

Des itérations en logique modale... et EXPTIME

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Référence : Dynamic logic, Harel, Kozen, Tiuryn.

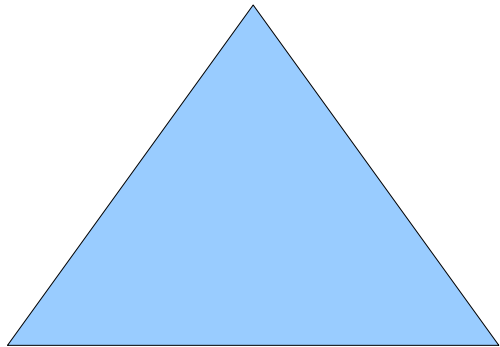
Plan du cours

- Un musée de logiques
 - Propositional dynamic logic
 - Common knowledge
 - Conséquence globale
- Machines alternantes (pour avoir un problème EXPTIME-hard intéressant)
 - Définition
 - Comparaison des classes de complexité
- EXPTIME en construisant un modèle canonique filtré

Comparaison K, S4 et LTL

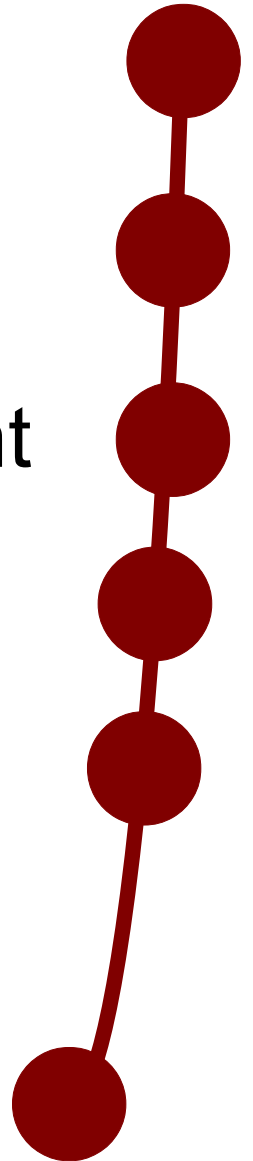
K, S4

- Profondeur polynomiale
- Branchement



LTL

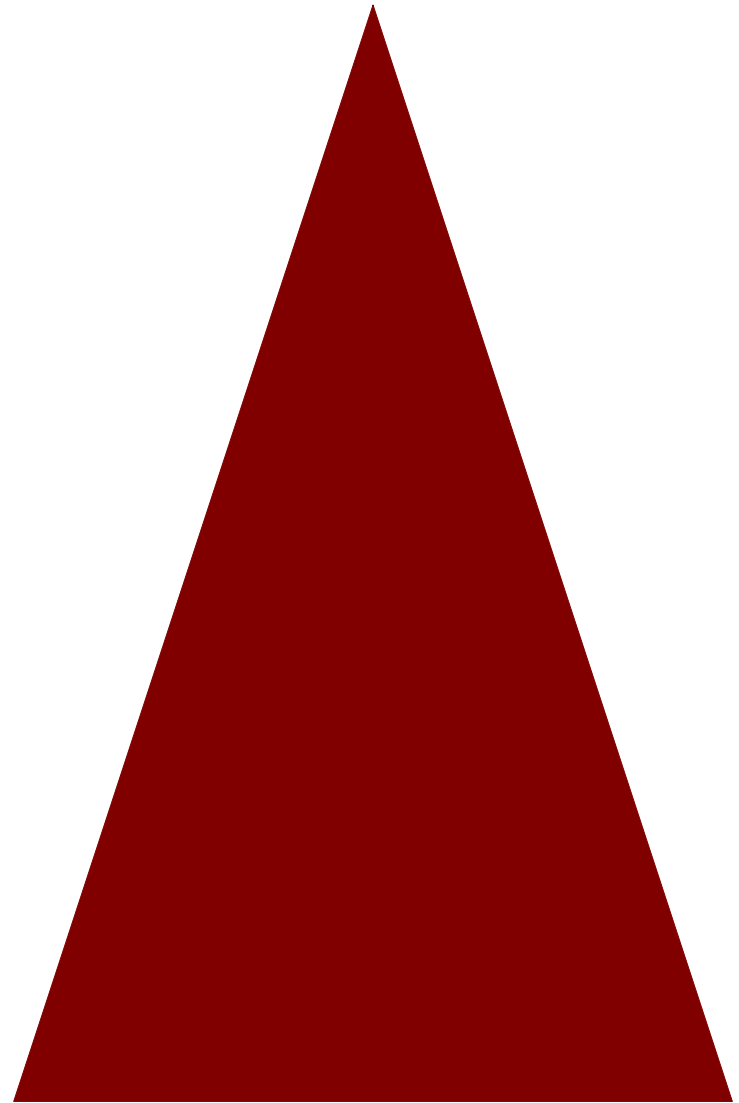
- Profondeur exponentielle
- Pas de branchement



