On statistical change detection for FDI

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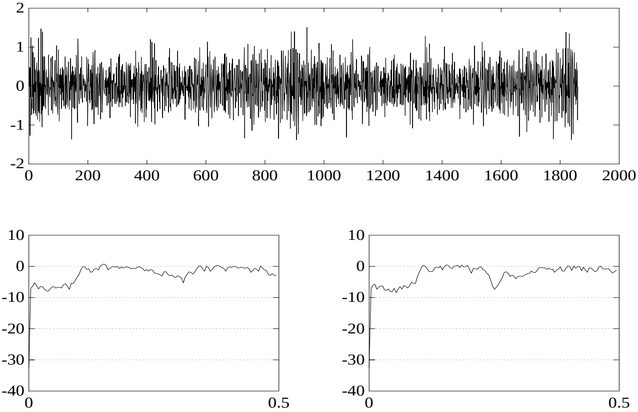
Simulated data - One change !



Problems and issues

Problems

- Detection of changes
  - Stochastic models (static, dynamic) $\longleftrightarrow$ uncertainties
  - Parameterized models (physical interpretation, diagnostics)
  - Damage $\longleftrightarrow$ change in the parameter vector : $\theta_0 \longrightarrow \theta_1$
- Many changes of interest are small
- Early detection of (small) deviations is useful

Key issues

1. Which function of the data should be handled ?
2. How to handle that function to make the decision ?

Content

Theory and algorithms

- Changes in the mean
- Changes in the spectrum
- Changes in the system dynamics and vibration-based SHM

Application examples

1. Handling the thermal effect in SHM
2. Aircraft flutter monitoring

Likelihood ratio

Likelihood	$p_\theta(y_i)$		
Log-likelihood ratio	s_i	$\triangleq$	$\ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}$
	$\mathbf{E}_{\theta_0}(s_i)$	$<$	0
	$\mathbf{E}_{\theta_1}(s_i)$	$>$	0
Likelihood ratio	Λ_N	$\triangleq$	$\frac{p_{\theta_1}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} = \frac{\prod_i p_{\theta_1}(y_i)}{\prod_i p_{\theta_0}(y_i)}$
Log-likelihood ratio	S_N	$\triangleq$	$\ln \Lambda_N = \sum_{i=1}^N s_i$

Hypothesis testing

Hypotheses	$\mathbf{H}_0$	$\mathbf{H}_1$	
Simple	θ_0	θ_1	Known parameter values
Composite	Θ_0	Θ_1	Unknown parameter values

Simple hypotheses: Likelihood ratio test

If $\Lambda_N \geq \lambda$ or equivalently $S_N \geq h$: decide $\mathbf{H}_1$; $\mathbf{H}_0$ otherwise

Composite hypotheses: Generalized likelihood ratio (GLR) test

$$\widehat{\Lambda}_N = \frac{\sup_{\theta_1 \in \Theta_1} \; p_{\theta_1}(\mathcal{Y}_1^N)}{\sup_{\theta_0 \in \Theta_0} \; p_{\theta_0}(\mathcal{Y}_1^N)} = \frac{p_{\widehat{\theta}_1}(\mathcal{Y}_1^N)}{p_{\widehat{\theta}_0}(\mathcal{Y}_1^N)}$$

Rule: Maximize the likelihoods w.r.t. unknown values of θ_0 and θ_1

On-line change detection

Hypothesis $\mathbf{H}_0$ $\qquad \theta = \theta_0$ known $(1 \leq i \leq k)$

Hypothesis $\mathbf{H}_1$ $\qquad \exists\ t_0$ unknown s.t. $\theta = \begin{cases} \theta_0 & (1 \leq i < t_0) \\ \theta_1 & (t_0 \leq i \leq k) \end{cases}$

Alarm time t_a : $\qquad t_a = \min\{k \geq 1 : g_k \geq h\}$

Wanted: decision function g_k, onset time estimate $(\widehat{t_0})_k$

CUSUM algorithm

Ratio of likelihoods under $\mathbf{H}_0$ and $\mathbf{H}_1$:

$$\frac{\prod_{i=1}^{t_0-1} p_{\theta_0}(y_i) \cdot \prod_{i=t_0}^{k} p_{\theta_1}(y_i)}{\prod_{i=1}^{k} p_{\theta_0}(y_i)} = \frac{\prod_{i=t_0}^{k} p_{\theta_1}(y_i)}{\prod_{i=t_0}^{k} p_{\theta_0}(y_i)} = \Lambda_{t_0}^k$$

