Modal filtering data reduction and subspace detection for handling the temperature effect in SHM

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Introduction

- Usefulness of global vibration-based SHM methods
- Limitations due to temperature effects on the dynamics of civil engineering structures
- A statistical subspace-based damage detection algorithm: null space of a matrix built on reference modes/modeshapes

  Non parametric version: null space of a matrix built on reference data set
- Limitations: large sensors arrays
- For handling large sensors arrays and temperature effect: no temperature measurement, data reduction using modal filtering, empirical merging of non parametric null spaces

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Parametric subspace-based damage detection

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases} \qquad F\,\varphi_\lambda = \lambda\,\varphi_\lambda, \quad \phi_\lambda \triangleq H\,\varphi_\lambda$$

$$R_i \triangleq \mathbf{E}\left(Y_k\, Y_{k-i}^T\right) \ , \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\ F^i\ G \Longrightarrow \mathcal{H} = \mathcal{O}\ \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \ , \quad \mathcal{C} \triangleq \left(G \quad FG \quad F^2G \quad \dots \right), \quad G \triangleq \mathbf{E}\left(X_k\, Y_k^T\right)$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \phi_\lambda)$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$ modes / mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

θ_0 : reference parameter for safe structure

Left null space: $S^T S = I_s, \;\; S^T \mathcal{O}_{p+1}(\theta_0) = 0$

Y_k: N-size sample of new measurements

Residual for SHM:

$$\zeta_N(\theta_0) \triangleq \text{vec}(\; S^T(\theta_0)\; \hat{\mathcal{H}} \;)$$

$\mathcal{J}(\theta_0)$: sensitivity of residual ζ w.r.t. modal changes

χ^2-test: $\zeta_N^T \, \Sigma^{-1} \, \mathcal{J}(\mathcal{J}^T \, \Sigma^{-1} \, \mathcal{J})^{-1} \, \mathcal{J}^T \, \Sigma^{-1} \zeta_N \;\geq h$

Non parametric version: empirical null space

Reference data set for safe structure

Left null space: $\hat{S}_0^T \; \hat{S}_0 = I_s, \;\; \hat{S}_0{}^T \; \hat{\mathcal{H}}^{(0)} = 0$

Y_k: N-size sample of new measurements

Residual for SHM:

$$\zeta_N \triangleq \text{vec}(\; \hat{S}_0{}^T \; \hat{\mathcal{H}} \;)$$

Σ: covariance of ζ

χ^2-test: $\zeta_N^T \Sigma^{-1} \zeta_N \geq h$ Non param. subspace detection

Merging multiple data sets at different temperatures

J reference data sets : $\overline{\mathcal{H}}_{p+1,q}^{(0)} \triangleq 1/J \;\; \Sigma_{j=1}^{J} \;\; \overline{\mathcal{H}}_{p+1,q}^{(0),j}$

Global empirical null space: $\overline{S}_0{}^T \; \overline{\mathcal{H}}_{p+1,q}^{(0)} = 0$

Y_k: N-size sample of new measurements

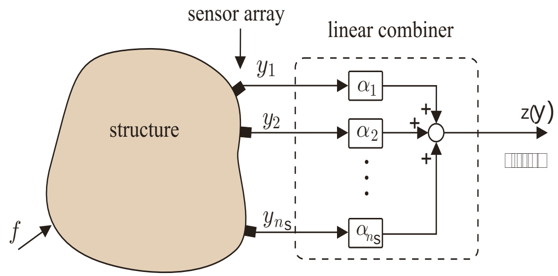
Residual for SHM:

$$\overline{\zeta}_N \triangleq \text{vec}\left(\overline{S}_0{}^T \; \hat{\mathcal{H}}\right)$$

$\overline{\Sigma}$: covariance of $\overline{\zeta}$

χ^2-test: $\overline{\zeta}_N^T \; \overline{\Sigma}^{-1} \; \overline{\zeta}_N \;\geq h$ Robust subspace detection

