Lessons learned from the theory and practice of

change detection

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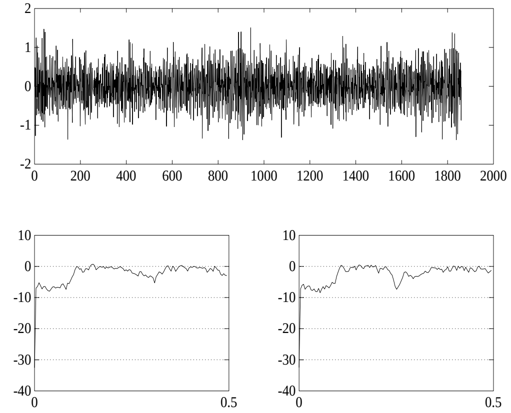
Simulated data - One change (Signal and spectral densities)



Introduction

- Detection of changes
  - Stochastic models (static, dynamic) $\longleftrightarrow$ uncertainties
  - Parameterized models (physical interpretation, diagnostics)
  - Damage $\longleftrightarrow$ deviation in the parameter vector
- Many changes of interest are small
- Early detection of (small) deviations is useful
- Key issue (1): which function of the data should be handled?
- Key issue (2): how to make the decision?

Content

Changes in the mean

Changes in the spectrum

Changes in the system dynamics and vibration-based SHM

Example: flutter monitoring

Changes in the mean

Key concepts - Independent data

Likelihood $p_\theta(y_i)$

Log-likelihood ratio $s_i \triangleq \ln \dfrac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}$

$$\mathrm{E}_{\theta_0}(s_i) < 0$$

$$\mathrm{E}_{\theta_1}(s_i) > 0$$

Likelihood ratio $\Lambda_N \triangleq \dfrac{p_{\theta_1}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} = \dfrac{\prod_i \; p_{\theta_1}(y_i)}{\prod_i \; p_{\theta_0}(y_i)}$

Log-likelihood ratio $S_N \triangleq \ln \; \Lambda_N = \sum_{i=1}^N s_i$

Hypothesis testing

Hypotheses	H_0	H_1	
Simple	θ_0	θ_1	Known parameter values
Composite	Θ_0	Θ_1	Unknown parameter values

Simple hypotheses : Likelihood ratio test

If $\Lambda_N \geq \lambda \; / \; S_N \geq h$: decide H_1, H_0 otherwise

Composite hypotheses : Generalized likelihood ratio (GLR)

$$\widehat{\Lambda}_N = \frac{\sup_{\theta_1 \in \Theta_1} \; p_{\theta_1}(\mathcal{Y}_1^N)}{\sup_{\theta_0 \in \Theta_0} \; p_{\theta_0}(\mathcal{Y}_1^N)} = \frac{p_{\widehat{\theta_1}}(\mathcal{Y}_1^N)}{p_{\widehat{\theta_0}}(\mathcal{Y}_1^N)}$$

Maximize the likelihoods w.r.t. unknown values of θ_0 and θ_1

On-line change detection, unknown onset time

Hypothesis H_0 $\quad \theta = \theta_0$ known $(1 \leq i \leq k)$

Hypothesis H_1 $\quad \exists \; t_0$ s.t. $\theta = \begin{cases} \theta_0 & (1 \leq i < t_0) \\ \theta_1 & (t_0 \leq i \leq k) \end{cases}$

Alarm time t_a : $\quad t_a = \min \{k \geq 1 : g_k \geq h\}$ decision function

Estimated onset time: $\widehat{t_0}$

Simple case: Known θ_1 - CUSUM algorithm

Ratio of likelihoods under H_0 and H_1:

$$\frac{\prod_{i=1}^{t_0-1} \; p_{\theta_0}(y_i) \; \cdot \; \prod_{i=t_0}^{k} \; p_{\theta_1}(y_i)}{\prod_{i=1}^{k} \; p_{\theta_0}(y_i)} = \frac{\prod_{i=t_0}^{k} \; p_{\theta_1}(y_i)}{\prod_{i=t_0}^{k} \; p_{\theta_0}(y_i)} = \Lambda_{t_0}^k$$

Maximize over the unknown onset time t_0:

$$\begin{aligned} (\widehat{t_0})_k &\triangleq \arg \max_{1 \leq j \leq k} \prod_{i=1}^{j-1} p_{\theta_0}(y_i) \; \cdot \; \prod_{i=j}^{k} p_{\theta_1}(y_i) \\ &= \arg \max_{1 \leq j \leq k} \Lambda_j^k \\ &= \arg \max_{1 \leq j \leq k} S_j^k, \qquad S_j^k = \ln \; \Lambda_j^k \end{aligned}$$

$$g_k \triangleq \max_{1 \leq j \leq k} S_j^k = \ln \Lambda_{\widehat{t_0}}^k$$

CUSUM algorithm (Contd.)