Rule: Maximize the likelihood ratio w.r.t. the unknown onset time t_0

$$\begin{aligned}
(\widehat{t_0})_k &\triangleq \arg\max_{1\leq j\leq k} \prod_{i=1}^{j-1} p_{\theta_0}(y_i) \cdot \prod_{i=j}^{k} p_{\theta_1}(y_i) \\
&= \arg\max_{1\leq j\leq k} \Lambda_j^k \\
&= \arg\max_{1\leq j\leq k} S_j^k, \qquad S_j^k \triangleq \ln \Lambda_j^k
\end{aligned}$$

$$g_k \triangleq \max_{1\leq j\leq k} S_j^k = \ln \Lambda_{\hat{t}_0}^k$$

CUSUM algorithm (Contd.) **Lesson 1**

$$g_k \triangleq \max_{1\leq j\leq k} S_j^k$$

$$= S_1^k - \min_{1\leq j\leq k} S_1^j = S_1^k - m_k, \qquad m_k \triangleq \min_{1\leq j\leq k} S_1^j$$

$$t_a = \min\{k \geq 1 : S_1^k \geq m_k + h\}$$

Adaptative threshold

$$g_k = (g_{k-1} + s_k)^+$$

$$g_k = \left(S_{k-N_k+1}^k\right)^+, \qquad N_k \triangleq N_{k-1} \cdot I(g_{k-1}) + 1$$

$$(\widehat{t_0})_k = t_a - N_{t_a} + 1$$

Sliding window with adaptive size

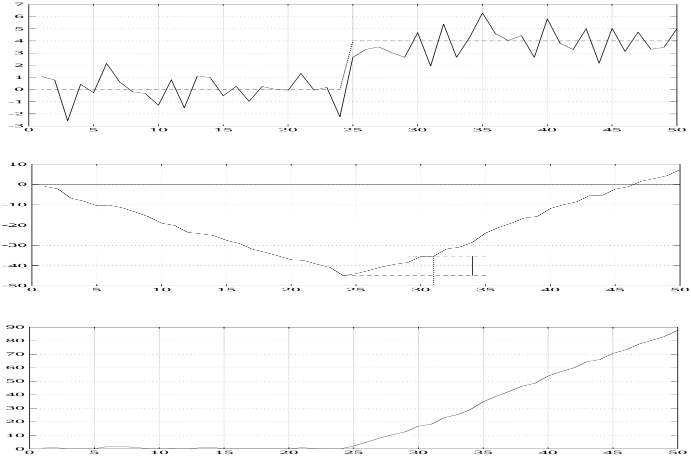
CUSUM algorithm - Gaussian example

$$\mathcal{N}(\mu, \sigma^2), \quad \theta \triangleq \mu, \quad p_\theta(y) \triangleq \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y_i-\mu)^2}{2\sigma^2}\right)$$

$$\begin{aligned} s_i &= \ln \frac{p_{\mu_1}(y_i)}{p_{\mu_0}(y_i)} \\ &= \frac{1}{2\,\sigma^2}\left((y_i-\mu_0)^2 - (y_i-\mu_1)^2\right) \\ &= \frac{\nu}{\sigma^2}\left(y_i - \mu_0 - \frac{\nu}{2}\right), \quad \nu \triangleq \mu_1 - \mu_0 \text{ change magnitude} \end{aligned}$$

S_1^k involves $\sum_{i=1}^{k} y_i$: Integrator (with adaptive threshold)

CUSUM algorithm - Gaussian example (Contd.)