Dans ce cours...

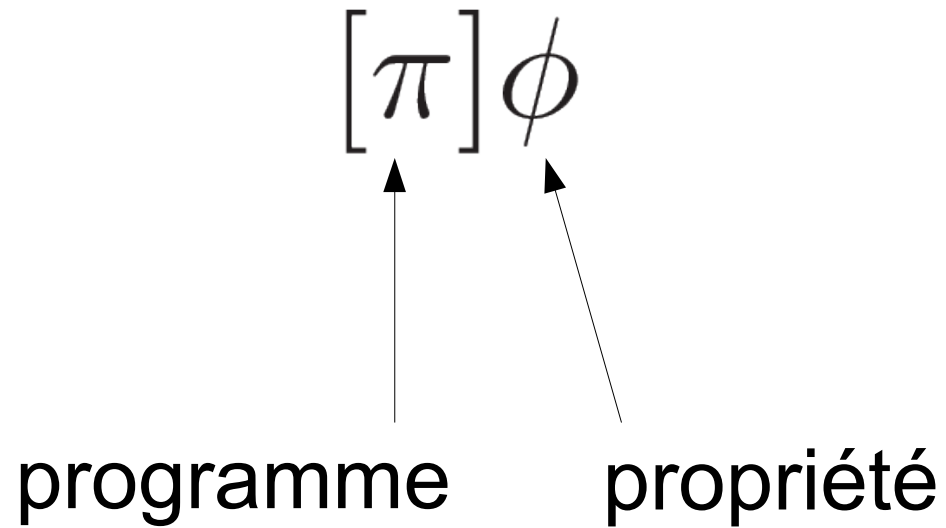
CTL, PDL,
connaissance
commune

- Profondeur
exponentielle
- Branchement



Propositional Dynamic Logic

Idée de Pratt



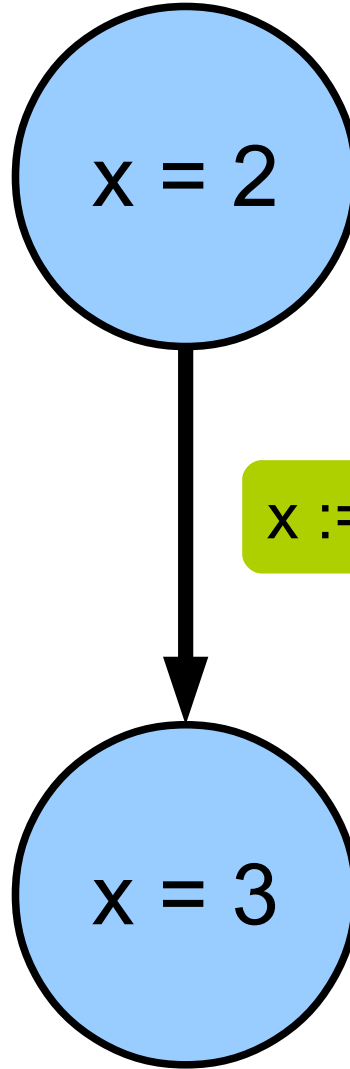
$$(x = 2) \rightarrow [x := x + 1](x = 3)$$

