Local approach to FDI in nonlinear dynamical systems

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Introduction

Parametrized nonlinear state-space systems

$$\begin{cases} \dot{x}(t) = f(\theta, x(t), u(t)) & x(t): \text{ unknown states} \\ y(t) = h(\theta, x(t), u(t)) & u(t): \text{ known inputs} \\ y_k = y(k\tau) + \epsilon_k & y_k: \text{ measured outputs} \end{cases}$$

θ : **unknown parameters**

Detect and diagnose (small) changes in θ.

Local approach to FDI

θ_0 : **known (or identified) reference value**

N**-size sample of new measurements** u_k, y_k

$$\text{Test} \quad \mathcal{H}_0 : \quad \theta = \theta_0 \quad \text{against} \quad \mathcal{H}_1 : \quad \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$$

Residual generation: Auxiliary process

$$\begin{cases} \xi_{k+1} = \varphi(\theta, \xi_k, u_k, y_k) \\ Z_k = \vartheta(\theta, \xi_k, u_k, y_k) \end{cases}$$

Residual generation: Parameter estimating function

$$\text{Function } H \text{ s.t.}: \quad \mathbf{E}_{\theta_0} H(\theta, Z_k) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$$

$$\text{Residual}: \quad \zeta_N(\theta_0) \triangleq \frac{1}{\sqrt{N}} \sum_{k=1}^{N} H(\theta_0, Z_k)$$

Example: LS-score (conditional mean-squares)

$$H(\theta, Z_k) \triangleq - \left(\frac{\partial \epsilon_k(\theta)}{\partial \theta}\right)^T \varepsilon_k(\theta)$$

$$\varepsilon_k(\theta) \triangleq y_k - \hat{y}_k(\theta)$$

Prediction error $\varepsilon_k(\theta)$: for sensor faults ONLY

Fault detection

Mean sensitivity and covariance matrices

$$M(\theta_0) \triangleq - \mathbf{E}_{\theta_0} \left. \frac{\partial H(\theta, Z_k)}{\partial \theta} \right|_{\theta=\theta_0}, \qquad \Sigma(\theta_0) \triangleq \lim_{N\to\infty} \mathbf{E}_{\theta_0} \, \zeta_N(\theta_0) \, \zeta_N^T(\theta_0)$$

Asymptotic Gaussianity

$$\zeta_N \;\to\; \begin{cases} \mathcal{N}(\quad 0, \;\Sigma) & \text{under} \quad \mathbf{P}_{\theta_0} \\ \mathcal{N}(\; M \,\delta\theta, \;\Sigma) & \text{under} \quad \mathbf{P}_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

Global test

$$\zeta_N^T \;\; \Sigma^{-1} \, M \, \left(M^T \, \Sigma^{-1} \, M\right)^{-1} \, M^T \, \Sigma^{-1} \;\; \zeta_N \;\; \geq \lambda$$

Fault isolation

$$\zeta \sim \mathcal{N}(M \,\eta, \;\Sigma), \quad \eta = \begin{pmatrix} \eta_a \\ \eta_b \end{pmatrix}, \quad M = \begin{pmatrix} M_a & M_b \end{pmatrix}, \quad p_{\eta_a,\eta_b}(\zeta)$$

Decide between $\eta_a = 0$ and $\eta_a \neq 0$; η_b unknown

$$\mathbf{F} \triangleq M^T \, \Sigma^{-1} \, M \triangleq \begin{pmatrix} \mathbf{F}_{aa} & \mathbf{F}_{ab} \\ \mathbf{F}_{ba} & \mathbf{F}_{bb} \end{pmatrix}$$

$\mathbf{F}_a^{*-1}$: upper-left block of $\mathbf{F}^{-1}$; $\quad \mathbf{F}_a^* = \mathbf{F}_{aa} - \mathbf{F}_{ab} \, \mathbf{F}_{bb}^{-1} \, \mathbf{F}_{ba}$

