Handling the temperature effect in SHM: combining a subspace-based statistical test and a temperature-adjusted null space

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Introduction

- Usefulness of global vibration-based SHM methods
- Limitations due to temperature effects on the dynamics of civil engineering structures
- Wanted: discriminate between changes in modal parameters due to damages and changes due to temperature effects
- A statistical subspace-based damage detection algorithm: null space of a matrix built on reference modes/modeshapes at a known temperature
- Proposed solution to temperature handling: measured temperatures, thermal effect modeling: analytical null space updating

Content

Parametric subspace-based damage detection

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases} \qquad F\,\varphi_\lambda = \lambda\,\varphi_\lambda, \quad \phi_\lambda \triangleq H\,\varphi_\lambda$$

$$R_i \triangleq \mathbf{E}\left(Y_k\, Y_{k-i}^T\right)\ ,\quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\ F^i\ G \Longrightarrow \mathcal{H} = \mathcal{O}\ \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}\ ,\quad \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2G & \dots \end{pmatrix},\quad G \triangleq \mathbf{E}\left(X_k\, Y_k^T\right)$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \phi_\lambda)$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$ modes / mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

θ_0 : reference parameter for safe structure

Left null space: $S^T S = I_s$, $S^T\,\mathcal{O}_{p+1}(\theta_0) = 0$

Y_k: N-size sample of new measurements

Residual for SHM:

$$\zeta_N(\theta_0) \triangleq \text{vec}(\ S^T(\theta_0)\ \hat{\mathcal{H}}\ )$$

$\mathcal{J}(\theta_0)$: sensitivity of ζ w.r.t. modal changes; $\Sigma(\theta_0)$: covariance

χ^2-test: $\zeta_N^T\ \Sigma^{-1}\ \mathcal{J}(\mathcal{J}^T\ \Sigma^{-1}\ \mathcal{J})^{-1}\ \mathcal{J}^T\ \Sigma^{-1}\zeta_N\ \geq h$

Modeling the temperature effect

Clamped, planar, axially pre-stressed, Euler-Bernoulli beam

Eigen problem

$$\begin{cases} EI\ \dfrac{d^4 w(x)}{dx^4} - N_0\ \dfrac{d^2 w(x)}{dx^2} - \rho A\ \omega^2\ w(x) = 0 \\ w(x)|_{x=0,L} = 0\ ;\quad \left.\dfrac{dw(x)}{dx}\right|_{x=0,L} = 0 \end{cases}$$

with $\omega \triangleq \ln|\lambda|$, $w = w_\omega \triangleq \phi_\lambda$, $w(x)$: transversal displacement.

E, I, ρ, A: Young's modulus, cross-section inertia momentum, density and cross-sectional area.

N_0: quasi-static axial preload in Newton:

$$N_0 = EA\ \varepsilon(x)\ ,\quad \varepsilon(x) \triangleq \varepsilon_0(x) - \alpha\ \delta T\ ,\quad \delta T \triangleq T_0(x) - T_{\text{ref}}$$

ε_0: mechanical strain, T_0: current temperature, T_{ref}: reference (no stress) temperature, α: thermal expansion coefficient.

N_0 remains spatially constant
(no external axial body forces, no axial surface tractions, gravity effects negligible).

Only one measure at a given point is necessary:
Thermally compensated strain gauges measure $\varepsilon(x) = N_0/EA$.

Solutions of the eigen problem:

$$2\gamma_n^-\gamma_n^+(1-\cos(\gamma_n^- L)\cosh(\gamma_n^+ L)) + ({\gamma_n^+}^2 - {\gamma_n^-}^2)\sin(\gamma_n^- L)\sinh(\gamma_n^+ L) = 0$$

with :

$$\gamma_n^{\pm} = \left(\sqrt{\frac{\rho A}{EI}\ \omega_n^2 + \left(\frac{N_0}{2EI}\right)^2} \pm \frac{N_0}{2EI}\right)^{1/2}$$

Solved numerically (no analytical solutions for the clamped case).

Analytic expression for mode shapes.

Pre-stress effects on mode shapes are negligible (numerical results).

Temperature-adjusted null space detection

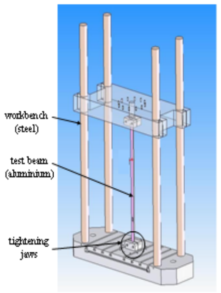
- θ_0 : reference modal parameter for safe structure
- Y_k: N-size sample of new measurements;
  T recorded
- Update the modal parameter θ_T:

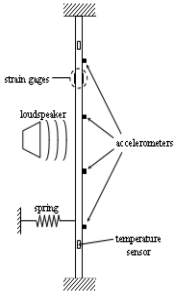
$$\delta T \longrightarrow \varepsilon(x) \longrightarrow (\omega_n)_n \longrightarrow (\lambda_n, \varphi_n)_n \longrightarrow \theta_T$$

