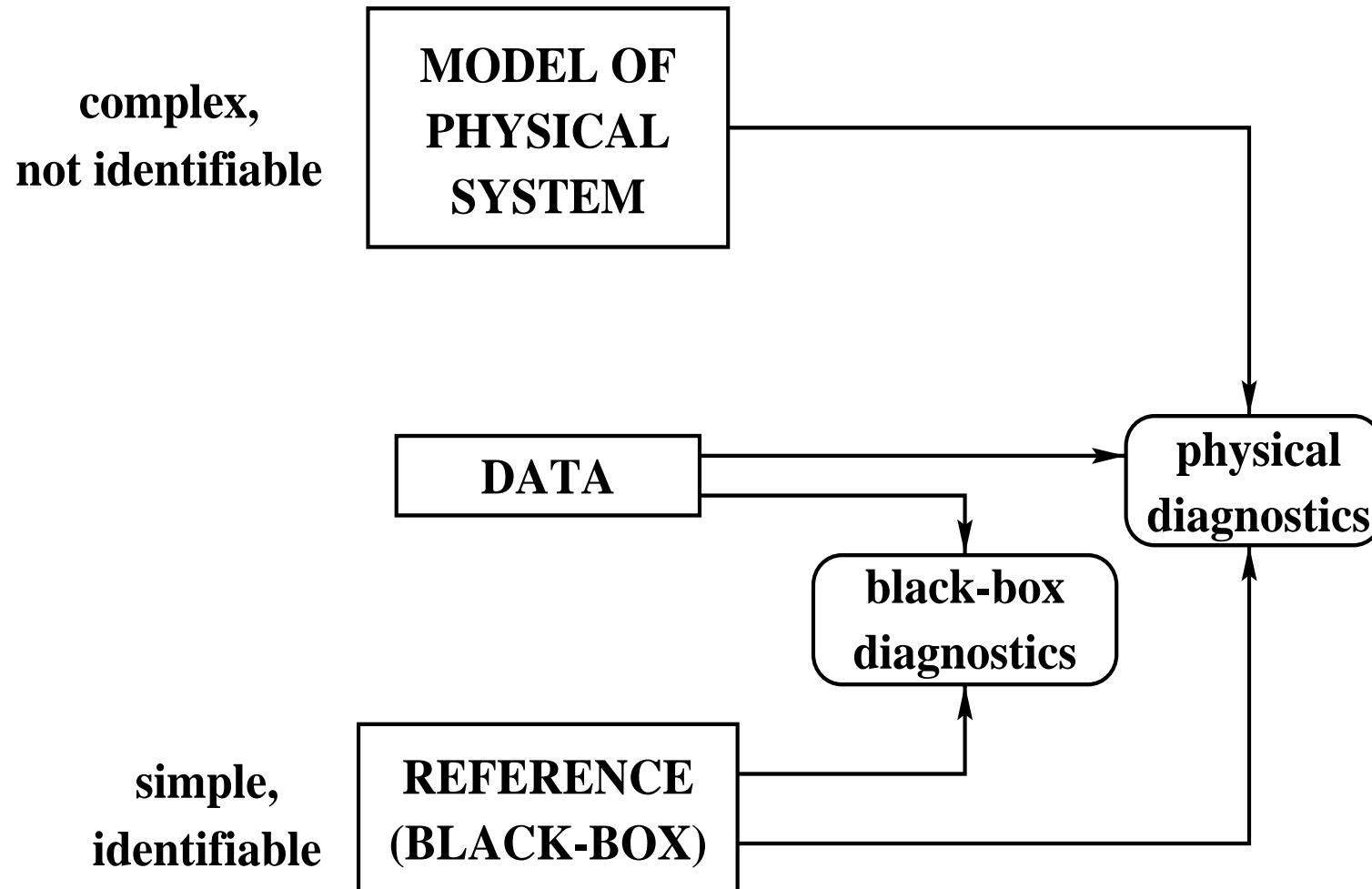
$$\theta_0 \leftrightarrow (R_i^0)_i \text{ characterized by: } S^T(\theta_0) \hat{\mathcal{H}}_{p+1,q}^0 = 0$$

## **Residual** for structural damage monitoring

$$\zeta_N(\theta_0) \triangleq \text{vec}( S^T(\theta_0) \hat{\mathcal{H}}_{p+1,q} )$$

