Variations on CUSUM tests for flutter monitoring

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Introduction - (1)

- Flutter: critical aircraft instability phenomenon
  unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- Flight flutter testing, very expensive and time consuming :
  Design the flutter free flight envelope
- Flutter clearance techniques:
  In-flight identification: output-only, or using input excitations
  Data processing: time-frequency, wavelet, envelope function

  Flutter prediction based on model-based approaches:
  flutterometer (μ-robustness), physical model updating
- Some challenges:
  Real time on-board monitoring,
  Handling transients between steady flight test points
- Our approach:
  Statistical detection for monitoring instability indicators

Introduction - (2)

- Aim of in-flight online flutter monitoring:
  Early detection of a deviation in the aircraft modal parameters before it develops into flutter.
- Change-point detection: natural approach
- For a scalar instability criterion ψ and a critical value ψ_c, online hypotheses testing:

$$\mathrm{H}_0 \ : \ \psi > \psi_c \text{ and } \mathrm{H}_1 \ : \ \psi \leq \psi_c$$

- CUSUM test as an approximation to the optimal test
- Variations on the CUSUM test

Subspace-based residual for modal monitoring

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases} \qquad F\ \phi_\lambda = \lambda\ \phi_\lambda \qquad \varphi_\lambda \triangleq H\ \phi_\lambda$$

$$R_i \triangleq \mathrm{E}\left(Y_k\ Y_{k-i}^T\right)\ ,\quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\ F^i\ G \Longrightarrow \mathcal{H} = \mathcal{O}\ \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}\ ,\quad \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2G & \dots \end{pmatrix}\ ,\quad G \triangleq \mathrm{E}\left(X_k\ Y_k^T\right)$$

Output-only covariance-driven subspace identification

SVD of $\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \mathrm{vec}\,\Phi \end{pmatrix}$ modes / mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

Given:

- a reference parameter $\theta_\star$, by SVD of $\widehat{\mathcal{H}}_{p+1,q}^\star$ (reference data)

$$U(\theta_\star)^T\ \widehat{\mathcal{H}}_{p+1,q}^\star = 0 \qquad \text{parameter estimating function}$$

$$U(\theta_\star)^T\ \mathcal{O}_{p+1}(\theta_\star) = 0\ ,\quad U(\theta_\star)^T U(\theta_\star) = I$$

- a n-size sample of new data; $\widehat{\mathcal{H}}_{p+1,q}$

For testing $\theta = \theta_\star$, statistics (residual) :

$$\zeta_n(\theta_\star) \triangleq \sqrt{n}\ \mathrm{vec}\left(U(\theta_\star)^T\ \widehat{\mathcal{H}}_{p+1,q}\right)$$

Local approach to testing

$$\widetilde{\mathrm{H}}_0 :\ \theta = \theta_\star \quad \text{and} \quad \widetilde{\mathrm{H}}_1 :\ \theta = \theta_\star + \Upsilon/\sqrt{n}$$

Mean sensitivity and covariance matrices:

$$\mathcal{J}_n(\theta_\star, \theta) \triangleq 1/\sqrt{n}\ \partial/\partial\tilde{\theta}\ \mathrm{E}_\theta\ \zeta_n(\tilde{\theta})\big|_{\tilde{\theta}=\theta_\star}\ ,\quad \Sigma_n(\theta_\star, \theta) \triangleq \mathrm{E}_\theta\left(\zeta_n(\theta_\star)\ \zeta_n(\theta_\star)^T\right)$$

If $\Sigma_n(\theta_\star, \theta)$ is positive definite, and for all Υ, under both hypoth:

$$\Sigma_n(\theta_\star, \theta)^{-1/2}\left(\zeta_n(\theta_\star) - \mathcal{J}_n(\theta_\star, \theta)\ \Upsilon\right) \xrightarrow[n \to \infty]{} \mathcal{N}(0, I)$$

Normalized residual:

$$\overline{\zeta}_n(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star, \theta)\ \zeta_n(\theta_\star)$$

$$\mathcal{K}_n(\theta_\star, \theta) \triangleq \overline{\Sigma}_n^{-1/2}\mathcal{J}_n^T\Sigma_n^{-1}\ ,\quad \overline{\Sigma}_n(\theta_\star, \theta) \triangleq \mathcal{J}_n^T\ \Sigma_n^{-1}\mathcal{J}_n$$

$$\left(\overline{\zeta}_n(\theta_\star) - \overline{\Sigma}_n(\theta_\star, \theta)^{1/2}\ \Upsilon\right) \xrightarrow[n \to \infty]{} \mathcal{N}(0, I)$$