$$
\begin{aligned}
g_k &\triangleq \max_{1\leq j\leq k} S_j^k \\
&= S_1^k - \min_{1\leq j\leq k} S_1^j = S_1^k - m_k, \qquad m_k \triangleq \min_{1\leq j\leq k} S_1^j
\end{aligned}
$$

$$t_a = \min\{k \geq 1 : S_1^k \geq m_k + h\}$$ **Adaptative threshold**

$$
\begin{aligned}
g_k &= (g_{k-1} + s_k)^+ \\
g_k &= \left(S_{k-N_k+1}^k\right)^+, \qquad N_k \triangleq N_{k-1} \cdot I(g_{k-1}) + 1 \\
(\widehat{t_0})_k &= t_a - N_{t_a} + 1
\end{aligned}
$$

Sliding window with adaptive size

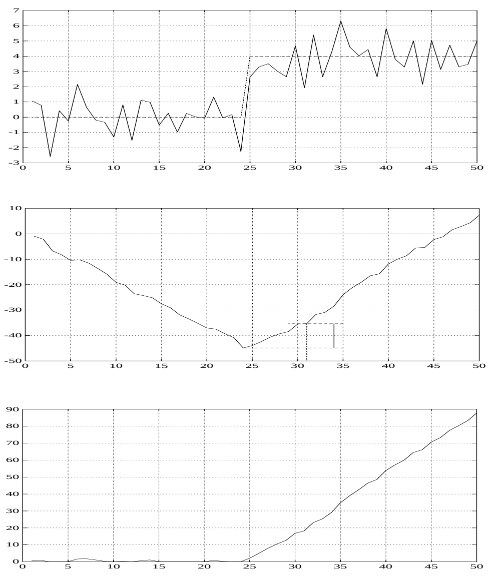
CUSUM algorithm (Contd.) - Gaussian example

$$\mathcal{N}(\mu,\sigma^2), \quad \theta \triangleq \mu, \quad p_\theta(y) \triangleq \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y_i-\mu)^2}{2\sigma^2}\right)$$

$$
\begin{aligned}
s_i &= \ln \frac{p_{\mu_1}(y_i)}{p_{\mu_0}(y_i)} \\
&= \frac{1}{2\,\sigma^2}\left((y_i-\mu_0)^2 - (y_i-\mu_1)^2\right) \\
&= \frac{\nu}{\sigma^2}\left(y_i - \mu_0 - \frac{\nu}{2}\right), \qquad \nu = \mu_1 - \mu_0
\end{aligned}
$$

S_1^k involves $\sum_{i=1}^{k} y_i$: **Integrator** (with adaptive threshold)

CUSUM algorithm - Gaussian example (Contd.)



y_k

S_1^k

g_k

Composite case: Unknown θ_1

Modified CUSUM algorithms

- **Minimum magnitude** of change
- **Weighted** CUSUM

GLR algorithm

- Double **maximization**

$$g_k = \max_{1\leq j\leq k} \sup_{\theta_1} S_j^k(\theta_1)$$

Gaussian case, additive faults: second maximization explicit.

Unknown θ_1 - Gaussian example (Contd.)

Minimum magnitude of change ν_m Lesson 2

Decreasing mean	**Increasing** mean
$T_1^k \triangleq \Sigma_{i=1}^k \left(y_i - \mu_0 + \frac{\nu_m}{2}\right)$	$U_1^k \triangleq \Sigma_{i=1}^k \left(y_i - \mu_0 - \frac{\nu_m}{2}\right)$
$M_k \triangleq \max_{1\leq j\leq k} T_1^j$	$m_k \triangleq \min_{1\leq j\leq k} U_1^j$
$t_a = \min\{k \geq 1 : M_k - T_1^k \geq h\}$	$t_a = \min\{k \geq 1 : U_1^k - m_k \geq h\}$

Changes in the spectrum

Key concepts - Dependent data

Conditional likelihood $p_\theta(y_i|\mathcal{Y}_1^{i-1})$

Log-likelihood ratio $\quad s_i \triangleq \ln \frac{p_{\theta_1}(y_i|\mathcal{Y}_1^{i-1})}{p_{\theta_0}(y_i|\mathcal{Y}_1^{i-1})}$

$$\mathbf{E}_{\theta_0}(s_i) < 0$$

$$\mathbf{E}_{\theta_1}(s_i) > 0$$

Likelihood ratio $\quad \Lambda_N \triangleq \frac{p_{\theta_1}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} = \frac{\Pi_i \; p_{\theta_1}(y_i|\mathcal{Y}_1^{i-1})}{\Pi_i \; p_{\theta_0}(y_i|\mathcal{Y}_1^{i-1})}$

Log-likelihood ratio $\quad S_N \triangleq \ln \Lambda_N = \Sigma_{i=1}^N \; s_i$

Key concepts - Dependent data (Contd.)

Which residuals? Lesson 3

- Likelihood ratio computationally complex
- Efficient score: likelihood sensitivity w.r.t. parameter vector
- Other estimating fonctions

Innovation not sufficient for monitoring the dynamics.

Key concepts - Dependent data (Contd.)