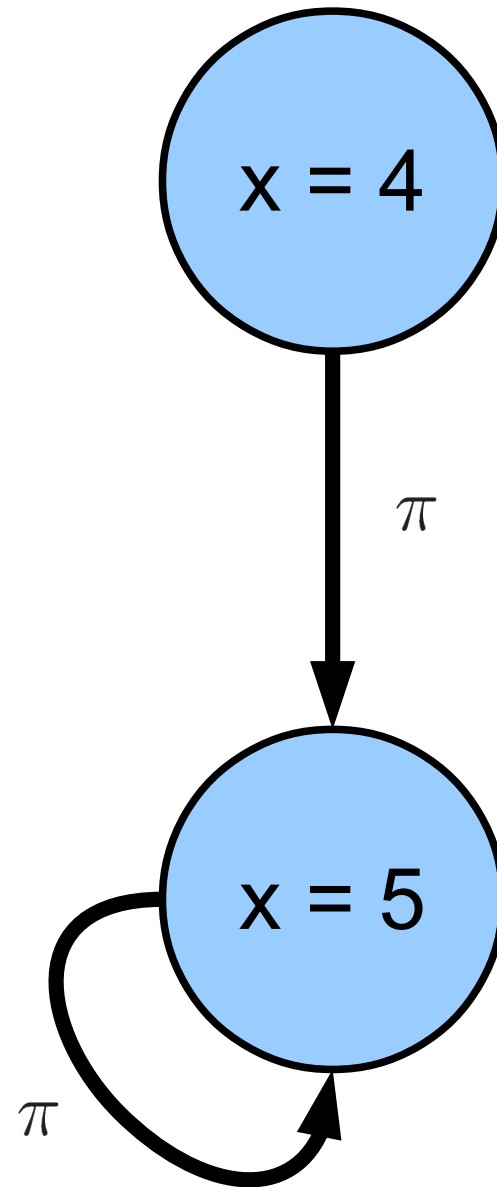
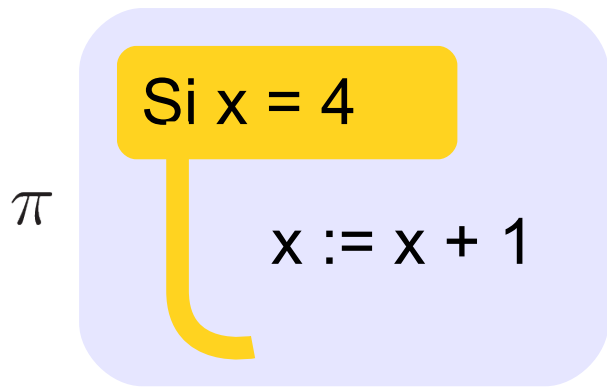
$x := x + 1$