Data-driven computation for online detection

$$\overline{\zeta}_n(\theta_\star) = \sum_{k=q}^{n-p} Z_k(\theta_\star)/\sqrt{n}$$

$$Z_k(\theta_\star) \triangleq \mathcal{K}_k(\theta_\star, \theta)\ \mathrm{vec}\left(U(\theta_\star)^T\ \mathcal{Y}_{k,p+1}^+\ \mathcal{Y}_{k,q}^{-T}\right)$$

Another approximation

For n large enough, and $k = 1, \dots, n$,
$Z_k(\theta_\star) \approx$ Gaussian i.i.d., mean 0 before change and $\neq 0$ after.

Monitoring any function $\psi(\theta)$

Replace $\mathcal{J}_n(\theta_\star, \theta)$ with $\mathcal{J}_n(\theta_\star, \theta)\ \mathcal{J}_{\theta\psi}^\star$, where $\mathcal{J}_{\theta\psi}^\star = \partial\theta/\partial\psi|_{\theta=\theta_\star}$.

CUSUM test for monitoring a scalar index

The crossing of a critical ψ_c by ψ is reflected into a change with the same sign in the mean ν of the i.i.d. Gaussian $Z_k(\theta_\star)$.

The CUSUM test may be used for testing between:

$$\mathrm{H}_0 : \nu > 0 \quad \text{and} \quad \mathrm{H}_1 : \nu \leq 0$$

Procedure for unknown sign and magnitude of change in ψ:

i) Set a min. change magnitude $\nu_m > 0$, and test between:

$$\mathrm{H}_0 : \nu > \nu_m/2 \text{ and } \mathrm{H}_1 : \nu \leq -\nu_m/2$$

$$S_n(\theta_\star) \triangleq \sum_{k=q}^{n-p} (Z_k(\theta_\star) + \nu_m), \;\; T_n(\theta_\star) \triangleq \max_{k=q,\ldots,n-p} S_k(\theta_\star)$$

$$g_n(\theta_\star) \triangleq T_n(\theta_\star) - S_n(\theta_\star) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \varrho \quad \text{threshold}$$

ii) Run 2 tests in parallel, for decreasing and increasing ψ;
iii) Make a decision from the first test which fires;
iv) Reset all sums and extrema to 0, switch to the other test.

Variations on the CUSUM test - (1)

For detecting aircraft instability precursors, select:

a) An instability criterion ψ and a critical value ψ_c;

b) A left kernel matrix $U(.)$;

c) A reference $\theta_\star$ for estimating $\mathcal{J}_n(\theta_\star)$ and $\Sigma_n^{-1}(\theta_\star)$;

d) A min. change magnitude ν_m and a threshold ϱ.

Variations on the CUSUM test - (2)

Three solutions for b)-c):

1. $\theta_\star \triangleq \theta_0$ identified on reference data for the stable system;
$U(\theta_\star)$ computed,
$\mathcal{J}_n(\theta_0), \Sigma_n^{-1}(\theta_0)$ estimated recursively with the test data.

2. $\theta_\star \triangleq \theta_c$, critical parameter closer to instability, computed at each flight point using θ_0 and an aeroelastic model;
$U(\theta_\star)$ computed,
$\mathcal{J}_n(\theta_c), \Sigma_n^{-1}(\theta_c)$ estimated recursively with the test data.

3. $U(.) \triangleq \widehat{U}_n$ estimated on test data,
$\mathcal{J}_n(\theta_0), \Sigma_n^{-1}(\theta_0)$ estimated recursively with the test data.

Solution 2. - Details

i) Compute the critical eigenvalues λ_c at flight point t using identified modal signatures $(\theta_1, \ldots, \theta_t)$ and extrapolation of the characteristic polynomial associated with the quasi-steady aeroelastic model $M\ddot{q} + (D + VB)\dot{q} + (K + V^2C)q = 0$;

ii) Build the critical modal signature θ_c from λ_c and the mode-shapes φ_λ identified at flight point t;

iii) Compute the $Z_k(\theta_c)$'s and $S_n(\theta_c)$;
Compute $\hat{\mathcal{J}}_n(\theta_c)$ and $\hat{\Sigma}_n^{-1}(\theta_c)$ with the test data;

iv) Run the CUSUM test between flight points t and $t+1$;

v) Repeat these steps for flight point $t+1$:
modal identification of θ_{t+1} to update the prediction of θ_c,
computation of $\hat{\mathcal{J}}_n(\theta_c)$ and $\hat{\Sigma}_n^{-1}(\theta_c)$,
CUSUM test between $t+1$ and $t+2$.