θ_0 : reference parameter, known (or identified)

Y_k: N-size sample of new measurements

Build a residual ζ **significantly non zero** when damage

Residual $\leftrightarrow$ **Estimating function** $\zeta_N(\theta, \mathcal{Y}_1^N)$

Characterized by: $\mathbf{E}_{\theta_0} \, \zeta_N(\theta, \mathcal{Y}_1^N) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$

Mean sensitivity $\mathcal{J}(\theta_0)$ and covariance $\Sigma(\theta_0)$ of $\zeta_N(\theta_0)$

The residual is asymptotically Gaussian

$$\zeta_N(\theta_0) \;\rightarrow\; \begin{cases} \mathcal{N}(\quad 0, \; \Sigma(\theta_0)) & \text{if } \mathrm{P}_{\theta_0} \\ \mathcal{N}(\; \mathcal{J}(\theta_0)\,\delta\theta, \; \Sigma(\theta_0)) & \text{if } \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \text{ small deviations} \end{cases}$$

(On-board) χ^2-test in the residual

$$\zeta_N^T \, \Sigma^{-1} \, \mathcal{J} \, (\mathcal{J}^T \, \Sigma^{-1} \, \mathcal{J})^{-1} \, \mathcal{J}^T \, \Sigma^{-1} \, \zeta_N \quad \geq h$$

Noises and uncertainty on θ_0 taken into account.

Changes in the dynamics and vibration-based SHM

Structural monitoring : Eigenstructure monitoring

$$\begin{cases} X_{k+1} = F \; X_k \; + \; V_k \\ Y_k = H \; X_k \end{cases} \qquad \begin{array}{l} F \, \Phi_\lambda = \lambda \, \Phi_\lambda \\ \varphi_\lambda \triangleq H \, \Phi_\lambda \end{array}$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \mathrm{vec}\,\Phi \end{pmatrix}$ $\begin{array}{l} \text{modes} \\ \text{mode shapes} \end{array}$

Detecting structural changes

- Reference data $\rightarrow$ covariances $\rightarrow$ Hankel matrix $\mathcal{H}_0$

  Left null space S s.t. $S^T \; \mathcal{H}_0 = 0$ $\qquad (S^T \; \mathcal{O}(\theta_0) = 0)$

- Fresh data $\rightarrow$ covariances $\rightarrow$ Hankel matrix $\mathcal{H}_1$

  Check if $\zeta \triangleq S^T \; \mathcal{H}_1 \neq 0$

  ζ asympt. Gaussian, test: χ^2 in ζ

Example: flutter monitoring

Monitoring a damping coefficient

- Test $\rho \geq \rho_c$ against $\rho < \rho_c$
- Write the subspace-based residual ζ as a cumulative sum
- Introduce a minimum change magnitude (actual change magnitude unknown)
- Run two CUSUM tests in parallel (actual change direction unknown)

Flutter monitoring (Contd)

Application to real datasets

Ariane booster launcher during a launch scenario

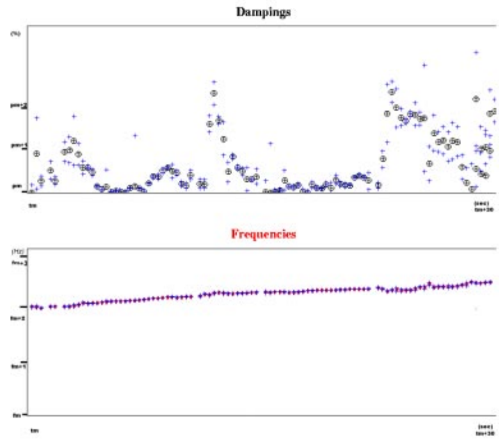
on the ground (not during a real flight test)

Running the test with 2 critical values

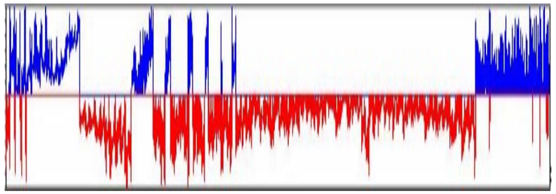
With Laurent Mével, Maurice Goursat, Albert Benveniste

Free COSMAD Toolbox to used with Scilab

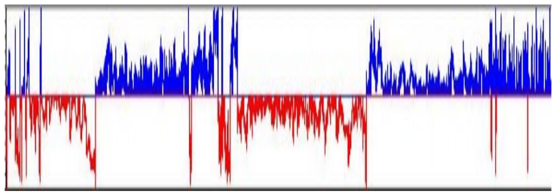
http://www.irisa.fr/sisthem/cosmad/



Automatic identification. Each symbol: processing 5 sec. data.



On-line test for $\rho_c = \rho_0 = \rho_c^{(1)}$.



On-line test for $\rho_c = \rho_0 = \rho_c^{(2)} < \rho_c^{(1)}$.

Bottom: $-g_n^-$ reflects $\rho < \rho_c$. Top: g_n^+ reflects $\rho > \rho_c$.

Conclusion

Advanced statistical signal processing mandatory for SHM

A statistical framework enlightens the meaning and increases the power of a number of familiar operations

integration, averaging, sensitivity, adaptive thresholds & windows

Change detection useful for (vibration-based) SHM

Current investigations

- flutter monitoring (with Dassault, Airbus),
- handling the temperature effect on civil structures (with LCPC)