Statistical projection (sensitivity)

$$2 \, \ln \frac{\max_{\eta_a} \, p_{\eta_a,0}(\zeta)}{p_{0,0}(\zeta)} \;=\; \zeta_a^T \, \mathbf{F}_{aa}^{-1} \, \zeta_a \,, \qquad \zeta_a \;:\text{partial score}$$

$$\zeta_a \;=\; M_a^T \, \Sigma^{-1} \, \zeta$$

Statistical rejection (minmax)

$$2 \, \ln \frac{\max_{\eta_a,\eta_b} \, p_{\eta_a,\eta_b}(\zeta)}{\max_{\eta_b} \, p_{0,\eta_b}(\zeta)} \;=\; \zeta_a^{*T} \, \mathbf{F}_a^{*\,-1} \, \zeta_a^* \,, \qquad \zeta_a^* \;:\text{effective score}$$

$$\zeta_a^* \;=\; \zeta_a - \mathbf{F}_{ab} \, \mathbf{F}_{bb}^{-1} \, \zeta_b$$

Regression of partial score on nuisance score

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Nonlinear systems: State elimination

DAE systems - Ritt's algorithm (Ljung-Glad)

$$\phi_i(\theta, x, u, y, p) = 0 \iff g_i(\theta, u, y, p) = 0 \qquad (1 \leq i \leq r)$$

Stochastic model - LS-score

$$g(\theta, u_k, y_k, \delta) = \epsilon_k \qquad (\dim g = r)$$

$$H(\theta, u_k, y_k, \delta) = \left(\frac{\partial}{\partial \theta} \, g(\theta, u_k, y_k, \delta)\right)^T \; g(\theta, u_k, y_k, \delta)$$

State estimation: (full order) observers

$$\begin{cases} \dot{\hat{x}} &= \hat{f}(\theta, \hat{x}, u, y) \\ \hat{y}_k(\theta) &= h(\theta, \hat{x}(k\tau), u(k\tau)) \end{cases}$$

$$\hat{f}(\theta, \hat{x}, u, y) \triangleq f(\theta, \hat{x}, u) + K_o \; (y - h(\theta, \hat{x}, u))$$

$$H(\theta_0, Z_k) = \left(\left. \frac{\partial \hat{y}_k(\theta)}{\partial \theta} \right|_{\theta=\theta_0} \right)^T (y_k - \hat{y}_k(\theta_0))$$

$$\begin{cases} \dfrac{\partial \dot{\hat{x}}}{\partial \theta} &= \dfrac{\partial}{\partial \hat{x}} \hat{f}(\theta_0, \hat{x}, u, y) \dfrac{\partial \hat{x}}{\partial \theta} + \dfrac{\partial}{\partial \theta} \hat{f}(\theta_0, \hat{x}, u, y) \\ \dfrac{\partial \hat{y}_k(\theta)}{\partial \theta} &= \dfrac{\partial}{\partial \hat{x}} h(\theta_0, \hat{x}(k\tau), u(k\tau)) \dfrac{\partial \hat{x}_k}{\partial \theta} + \dfrac{\partial}{\partial \theta} h(\theta_0, \hat{x}(k\tau), u(k\tau)) \end{cases}$$

Numerical example: electrical circuit

$$\begin{aligned} L \; \dot{I} &= -\,(R\; I - u), \qquad R = a\; T \\ \dot{T} &= \alpha\; R\; I^2 - \beta\;(T - T_e) \end{aligned}$$

$$X \triangleq (I \quad T)^T, \qquad \theta \triangleq (a \quad b)^T, \qquad a \triangleq R/T, \qquad b \triangleq 1/L$$

$$\begin{cases} \dot{I} &= -\, b\; (a\; I\; T - u) \\ \dot{T} &= \alpha\; a\; I^2\; T - \beta\;(T - T_e) \\ y &= I \end{cases}$$

Faults: (1) Resistance $R + 20\%$, (2) Inductance L - 9%.

