On sensors positioning

for Structural Health Monitoring

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Motivations

- **Monitoring** (the eigenstructure of) **a** (linear) **system**: identification, damage detection and localization.
- For given monitoring requirements: How to achieve optimum sensors positioning ?
- For a given sensors set: Which damages can be efficiently monitored ?
- Criteria to assess sensors sets, handling eigenstructure, monitoring requirements, noise and uncertainties, excitation type.

Contents

- Sensors positioning issues for SHM
- Sensors positioning criteria
- Scalar functions of a matrix
- Exploiting a distance between two matrices
- A criterion: the power of a damage detection test

Sensors positioning issues for SHM

System models and parameters

Invariant parameterizations

Different numbers of sensors

Frequency content and geometry of the excitation

System models and invariant parameterizations

FEM:

$$\begin{cases} M \ \ddot{\mathcal{Z}}(s) + C \ \dot{\mathcal{Z}}(s) + K \ \mathcal{Z}(s) \ = \ \varepsilon(s) \\ Y(s) \ = \ L \ \mathcal{Z}(s) \end{cases}$$

$$(M \ \mu^2 + C \ \mu + K) \ \Psi_\mu = 0 \ , \quad \psi_\mu = L \ \Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} \ = \ F \ X_k + V_k \\ Y_k \ = \ H \ X_k \end{cases}$$

$$F \ \Phi_\lambda = \lambda \ \Phi_\lambda \ , \ \varphi_\lambda \stackrel{\Delta}{=} H \ \Phi_\lambda \ , \ \underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \ , \ \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode-shapes}}$$

ARMA:

$$Y_k = \sum_{i=1}^{p} A_i \ Y_{k-i} + \sum_{j=0}^{p-1} B_j \ W_{k-j}$$

$$\left(A_p \ \lambda^p + \ldots + A_1 \ \lambda - I\right) \ \varphi_\lambda = 0$$

Structural monitoring and sensors positioning problems statement

Structural monitoring

For a given sensor positioning L:
monitor the modes and modeshapes $(\lambda, \varphi_\lambda)$.

Sensors positioning

For a given excitation level and profile:
Optimize an objective function w.r.t. matrix L:

- Using a parameterization invariant w.r.t. L !
- Handling different numbers of sensors.

Sensors positioning criteria

Matrix criteria

Observability, controllability, estimation error covariance, MAC matrix, Fisher information, ...

Scalar functions of a matrix

Determinant, trace, extremal eigenvalues, minimizing off-diagonal terms (e.g. of MAC), ...

Invariance properties

Measurements scaling, mode-shapes normalization, ...

Scalar functions of common use

For a q-dimensional matrix M and $z < 0$:

$$c_z = \left(\frac{\text{Trace}\,(M^z)}{q} \right)^{1/z}$$

Determinant, trace, extremal eigenvalue:

$$\lim_{z \nearrow 0} \ c_z = |M|^{1/q} \ , \qquad c_{-1} = \frac{q}{\text{Trace}\left(M^{-1}\right)}$$

$$\lim_{z \searrow -\infty} \ c_z = \lambda_{\min}(M)$$

Exploiting a distance between matrices

A distance between two matrices

Distance to a diagonal matrix

Scalar functions of potential interest

Kullback distance between two symmetric matrices

$$2\ \mathrm{K}(M_1, M_2) \triangleq \mathrm{Trace}(M_1\ M_2^{-1} - I_q) - \ln |M_1\ M_2^{-1}|$$

Invariance: $\mathrm{K}(A\ M_1\ A^T, A\ M_2\ A^T) = \mathrm{K}(M_1, M_2)$

Distance to a diagonal matrix

$$2\ \mathrm{K}(M, \Delta_q) = \sum_{i=1}^{q} \frac{m_{ii}}{\delta_i} + \sum_{i=1}^{q} \ln \delta_i - \ln |M| - q$$

Approximation

$$4\ \mathrm{K}(M, I_q) \approx \|M - I_q\|_F^2 \approx \|M^{-1} - I_q\|_F^2$$

Scalar functions of potential interest, with different invariance properties

$$C_1(M) \triangleq \mathrm{K}(M, I_q) = (\mathrm{Trace}\, M - \ln |M| - q)/2$$

$$C_2(M) \triangleq \min_{\delta > 0} \mathrm{K}(M, \delta\ I_q) = \max_{AA^T = I} C_0\left(A\ M\ A^T\right)$$

$$C_3(M) \triangleq \min_{\Delta_q > 0} \mathrm{K}\left(M, \Delta_q\right) = C_0(M)$$

$$C_0(M) \triangleq -1/2\ \ln\left(|M| / \prod_{i=1}^{q} M_{ii}\right) \quad \text{mutual info}$$

Useful criterion: power of a damage detection test

q-dimensional Gaussian residual ζ

Sensor set L reflected into matrices $\mathcal{J}, \Sigma$ in:

$$\zeta \sim \mathcal{N}(\mathcal{J}\,\Upsilon, \Sigma) \ : \begin{cases} \mathcal{H}_0 \ : \ \Upsilon = 0 \\ \mathcal{H}_1 \ : \ \Upsilon \neq 0 \quad \text{damage} \end{cases}$$

How to compare different sensor sets,
possibly with different numbers, thus different q?

Use the test power

The power criterion

χ^2-test:

$$\chi^2 = \zeta^T \Sigma^{-1} \mathcal{J} (\mathcal{J}^T \Sigma^{-1} \mathcal{J})^{-1} \mathcal{J}^T \Sigma^{-1} \zeta$$

Noncentrality parameter:

$$\gamma^2(\Upsilon) = \Upsilon^T \Gamma \Upsilon, \quad \Gamma = \mathcal{J}^T \Sigma^{-1} \mathcal{J} \quad \text{Fisher info}$$

Test power : function of γ^2 only. Hence the criterion:

$$\int_{\Upsilon \in \mathrm{R}^m, \|\Upsilon\|^2=1} \gamma^2(\Upsilon)\, d\Upsilon = \frac{\text{Area}(S_m)}{m} \quad \text{Trace}(\Gamma)$$

But, for a fixed false alarms rate, the test power depends on $q \triangleq \dim(\zeta)$!

Compensating for q

For q large and small Υ (damage):

$$\text{(level) } \mathrm{P}_0(\chi^2 < \lambda) \leq \alpha \; , \; \beta = \mathrm{P}_1(\chi^2 \geq \lambda) \text{ (power)}$$

$$\beta \approx \alpha + \frac{\gamma^2}{2} \frac{e^{-\delta^2/2}}{\sqrt{2\pi q}} \; , \quad \delta = \phi^{-1}(1-\alpha)$$

δ does not depend upon q. Hence use:

$$(\beta - \alpha) e^{\delta^2/2} = \gamma^2 / 2\sqrt{2\pi q}$$

Integrating over unit sphere: $\dfrac{\text{Trace}(\Gamma)}{\sqrt{q}}$

(implemented within the COSMAD toolbox)

Conclusion

- Trade-off between instrumentation costs and information and efficiency of SHM algorithms
- Criteria to be optimized for sensors positioning
- Relevance of a given sensors set (number, positions) often summarized in a matrix
- Invariance properties
- Scalar functions of matrices : distances, test power