Online detection of aircraft instability precursors

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Content

Introduction - (1)

- Flutter: critical aircraft instability phenomenon
  unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- Flight flutter testing, very expensive and time consuming :
  Design the flutter free flight envelope
- Flutter clearance techniques:
  In-flight identification: output-only, or using input excitations
  Data processing: time-frequency, wavelet, envelope function

  Flutter prediction based on model-based approaches:
  flutterometer (μ-robustness), physical model updating
- Some challenges:
  Real time on-board monitoring,
  robustness to noise and uncertainties
- Our approach:
  Statistical detection for monitoring instability indicators

Introduction - (2)

- Aim of in-flight online flutter monitoring:
  Early detection of a deviation in the aircraft modal parameters before it develops into flutter.
- Change-point detection: natural approach
- For a scalar instability criterion ψ and a critical value ψ_c, online hypotheses testing:

$$\mathrm{H}_0 \ : \ \psi > \psi_c \text{ and } \mathrm{H}_1 \ : \ \psi \leq \psi_c$$

- CUSUM test as an approximation to the optimal test
- Performances of the CUSUM test for different statements of H_0

Subspace-based residual for modal monitoring

$$\begin{cases} X_{k+1} = F\, X_k + V_k \\ Y_k = H\, X_k \end{cases} \qquad \begin{array}{l} F\,\phi_\lambda = \lambda\,\phi_\lambda \\ \varphi_\lambda \triangleq H\,\phi_\lambda \end{array}$$

$$R_i \triangleq \mathbf{E}\left(Y_k\, Y_{k-i}^T\right) \;,\quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\, F^i\, G \Longrightarrow \mathcal{H} = \mathcal{O}\,\mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \;,\quad \begin{array}{l} \mathcal{C} \triangleq (\, G \quad FG \quad F^2G \quad \dots\,) \\ G \triangleq \mathbf{E}\left(X_k\, Y_k^T\right) \end{array}$$

Output-only covariance-driven subspace identification

SVD of $\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$ modes / mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

Given:

- a reference parameter $\theta_\star$, by SVD of $\widehat{\mathcal{H}}^\star_{p+1,q}$ (reference data)

$$U(\theta_\star)^T\, \widehat{\mathcal{H}}^\star_{p+1,q} = 0 \qquad \text{parameter estimating function}$$

$$U(\theta_\star)^T\, \mathcal{O}_{p+1}(\theta_\star) = 0 \;,\quad U(\theta_\star)^T U(\theta_\star) = I$$

- a n-size sample of new data; $\widehat{\mathcal{H}}_{p+1,q}$

For testing $\theta = \theta_\star$, statistics (residual) :

$$\zeta_n(\theta_\star) \triangleq \sqrt{n}\ \text{vec}\left(U(\theta_\star)^T\ \widehat{\mathcal{H}}_{p+1,q}\right)$$

Local approach to testing

$$\widetilde{\mathrm{H}}_0 :\ \theta = \theta_\star \quad \text{and} \quad \widetilde{\mathrm{H}}_1 :\ \theta = \theta_\star + \Upsilon/\sqrt{n}$$

Mean sensitivity and covariance matrices:

$$\mathcal{J}_n(\theta_\star,\theta) \triangleq 1/\sqrt{n}\ \partial/\partial\tilde\theta\ \mathbf{E}_\theta\ \zeta_n(\tilde\theta)\big|_{\tilde\theta=\theta_\star} \;,\quad \Sigma_n(\theta_\star,\theta) \triangleq \mathbf{E}_\theta\left(\zeta_n(\theta_\star)\ \zeta_n(\theta_\star)^T\right)$$

If $\Sigma_n(\theta_\star,\theta)$ is positive definite, and for all Υ, under both hypoth:

$$\Sigma_n(\theta_\star,\theta)^{-1/2}\left(\zeta_n(\theta_\star) - \mathcal{J}_n(\theta_\star,\theta)\ \Upsilon\right) \xrightarrow[n\to\infty]{} \mathcal{N}(0, I)$$

Normalized residual:

$$\bar\zeta_n(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star,\theta)\ \zeta_n(\theta_\star)$$

$$\mathcal{K}_n(\theta_\star,\theta) \triangleq \bar\Sigma_n^{-1/2}\mathcal{J}_n^T\Sigma_n^{-1} \;,\quad \bar\Sigma_n(\theta_\star,\theta) \triangleq \mathcal{J}_n^T\ \Sigma_n^{-1}\mathcal{J}_n$$

$$\left(\bar\zeta_n(\theta_\star) - \bar\Sigma_n(\theta_\star,\theta)^{1/2}\ \Upsilon\right) \xrightarrow[n\to\infty]{} \mathcal{N}(0, I)$$

Data-driven computation for online detection

$$\bar\zeta_n(\theta_\star) = \sum_{k=q}^{n-p} Z_k(\theta_\star)/\sqrt{n}$$

$$Z_k(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star,\theta)\ \text{vec}\left(U(\theta_\star)^T\ \mathcal{Y}^+_{k,p+1}\ \mathcal{Y}^{-T}_{k,q}\right)$$

$\sum_{k=q}^{n-p} Z_k(\theta_\star)/\sqrt{n}$ asymptotically Gaussian distributed, with mean zero under $\widetilde{\mathrm{H}}_0$ and $\bar\Sigma(\theta_\star,\theta)^{1/2}\ \Upsilon$ under $\widetilde{\mathrm{H}}_1$.

Another approximation

For n large enough, and $k = 1, \dots, n$,
$Z_k(\theta_\star) \approx$ Gaussian i.i.d., mean 0 before change and $\neq 0$ after.

