A Polyreference Least Square Complex Frequency domain based statistical test for damage detection

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Content

Introduction

- Damage detection
- Local approach to change detection $\longleftrightarrow$ parameter estimating function
- Time domain: subspace-based χ^2-tests, input-output or output-only
- Limitation: number of outputs
- Frequency domain: new input-output identification algorithm (Polyreference LSCF)
- Wanted: associated damage detection test

  Nominal input/output transfer functions available

Modal model and parameters

$$\mathrm{Y}(s) = \mathrm{H}(s)\mathrm{F}(s), \quad \mathrm{H}(s) = \left(\mathrm{M}s^2 + \mathrm{C}s + K\right)^{-1}, \quad \mathrm{M}, \mathrm{C}, \mathrm{K} \in \mathbb{R}^{N_m \times N_m}$$

Full modal model:
λ_m : m-th mode, $\Phi_m \in \mathbb{C}^{N_m}$ associated modeshape

$$\mathrm{H}(s) = \sum_{m=1}^{N_m} \left(Q_m \frac{\Phi_m \ \Phi_m^T}{s - \lambda_m} + Q_m^* \frac{\Phi_m^* \ \Phi_m^{*T}}{s - \lambda_m^*} \right)$$

Limited modal model: N_i inputs, N_o outputs, $N_i \ll N_o$

$$\mathrm{H}(s) = \sum_{m=1}^{N_m} \left(\frac{\Phi_m \ L_m^T}{s - \lambda_m} + \frac{\Phi_m^* \ L_m^{*T}}{s - \lambda_m^*} \right)$$

$L_m \in \mathbb{C}^{N_i}$: modal participation factors

Common denominator transfer function model

Polynomial basis function $(\Omega_l)_{1 \leq l \leq N_f}$, $\Omega_l = e^{i\omega_l T_s}$, T_s sampling

Common denominator transfer function model

$$\mathrm{H}(\Omega_l) = \mathrm{B}(\Omega_l)\ \mathrm{A}^{-1}(\Omega_l)\ , \quad \mathrm{B}, \mathrm{A} \text{ polynomials}$$

FRF between all the inputs and any output o

$$\mathrm{H}_o(\Omega_l) = \mathrm{B}_o(\Omega_l)\ \mathrm{A}^{-1}(\Omega_l)$$

Modal analysis algorithm (Guillaume, 2006)

Measured FRFs $\left(\widehat{\mathrm{H}}_o(\omega_l)\right)_{o,k}$

Minimize the LS cost function

$$C = \sum_o \sum_l \mathrm{trace}\left(\mathrm{E}_o^H(\omega_l)\ \mathrm{E}_o(\omega_l)\right), \quad \mathrm{E}_o(\omega_l) = \widehat{\mathrm{H}}_o(\omega_l)\ \mathrm{A}(\Omega_l) - \mathrm{B}_o(\Omega_l)$$

Scalar frequency domain test for change detection

Local approach to testing $G = G_0$ for the input/output transfer function (Benveniste-Delyon, 2000)

$$y_n = G(z)\ u_n + v_n\ , \quad G - G_0 = \frac{1}{\sqrt{K}}\ \widetilde{G}$$

DFT on K blocks with size N: $(U_k^N(\omega))_{k=1\ldots K}, (Y_k^N(\omega))_{k=1\ldots K}$
$K, N \to \infty$, $\frac{\sqrt{K}}{N} \to 0$

$$\zeta_K^N(G_0, \omega) \triangleq \frac{1}{\sqrt{K}} \sum_{k=1}^K U_k^N(-\omega) \left(Y_k^N(\omega) - G_0(\Omega)\ U_k^N(\omega) \right)$$

$$\sim \mathcal{N}\left(S^{uu}(\omega)\ \widetilde{G}(\Omega),\ S^{uu}(\omega)\ S^{vv}(\omega) \right)$$

Test $\widetilde{G}(\Omega) = 0\ /\ \widetilde{G}(\Omega) \neq 0$: $\chi_K^N(G_0, \omega) = \dfrac{|\zeta_K^N(G_0, \omega)|^2}{\widehat{S}_0^{uu}(\omega)\ \widehat{S}_0^{vv}(\omega)}$

Multidimensional frequency domain local test

Use numerator and denominator of common-denominator TF

$$\mathrm{B}(\omega) = \mathrm{H}(\omega)\ \mathrm{A}(\omega) + \mathrm{V}(\omega)\ , \quad H - H_0 = \frac{1}{\sqrt{K}}\ \tilde{H}$$