$x = 2$

$x := x + 1$

$x = 3$

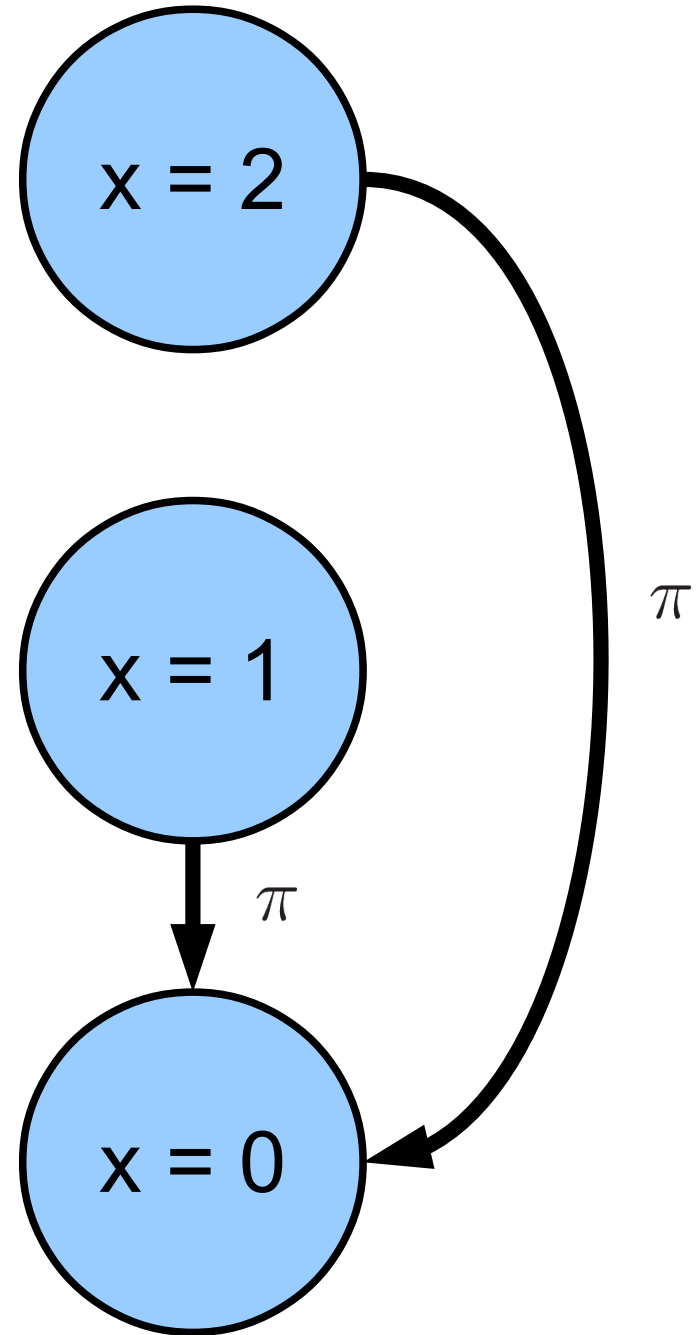




π

Pendant que $x > 0$

$x := x - 1$



Langage des programmes en logique dynamique

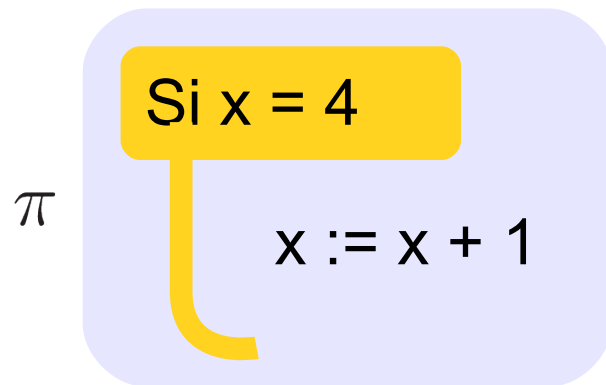
Un programme = une expression régulière :

