# Handling the temperature effect in vibration monitoring of civil structures: a combined subspace-based and nuisance rejection approach


*Étienne Balmès*

MssMat, Ecole Centrale Paris, France

*Michèle Basseville, Laurent Mevel, Houssein Nasser*

IRISA (CNRS & INRIA & Univ.), Rennes, France


National Computer & Security project Constructif


houssein.nasser@irisa.fr -- http://www.irisa.fr/sisthem/




## Introduction

- Usefulness of global vibration-based SHM methods
- Limitations due to temperature effects on the dynamics of civil engineering structures
- Wanted: discriminate between changes in modal parameters due to damages and changes due to temperature effects
- A statistical subspace-based damage detection algorithm: null space of a matrix built on reference modes/modeshapes at a known temperature
- Proposed solution to temperature handling: no temperature measurement, thermal effect modeling, statistical nuisance rejection



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The temperature effect - Examples

The temperature effect - Modeling

The temperature effect - Rejection

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## Modeling

**FE model:**
$$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\phi_\mu = 0 \;, \quad \psi_\mu = L\phi_\mu$$

**State space:**
$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda\, \Phi_\lambda \;, \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\tau\mu} = \lambda}_{\text{modes}} \;, \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{modeshapes}}$$

## Parametric subspace-based damage detection

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases} \qquad F\ \Phi_\lambda = \lambda\ \Phi_\lambda, \quad \varphi_\lambda \triangleq H\ \Phi_\lambda$$

$$R_i \triangleq \mathbf{E}\left(Y_k\ Y_{k-i}^T\right), \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H\ F^i\ G \Longrightarrow \mathcal{H} = \mathcal{O}\ \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (\ G \quad FG \quad F^2G \quad \dots\ ), \quad G \triangleq \mathbf{E}\left(X_k\ Y_k^T\right)$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \phi_\lambda)$$



Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\ \Phi \end{pmatrix}$ modes, mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

$\theta_0$ : reference parameter for safe structure

Left null space: $S^T S = I_s,\ \ S^T\ \mathcal{O}_{p+1}(\theta_0) = 0$

$Y_k$: $N$-size sample of new measurements

Residual for SHM:

$$\zeta_N(\theta_0) \triangleq \sqrt{N}\ \ \text{vec}(\ S^T(\theta_0)\ \hat{\mathcal{H}}\ )$$

$\mathcal{J}(\theta_0)$: sensitivity of $\zeta$ w.r.t. modal changes; $\Sigma(\theta_0)$: covariance

$\chi^2$-test: $\zeta_N^T\ \Sigma^{-1}\ \mathcal{J}(\mathcal{J}^T\ \Sigma^{-1}\ \mathcal{J})^{-1}\ \mathcal{J}^T\ \Sigma^{-1}\zeta_N\ \geq h$



## Focussed monitoring

Fisher information $\mathbf{F}(\theta_0) \triangleq \mathcal{J}(\theta_0)^T\ \Sigma(\theta_0)^{-1}\ \mathcal{J}(\theta_0)$

$$\theta = \begin{pmatrix} \theta_a \\ \theta_b \end{pmatrix}, \ \mathcal{J} = (\mathcal{J}_a\ \ \mathcal{J}_b), \ \mathbf{F} = \begin{pmatrix} \mathbf{F}_{aa} & \mathbf{F}_{ab} \\ \mathbf{F}_{ba} & \mathbf{F}_{bb} \end{pmatrix} = \begin{pmatrix} \mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_a & \mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_b \\ \mathcal{J}_b^T\Sigma^{-1}\mathcal{J}_a & \mathcal{J}_b^T\Sigma^{-1}\mathcal{J}_b \end{pmatrix}$$

$$\mathbf{F}_a^\star \triangleq \mathbf{F}_{aa} - \mathbf{F}_{ab}\ \mathbf{F}_{bb}^{-1}\ \mathbf{F}_{ba}$$

### Sensitivity approach - Partial residual

$$\tilde{\zeta}_a \triangleq \mathcal{J}_a^T\ \Sigma^{-1}\ \zeta, \quad \tilde{\chi}_a^2 \triangleq \tilde{\zeta}_a^T\ \mathbf{F}_{aa}^{-1}\ \tilde{\zeta}_a$$