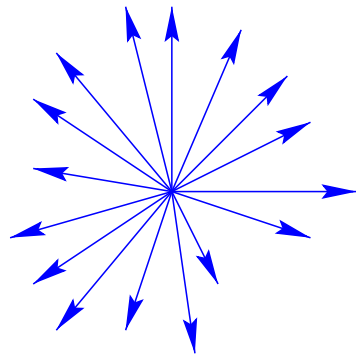


# On-board damage diagnostics without solving an inverse problem

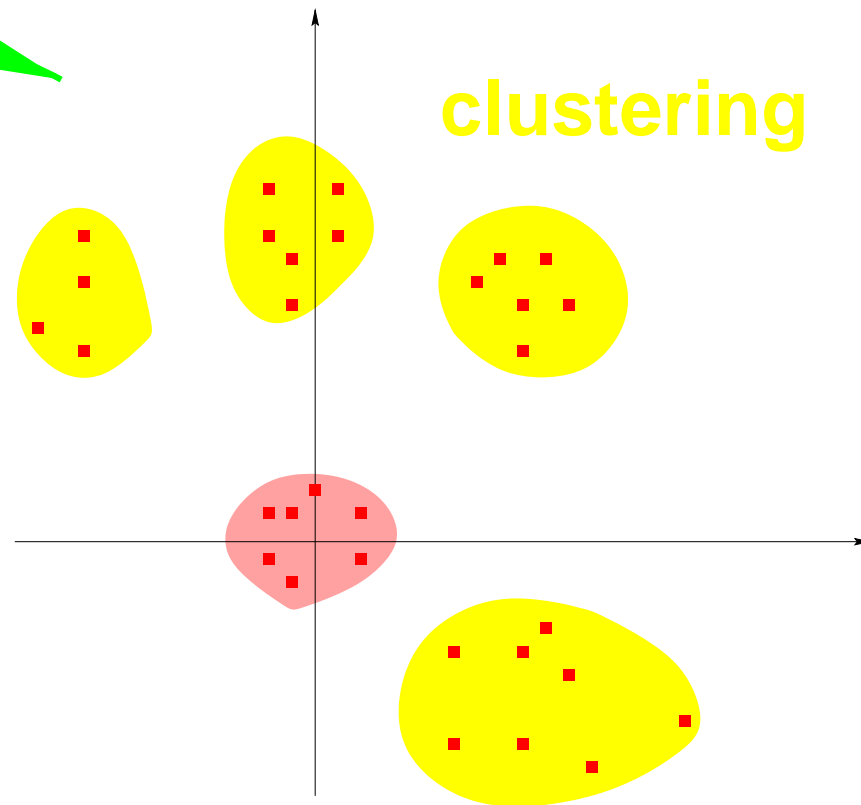
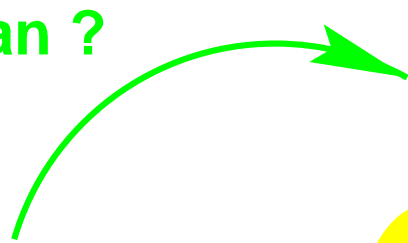


# Damage **diagnostics**: projecting changes

Jacobian ?



FE domain changes



modal domain changes

Damage **diagnostics**: (local) **sensitivity** approach

$$\zeta \sim \mathcal{N}(\mathcal{M} \delta\theta, \Sigma), \quad \delta\theta = \mathcal{I} \mathcal{J}_{(M_0^*, K_0^*)} \begin{pmatrix} \delta M \\ \delta K \end{pmatrix}$$

$(M_0^*, K_0^*)$  : **design** model

$$\text{Jacobian} : (\delta M, \delta K) \xrightarrow{\mathcal{J}_{(M_0^*, K_0^*)}} (\delta\mu, \delta\psi_\mu)$$

Reduction:  $\mathcal{I}$  matching computed/identified modes

$$\text{Problem} : \dim \begin{pmatrix} M \\ K \end{pmatrix} \gg \dim \theta$$

## Computing Jacobian

1.  $(\delta M, \delta K) \xrightarrow[\text{mode selection}]{\mathcal{IJ}_{(M_0^*, K_0^*)}} (\delta \mu, \delta \psi_\mu)$

2. Apply  $\mathcal{IJ}$  to **unit** vectors  $(\delta M, \delta K)$

3. **Truncate small** vectors  $(\delta \mu, \delta \psi_\mu)$

4. **Cluster** the remaining vectors  $(\delta \mu, \delta \psi_\mu)$ ,  
using the  $\chi^2$ -**metric**.

? Better reduction techniques, instead of 1., 2., 3.

## Computing sensitivities

$$M \ddot{\mathbf{Z}}(t) + C \dot{\mathbf{Z}}(t) + K \mathbf{Z}(t) = \boldsymbol{\nu}(t) ; \quad \boldsymbol{\phi}^T (\mu^2 M + \mu C + K) = 0$$

$$\delta \mu (2\mu M + C) \boldsymbol{\Phi} + (\mu^2 M + \mu C + K) \delta \boldsymbol{\Phi} + \mu^2 \delta M + \mu \delta C + \delta K = 0$$

$$\delta \mu = - \frac{\boldsymbol{\phi}^T (\mu^2 \delta M + \mu \delta C + \delta K) \boldsymbol{\phi}}{\boldsymbol{\phi}^T (2\mu M + C) \boldsymbol{\phi}}$$

$$(\mu^2 M + \mu C + K) \delta \boldsymbol{\phi} = -\delta \mu (2\mu M + C) \boldsymbol{\phi} - (\mu^2 \delta M + \mu \delta C + \delta K) \boldsymbol{\phi}$$

$$\boldsymbol{\phi}^T \delta \boldsymbol{\phi} = 0$$

$$\delta \Phi_c = \mathcal{J}_{\Phi_c \psi} \delta \psi ; \quad \delta \Phi_i = \delta \Phi_c (\Phi_i \neq \Phi_c)$$

## Numerical results - **Steelquake**

Sensors 2, 8, 10, 13.

8 modes identified on  $Q_{10}$ .

Theoretically, under no damage assumption:  $\chi^2 = 80$ .

Experimentally, for undamaged scenarios  $Q_{09} - Q_{12}$ :  $\chi^2 = 8 \cdot 10^3$ .

For damaged scenarios  $Q_{39} - Q_{42}$ :  $\chi^2 = 8 \cdot 10^8$ .

## Numerical results - Z24 bridge

	Mode	1	2	3	4	$\chi^2$
Safe	Freq.(Hz)	3.88	5.01	9.8	10.3	25
DS1	Freq.(Hz)	3.87	5.06	9.79	10.32	70000
DS2	Freq.(Hz)	3.76	4.93	9.74	10.25	70000

# Damage localization - Z24 bridge