$\pi_1; \pi_2$

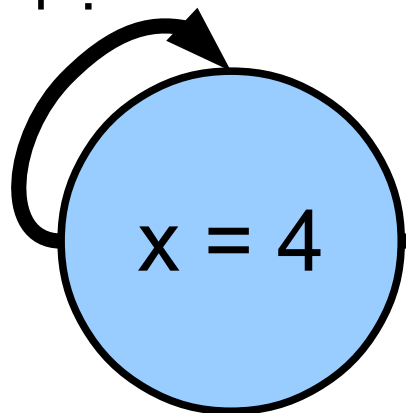
$\pi_1 \cup \pi_2$

π^*

$\varphi? \longleftarrow \text{test}$

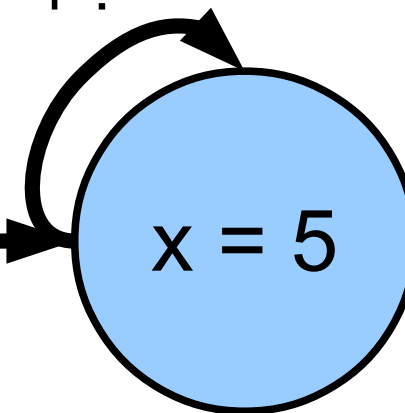


$x = 4 ?$

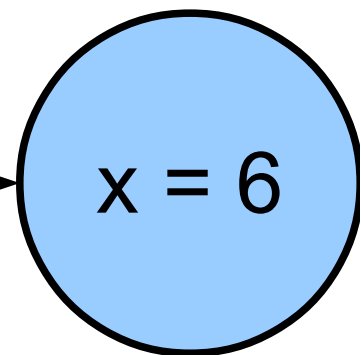


$x := x + 1$

$x \neq 4 ?$

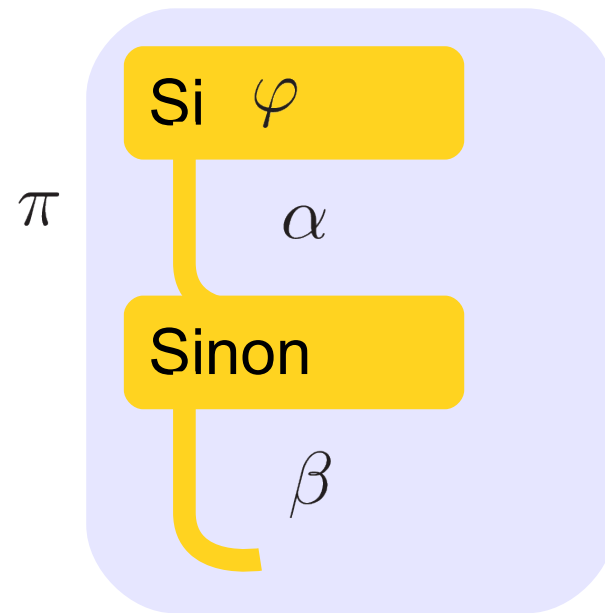


$x := x + 1$



π

$((x = 4)?; x := x + 1) \cup (x \neq 4)?$



$$(\varphi?; \alpha) \cup ((\neg\varphi)?; \beta)$$

Pendant que φ

α

$(\varphi?; \alpha)^* ; (\neg\varphi)?$

Fisher, Ladner 1979 : Propositional Dynamic logic

$$[\pi]\phi$$

Programme
abstrait

Des propositions...