### Min-max approach - Robust residual

$$\zeta_a^\star \triangleq \tilde{\zeta}_a - \mathbf{F}_{ab}\ \mathbf{F}_{bb}^{-1}\ \tilde{\zeta}_b, \quad \chi_a^{\star\,2} = \zeta_a^{\star T}\ \mathbf{F}_a^{\star\,-1}\ \zeta_a^\star$$



## The temperature effect - Example 1

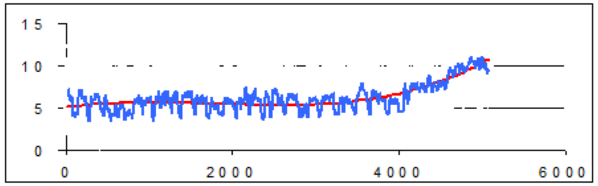
## Z24 bridge

- A benchmark of the BRITE/EURAM project SIMCES and of the European COST action F3
- Response to traffic excitation under the bridge measured over one year in 139 points
- Two damage scenarios (DS1 and DS2): pier settlements of 20mm and 80mm.

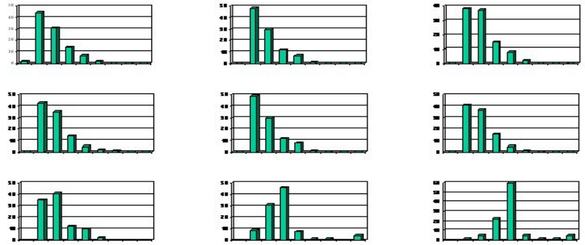
Identified first four natural frequencies / Test values
(Results with four sensors)

| | Mode | 1 | 2 | 3 | 4 | $\chi^2$ |
|---|---|---|---|---|---|---|
| Undamaged | Freq.(Hz) | 3.88 | 5.01 | 9.80 | 10.30 | $8.80 \cdot 10e2$ |
| Damaged (1) | Freq.(Hz) | 3.87 | 5.06 | 9.79 | 10.32 | $8.00 \cdot 10e5$ |
| Damaged (2) | Freq.(Hz) | 3.76 | 4.93 | 9.74 | 10.25 | $3.96 \cdot 10e6$ |






Evolution of the test values over nine months (log-scale).



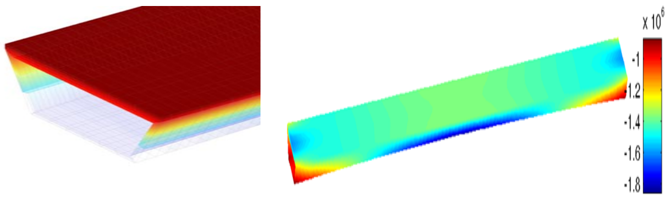
Distribution of the test values for each of the nine months.



## The temperature effect - Example 2

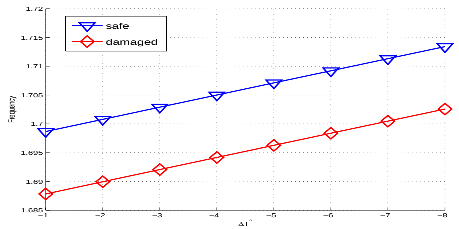
### Simulated bridge deck

Finite elements toolbox OpenFEM (with Matlab or Scilab).
60 m span, 9600 volume elements, 13668 nodes.
Temperature variations: either a uniform temperature elevation or a linear variation with $z$.

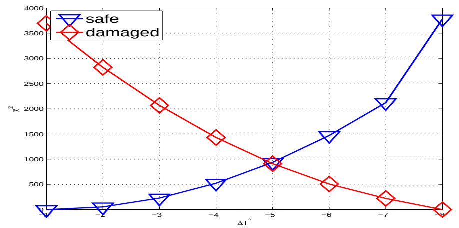



Linear thermal field (Left) and induced axial stress (Right).
The warmer deck expands while the cooler bottom contracts.






Decreasing temperature effect on the first frequency.




Partial $\chi^2$-test values.

