Merging sensor data from multiple temperature scenarios for vibration-based monitoring of civil structures

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Introduction

- Usefulness of global vibration-based SHM methods
- Limitations due to temperature effects on the dynamics of civil engineering structures
- A statistical subspace-based damage detection algorithm: null space of a matrix built on reference modes/modeshapes
- Non parametric version: null space of a matrix built on reference data set
- Proposed solution to temperature handling: no temperature measurement, empirical merging of non parametric null spaces

Content

Parametric subspace-based damage detection

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases} \qquad \begin{array}{l} F\,\varphi_\lambda = \lambda\,\varphi_\lambda \\ \phi_\lambda \triangleq H\,\varphi_\lambda \end{array}$$

$$R_i \triangleq \mathbf{E}\left(Y_k\ Y_{k-i}^T\right)\ ,\quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\ F^i\ G \Longrightarrow \mathcal{H} = \mathcal{O}\ \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}\ ,\quad \begin{array}{l} \mathcal{C} \triangleq (\ G \quad FG \quad F^2G \quad \dots\ ) \\ G \triangleq \mathbf{E}\left(X_k\ Y_k^T\right) \end{array}$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \phi_\lambda)$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \mathrm{vec}\,\Phi \end{pmatrix}$ modes / mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

θ_0 : reference parameter for safe structure

Left null space: $S^T S = I_s, \;\; S^T \, \mathcal{O}_{p+1}(\theta_0) = 0$

Y_k: N-size sample of new measurements

Residual for SHM:

$$\zeta_N(\theta_0) \triangleq \mathrm{vec}(\; S^T(\theta_0) \; \hat{\mathcal{H}} \;)$$

$\mathcal{J}(\theta_0)$: sensitivity of residual ζ w.r.t. modal changes

χ^2-test: $\zeta_N^T \, \Sigma^{-1} \, \mathcal{J}(\mathcal{J}^T \, \Sigma^{-1} \, \mathcal{J})^{-1} \, \mathcal{J}^T \, \Sigma^{-1} \zeta_N \; \geq h$

Non parametric version: empirical null space

Reference data set for safe structure

Left null space: $\hat{S}_0^T \, \hat{S}_0 = I_s, \;\; \hat{S}_0{}^T \, \hat{\mathcal{H}}^{(0)} = 0$

Y_k: N-size sample of new measurements

Residual for SHM:

$$\zeta_N \triangleq \mathrm{vec}(\; \hat{S}_0{}^T \; \hat{\mathcal{H}} \;)$$

Σ: covariance of ζ

χ^2-test: $\zeta_N^T \, \Sigma^{-1} \, \zeta_N \; \geq h$

Merging multiple data sets at different temperatures

J reference data sets : $\overline{\mathcal{H}}^{(0)}_{p+1,q} \triangleq 1/J \; \sum_{j=1}^{J} \; \overline{\mathcal{H}}^{(0),j}_{p+1,q}$

Global empirical null space: $\overline{S}_0{}^T \, \overline{\mathcal{H}}^{(0)}_{p+1,q} = 0$

Y_k: N-size sample of new measurements

Residual for SHM:

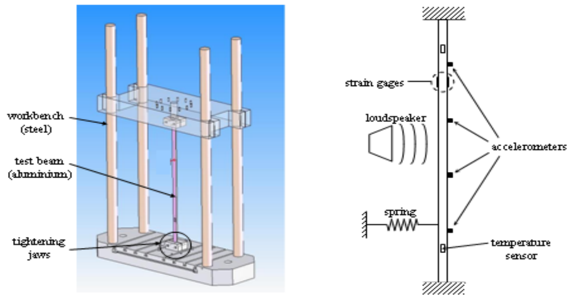
$$\overline{\zeta}_N \triangleq \mathrm{vec}\left(\overline{S}_0{}^T \; \hat{\mathcal{H}}\right)$$

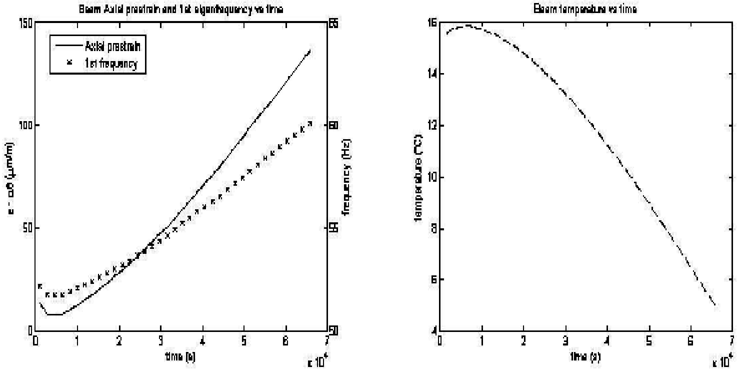
$\overline{\Sigma}$: covariance of $\overline{\zeta}$