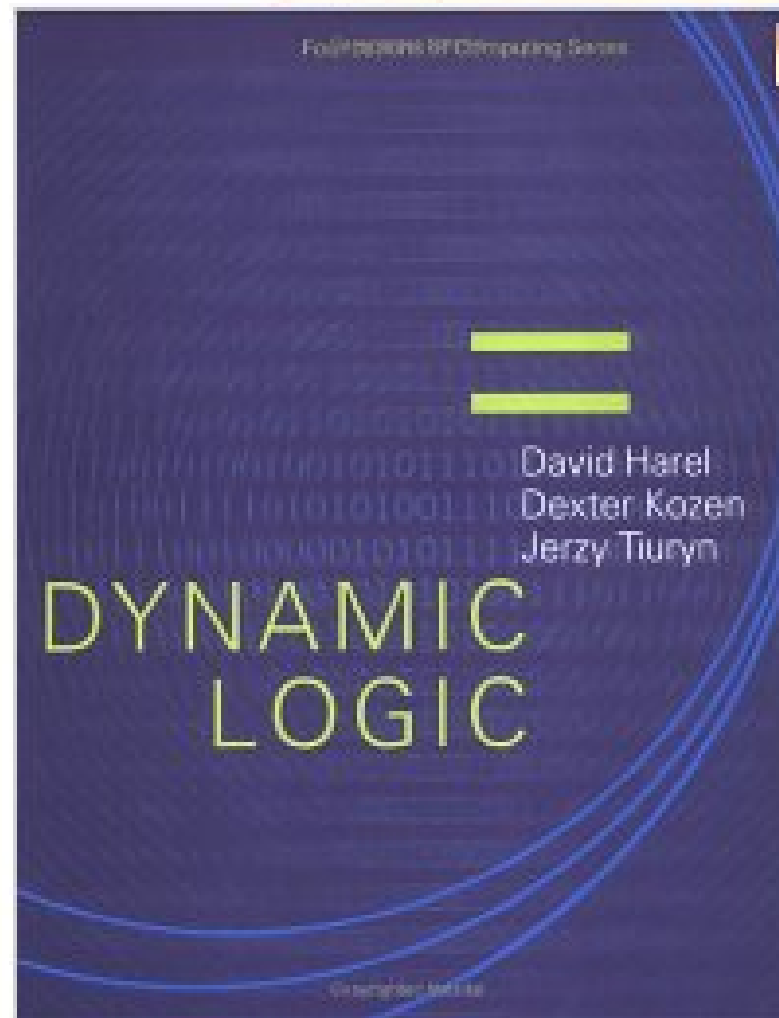
$$[a^*](p \vee \neg [ba^*]q)$$

De Pratt 1980

- Le problème de satisfiabilité de PDL est dans EXPTIME.
- Pleins de variantes :
 - Berman and Paterson (1981) : pas de test
 - Vakarelov (1983)
De Giacomo and Massacci (2000) : le converse
 - 1982 : Streett : cycles infinis
 - 2005 : Lange and Lutz : avec intersection
 - Intersection