y_k

S_1^k

g_k

Unknown θ_1 - Algorithms

Modified CUSUM algorithms

- Minimum magnitude of change
- Weighted CUSUM

GLR algorithm

- Double maximization

$$g_k = \max_{1 \leq j \leq k} \sup_{\theta_1} S_j^k(\theta_1)$$

- Gaussian case, additive faults: second maximization explicit

Unknown θ_1 - Gaussian example **Lesson 2**

Introducing a minimum magnitude of change ν_m

Decreasing mean

$$T_1^k \triangleq \sum_{i=1}^k \left(y_i - \mu_0 + \frac{\nu_m}{2}\right)$$

$$M_k \triangleq \max_{1\leq j\leq k} T_1^j$$

$$t_a = \min\left\{k \geq 1 : M_k - T_1^k \geq h\right\}$$

Increasing mean

$$U_1^k \triangleq \sum_{i=1}^k \left(y_i - \mu_0 - \frac{\nu_m}{2}\right)$$

$$m_k \triangleq \min_{1\leq j\leq k} U_1^j$$

$$t_a = \min\left\{k \geq 1 : U_1^k - m_k \geq h\right\}$$

Conditional likelihood ratio

Cond. likelihood $p_\theta(y_i | \mathcal{Y}_1^{i-1})$

Log-likelihood ratio $s_i \triangleq \ln \dfrac{p_{\theta_1}(y_i | \mathcal{Y}_1^{i-1})}{p_{\theta_0}(y_i | \mathcal{Y}_1^{i-1})}$

$\mathbf{E}_{\theta_0}(s_i) < 0$

$\mathbf{E}_{\theta_1}(s_i) > 0$

Likelihood ratio $\Lambda_N \triangleq \dfrac{p_{\theta_1}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} = \dfrac{\prod_i p_{\theta_1}(y_i | \mathcal{Y}_1^{i-1})}{\prod_i p_{\theta_0}(y_i | \mathcal{Y}_1^{i-1})}$

Log-likelihood ratio $S_N \triangleq \ln \Lambda_N = \sum_{i=1}^N s_i$

Which residuals ? Lesson 3

Residuals based on statistical inference

- Likelihood ratio : may be computationally complex
- Efficient score $\triangleq$ likelihood sensitivity w.r.t. parameter
- Any other parameter estimating fonction

Warning

- The innovation is OK for additive faults but NOT for multiplicative faults
- The innovation is NOT sufficient for monitoring the system dynamics

Building a residual

Given

- θ_0 : reference parameter, known (or identified)
- Y_k: N-size sample of new measurements

Wanted

- A residual ζ significantly non zero when a change occurs

Solution

- Residual $\leftrightarrow$ Estimating function $\zeta_N(\theta, \mathcal{Y}_1^N)$
- Characterized by: $\mathbf{E}_{\theta_0}\ \zeta_N(\theta, \mathcal{Y}_1^N) = 0 \iff \theta = \theta_0$

Designing the test

Residual behavior

- Mean sensitivity $\mathcal{J}(\theta_0)$ and covariance $\Sigma(\theta_0)$ of $\zeta_N(\theta_0)$
- The residual is asymptotically Gaussian

$$\zeta_N(\theta_0) \rightarrow \begin{cases} \mathcal{N}(0, & \Sigma(\theta_0)) \text{ if } \mathbf{P}_{\theta_0} \\ \mathcal{N}(\mathcal{J}(\theta_0)\,\delta\theta, & \Sigma(\theta_0)) \text{ if } \mathbf{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \text{ small change} \end{cases}$$

(On-board) χ^2-test

$$\zeta_N^T\, \Sigma^{-1}\, \mathcal{J}\, (\mathcal{J}^T\, \Sigma^{-1}\, \mathcal{J})^{-1}\, \mathcal{J}^T\, \Sigma^{-1}\, \zeta_N \quad \geq h$$

- Invariant / pre-multiplication of ζ with invertible gain.
- Noises and uncertainty on θ_0 taken into account

Vibration-based monitoring problem

The excitation

- natural, not controlled
- not measured:
  - buildings, bridges, offshore structures,
  - rotating machinery,
  - cars, trains, aircrafts
- nonstationary (e.g., turbulent)

Questions

- How to detect and localize small damages ?
  Early ?
  On-board ?

Modelling

FE model:

$$\begin{cases} M\ \ddot{\mathcal{Z}}(s) + C\ \dot{\mathcal{Z}}(s) + K\ \mathcal{Z}(s) \ = \ \epsilon(s) \\ Y(s) \ = \ L\ \mathcal{Z}(s) \end{cases}$$

$$(M\ \mu^2 + C\ \mu + K)\ \Psi_\mu = 0\ , \quad \psi_\mu = L\ \Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} \ = \ F\ X_k + V_k \\ Y_k \ = \ H\ X_k \end{cases}$$

$$F\ \Phi_\lambda = \lambda\ \Phi_\lambda\ , \quad \varphi_\lambda \triangleq H\ \Phi_\lambda$$

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}}\ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}$$