χ^2-test: $\overline{\zeta}_N^T \, \overline{\Sigma}^{-1} \, \overline{\zeta}_N \; \geq h$

Example - Beam within a climatic chamber

- A laboratory test-case provided by LCPC Climatic chamber in Nantes
- Vertical clamped beam subject to decreasing temperatures
- Small local damage: horizontal clamped spring attached to the beam, with tunable stiffness and height

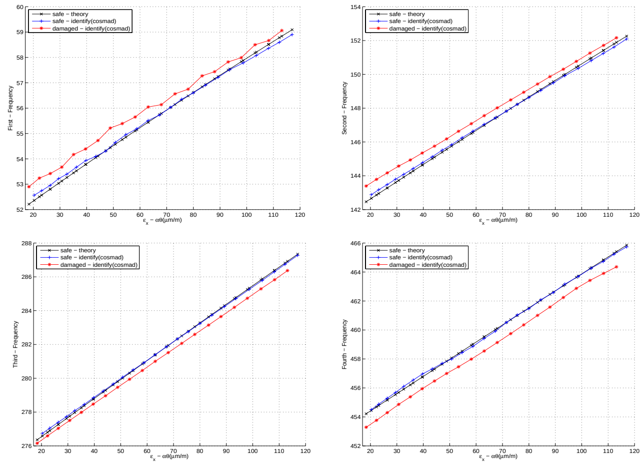




Beam axial prestress and 1st frequency (left)

and temperature (right) vs time.

Decreasing temperature effect on the first 4 frequencies

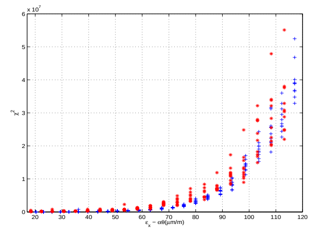


First 4 frequencies vs. thermal constraint.
Computed (black) and identified (safe, damaged)

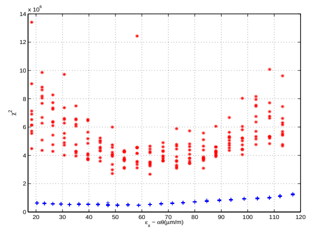
Implementation issues

- Compute the key matrix Σ only once
- For robustness w.r.t. changes in the excitation: compute Σ for each scenario

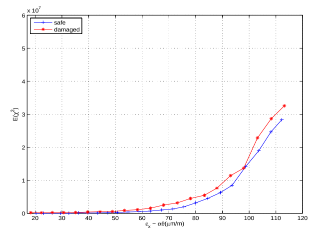
Handling the temperature effect



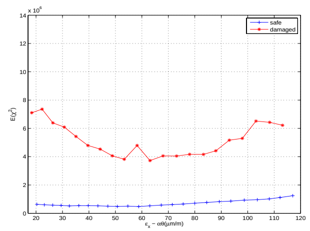
Original χ^2-test



New χ^2-test



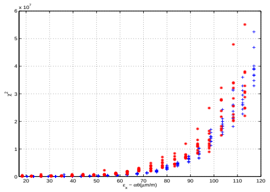
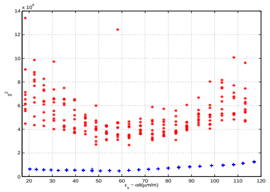
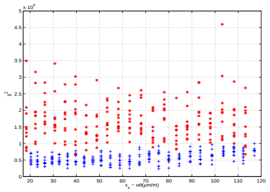
Original - Average



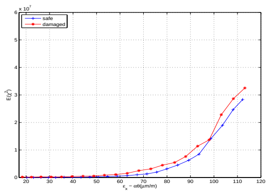
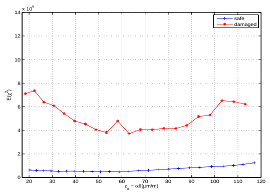
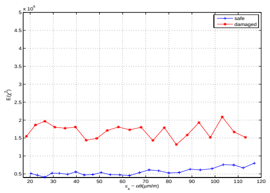
New - Average

Safe and damaged

Comparison with a parametric model-based approach: temperature-adjusted null space

Original χ^2-test | Empirical merging | Analytical updating

Average | Average | Average

Safe and damaged

Conclusion

Temperature effect in vibration-based SHM

Statistical non-parametric approach

- Statistical subspace-based damage detection algorithm
- Empirical null space merging data at # temperatures

Example: clamped beam within climatic chamber

Comparison with a parametric model-based approach (temperature-adjusted null space)

Ongoing: statistical nuisance rejection

Future: in-operation examples,
extension to nonstationary case