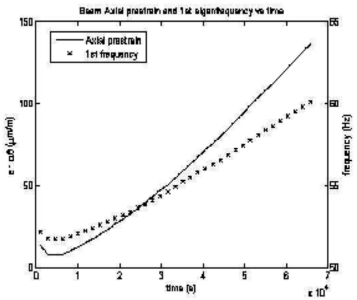
- Update the null space $S(\theta_T)$
- Compute the residual $\zeta_N(\theta_T) \triangleq \text{vec}(\ S^T(\theta_T)\ \hat{\mathcal{H}}\ )$
- Compute the χ^2-test

Example - Beam within a climatic chamber

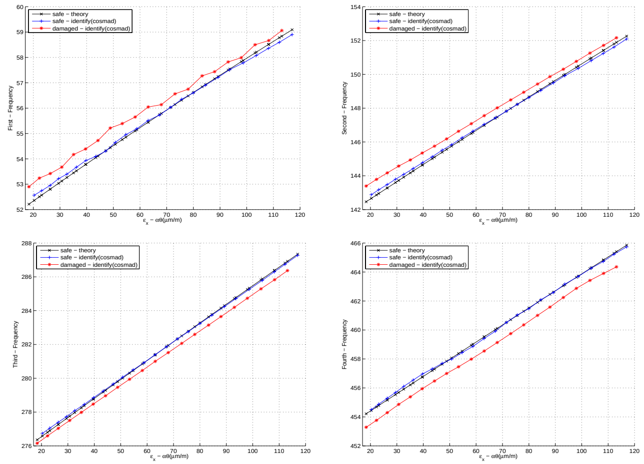
- A laboratory test-case provided by LCPC
  Climatic chamber in Nantes
- Vertical clamped beam
  subject to decreasing temperatures
- Small local damage: horizontal clamped spring attached
  to the beam, with tunable stiffness and height







Decreasing temperature effect on the first 4 frequencies



First 4 frequencies vs. thermal constraint.
Computed (black) and identified (safe, damaged)

Implementation issues

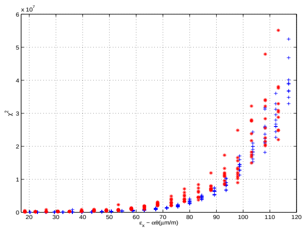
- Frequencies : computed $\neq$ identified
  (the clamped boundary condition, obtained with tightening jaws, is not perfect)

  De-biased temperature-adjusted modal parameter:

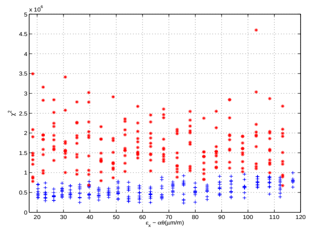
$$(\forall \varepsilon) \quad \widetilde{\theta}_T(\varepsilon) \triangleq \theta_T(\varepsilon) + \widetilde{\theta}_0(\varepsilon_0) - \theta_T(\varepsilon_0)$$

- Compute the key matrices:
  residual sensitivity $\mathcal{J}$ and covariance Σ
  for every realization of each scenario

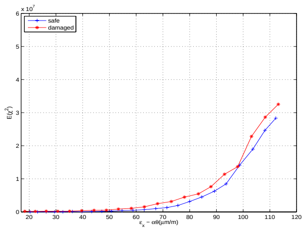
Handling the temperature effect



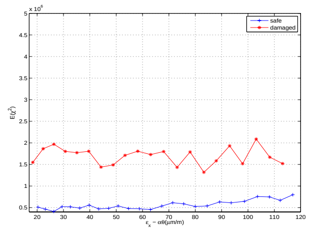
Original χ^2-test



New χ^2-test



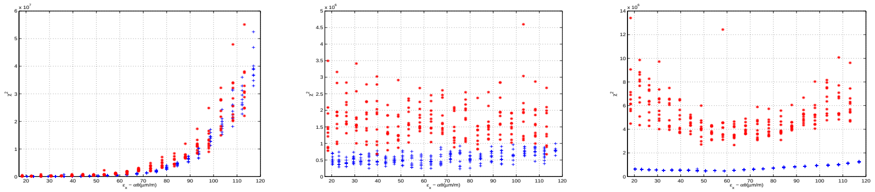
Original - Average



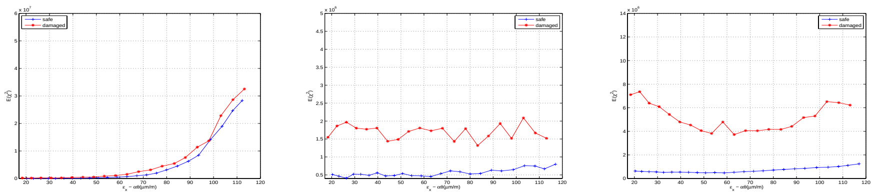
New - Average

Safe and damaged

Comparison with a non parametric approach:
empirical null space merging data at # temperatures



Original χ^2-test   Analytical updating   Empirical merging



Average   Average   Average

Safe and damaged

Conclusion

Temperature effect in vibration-based SHM

Statistical parametric model-based approach

- Statistical subspace-based damage detection algorithm

- Temperature-adjusted null space

Example: clamped beam within climatic chamber

Comparison with a non parametric approach
(empirical null space, merging data at # temperatures)

Ongoing: statistical nuisance rejection

Future: in-operation examples,
extension to 3D temperature fields,
thermal model parameterization