Référence

Cliquez pour **Feuilleter!**



Référence



Ronald Fagin
Joseph Y. Halpern
Yoram Moses
Moshe Y. Vardi

Reasoning About Knowledge

Application : jeu

	α	β	γ
A	5, 2	4, 1	6, 3
B	2, 7	3, 4	2, 6
C	4, 0	9, 6	3, 8

Joueurs rationnels



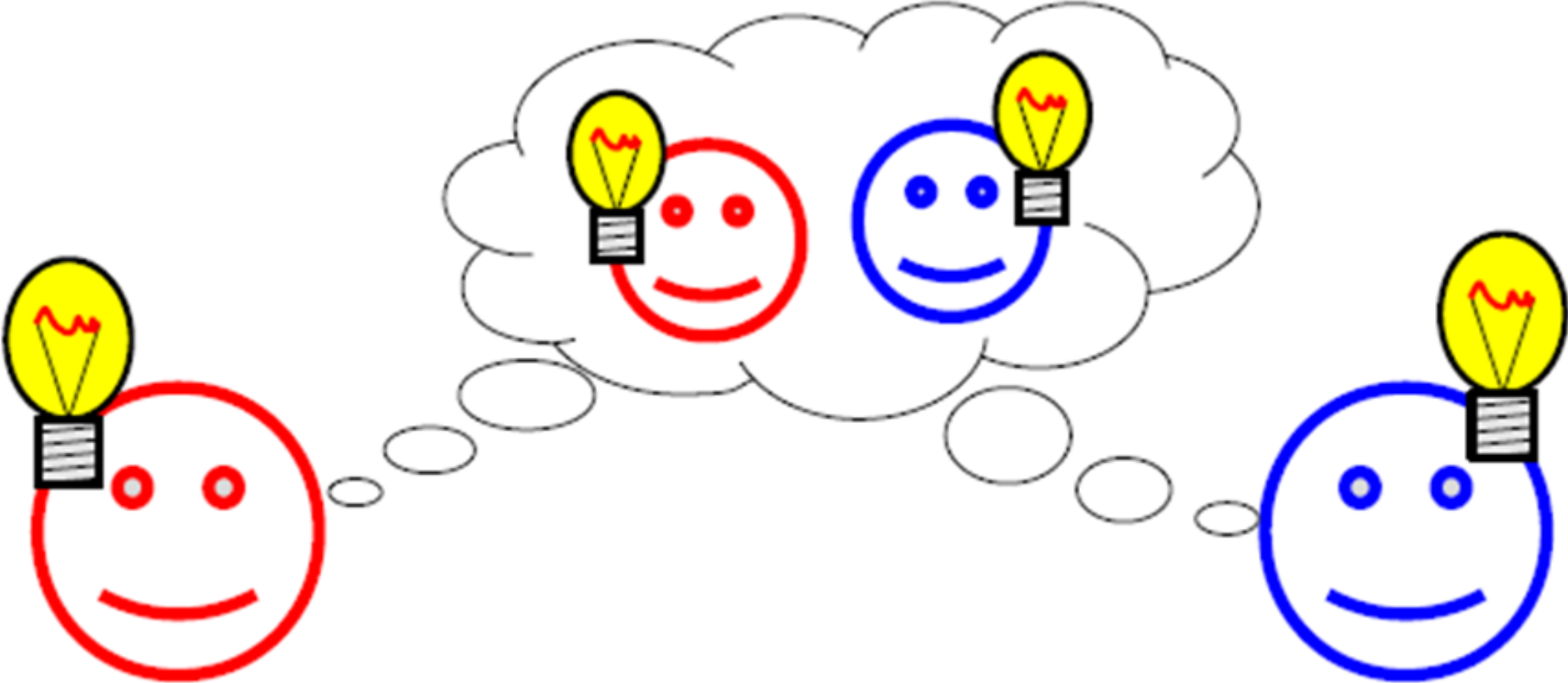
Stratégie dominée pour le joueur grec

	α	β	γ
A	5, 2	4, (1) < (6), 3	
B	2, 7	3, (4) < (2), 6	
C	4, 0	9, (6) < (3), 8	

Stratégie dominée pour le joueur latin

	α	β	γ
A	5 2 Y	4 1 V	6 3 Y
B	2 7	3 4	2 6
C	4, 0	9, 6	3, 8

Connaissance commune au degré 1



Nouveau jeu

	α	β	γ
A	5, 2	4, 1	6, 3
B	2, 7	3, 4	2, 6
C	4, 0	9, 6	3, 8

Re-stratégie dominée !

	α	β	γ
A	5, 2	4, 1	6, 3
B	2, 7	3, 4	2, 6
C	4, 0	9, 6	3, 8

The table illustrates a game with three strategies for Player 1 (A, B, C) and three strategies for Player 2 (α , β , γ). The payoffs are shown as (Player 1, Player 2). The column for strategy β is shaded gray, indicating it is dominated. Hand-drawn annotations show that strategy β is dominated by strategy α (indicated by a circle around the β column header and a circle around the α column header) and by strategy γ (indicated by a circle around the γ column header and a circle around the β column header). The arrows point from the α and γ cells to the β cell, with a less-than sign (<) in the middle of each arrow, signifying that the payoffs in the α and γ columns are strictly greater than the payoffs in the β column.

Re-stratégie dominée !

	α	β	γ
A	5, 2	4, 1	6, 3
B	2, 7	3, 4	2, 6
C	4, 0	9, 6	3, 8

The table illustrates a game matrix where the middle column (strategy β) is dominated. Hand-drawn annotations show that strategy β is dominated by strategy α (since 5 > 4 and 2 > 1) and by strategy γ (since 6 > 4 and 3 > 1). The entire β column is shaded gray to indicate it is dominated. Additionally, the first and third rows (strategies A and C) are shaded gray, indicating they are dominated by strategy B.

