On-line monitoring

of slow to fast evolving aeronautic structures

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COSMAD : Free Scilab Toolbox for modal analysis and SHM

http://www.irisa.fr/sisthem/cosmad/

Problems : In-operation on-line structural identification and monitoring

- In-operation modal analysis:

  The excitation is typically not controlled (natural), not measured, and nonstationary (e.g., turbulent)

- Objectives :
  - On-line in-flight exploitation of test data
  - On-line flight flutter monitoring
- Monitoring as an identification problem
- Monitoring as a detection problem

Aircraft flutter monitoring

- Aero-elastic flutter: critical instability phenomenon
- Flight flutter testing procedure
- Objective: on-line in-flight exploitation of test data
- On-line flight flutter monitoring problem: monitoring some specific damping coefficient
- Decide whether $\rho < \rho_c$, critical value

Contents

- Modelling
- Covariance-driven subspace identification
- Automated modal analysis for monitoring
- Subspace-based damage detection test
- On-line test for flutter monitoring
- Application to real datasets

Modelling

FE model:
$$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \;, \quad \psi_\mu = L\Psi_\mu$$

State space:
$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \; \Phi_\lambda \;, \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \;, \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}$$

Output-only covariance-driven subspace identification

$$\begin{cases} X_{k+1} = F \; X_k + V_k \\ Y_k = H \; X_k \end{cases}$$

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k \; Y_{k-i}^T\right)}_{\text{ok if stationary !}} \;, \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \;, \quad \begin{array}{l} \mathcal{C} \triangleq \left(G \quad FG \quad F^2G \quad \dots \right) \\ G \triangleq \mathbf{E}\left(X_k \; Y_k^T\right) \end{array}$$

$$R_i = H \; F^i \; G \Longrightarrow \mathcal{H} = \mathcal{O} \; \mathcal{C} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

With input/output data: Handle projections $Y/\mathcal{U}, Y/\mathcal{U}^\perp$

Implementation

$$\underbrace{\hat{R}_i \triangleq 1/N \sum_{k=1}^{N} Y_k \; Y_{k-i}^T}_{\text{ok when nonstationary !}} \;, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\hat{\mathcal{H}} \approx \hat{\mathcal{O}} \; \hat{\mathcal{C}}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

Two steps:

- **Stabilization diagrams**: increasing SVD trunc. order
- **MAC plots**: mode-shapes in a selected freq. band

Implementation - Details

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = \left(U_1 \;\; U_2 \right) \Delta \; W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T \;; \quad \hat{\mathcal{O}} = U_1 \; \Delta_1^{1/2}$$

$$\mathcal{O}_p^\uparrow(H, F) = \mathcal{O}_p(H, F) \; F$$

$$\det(F - \lambda \; I) = 0 \;, \quad F \; \Phi_\lambda = \lambda \; \Phi_\lambda, \quad \varphi_\lambda = H \; \Phi_\lambda$$

Automated modal analysis for monitoring

Automatic post-processing of stabilization diagrams, GUI within COSMAD toolbox.

- Extracted frequencies and damping coefficients plotted over time
- Zoom on a selected frequency
- Interactive graphical output
- Time evolution of frequencies and damping coeff. and associated confidence intervals

Structural monitoring : Eigenstructure monitoring

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases} \qquad \begin{array}{l} F\ \Phi_\lambda = \lambda\ \Phi_\lambda \\ \varphi_\lambda \triangleq H\ \Phi_\lambda \end{array}$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix} \begin{array}{l} \text{modes} \\ \text{mode shapes} \end{array}$

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

Damage detection

θ_0 : reference parameter, known (or identified)

Y_k: N-size sample of new measurements

Build a residual ζ significantly non zero when damage

Local approach (small deviations)

Test $\mathcal{H}_0 : \ \theta = \theta_0$ against $\mathcal{H}_1 : \ \theta = \theta_0 + \delta\theta/\sqrt{N}$

Subspace model/data correlation (1)

Fresh data $\longrightarrow \hat{R}_i \longrightarrow \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$

Nominal model : $\mathcal{O}(\theta_0) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \Phi\Delta^2 \\ \vdots \end{pmatrix}$ Observability in modal basis

$! \ \mathcal{H} = \mathcal{O}\ \mathcal{C} \ ! \qquad \ker \hat{\mathcal{H}}^T \stackrel{?}{=} \ker \mathcal{O}^T(\theta_0)$

Subspace model/data correlation (2)

$$\exists U, \quad U^T\, U = I_s, \qquad U^T\, \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } U(\theta_0)$$

Check if : $U^T(\theta_0)\, \hat{\mathcal{H}} \;\approx\; 0$

Residual for structural damage monitoring

$$\zeta_N(\theta_0) \triangleq \sqrt{N}\ \operatorname{vec}(\ U^T(\theta_0)\ \hat{\mathcal{H}}\ )$$

? How to assess the significance of : $U^T(\theta_0)\ \hat{\mathcal{H}} \approx 0$?