Solution 3. - Details

i) Initialization
For an initial airspeed:
Estimate a reference θ_0 and compute the constant terms in $\hat{\mathcal{J}}_n$;

Select data sample size L, lag τ, block size K, ν_m, and ϱ;

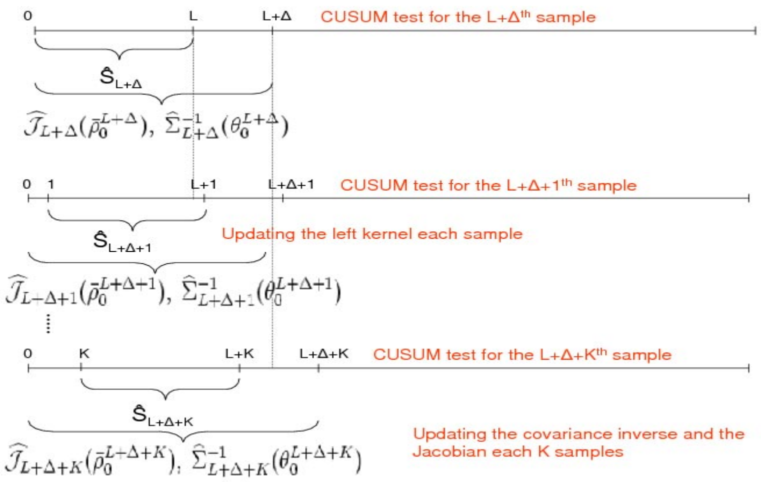
Compute $\hat{\Sigma}_{L+\tau}^{-1}$ and $\hat{\mathcal{J}}_{L+\tau}$ with the first $L+\tau$ samples;
Compute $\hat{U}_{L+\tau}$ with $(Y_1, ..., Y_L)$;
Compute the Z_k's and $S_{L+\tau}$.

ii) Recursive loop
Running the CUSUM test: for each $n \geq L+\tau$:

Compute recursively $\widehat{U}_n$ with $(Y_{n-\tau-L+1}, ..., Y_{n-\tau})$;
Use $\widehat{U}_n$ with $\widehat{\Sigma}_n^{-1}$ and $\widehat{\mathcal{J}}_n$ to compute S_n and g_n until $g_n \geq \varrho$.

Update recursively $\widehat{\Sigma}_n^{-1}$ and $\widehat{\mathcal{J}}_n$ every K samples.

Solution 3. - Details (Contd.)

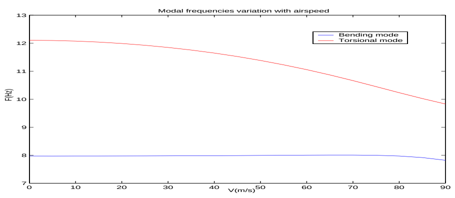


Example - Aeroelastic Hancock wing model

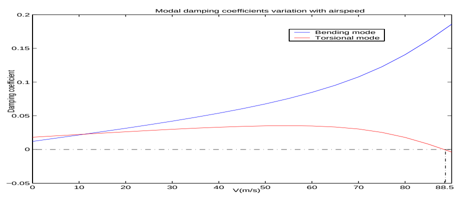
Rigid wing with constant chord; 2 d.o.f. in bending and torsion.

Matrix F, and eigenvalues λ: functions of airspeed V.
Flutter airspeed: $V_f = 88.5m/s$.

Stability indicator ψ: Damping coefficient



Frequencies



Damping coefficients

Bending & torsion modes

20700-size 2D-samples simulated (300 for each V=20:1:88m/s).