Monitoring any function $\psi(\theta)$

Replace $\mathcal{J}_n(\theta_\star,\theta)$ with $\mathcal{J}_n(\theta_\star,\theta)\ \mathcal{J}^\star_{\theta\psi}$, where $\mathcal{J}^\star_{\theta\psi} = \partial\theta/\partial\psi|_{\theta=\theta_\star}$.

CUSUM test for monitoring a scalar index

The crossing of a critical ψ_c by ψ is reflected into a change with the same sign in the mean ν of the i.i.d. Gaussian $Z_k(\theta_\star)$.

The CUSUM test may be used for testing between:

$$\mathrm{H}_0 : \nu > 0 \quad \text{and} \quad \mathrm{H}_1 : \nu \leq 0$$

Procedure for unknown sign and magnitude of change in ψ

i) Set a min. change magnitude $\nu_m > 0$, and test between:

$$\mathrm{H}_0 : \nu > \nu_m/2 \text{ and } \mathrm{H}_1 : \nu \leq -\nu_m/2$$

$$S_n(\theta_\star) \triangleq \sum_{k=q}^{n-p} (Z_k(\theta_\star) + \nu_m), \; T_n(\theta_\star) \triangleq \max_{k=q,\ldots,n-p} S_k(\theta_\star)$$

$$g_n(\theta_\star) \triangleq T_n(\theta_\star) - S_n(\theta_\star) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\lessgtr}} \varrho \quad \text{threshold}$$

ii) Run 2 tests in parallel, for decreasing and increasing ψ;
iii) Make a decision from the first test which fires;
iv) Reset all sums and extrema to 0, switch to the other test.

Using and tuning the CUSUM test

For detecting aircraft instability precursors, select:

a) An instability criterion ψ and a critical value ψ_c;
b) A reference state for the system, for identifying (or computing) $\theta_\star$ and computing $U(\theta_\star)$;
c) Estimates of $\mathcal{J}_n(\theta_\star, \theta)$ and $\Sigma_n(\theta_\star, \theta)$;
d) A min. change magnitude ν_m and a threshold ϱ.

Two solutions for b)-c):

1. $\theta_\star \triangleq \theta_0$ identified on reference data for the stable system; $\mathcal{J}_n, \Sigma_n$ estimated once for all with those reference data.

2. $\theta_\star \triangleq \theta_c$, critical parameter closer to instability, computed using θ_0 and an aeroelastic model; $\mathcal{J}_n, \Sigma_n$ estimated recursively with the test data.

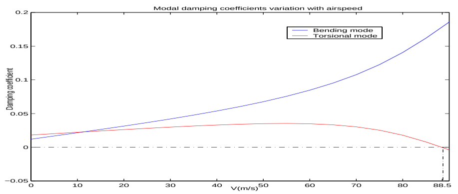
Example - Aeroelastic Hancock wing model

Rigid wing with constant chord; 2 d.o.f. in bending and torsion.

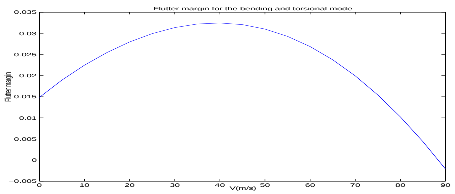
Matrix F, and eigenvalues λ: functions of airspeed V.
Flutter airspeed: $V_f = 88.5m/s$.

Flutter margin: stability indicator (from Routh's criterion)

$$\psi_{jk} = \frac{|1 - \lambda_j \; \lambda_k|^2 \; |1 - \lambda_j \; \bar{\lambda}_k|^2 \; (1 - |\lambda_j|^2) \; (1 - |\lambda_k|^2)}{1 - |\lambda_j|^2 \; |\lambda_k|^2}$$



Damping coefficient



Flutter margin

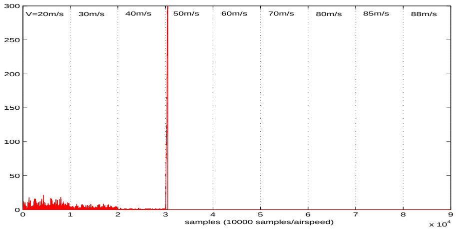
10000-size 2D-samples for $V = 20, 30, 40, 50, 60, 70, 80, 85, 88m/s$.

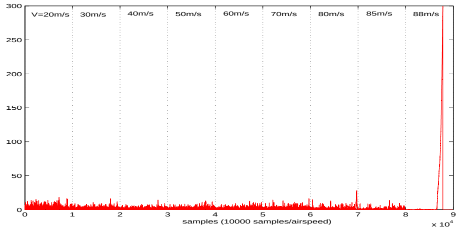
Example - Numerical results

CUSUM test run with $\nu_m = 0.1$, and the flutter margin as ψ.

Solution 1. with $\theta_\star = \theta_0$ at $V = 40m/s$ and fixed $\mathcal{J}, \Sigma$.

Solution 2. with $\theta_\star = \theta_c$ at $V = 85m/s$ and online $\widehat{\mathcal{J}}_n, \widehat{\Sigma}_n$.





Solution 1. ($\theta_\star = \theta_0$ too far from the instability):
alarm at $V = 50m/s$ since $\psi(\theta_0) = 0.0332 > \psi(\theta)|_{V=50}$

Solution 2. ($\theta_\star = \theta_c$ close to instability):
alarm at $V = 88m/s$ since $\psi(\theta_c) = \psi_c = 0.0037 \approx \psi(\theta)|_{V=89} = 0$

Conclusion

online detection of instability precursors

- Model-free subspace statistics, local approach, CUSUM
- Analytical model for flutter prediction
- Recursive computation of covariance matrix

Relevance on a small simulated structure

Limitations: cost of online covariance computation

- Availability of flutter prediction model in real cases

Major issues: dimension of θ, large number of correlated criteria