Safe and damaged

## The temperature effect - Modeling

- Thermal field $\rightarrow$ materials expansion $\rightarrow$ thermal stress field.
- Temperature modification = external load.
- Static equilibrium under thermal loading $\leftrightarrow$ pre-stress.
- Stiffness $K$, and thus modal frequencies, affected.
  Mass $M$ and damping $C$ assumed not affected.
- Assuming thermal loads inducing small perturbations:
  stiffness = linear function of the thermal field $T(x)$.
- If $T(x)$ = linear combination of constant thermal fields,
  temperature effect on $K$:

$$K = K_0 + K_T \triangleq K_0 + \sum_i \alpha_i \; K_{T,i}$$



## Computing the residual sensitivity w.r.t. $T$

Small deviations

$$\mathrm{E}(\zeta_N) = \mathcal{J}(\theta_0)\ \delta\theta + \mathcal{J}_T\ \delta T, \quad \mathcal{J}_T \triangleq \mathcal{J}(\theta_0)\ \mathcal{J}_{\theta T}$$

$\mathcal{J}_{\theta T}$ involves computing $\delta\mu$ and $\delta\phi$ for $\delta K = K_{T,i}, \delta M = \delta C = 0$,
based on the differentiation of:

$$(M\mu^2 + C\mu + K)\phi_\mu = 0$$

that is:

$$\delta\mu = -\frac{\phi^T\ (\mu^2\ \delta M + \mu\ \delta C + \delta K)\ \phi}{\phi^T\ (2\ \mu\ M + C)\ \phi}$$

and

$$(\mu^2\ M + \mu\ C + K)\ \delta\phi = -\ \delta\mu\ (2\ \mu\ M + C)\ \phi - (\mu^2\ \delta M + \mu\ \delta C + \delta K)\ \phi$$

with $\phi^T \delta\phi = 0$.



## The temperature effect - Rejection

- $\theta_0$ : reference modal parameter for safe structure

  Compute the null space $S(\theta_0)$,

  the sensitivities $\mathcal{J}(\theta_0)$ and $\mathcal{J}_T$,

  the covariance $\Sigma(\theta_0)$ and Fisher matrix $\mathrm{F} \triangleq \mathrm{F}(\theta_0, T)$
- $Y_k$: $N$-size sample of new measurements
- Compute the residual $\zeta_N(\theta_0) \triangleq \sqrt{N}\ \ \mathrm{vec}(\ S^T(\theta_0)\ \hat{\mathcal{H}}\ )$
- Compute the robust residual $\zeta^\star_{\theta_0} \triangleq \tilde{\zeta}_{\theta_0} - \mathrm{F}_{\theta_0,T}\ \mathrm{F}^{-1}_{T,T}\ \tilde{\zeta}_T$
- Compute the $\chi^2$-test : $\zeta^{\star\,T}_{\theta_0}\ \mathrm{F}^{\star\,-1}_{\theta_0}\ \zeta^\star_{\theta_0}$



## Implementation issues

- Compute the key matrices:

  sensitivities $\mathcal{J}(\theta_0)$ and $\mathcal{J}_T$ and covariance $\Sigma$

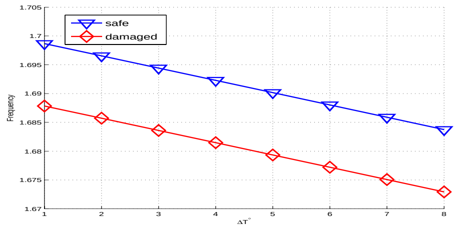
  on a long data sample for the safe structure.

  In case of nonstationary excitation, computing $\Sigma$

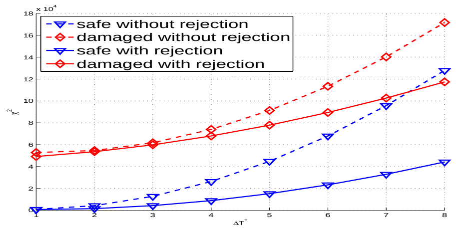
  on current data might be preferable.
- $\Sigma$ computed with QR (Zhang, 2003).

  Null space $S$ computed with QR (Nasser, 2006).

Example - Back to the simulated bridge deck




Increasing temperature effect on the first frequency.




Global (dotted) and minmax (solid) $\chi^2$-test values.
Minmax operating range: $8\ C \times 2$

Safe and damaged



# Conclusion

Temperature effect in vibration-based SHM

Statistical parametric model-based approach

Subspace-based damage detection algorithm

Local rejection of the temperature seen as nuisance

Example: simulated bridge deck

Ongoing: empirical null space merging data at # temperatures,
analytical temperature-adjusted null space

Future: in-operation examples,
extension to 3D temperature fields,
thermal model parameterization