State elimination

$$\begin{aligned} g(\theta, u, y, p) &= \alpha\; a\; b\; u y^3 - \alpha\; a\; \dot{y} y^3 + \beta T_e\; a\; b\; y^2 \\ &+ \beta\; \dot{y} y + \ddot{y} y - b\; y \dot{u} - \beta\; b\; u y + b\; u \dot{y} - \dot{y}^2 \end{aligned}$$

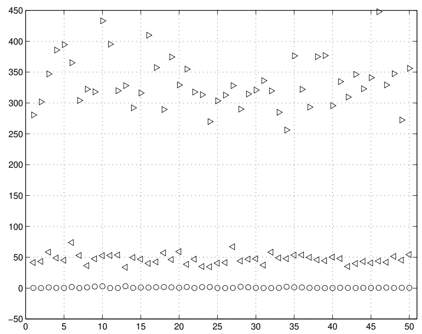
State estimation

$$\begin{cases} \dot{\hat{I}} &= -\, b\; (a\; \hat{I}\; \hat{T} - u) \; + \; k_I\; (y - \hat{I}) \\ \dot{\hat{T}} &= \alpha\; a\; \hat{I}^2\; \hat{T} \; - \; \beta\; (\hat{T} - T_e) \; + \; k_T\; (y - \hat{I}) \end{cases}$$

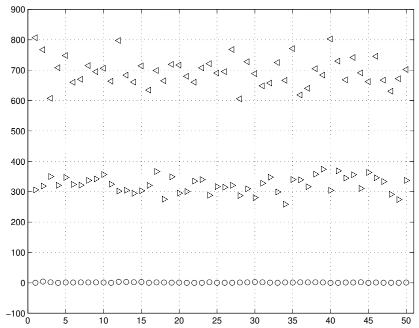
Faults

$$\theta_0 = (0.1 \quad 0.7)^T, \qquad \theta_1 = (0.12 \quad 0.7)^T, \qquad \theta_2 = (0.1 \quad 0.77)^T$$

Global test



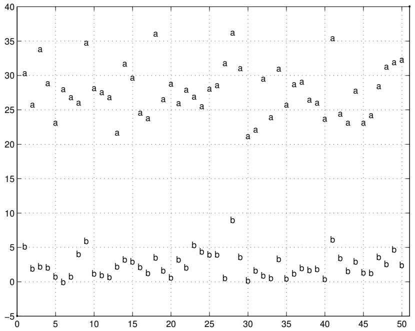
State elimination



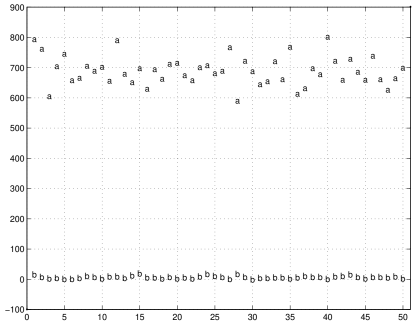
State estimation

Simulations with: ○: θ_0, ◁: θ_1, ▷: θ_2.

Isolation: fault in a



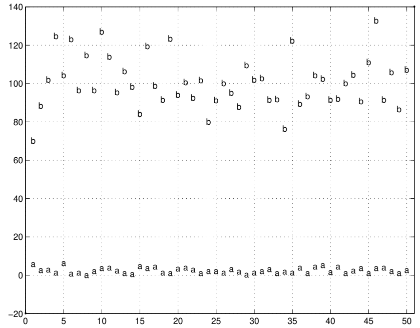
State elimination



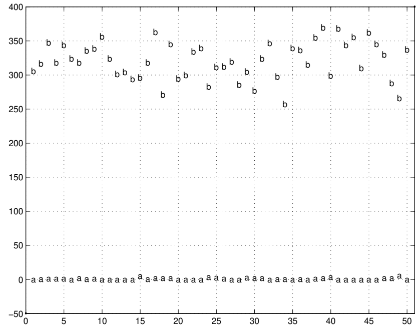
State estimation

Simulations with θ_1 - `a`: t_a^*, `b`: t_b^*.

Isolation: fault in b



State elimination



State estimation

Simulations with θ_2 - `a`: t_a^*, `b`: t_b^*.

Conclusion: state estimation versus elimination

	Estimation	Elimination
Advantages	Noise robustness	Systematic method for DAE systems Model robustness
Drawbacks	Design, tuning Stability	Complex method Noise sensitive (deriv.) Numerical conditioning

Future work: Implicit observers