Élimination des stratégies dominées

	α	β	γ
A	5, 2	4, 1	6, 3
B	2, 7	3, 4	2, 6
C	4, 0	9, 6	3, 8

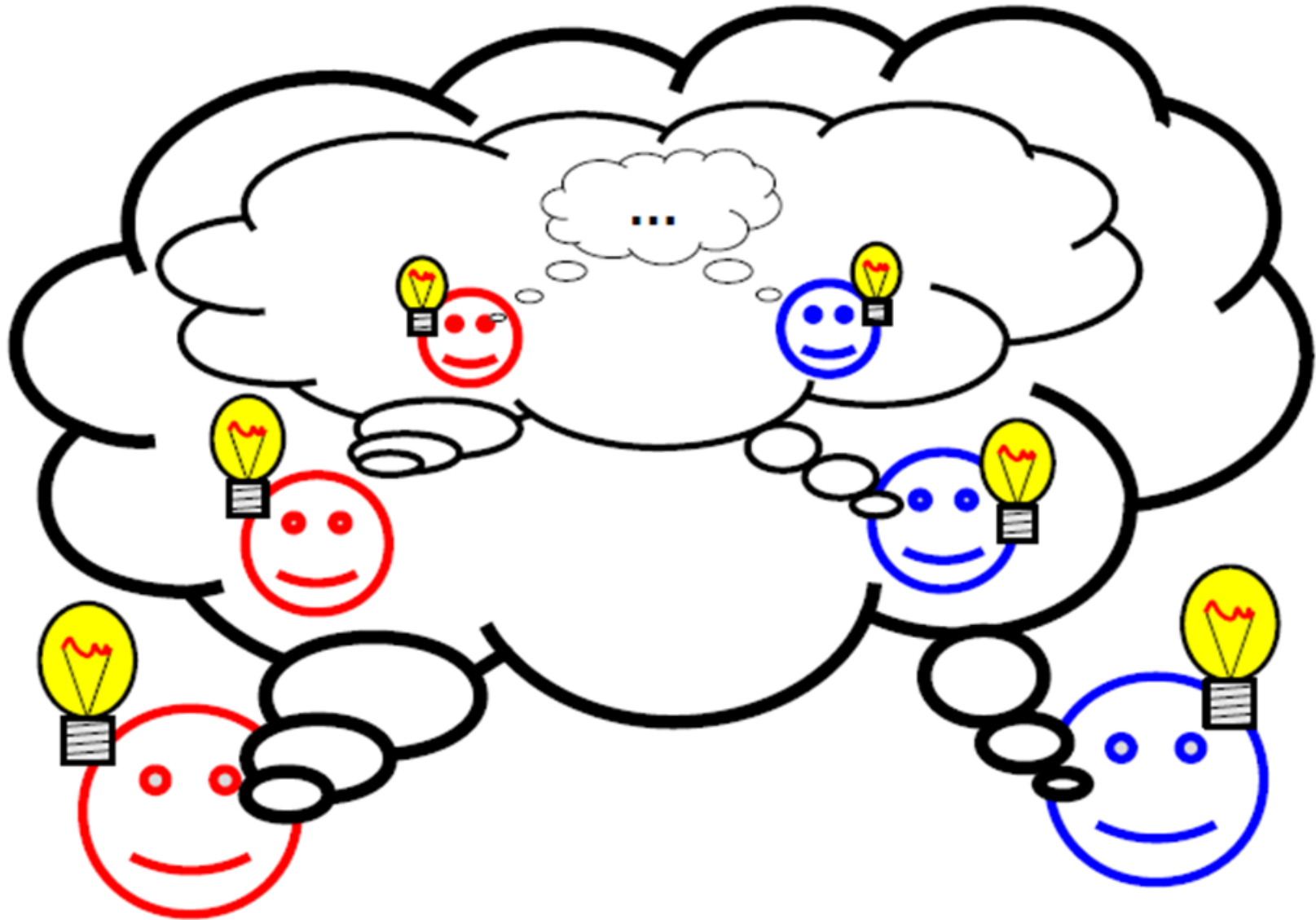


	α	β	γ
A	5, 2	4, 1	6, 3
B	2, 7	3, 4	2, 6
C	4, 0	9, 6	3, 8



	α	β	γ
A	5, 2	4, 1	6, 3
B	2, 7	3, 4	2, 6
C	4, 0	9, 6	3, 8

Connaissance commune



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