Data reduction using modal filtering



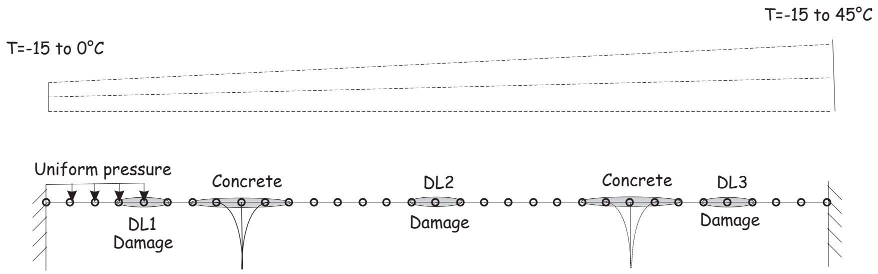
z_l orthogonal to the N modes in a frequency band of interest except mode l. More sensors than modes.

$$\Phi^T \; \alpha = I \;\; ; \;\; \Phi^T \text{ rank deficient } \longrightarrow \text{SVD}$$
$$Z = \alpha \; Y \;\; , \;\; \dim(Z) < \dim(Y)$$

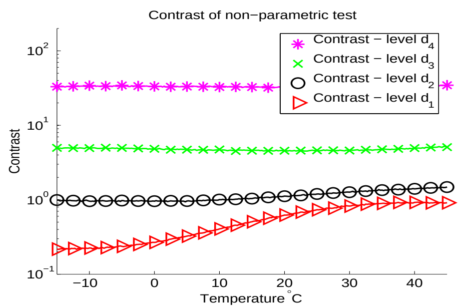
Non parametric & robust subspace detection on Z.

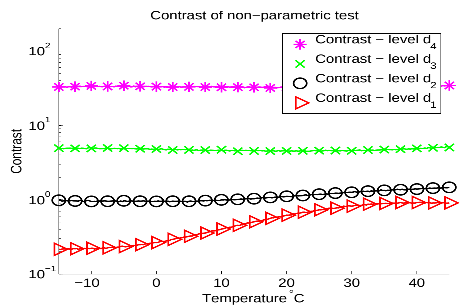
Example - Simulated three-span bridge

- A simulator provided by ULB
  Two materials: steel and concrete
  Excitation: uniform pressure on the first span
  Motion restricted to in-plane vibrations
- Both hand sides subject to different temperature gradients
- Four damage scenarios: stiffness reduction at three locations



Non parametric subspace detection

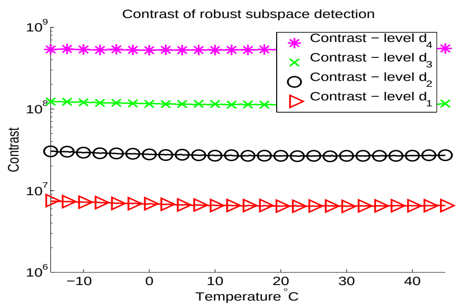


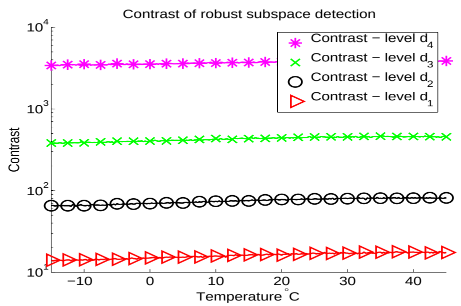


Contrast between the undamaged and the four damage levels.

Using 29 sensors (left). Using 10 filters (right).

Robust subspace detection





Contrast between the undamaged and the four damage levels.

Using 29 sensors (left). Using 10 filters (right).

Conclusion

Temperature effect & large sensors arrays in vibration-based SHM

Statistical non parametric approach

Statistical subspace-based damage detection algorithm

Empirical null space merging data at # temperatures

Modal filters for data reduction

Example: simulated three span bridge

Ongoing: sensor noise effect

Future: in-operation examples,
comparison with other methods