Example - Numerical results

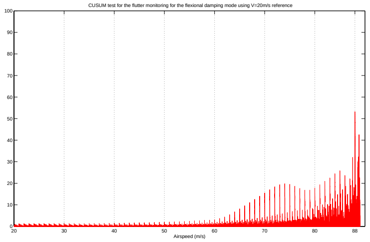
CUSUM test run with $\nu_m = 0.1$, $\varrho = 100$, and the damping as ψ.
Solution 1. with $\theta_\star = \theta_0$ at $V = 20m/s$, online recursive $\hat{\mathcal{J}}_n, \hat{\Sigma}_n$.
Solution 2. with $\theta_\star = \theta_c$ at $V = 85m/s$, online recursive $\hat{\mathcal{J}}_n, \hat{\Sigma}_n$.
Solution 3. with online recursive $\hat{U}_n, \hat{\mathcal{J}}_n, \hat{\Sigma}_n$.

Alarm onset times depend on threshold; $\hat{V}_f$ is more important.

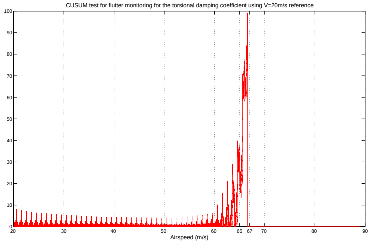
Solution 1. $\theta_\star$ far from instability, alarm at V=67m/s, $\hat{V}_f$=65m/s.
The test detects that torsional damping decreases under ψ_c.

Solution 2. $\theta_\star$ close to instability, alarm at V=88m/s, $\hat{V}_f$=85m/s.
The test detects that flutter is happening between two steady points, and confirms the flutter prediction.

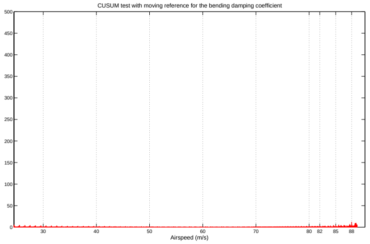
Solution 3. Alarm at V=88m/s, $\hat{V}_f$=78m/s much closer to flutter.
Good behavior for light damping decrease before alarm.
Detection (before flutter) of torsional damping drop.



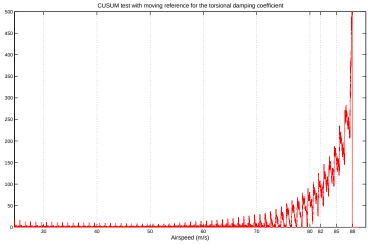
Solution 1: Bending mode



Solution 1: Torsion mode



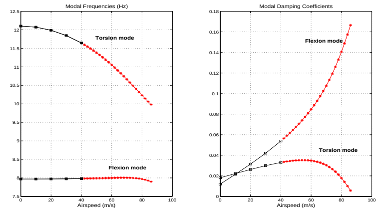
Solution 3: Bending mode



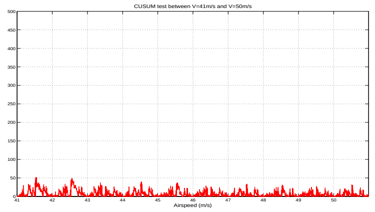
Solution 3: Torsion mode

No alarm

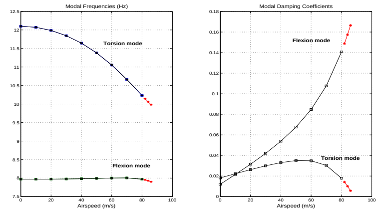
Alarm of Sol.3 closer to flutter



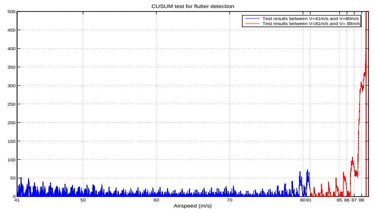
θ's estimated up to $V = 40m/s$, θ_c predicted



Sol. 2 between flight points, $V = 40m/s$ and $50m/s$



θ's estimated up to $V = 80m/s$ θ_c predicted



Sol. 2 between flight points $V = 40m/s$ and $80m/s$ and $V = 81m/s$ and $90m/s$

Drop of the torsion mode damping

Conclusion

Online detection for flutter monitoring

- Model-free subspace statistics, local approach, CUSUM
- Analytical model for flutter prediction
- Recursive computation of Jacobian and covariance matrices

Three variants of CUSUM

- Algo 1: detection of $\psi \leq \psi_0$
- Algo 2: flutter detection
- Algo 3: detection of abrupt drop in ψ

Relevance on a small simulated structure

Limitations: cost of online covariance computation

- Availability of flutter prediction model in real cases

Major issues: dimension of θ, large number of correlated criteria