Output-only subspace-based structural identification and damage detection and localization

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Toolboxes: LMS CADA-X, and Scilab

ftp://ftp.inria.fr/INRIA/Projects/Meta2/Scilab/contrib/MODAL/

Problems : In-operation modal identification and damage detection and localization

- The excitation is typically:
  - natural, not controlled.
  - not measured:
    - buildings, bridges, offshore structures,
    - rotating machinery,
    - cars, trains, aircrafts.
  - nonstationary (e.g., turbulent).
- How to merge multiple measurements setups e.g. in case of moving sensors?
- How to detect and localize small damages?

Identification and merging

- Output-only eigenstructure identification,
- In the presence of nonstationary excitation,
- Handling moving sensor pools, with some reference sensors.

Wanted:

- Avoid merging identification results from the different pools,
- Merge the data instead, and process them globally,
- Use a standard subspace algorithm.

Damage detection and localization

- Output-only damage detection and localization,
- In the presence of nonstationary excitation,
- On-board handling of small damages.

Wanted:

- Early warning and interpretation of damages,
- Avoid re-identification prior to detection,
- Avoid inverse problem solving prior to damage localization.

Contents

- Modelling
- Output-only covariance-based subspace identification
- Merging multiple measurements setups
- Robustness to nonstationary excitation
- Damage detection
- Back to eigenstructure monitoring
- Damage diagnostics

Modelling

FE model: $$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \ , \quad \psi_\mu = L\Psi_\mu$$

State space: $$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \ \Phi_\lambda \ , \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}$$

Output-only covariance-based subspace identification

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k \ Y_{k-i}^T\right)}_{\text{ok if stationary !}} \ , \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H \ F^i \ G \ , \quad G \triangleq \mathbf{E}\left(X_k \ Y_k^T\right)$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \ , \ \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2G & \dots \end{pmatrix}$$

$$\mathcal{H} = \mathcal{O} \ \mathcal{C} \ , \ \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Implementation

$$\underbrace{\hat{R}_i \triangleq \frac{1}{N} \sum_{k=1}^{N} Y_k \ Y_{k-i}^T}_{\text{ok when nonstationary !}} \ , \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = U \ \Delta \ W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T \ ; \quad \hat{\mathcal{O}} = U \ \Delta_1^{1/2}$$

$$\mathcal{O}_p^\uparrow(H, F) = \mathcal{O}_p(H, F) \ F$$

$$\det(F - \lambda \ I) = 0 \ , \quad F \ \Phi_\lambda = \lambda \ \Phi_\lambda, \quad \varphi_\lambda = H \ \Phi_\lambda$$

Merging multiple measurements setups

$$\underbrace{\begin{bmatrix} Y_k^{(0,1)} \\ Y_k^{(1)} \end{bmatrix}}_{\text{Record 1}} \quad \underbrace{\begin{bmatrix} Y_k^{(0,2)} \\ Y_k^{(2)} \end{bmatrix}}_{\text{Record 2}} \quad \cdots \quad \underbrace{\begin{bmatrix} Y_k^{(0,J)} \\ Y_k^{(J)} \end{bmatrix}}_{\text{Record J}}$$

$$\begin{cases} X_{k+1}^{(j)} = F \; X_k^{(j)} + V_k^{(j)} \\ Y_k^{(0,j)} = H_0 \; X_k^{(j)} & \text{(the reference)} \\ Y_k^{(j)} = H_j \; X_k^{(j)} & \text{(sensor pool n}^o j\text{)} \end{cases}$$

$$R_i^{0,j} \triangleq \mathrm{E} \; Y_k^{(0,j)} \; {Y_{k-i}^{(0,j)}}^T \; , \quad R_i^j \triangleq \mathrm{E} \; Y_k^{(j)} \; {Y_{k-i}^{(0,j)}}^T$$

$\mathrm{E} \; Y_k^{(j)} \; {Y_{k-i}^{(j)}}^T$ not used, $\mathrm{E} \; Y_k^{(j')} \; {Y_{k-i}^{(j)}}^T$ $(j \neq j')$ not available

Stationary excitation

$$\mathrm{cov} V_k^{(j)} = Q \; , \quad G \triangleq \mathrm{E} \; X_k^{(j)} \; {Y_k^{(0,j)}}^T$$

$$R_i^{0,j} = H_0 \; F^i \; G \triangleq R_i^0 \; , \quad R_i^j = H_j \; F^i \; G$$

$$R_i^{\pi} \triangleq \begin{bmatrix} R_i^0 \\ R_i^1 \\ \vdots \\ R_i^J \end{bmatrix} = H \; F^i \; G \; , \quad H \triangleq \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_J \end{bmatrix}$$

Nonstationary excitation

$$\mathrm{cov} V_k^{(j)} = Q_j \; , \quad G_j \triangleq \mathrm{E} \; X_k^{(j)} \; {Y_k^{(0,j)}}^T$$

$$R_i^{0,j} = H_0 \; F^i \; G_j \; , \quad R_i^j = H_j \; F^i \; G_j$$

Hint: right renormalization of the covariances.

Robustness to nonstationary excitation

Time-varying excitation within each record

$$\mathrm{cov} V_k^{(j)} = Q_k$$

Approximate factorization of covariances

$$\hat{R}_i \approx H \; F^i \; \hat{G}$$

Consistency : $T^{-1} \; \hat{F} \; T \to F, \quad \hat{H} \to H$

Combination of:

- the key factorization property of the covariances,
- the averaging operation underlying covariance computation,

allows to cancel out nonstationarities in the excitation.