Reference FRFs $\to$ on K N-size blocks: $(A_{0,k}^N(\Omega), B_{0,k}^N(\Omega))_{k=1,\ldots,K}$

$$\mathrm{B}_{k,0}^N(\Omega) = \mathrm{H}_0(\omega)\ \mathrm{A}_{k,0}^N(\Omega) + \mathrm{V}_{k,0}^N(\omega)$$

New FRFs $(\mathrm{H}(\omega_\ell))_{\ell=1,\ldots,N_f}$. For each $\omega = \omega_\ell$:

$$\zeta_K^N(\mathrm{B}_{k,0}^N, \mathrm{A}_{k,0}^N, \omega) \triangleq \frac{1}{\sqrt{K}} \sum_{k=1}^K \left(\mathrm{B}_{k,0}^N(\Omega) - \mathrm{H}(\omega)\ \mathrm{A}_{k,0}^N(\Omega)\right) \left(\mathrm{A}_{k,0}^N(\Omega)\right)^H$$

$$\sim \mathcal{N}\left(-\tilde{\mathrm{H}}(\omega)\ S_0^{aa}(\omega),\ S_0^{aa}(\omega)\ S_0^{vv}(\omega) \right)$$

Test $\widetilde{H}(\omega) = 0 / \widetilde{H}(\omega) \neq 0$: $\chi_K^N(\mathrm{B}_{k,0}^N, \mathrm{A}_{k,0}^N, \omega) = \dfrac{\zeta_K^N(\mathrm{B}, \mathrm{A}, \omega)\ (\zeta_K^N(\mathrm{B}, \mathrm{A}, \omega)}{\widehat{S}_0^{aa}(\omega)\ \widehat{S}_0^{vv}(\omega)}$

Model validation

- One data set
- Does it match the reference modal model ?

  Does it match slight modifications of the modal model ?
- $\longrightarrow$ Optimizing the χ^2-test criterion
- Implementation: Rule of thumb $K \sim \sqrt{N}$

Example - Aircraft in-flight test data

(Cauberghe PhD, 2004)

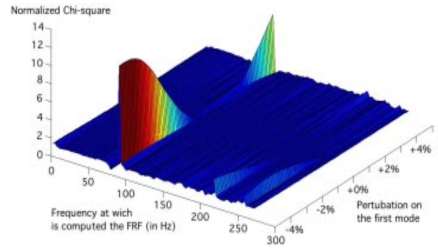
- $N_i = 1,\ N_o = 7$ - Artificial excitation
- Reference : Modal analysis on temporal data set, $n = 24000$

First mode	:	98.7 Hz
Second mode	:	201.3 Hz
Third mode	:	275.7 Hz

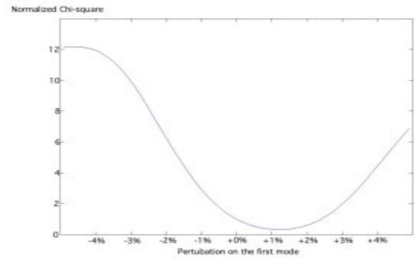
- $K = 28$ blocks with size $N = 784 \longrightarrow \mathrm{B}_{k,0}^{N}, \mathrm{A}_{k,0}^{N}$
- 1st mode changed from 95% to 105%

FRFs re-built under every change condition

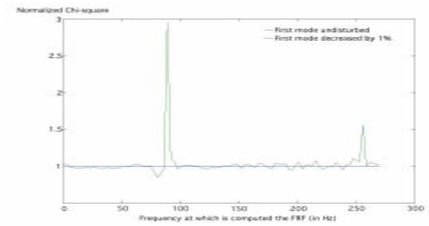
Example - Numerical results



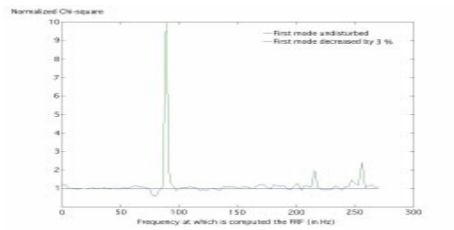
χ^2-test, entire frequency band
Varying perturbation on mode 1



χ^2-test, at the 1st frequency
Section along perturbation axis



Entire band, -1 % perturbation



Entire band, -3 % perturbation

Conclusion

Frequency domain test for change detection

Polyreference LSCF

Local approach to change detection

Multidimensional test

Relevance for model validation on a real aircraft

Ongoing and future issues:

Output-only detection algorithm (OMAX)

Damage localization

Large number of outputs