Subspace model/data correlation (3)

The residual is asymptotically Gaussian

$$\zeta_N(\theta_0) \ \to \ \begin{cases} \mathcal{N}(\quad 0,\ \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\ \mathcal{J}(\theta_0)\,\delta\theta,\ \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0+\delta\theta/\sqrt{N}} \end{cases}$$

$\mathcal{J}(\theta_0)$: mean sensitivity (Jacobian) of residual ζ w.r.t. modal changes

(On-board) χ^2-test in the residual

$$\zeta_N^T\ \Sigma^{-1}\,\mathcal{J}\,(\mathcal{J}^T\,\Sigma^{-1}\,\mathcal{J})^{-1}\,\mathcal{J}^T\,\Sigma^{-1}\ \zeta_N \ \ \geq h$$

(On-board) modal χ^2-test

$$\zeta_N^T\ \Sigma^{-1}\,\mathcal{J}_i\,(\mathcal{J}_i^T\,\Sigma^{-1}\,\mathcal{J}_i)^{-1}\,\mathcal{J}_i^T\,\Sigma^{-1}\ \zeta_N \ \ \geq h$$

Monitoring a damping coefficient

- Write the subspace-based residual ζ
  as a cumulative sum
- Test $\rho \geq \rho_c$ against $\rho < \rho_c$
  Non local ! Use a different asymptotics for ζ
- Introduce a minimum change magnitude
  (actual change magnitude unknown)
- Run two CUSUM tests in parallel
  (actual change direction unknown)

Monitoring a damping coefficient - Details

$$\overline{\zeta}_n(\rho_0) \triangleq \mathcal{J}(\rho_0)^T\ \Sigma(\theta_0)^{-1}\ \zeta_n(\theta_0) \ \ = \ \sum_{k=q}^{n-p} Z_k(\rho_0)\ /\sqrt{n}$$

$$Z_k(\rho_0) \triangleq \mathcal{J}(\rho_0)^T\ \Sigma(\theta_0)^{-1}\ \operatorname{vec}\left(U(\theta_0)^T\ \mathcal{Y}_{k,p+1}^{+}\ \mathcal{Y}_{k,q}^{-\,T}\right)$$

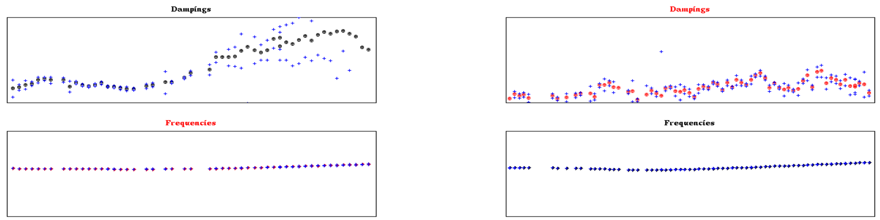
$$\nu \triangleq \mathrm{E}(Z_k), \quad \widetilde{\mathrm{H}}_0 :\ \nu \geq +\,\nu_m/2 \quad \text{and} \quad \widetilde{\mathrm{H}}_1 :\ \nu < -\,\nu_m/2$$

Decreasing mean	Increasing mean
$S_n^{(-)} \triangleq \overline{\Sigma}^{-1/2}\ \sum_{k=q}^{n-p}\ (Z_k + \nu_m)$	$S_n^{(+)} \triangleq \overline{\Sigma}^{-1/2}\ \sum_{k=q}^{n-p}\ (Z_k - \nu_m)$
$T_n^{(-)} \triangleq \max_{q\leq k\leq n-p} S_k^{(-)}$	$T_n^{(+)} \triangleq \min_{q\leq k\leq n-p} S_k^{(+)}$
$g_n^- \triangleq T_n^{(-)} - S_n^{(-)}$	$g_n^+ \triangleq S_n^{(+)} - \ T_n^{(+)}$
$g_n^- \geq \gamma^-$	$g_n^+ \geq \gamma^+$