Subspace identification of the dynamics - Theory

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k \; Y_{k-i}^T\right)}_{\text{ok if stationary !}} \;,\quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H \, F^i \, G \;, \quad G \triangleq \mathbf{E}\left(X_k \; Y_k^T\right)$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \;,\; \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2G & \dots \end{pmatrix}$$

$$\mathcal{H} = \mathcal{O}\,\mathcal{C} \quad , \quad \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Subspace identification of the dynamics - Implementation

$$\underbrace{\hat{R}_i \triangleq \frac{1}{N} \sum_{k=1}^{N} Y_k \, Y_{k-i}^T}_{\text{ok when nonstationary !}} \quad , \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi_\lambda})$$

$$\hat{\mathcal{H}} = U \, \Delta \, W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T \; ; \quad \hat{\mathcal{O}} = U \, \Delta_1^{1/2}$$

$$\mathcal{O}_p^{\uparrow}(H, F) = \mathcal{O}_p(H, F) \, F$$

$$\det(F - \lambda \, I) = 0 \, , \quad F \, \Phi_\lambda = \lambda \, \Phi_\lambda, \quad \varphi_\lambda = H \, \Phi_\lambda$$

Introducing the parameter vector

FE model:
$$\begin{cases} M\ \ddot{\mathcal{Z}}(s) + C\ \dot{\mathcal{Z}}(s) + K\ \mathcal{Z}(s) = \epsilon(s) \\ Y(s) = L\ \mathcal{Z}(s) \end{cases}$$

$$(M\ \mu^2 + C\ \mu + K)\ \Psi_\mu = 0\ , \quad \psi_\mu = L\ \Psi_\mu$$

State space:
$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases}$$

$$F\ \Phi_\lambda = \lambda\ \Phi_\lambda\ , \quad \varphi_\lambda \triangleq H\ \Phi_\lambda$$

Parameter:
$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}}\ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}\ ; \quad \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\ \Phi \end{pmatrix}$$

Eigenstructure monitoring

$$\begin{cases} X_{k+1} = F\, X_k + V_k \\ Y_k = H\, X_k \end{cases} \qquad \begin{array}{l} F\,\varphi_\lambda = \lambda\,\varphi_\lambda \\ \Phi_\lambda \triangleq H\,\varphi_\lambda \end{array}$$

Canonical parametrization : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

System parameter characterization

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel

Detecting structural changes

System parameter characterization

- $\exists U, \quad U^T\ U = I_s, \quad U^T\ \mathcal{O}_{p+1}(\theta_0) = 0;$ say $U(\theta_0)$
- $\theta_0 \leftrightarrow (R_i^0)_i$ characterized by: $U^T(\theta_0)\ \hat{\mathcal{H}}_{p+1,q}^0 = 0$

Residual for structural monitoring

- $\zeta_N(\theta_0) \triangleq \mathrm{vec}(\ U^T(\theta_0)\ \hat{\mathcal{H}}_{p+1,q}\ )$

(On-board) χ^2-test

$$\zeta_N^T\ \Sigma^{-1}\ \mathcal{J}\ (\mathcal{J}^T\ \Sigma^{-1}\ \mathcal{J})^{-1}\ \mathcal{J}^T\ \Sigma^{-1}\ \zeta_N \quad \geq h$$

Relation to parity space

$$\zeta_{\text{parity}} = \mathcal{G}^T \ \mathcal{Y}^+_{k,p+1}, \quad \mathcal{G}^T \ \mathcal{O}_{p+1} = 0$$

$$\zeta_{\text{subspace}} = U^T \ \hat{\mathcal{H}}_{p+1,q}, \quad U^T \ \mathcal{O}_{p+1} = 0$$

First order statistics $\longleftrightarrow$ Second order statistics

Handling the thermal effect in SHM

The problem

- The temperature T modifies the eigenfrequencies $\longrightarrow$ $T \triangleq$ nuisance parameter
- Model of thermal effect on stiffness matrix K

Three solutions

1. Analytic updating of the left kernel U
2. Statistical rejection of the nuisance T
3. Data fusion: empirical mean of Hankel matrices (reference data sets at different unknown T)

Applications

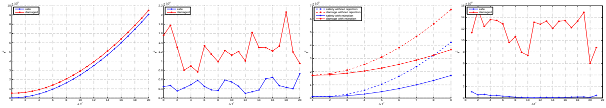
Simulated bridge deck

- Provided by É. Balmès, Ecole Centrale Paris
- Finite elements toolbox OpenFEM (with Matlab or Scilab)
- 60 m span, 9600 volume elements, 13668 nodes
- Temperature variations: either uniform or linear with z