Introducing the parameter vector

FE model:
$$\begin{cases} M \ddot{\mathcal{Z}}(s) + C \dot{\mathcal{Z}}(s) + K \mathcal{Z}(s) = \nu(s) \\ Y(s) = L \mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \; , \quad \psi_\mu = L\Psi_\mu$$

State space:
$$\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases}$$

$$F\Phi_\lambda = \lambda \; \Phi_\lambda \; , \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

Parameter: $\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \; , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}} \; ; \; \theta \triangleq \begin{pmatrix} \Lambda \\ \mathrm{vec} \, \Phi \end{pmatrix}$

Damage detection

Local approach (small deviations)

θ_0 : reference parameter, known (or identified)

Y_k: N-size sample of new measurements

Build a residual ζ significantly non zero when damage

Test $\quad \mathcal{H}_0 : \quad \theta = \theta_0 \quad$ against $\quad \mathcal{H}_1 : \quad \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$

Residual $\leftrightarrow$ Estimating function

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} K(\theta_0, Y_k)$$

Characterized by: $\mathbf{E}_{\theta_0} K(\theta, Y_k) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$

Warning: Prediction error for sensor faults ONLY !

Mean sensitivity (Jacobian) and covariance

$$\mathcal{M}(\theta_0) \triangleq -\mathbf{E}_{\theta_0} \frac{\partial K(\theta_0, Y_k)}{\partial \theta}, \quad \Sigma(\theta_0) \triangleq \lim_{N \to \infty} \mathbf{E}_{\theta_0} \zeta_N(\theta_0) \zeta_N^T(\theta_0)$$

The residual is asymptotically Gaussian

$$\zeta_N(\theta_0) \rightarrow \begin{cases} \mathcal{N}(\quad 0, \ \Sigma(\theta_0)) & \text{under} \quad \mathbf{P}_{\theta_0} \\ \mathcal{N}(\ \mathcal{M}(\theta_0)\, \delta\theta, \ \Sigma(\theta_0)) & \text{under} \quad \mathbf{P}_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

(On-board) χ^2-test

$$\zeta_N^T \ \Sigma^{-1} \ \mathcal{M} \ (\mathcal{M}^T \ \Sigma^{-1} \ \mathcal{M})^{-1} \ \mathcal{M}^T \ \Sigma^{-1} \ \zeta_N \quad \geq h$$

Invariant / pre-multiplication of ζ with invertible gain

Back to eigenstructure monitoring

$$\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases} \qquad F \varphi_\lambda = \lambda \varphi_\lambda, \quad \Phi_\lambda \triangleq H \varphi_\lambda$$

Canonical parametrization : $\theta \triangleq \begin{pmatrix} \Lambda \\ \operatorname{vec} \Phi \end{pmatrix}$

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

System parameter characterization:

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel.

Back to structural subspace identification

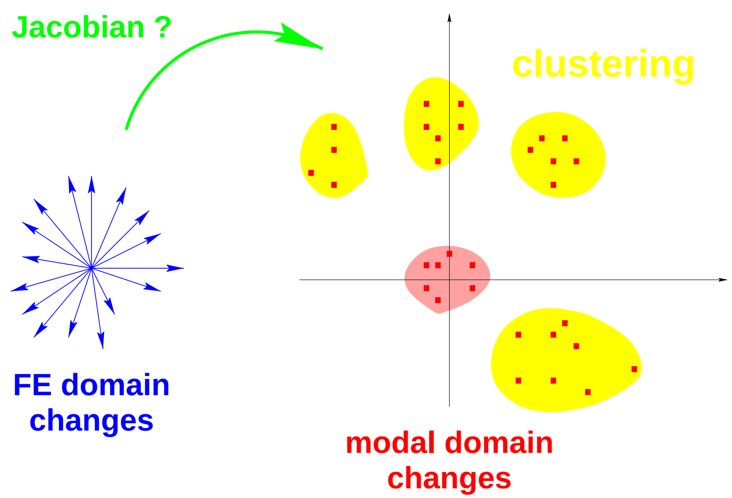
$$\exists S, \quad S^T\, S = I_s, \qquad S^T\, \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } S(\theta_0)$$

$$\theta_0 \leftrightarrow (R_i^0)_i \text{ characterized by: } S^T(\theta_0)\, \hat{\mathcal{H}}^0_{p+1,q} = 0$$

Residual for structural damage monitoring

$$\zeta_N(\theta_0) \triangleq \mathrm{vec}(\ S^T(\theta_0)\, \hat{\mathcal{H}}_{p+1,q}\ )$$

On-board damage diagnostics: projecting changes



Damage diagnostics: (local) sensitivity approach

$$\zeta \sim \mathcal{N}(\mathcal{M}\ \delta\theta, \ \ \Sigma), \qquad \delta\theta = \mathcal{I}\ \mathcal{J}_{(M_0^\star, K_0^\star)} \begin{pmatrix} \delta M \\ \delta K \end{pmatrix}$$

$(M_0^\star, K_0^\star)$: design model

Jacobian : $(\delta M, \delta K) \xrightarrow{\mathcal{J}_{(M_0^\star, K_0^\star)}} (\delta\mu, \delta\psi_\mu)$

Reduction: $\mathcal{I}$ matching computed/identified modes

Problem : $\dim \begin{pmatrix} M \\ K \end{pmatrix} \gg \dim \theta$

Computing Jacobian

1. $(\delta M, \delta K) \xrightarrow[\textit{mode selection}]{\mathcal{I}\mathcal{J}_{(M_0^\star, K_0^\star)}} (\delta\mu, \delta\psi_\mu)$
2. Apply $\mathcal{I}\mathcal{J}$ to unit vectors $(\delta M, \delta K)$
3. Truncate small vectors $(\delta\mu, \delta\psi_\mu)$
4. Cluster the remaining vectors $(\delta\mu, \delta\psi_\mu)$, using the χ^2-metric.

? Better reduction techniques, instead of 1., 2., 3.