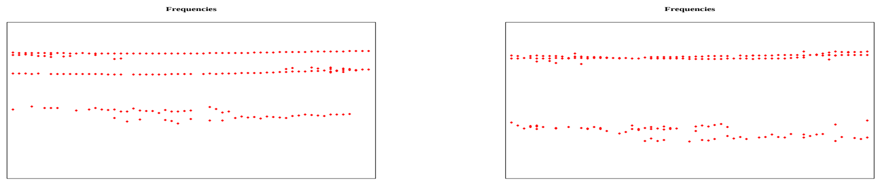
Application to real datasets

- **Automated modal analysis**
  AIRBUS France in-flight aircraft dataset
  EADS Launch Vehicles ground test dataset
  – Window size effect
  – Sensors selection influence
  – Data filtering effect

- **Detection test** for flutter monitoring
  Ariane booster launcher during a launch scenario on the ground (not during a real flight test)
  – Running the test with 2 critical values

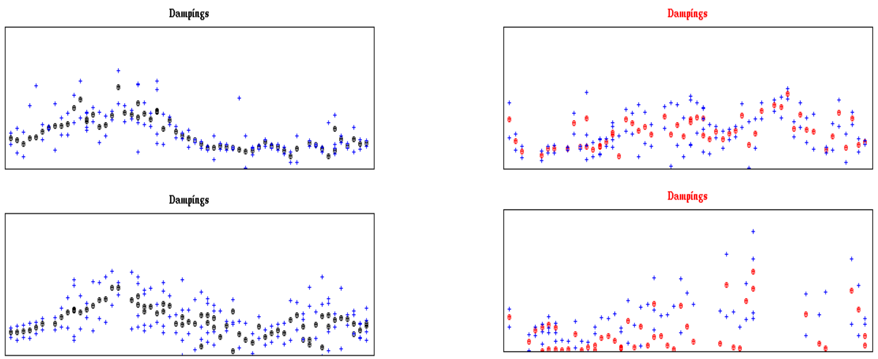


Window size: 4000 (left) and 2000 (right).

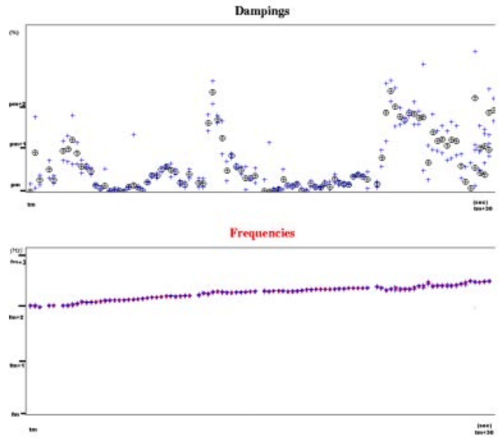


Time evolution of some extracted modes

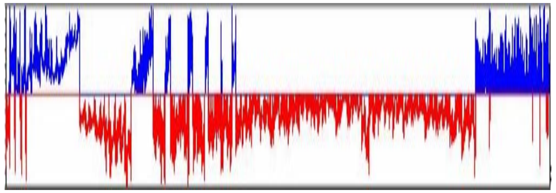
First sensor selection (left), 2nd sensor selection (right).



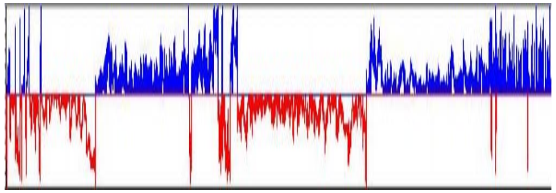
Damping evolution over time for Exp.1 - First (left) and second (right) mode. With (Top) and without (Bottom) filtering.



Automatic identification. Each symbol: processing 5 sec. data.



Test for $\rho_c = \rho_0 = \rho_c^{(1)}$.



Test for $\rho_c = \rho_0 = \rho_c^{(2)} < \rho_c^{(1)}$.

Bottom: $-g_n^-$ reflects $\rho < \rho_c$. **Top**: g_n^+ reflects $\rho > \rho_c$.

Drawbacks, advantages and conclusion

- Automated identification
  - Higher computing resources
  - Bias / variance trade-off
  - Good results, especially for frequencies
  - Mature approach
- Detection
  - Calibration (estimation of Σ)
  - Actual case $\rho_c < \rho_0$ not solved yet
  - Sample point-wise processing
  - Robustness to non-stationarity (unsteady aerodynamics)
- Complementary approaches
- Further investigations on flutter detection within FliTE2