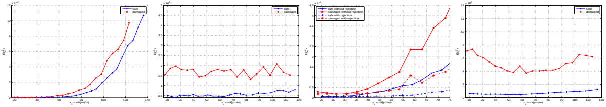
Beam within a climatic chamber

- Laboratory test-case provided by F. Treyssède, LCPC
- Vertical clamped beam subject to decreasing T
- Small local damage: horizontal clamped spring attached to the beam, with tunable stiffness and height

Numerical results



Bridge deck



Beam within a climatic chamber

Original test and 3 tests handling the thermal effect versus T: kernel updating, nuisance rejection, data fusion

Safe, damaged

Flutter monitoring

- Flutter: critical aircraft instability phenomenon
  unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- Flight flutter testing, very expensive and time consuming :
  Design the flutter free flight envelope
- Flutter clearance techniques:
  In-flight identification, flutter prediction
- Some challenges:
  Real time on-board monitoring,
  Handling transients between steady flight test points
- Our approach:
  Change detection for monitoring instability indicators

Using CUSUM for monitoring an instability criterion ψ

Hypotheses

- $\mathbf{H}_0$: $\psi \geq \psi_c$
- $\mathbf{H}_1$: $\psi < \psi_c$

ψ_c : critical value

Real-time constraint

- Write the subspace-based residual ζ as a cumulative sum

Proposed solution (Lesson 2)

- Introduce a minimum change magnitude (actual change magnitude unknown)
- Run two CUSUM tests in parallel (actual change direction unknown)

Implementing the CUSUM test

For detecting aircraft instability precursors, select

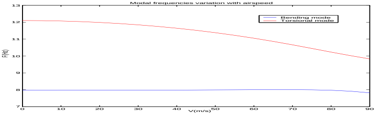
- *a)* An instability criterion ψ and a critical value ψ_c;
- *b)* A left kernel matrix $U(.)$;
- *c)* A reference $\theta_\star$ for estimating $\mathcal{J}_n(\theta_\star)$ and $\Sigma_n^{-1}(\theta_\star)$;
- *d)* A min. change magnitude ν_m and a threshold h.

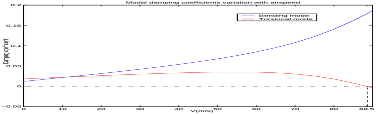
Three solutions

Three solutions for *b)-c)*

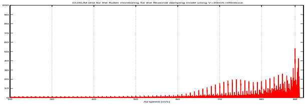
1. $\theta_\star \triangleq \theta_0$ identified on reference data for the stable system; $U(\theta_\star)$ computed, $\mathcal{J}_n(\theta_0), \Sigma_n^{-1}(\theta_0)$ estimated recursively with the test data.
2. $\theta_\star \triangleq \theta_c$, critical parameter closer to instability, computed at each flight point using θ_0 and an aeroelastic model; $U(\theta_\star)$ computed, $\mathcal{J}_n(\theta_c), \Sigma_n^{-1}(\theta_c)$ estimated recursively with the test data.
3. $U(.) \triangleq \widehat{U}_n$ estimated on test data, $\mathcal{J}_n(\theta_0), \Sigma_n^{-1}(\theta_0)$ estimated recursively with the test data.

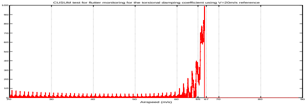
Results - Aeroelastic Hancock wing model



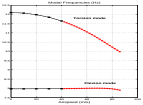


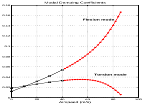
Frequencies (Left) and dampings (Right). Bending and torsion

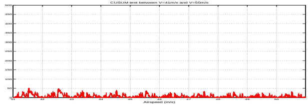




Sol **1** with θ_0 at V=20 m/s. Bending (Left) and torsion (Right)







θ_c predicted after t flight points. Sol **2** between flight points t and $t+1$.

Advanced statistical signal processing is mandatory for process monitoring and diagnostics

- A statistical framework enlightens the meaning and increases the power of a number of familiar operations: integration, averaging, sensitivity, adaptive thresholds, adaptive windows, ...

Change detection is useful for vibration-based SHM

- Handling the temperature effect in SHM of civil structures
- Aircraft flutter monitoring